

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise A, Question 1**

**Question:**

Simplify this expression:

$$4x - 5y + 3x + 6y$$

**Solution:**

$$\begin{aligned}4x - 5y + 3x + 6y \\&= 4x + 3x - 5y + 6y \\&= 7x + y\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise A, Question 2

**Question:**

Simplify this expression:

$$3r + 7t - 5r + 3t$$

**Solution:**

$$\begin{aligned}3r + 7t - 5r + 3t \\&= 3r - 5r + 7t + 3t \\&= -2r + 10t\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 3

**Question:**

Simplify this expression:

$$3m - 2n - p + 5m + 3n - 6p$$

**Solution:**

$$\begin{aligned} 3m - 2n - p + 5m + 3n - 6p \\ = 3m + 5m - 2n + 3n - p - 6p \\ = 8m + n - 7p \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 4

**Question:**

Simplify this expression:

$$3ab - 3ac + 3a - 7ab + 5ac$$

**Solution:**

$$\begin{aligned} 3ab - 3ac + 3a - 7ab + 5ac \\ = 3ab - 7ab - 3ac + 5ac + 3a \\ = 3a - 4ab + 2ac \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 5

**Question:**

Simplify this expression:

$$7x^2 - 2x^2 + 5x^2 - 4x^2$$

**Solution:**

$$\begin{aligned}7x^2 - 2x^2 + 5x^2 - 4x^2 \\ = 6x^2\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 6

**Question:**

Simplify this expression:

$$4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$$

**Solution:**

$$\begin{aligned} &4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2 \\ &= 4m^2n - 2m^2n + 5mn^2 + mn^2 - 3mn^2 \\ &= 2m^2n + 3mn^2 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 7

**Question:**

Simplify this expression:

$$5x^2 + 4x + 1 - 3x^2 + 2x + 7$$

**Solution:**

$$\begin{aligned}5x^2 + 4x + 1 - 3x^2 + 2x + 7 \\&= 5x^2 - 3x^2 + 4x + 2x + 1 + 7 \\&= 2x^2 + 6x + 8\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 8

**Question:**

Simplify this expression:

$$6x^2 + 5x - 12 + 3x^2 - 7x + 11$$

**Solution:**

$$\begin{aligned}6x^2 + 5x - 12 + 3x^2 - 7x + 11 \\&= 6x^2 + 3x^2 + 5x - 7x - 12 + 11 \\&= 9x^2 - 2x - 1\end{aligned}$$



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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 9

**Question:**

Simplify this expression:

$$3x^2 - 5x + 2 + 3x^2 - 7x - 12$$

**Solution:**

$$\begin{aligned} 3x^2 - 5x + 2 + 3x^2 - 7x - 12 \\ = 3x^2 + 3x^2 - 5x - 7x + 2 - 12 \\ = 6x^2 - 12x - 10 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 10

**Question:**

Simplify this expression:

$$4c^2d + 5cd^2 - c^2d + 3cd^2 + 7c^2d$$

**Solution:**

$$\begin{aligned} &4c^2d + 5cd^2 - c^2d + 3cd^2 + 7c^2d \\ &= 4c^2d - c^2d + 7c^2d + 5cd^2 + 3cd^2 \\ &= 10c^2d + 8cd^2 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 11

**Question:**

Simplify this expression:

$$2x^2 + 3x + 1 + 2(3x^2 + 6)$$

**Solution:**

$$\begin{aligned}2x^2 + 3x + 1 + 2(3x^2 + 6) \\&= 2x^2 + 3x + 1 + 6x^2 + 12 \\&= 8x^2 + 3x + 13\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise A, Question 12**

**Question:**

Simplify this expression:

$$4(a + a^2b) - 3(2a + a^2b)$$

**Solution:**

$$\begin{aligned}4(a + a^2b) - 3(2a + a^2b) \\&= 4a + 4a^2b - 6a - 3a^2b \\&= a^2b - 2a\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 13

**Question:**

Simplify this expression:

$$2(3x^2 + 4x + 5) - 3(x^2 - 2x - 3)$$

**Solution:**

$$\begin{aligned}2(3x^2 + 4x + 5) - 3(x^2 - 2x - 3) \\&= 6x^2 + 8x + 10 - 3x^2 + 6x + 9 \\&= 3x^2 + 14x + 19\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 14

**Question:**

Simplify this expression:

$$7(1 - x^2) + 3(2 - 3x + 5x^2)$$

**Solution:**

$$\begin{aligned} &7(1 - x^2) + 3(2 - 3x + 5x^2) \\ &= 7 - 7x^2 + 6 - 9x + 15x^2 \\ &= 8x^2 - 9x + 13 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise A, Question 15**

**Question:**

Simplify this expression:

$$4(a + b + 3c) - 3a + 2c$$

**Solution:**

$$\begin{aligned}4(a + b + 3c) - 3a + 2c \\&= 4a + 4b + 12c - 3a + 2c \\&= a + 4b + 14c\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 16

**Question:**

Simplify this expression:

$$4(c + 3d^2) - 3(2c + d^2)$$

**Solution:**

$$\begin{aligned}4(c + 3d^2) - 3(2c + d^2) \\&= 4c + 12d^2 - 6c - 3d^2 \\&= -2c + 9d^2\end{aligned}$$



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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise A, Question 17

**Question:**

Simplify this expression:

$$5 - 3(x^2 + 2x - 5) + 3x^2$$

**Solution:**

$$\begin{aligned}5 - 3(x^2 + 2x - 5) + 3x^2 \\&= 5 - 3x^2 - 6x + 15 + 3x^2 \\&= 20 - 6x\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 18

**Question:**

Simplify this expression:

$$(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$$

**Solution:**

$$\begin{aligned}(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4) \\= r^2 + 3t^2 + 9 - 2r^2 - 3t^2 + 4 \\= 13 - r^2\end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 1**

**Question:**

Simplify this expression:

$$x^3 \times x^4$$

**Solution:**

$$= x^{3+4}$$

$$= x^7$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 2**

**Question:**

Simplify this expression:

$$2x^3 \times 3x^2$$

**Solution:**

$$\begin{aligned} &= 2 \times 3 \times x^{3+2} \\ &= 6x^5 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise B, Question 3

**Question:**

Simplify this expression:

$$4p^3 \div 2p$$

**Solution:**

$$\begin{aligned} &= 4 \div 2 \times p^3 \div p \\ &= 2 \times p^{3-1} \\ &= 2p^2 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise B, Question 4

**Question:**

Simplify this expression:

$$3x^{-4} \div x^{-2}$$

**Solution:**

$$\begin{aligned} &= 3x^{-4 - -2} \\ &= 3x^{-2} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 5

**Question:**

Simplify this expression:

$$k^3 \div k^{-2}$$

**Solution:**

$$\begin{aligned} &= k^{3 - -2} \\ &= k^5 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 6

**Question:**

Simplify this expression:

$$(y^2)^5$$

**Solution:**

$$\begin{aligned} &= y^{2 \times 5} \\ &= y^{10} \end{aligned}$$



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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 7

**Question:**

Simplify this expression:

$$10x^5 \div 2x^{-3}$$

**Solution:**

$$\begin{aligned} &= 5x^{5 - -3} \\ &= 5x^8 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise B, Question 8

**Question:**

Simplify this expression:

$$(p^3)^2 \div p^4$$

**Solution:**

$$\begin{aligned} &= p^6 \div p^4 \\ &= p^{6-4} \\ &= p^2 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 9

**Question:**

Simplify this expression:

$$(2a^3)^2 \div 2a^3$$

**Solution:**

$$\begin{aligned} &= 4a^6 \div 2a^3 \\ &= 2a^{6-3} \\ &= 2a^3 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 10**

**Question:**

Simplify this expression:

$$8p^{-4} \div 4p^3$$

**Solution:**

$$\begin{aligned} &= 2p^{-4-3} \\ &= 2p^{-7} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 11**

**Question:**

Simplify this expression:

$$2a^{-4} \times 3a^{-5}$$

**Solution:**

$$\begin{aligned} &= 6a^{-4 + -5} \\ &= 6a^{-9} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 12**

**Question:**

Simplify this expression:

$$21a^3b^2 \div 7ab^4$$

**Solution:**

$$\begin{aligned} &= 3a^{3-1}b^{2-4} \\ &= 3a^2b^{-2} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 13**

**Question:**

Simplify this expression:

$$9x^2 \times 3 (x^2)^3$$

**Solution:**

$$\begin{aligned} &= 27x^2 \times x^2 \times 3 \\ &= 27x^{2+6} \\ &= 27x^8 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 14

**Question:**

Simplify this expression:

$$3x^3 \times 2x^2 \times 4x^6$$

**Solution:**

$$\begin{aligned} &= 24 \times x^{3+2+6} \\ &= 24x^{11} \end{aligned}$$



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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 15**

**Question:**

Simplify this expression:

$$7a^4 \times (3a^4)^2$$

**Solution:**

$$\begin{aligned} &= 7a^4 \times 9a^8 \\ &= 63a^{12} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 16

**Question:**

Simplify this expression:

$$(4y^3)^3 \div 2y^3$$

**Solution:**

$$\begin{aligned} &= 64y^9 \div 2y^3 \\ &= 32y^6 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise B, Question 17

**Question:**

Simplify this expression:

$$2a^3 \div 3a^2 \times 6a^5$$

**Solution:**

$$\begin{aligned} &= 4a^{3-2+5} \\ &= 4a^6 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise B, Question 18**

**Question:**

Simplify this expression:

$$3a^4 \times 2a^5 \times a^3$$

**Solution:**

$$\begin{aligned} &= 6a^{4+5+3} \\ &= 6a^{12} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 1

**Question:**

Expand and simplify if possible:

$$9(x - 2)$$

**Solution:**

$$= 9x - 18$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 2

**Question:**

Expand and simplify if possible:

$$x(x + 9)$$

**Solution:**

$$= x^2 + 9x$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 3**

**Question:**

Expand and simplify if possible:

$$-3y(4 - 3y)$$

**Solution:**

$$= -12y + 9y^2$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 4**

**Question:**

Expand and simplify if possible:

$$x(y + 5)$$

**Solution:**

$$= xy + 5x$$



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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise C, Question 5

**Question:**

Expand and simplify if possible:

$$-x(3x + 5)$$

**Solution:**

$$= -3x^2 - 5x$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise C, Question 6

**Question:**

Expand and simplify if possible:

$$- 5x ( 4x + 1 )$$

**Solution:**

$$= - 20x^2 - 5x$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 7

**Question:**

Expand and simplify if possible:

$$(4x + 5)x$$

**Solution:**

$$= 4x^2 + 5x$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 8**

**Question:**

Expand and simplify if possible:

$$- 3y ( 5 - 2y^2 )$$

**Solution:**

$$= - 15y + 6y^3$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 9

**Question:**

Expand and simplify if possible:

$$- 2x ( 5x - 4 )$$

**Solution:**

$$= - 10x^2 + 8x$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 10

**Question:**

Expand and simplify if possible:

$$(3x - 5)x^2$$

**Solution:**

$$= 3x^3 - 5x^2$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 11**

**Question:**

Expand and simplify if possible:

$$3(x + 2) + (x - 7)$$

**Solution:**

$$\begin{aligned} &= 3x + 6 + x - 7 \\ &= 4x - 1 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 12

**Question:**

Expand and simplify if possible:

$$5x - 6 - (3x - 2)$$

**Solution:**

$$\begin{aligned} &= 5x - 6 - 3x + 2 \\ &= 2x - 4 \end{aligned}$$



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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 13

**Question:**

Expand and simplify if possible:

$$x ( 3x^2 - 2x + 5 )$$

**Solution:**

$$= 3x^3 - 2x^2 + 5x$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 14**

**Question:**

Expand and simplify if possible:

$$7y^2 ( 2 - 5y + 3y^2 )$$

**Solution:**

$$= 14y^2 - 35y^3 + 21y^4$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 15

**Question:**

Expand and simplify if possible:

$$- 2y^2 ( 5 - 7y + 3y^2 )$$

**Solution:**

$$= - 10y^2 + 14y^3 - 6y^4$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 16**

**Question:**

Expand and simplify if possible:

$$7(x - 2) + 3(x + 4) - 6(x - 2)$$

**Solution:**

$$\begin{aligned} &= 7x - 14 + 3x + 12 - 6x + 12 \\ &= 4x + 10 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 17

**Question:**

Expand and simplify if possible:

$$5x - 3(4 - 2x) + 6$$

**Solution:**

$$\begin{aligned} &= 5x - 12 + 6x + 6 \\ &= 11x - 6 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 18**

**Question:**

Expand and simplify if possible:

$$3x^2 - x(3 - 4x) + 7$$

**Solution:**

$$\begin{aligned} &= 3x^2 - 3x + 4x^2 + 7 \\ &= 7x^2 - 3x + 7 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise C, Question 19

**Question:**

Expand and simplify if possible:

$$4x(x + 3) - 2x(3x - 7)$$

**Solution:**

$$\begin{aligned} &= 4x^2 + 12x - 6x^2 + 14x \\ &= 26x - 2x^2 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise C, Question 20**

**Question:**

Expand and simplify if possible:

$$3x^2 ( 2x + 1 ) - 5x^2 ( 3x - 4 )$$

**Solution:**

$$\begin{aligned} &= 6x^3 + 3x^2 - 15x^3 + 20x^2 \\ &= 23x^2 - 9x^3 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 1**

**Question:**

Factorise this expression completely:

$$4x + 8$$

**Solution:**

$$= 4 ( x + 2 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 2

**Question:**

Factorise this expression completely:

$$6x - 24$$

**Solution:**

$$= 6 ( x - 4 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 3**

**Question:**

Factorise this expression completely:

$$20x + 15$$

**Solution:**

$$= 5 ( 4x + 3 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 4**

**Question:**

Factorise this expression completely:

$$2x^2 + 4$$

**Solution:**

$$= 2 ( x^2 + 2 )$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise D, Question 5

**Question:**

Factorise this expression completely:

$$4x^2 + 20$$

**Solution:**

$$= 4 ( x^2 + 5 )$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise D, Question 6

**Question:**

Factorise this expression completely:

$$6x^2 - 18x$$

**Solution:**

$$= 6x ( x - 3 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 7

**Question:**

Factorise this expression completely:

$$x^2 - 7x$$

**Solution:**

$$= x ( x - 7 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 8**

**Question:**

Factorise this expression completely:

$$2x^2 + 4x$$

**Solution:**

$$= 2x ( x + 2 )$$



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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise D, Question 9

**Question:**

Factorise this expression completely:

$$3x^2 - x$$

**Solution:**

$$= x ( 3x - 1 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 10**

**Question:**

Factorise this expression completely:

$$6x^2 - 2x$$

**Solution:**

$$= 2x ( 3x - 1 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 11

**Question:**

Factorise this expression completely:

$$10y^2 - 5y$$

**Solution:**

$$= 5y ( 2y - 1 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 12**

**Question:**

Factorise this expression completely:

$$35x^2 - 28x$$

**Solution:**

$$= 7x ( 5x - 4 )$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise D, Question 13

**Question:**

Factorise this expression completely:

$$x^2 + 2x$$

**Solution:**

$$= x ( x + 2 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 14**

**Question:**

Factorise this expression completely:

$$3y^2 + 2y$$

**Solution:**

$$= y ( 3y + 2 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 15**

**Question:**

Factorise this expression completely:

$$4x^2 + 12x$$

**Solution:**

$$= 4x ( x + 3 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 16**

**Question:**

Factorise this expression completely:

$$5y^2 - 20y$$

**Solution:**

$$= 5y ( y - 4 )$$



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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 17

**Question:**

Factorise this expression completely:

$$9xy^2 + 12x^2y$$

**Solution:**

$$= 3xy ( 3y + 4x )$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise D, Question 18

**Question:**

Factorise this expression completely:

$$6ab - 2ab^2$$

**Solution:**

$$= 2ab ( 3 - b )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 19**

**Question:**

Factorise this expression completely:

$$5x^2 - 25xy$$

**Solution:**

$$= 5x ( x - 5y )$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise D, Question 20

#### Question:

Factorise this expression completely:

$$12x^2y + 8xy^2$$

#### Solution:

$$= 4xy ( 3x + 2y )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 21

**Question:**

Factorise this expression completely:

$$15y - 20yz^2$$

**Solution:**

$$= 5y ( 3 - 4z^2 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise D, Question 22

**Question:**

Factorise this expression completely:

$$12x^2 - 30$$

**Solution:**

$$= 6 ( 2x^2 - 5 )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 23**

**Question:**

Factorise this expression completely:

$$xy^2 - x^2y$$

**Solution:**

$$= xy ( y - x )$$

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## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise D, Question 24**

**Question:**

Factorise this expression completely:

$$12y^2 - 4yx$$

**Solution:**

$$= 4y ( 3y - x )$$



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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 1

**Question:**

Factorise:

$$x^2 + 4x$$

**Solution:**

$$= x ( x + 4 )$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 2

**Question:**

Factorise:

$$2x^2 + 6x$$

**Solution:**

$$= 2x ( x + 3 )$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 3

**Question:**

Factorise:

$$x^2 + 11x + 24$$

**Solution:**

$$\begin{aligned} &= x^2 + 8x + 3x + 24 \\ &= x(x + 8) + 3(x + 8) \\ &= (x + 8)(x + 3) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise E, Question 4**

**Question:**

Factorise:

$$x^2 + 8x + 12$$

**Solution:**

$$\begin{aligned} &= x^2 + 2x + 6x + 12 \\ &= x(x + 2) + 6(x + 2) \\ &= (x + 2)(x + 6) \end{aligned}$$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise E, Question 5**

**Question:**

Factorise:

$$x^2 + 3x - 40$$

**Solution:**

$$\begin{aligned} &= x^2 + 8x - 5x - 40 \\ &= x(x + 8) - 5(x + 8) \\ &= (x + 8)(x - 5) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 6

**Question:**

Factorise:

$$x^2 - 8x + 12$$

**Solution:**

$$\begin{aligned} &= x^2 - 2x - 6x + 12 \\ &= x(x - 2) - 6(x - 2) \\ &= (x - 2)(x - 6) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 7

**Question:**

Factorise:

$$x^2 + 5x + 6$$

**Solution:**

$$\begin{aligned} &= x^2 + 3x + 2x + 6 \\ &= x(x + 3) + 2(x + 3) \\ &= (x + 3)(x + 2) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 8

**Question:**

Factorise:

$$x^2 - 2x - 24$$

**Solution:**

$$\begin{aligned} &= x^2 - 6x + 4x - 24 \\ &= x(x - 6) + 4(x - 6) \\ &= (x - 6)(x + 4) \end{aligned}$$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 9

**Question:**

Factorise:

$$x^2 - 3x - 10$$

**Solution:**

$$\begin{aligned} &= x^2 - 5x + 2x - 10 \\ &= x(x - 5) + 2(x - 5) \\ &= (x - 5)(x + 2) \end{aligned}$$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise E, Question 10**

**Question:**

Factorise:

$$x^2 + x - 20$$

**Solution:**

$$\begin{aligned} &= x^2 - 4x + 5x - 20 \\ &= x(x - 4) + 5(x - 4) \\ &= (x - 4)(x + 5) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 11

**Question:**

Factorise:

$$2x^2 + 5x + 2$$

**Solution:**

$$\begin{aligned} &= 2x^2 + x + 4x + 2 \\ &= x(2x + 1) + 2(2x + 1) \\ &= (2x + 1)(x + 2) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 12

**Question:**

Factorise:

$$3x^2 + 10x - 8$$

**Solution:**

$$\begin{aligned} &= 3x^2 - 2x + 12x - 8 \\ &= x(3x - 2) + 4(3x - 2) \\ &= (3x - 2)(x + 4) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 13

**Question:**

Factorise:

$$5x^2 - 16x + 3$$

**Solution:**

$$\begin{aligned} &= 5x^2 - 15x - x + 3 \\ &= 5x(x - 3) - (x - 3) \\ &= (x - 3)(5x - 1) \end{aligned}$$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise E, Question 14**

**Question:**

Factorise:

$$6x^2 - 8x - 8$$

**Solution:**

$$\begin{aligned} &= 6x^2 - 12x + 4x - 8 \\ &= 6x(x - 2) + 4(x - 2) \\ &= (x - 2)(6x + 4) = 2(x - 2)(3x + 2) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 15

**Question:**

Factorise:

$$2x^2 + 7x - 15$$

**Solution:**

$$\begin{aligned} &= 2x^2 + 10x - 3x - 15 \\ &= 2x(x + 5) - 3(x + 5) \\ &= (x + 5)(2x - 3) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 16

**Question:**

Factorise:

$$2x^4 + 14x^2 + 24$$

**Solution:**

$$\begin{aligned} &= 2y^2 + 14y + 24 \\ &= 2y^2 + 6y + 8y + 24 \\ &= 2y(y + 3) + 8(y + 3) \\ &= (y + 3)(2y + 8) \\ &= (x^2 + 3)(2x^2 + 8) = 2(x^2 + 3)(x^2 + 4) \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 17

**Question:**

Factorise:

$$x^2 - 4$$

**Solution:**

$$\begin{aligned} &= x^2 - 2^2 \\ &= (x + 2)(x - 2) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 18

**Question:**

Factorise:

$$x^2 - 49$$

**Solution:**

$$\begin{aligned} &= x^2 - 7^2 \\ &= (x + 7)(x - 7) \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 19

**Question:**

Factorise:

$$4x^2 - 25$$

**Solution:**

$$\begin{aligned} &= (2x)^2 - 5^2 \\ &= (2x + 5)(2x - 5) \end{aligned}$$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 20

**Question:**

Factorise:

$$9x^2 - 25y^2$$

**Solution:**

$$\begin{aligned} &= (3x)^2 - (5y)^2 \\ &= (3x + 5y)(3x - 5y) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise E, Question 21

**Question:**

Factorise:

$$36x^2 - 4$$

**Solution:**

$$\begin{aligned} &= 4 ( 9x^2 - 1 ) \\ &= 4 [ ( 3x )^2 - 1 ] \\ &= 4 ( 3x + 1 ) ( 3x - 1 ) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 22

**Question:**

Factorise:

$$2x^2 - 50$$

**Solution:**

$$\begin{aligned} &= 2 ( x^2 - 25 ) \\ &= 2 ( x^2 - 5^2 ) \\ &= 2 ( x + 5 ) ( x - 5 ) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise E, Question 23

**Question:**

Factorise:

$$6x^2 - 10x + 4$$

**Solution:**

$$\begin{aligned} &= 2 ( 3x^2 - 5x + 2 ) \\ &= 2 ( 3x^2 - 3x - 2x + 2 ) \\ &= 2 [ 3x ( x - 1 ) - 2 ( x - 1 ) ] \\ &= 2 ( x - 1 ) ( 3x - 2 ) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise E, Question 24**

**Question:**

Factorise:

$$15x^2 + 42x - 9$$

**Solution:**

$$\begin{aligned} &= 3 ( 5x^2 + 14x - 3 ) \\ &= 3 ( 5x^2 - x + 15x - 3 ) \\ &= 3 [ x ( 5x - 1 ) + 3 ( 5x - 1 ) ] \\ &= 3 ( 5x - 1 ) ( x + 3 ) \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise F, Question 1

#### Question:

Factorise:

Simplify:

(a)  $x^3 \div x^{-2}$

(b)  $x^5 \div x^7$

(c)  $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

(d)  $(x^2)^{\frac{3}{2}}$

(e)  $(x^3)^{\frac{5}{3}}$

(f)  $3x^{0.5} \times 4x^{-0.5}$

(g)  $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$

(h)  $5x^1 \frac{2}{5} \div x^{\frac{2}{5}}$

(i)  $3x^4 \times 2x^{-5}$

#### Solution:

(a)  $= x^{3 - -2}$   
 $= x^5$

(b)  $= x^{5-7}$   
 $= x^{-2}$

(c)  $= x^{\frac{3}{2} + \frac{5}{2}}$   
 $= x^4$

(d)  $= x^{2 \times \frac{3}{2}}$   
 $= x^3$

(e)  $= x^{3 \times \frac{5}{3}}$   
 $= x^5$

(f)  $= 12x^{0.5 + -0.5}$   
 $= 12x^0$

$$= 12$$

$$(g) = 3x^{\frac{2}{3} - \frac{1}{6}}$$

$$= 3x^{\frac{1}{2}}$$

$$(h) = 5x^{1 \frac{2}{5} - \frac{2}{5}}$$

$$= 5x$$

$$(i) = 6x^{4 + -5}$$

$$= 6x^{-1}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise F, Question 2

**Question:**

Factorise:

Evaluate:

(a)  $25^{\frac{1}{2}}$

(b)  $81^{\frac{1}{2}}$

(c)  $27^{\frac{1}{3}}$

(d)  $4^{-2}$

(e)  $9^{-\frac{1}{2}}$

(f)  $(-5)^{-3}$

(g)  $\left(\frac{3}{4}\right)^0$

(h)  $1296^{\frac{1}{4}}$

(i)  $\left(1\frac{9}{16}\right)^{\frac{3}{2}}$

(j)  $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

(k)  $\left(\frac{6}{5}\right)^{-1}$

(l)  $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

**Solution:**

(a)  $= \sqrt{25}$   
 $= \pm 5$

(b)  $= \sqrt{81}$

$$= \pm 9$$

$$\begin{aligned} \text{(c)} &= \sqrt[3]{27} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{(d)} &= \frac{1}{4^2} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{(e)} &= \frac{1}{9^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{9}} \\ &= \pm \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{(f)} &= \frac{1}{(-5)^3} \\ &= \frac{1}{-125} \end{aligned}$$

$$\text{(g)} = 1$$

$$\begin{aligned} \text{(h)} &= \sqrt[4]{1296} \\ &= \pm 6 \end{aligned}$$

$$\begin{aligned} \text{(i)} &= \left( \frac{25}{16} \right)^{\frac{3}{2}} \\ &= \frac{(\sqrt{25})^3}{(\sqrt{16})^3} \\ &= \frac{5^3}{4^3} \\ &= \frac{125}{64} \end{aligned}$$

$$\begin{aligned} \text{(j)} &= \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2} \\ &= \frac{(3)^2}{(2)^2} \\ &= \frac{9}{4} \end{aligned}$$

$$\begin{aligned} \text{(k)} &= \left( \frac{5}{6} \right)^1 \\ &= \frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{(1)} \quad & \frac{(\sqrt[3]{512})^2}{(\sqrt[3]{343})^2} \\ &= \frac{(8)^2}{(7)^2} \\ &= \frac{64}{49} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 1

**Question:**

Simplify:

$$\sqrt{28}$$

**Solution:**

$$\begin{aligned} &= \sqrt{4} \times \sqrt{7} \\ &= 2\sqrt{7} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 2

**Question:**

Simplify:

$$\sqrt{72}$$

**Solution:**

$$\begin{aligned} &= \sqrt{8} \times \sqrt{9} \\ &= \sqrt{2} \times \sqrt{4} \times \sqrt{9} \\ &= \sqrt{2} \times 2 \times 3 \\ &= 6\sqrt{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 3

**Question:**

Simplify:

$$\sqrt{50}$$

**Solution:**

$$\begin{aligned} &= \sqrt{25} \times \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 4

**Question:**

Simplify:

$$\sqrt{32}$$

**Solution:**

$$\begin{aligned} &= \sqrt{16} \times \sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 5

**Question:**

Simplify:

$$\sqrt{90}$$

**Solution:**

$$\begin{aligned} &= \sqrt{9} \times \sqrt{10} \\ &= 3\sqrt{10} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 6

**Question:**

Simplify:

$$\frac{\sqrt{12}}{2}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{4 \times 3}}{2} \\ &= \frac{2 \times \sqrt{3}}{2} \\ &= \sqrt{3} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise G, Question 7

**Question:**

Simplify:

$$\frac{\sqrt{27}}{3}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{9 \times 3}}{3} \\ &= \frac{3 \times \sqrt{3}}{3} \\ &= \sqrt{3} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise G, Question 8**

**Question:**

Simplify:

$$\sqrt{20} + \sqrt{80}$$

**Solution:**

$$\begin{aligned} &= \sqrt{4} \sqrt{5} + \sqrt{16} \sqrt{5} \\ &= 2 \sqrt{5} + 4 \sqrt{5} \\ &= 6 \sqrt{5} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise G, Question 9**

**Question:**

Simplify:

$$\sqrt{200} + \sqrt{18} - \sqrt{72}$$

**Solution:**

$$\begin{aligned} &= \sqrt{100} \sqrt{2} + \sqrt{9} \sqrt{2} - \sqrt{9} \sqrt{4} \sqrt{2} \\ &= 10 \sqrt{2} + 3 \sqrt{2} - 6 \sqrt{2} \\ &= 7 \sqrt{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise G, Question 10**

**Question:**

Simplify:

$$\sqrt{175} + \sqrt{63} + 2\sqrt{28}$$

**Solution:**

$$\begin{aligned} &= \sqrt{25} \times \sqrt{7} + \sqrt{9} \times \sqrt{7} + 2 \times \sqrt{4} \times \sqrt{7} \\ &= 5\sqrt{7} + 3\sqrt{7} + 4\sqrt{7} \\ &= 12\sqrt{7} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise G, Question 11**

**Question:**

Simplify:

$$1 \sqrt{28} - 2 \sqrt{63} + \sqrt{7}$$

**Solution:**

$$\begin{aligned} &= \sqrt{4} \sqrt{7} - 2 \sqrt{9} \sqrt{7} + \sqrt{7} \\ &= 2 \sqrt{7} - 6 \sqrt{7} + \sqrt{7} \\ &= -3 \sqrt{7} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise G, Question 12

**Question:**

Simplify:

$$\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$$

**Solution:**

$$\begin{aligned} &= \sqrt{16}\sqrt{5} - 2\sqrt{4}\sqrt{5} + 3\sqrt{9}\sqrt{5} \\ &= 4\sqrt{5} - 4\sqrt{5} + 9\sqrt{5} \\ &= 9\sqrt{5} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 13

**Question:**

Simplify:

$$3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$$

**Solution:**

$$\begin{aligned} &= 3\sqrt{16}\sqrt{5} - 2\sqrt{4}\sqrt{5} + 5\sqrt{9}\sqrt{5} \\ &= 12\sqrt{5} - 4\sqrt{5} + 15\sqrt{5} \\ &= 23\sqrt{5} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise G, Question 14

**Question:**

Simplify:

$$\frac{\sqrt{44}}{\sqrt{11}}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{4} \sqrt{11}}{\sqrt{11}} \\ &= 2 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise G, Question 15**

**Question:**

Simplify:

$$\sqrt{12} + 3\sqrt{48} + \sqrt{75}$$

**Solution:**

$$\begin{aligned} &= \sqrt{4}\sqrt{3} + 3\sqrt{16}\sqrt{3} + \sqrt{25}\sqrt{3} \\ &= 2\sqrt{3} + 12\sqrt{3} + 5\sqrt{3} \\ &= 19\sqrt{3} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 1

#### Question:

Rationalise the denominator:

$$\frac{1}{\sqrt{5}}$$

#### Solution:

$$\begin{aligned} &= \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{\sqrt{5}}{5} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 2

**Question:**

Rationalise the denominator:

$$\frac{1}{\sqrt{11}}$$

**Solution:**

$$\begin{aligned} &= \frac{1 \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}} \\ &= \frac{\sqrt{11}}{11} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 3

**Question:**

Rationalise the denominator:

$$\frac{1}{\sqrt{2}}$$

**Solution:**

$$\begin{aligned} &= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 4

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{3}}{\sqrt{15}}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{3} \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}} \\ &= \frac{\sqrt{3 \times 15}}{15} \\ &= \frac{\sqrt{45}}{15} \\ &= \frac{\sqrt{9 \times 5}}{15} \\ &= \frac{\sqrt{9} \times \sqrt{5}}{15} \\ &= \frac{3 \times \sqrt{5}}{15} \\ &= \frac{\sqrt{5}}{5} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 5

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{12}}{\sqrt{48}}$$

**Solution:**

$$= \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{4}}$$

$$= \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 6

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{5}}{\sqrt{80}}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{16}} \\ &= \frac{1}{\sqrt{16}} \\ &= \frac{1}{4} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
**Exercise H, Question 7**

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{12}}{\sqrt{156}}$$

**Solution:**

$$= \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{13}}$$

$$= \frac{1}{\sqrt{13}}$$

$$= \frac{1 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}}$$

$$= \frac{\sqrt{13}}{13}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise H, Question 8

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{7}}{\sqrt{63}}$$

**Solution:**

$$\begin{aligned}\frac{\sqrt{7}}{\sqrt{7 \times 9}} \\ &= \frac{1}{\sqrt{9}} \\ &= \frac{1}{3}\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 9

**Question:**

Rationalise the denominator:

$$\frac{1}{1 + \sqrt{3}}$$

**Solution:**

$$\begin{aligned} &= \frac{1 \times (1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} \\ &= \frac{1 - \sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3} \\ &= \frac{1 - \sqrt{3}}{-2} \text{ or} \\ &= \frac{-1 + \sqrt{3}}{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise H, Question 10

**Question:**

Rationalise the denominator:

$$\frac{1}{2 + \sqrt{5}}$$

**Solution:**

$$\begin{aligned} &= \frac{1 \times (2 - \sqrt{5})}{(2 + \sqrt{5})(2 - \sqrt{5})} \\ &= \frac{2 - \sqrt{5}}{4 - 5} \\ &= \frac{2 - \sqrt{5}}{-1} \\ &= -2 + \sqrt{5} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise H, Question 11

**Question:**

Rationalise the denominator:

$$\frac{1}{3 - \sqrt{7}}$$

**Solution:**

$$\begin{aligned} &= \frac{3 + \sqrt{7}}{(3 - \sqrt{7})(3 + \sqrt{7})} \\ &= \frac{3 + \sqrt{7}}{9 - 7} \\ &= \frac{3 + \sqrt{7}}{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 12

**Question:**

Rationalise the denominator:

$$\frac{4}{3 - \sqrt{5}}$$

**Solution:**

$$\begin{aligned} &= \frac{4 \times (3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} \\ &= \frac{12 + 4\sqrt{5}}{9 - 5} \\ &= \frac{12 + 4\sqrt{5}}{4} \\ &= 3 + \sqrt{5} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 13

**Question:**

Rationalise the denominator:

$$\frac{1}{\sqrt{5} - \sqrt{3}}$$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{5} + \sqrt{3}}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \\ &= \frac{\sqrt{5} + \sqrt{3}}{5 - 3} \\ &= \frac{\sqrt{5} + \sqrt{3}}{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 14

**Question:**

Rationalise the denominator:

$$\frac{3 - \sqrt{2}}{4 - \sqrt{5}}$$

**Solution:**

$$\begin{aligned} &= \frac{(3 - \sqrt{2})(4 + \sqrt{5})}{(4 - \sqrt{5})(4 + \sqrt{5})} \\ &= \frac{(3 - \sqrt{2})(4 + \sqrt{5})}{16 - 5} \\ &= \frac{(3 - \sqrt{2})(4 + \sqrt{5})}{11} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebra and functions  
Exercise H, Question 15

**Question:**

Rationalise the denominator:

$$\frac{5}{2 + \sqrt{5}}$$

**Solution:**

$$\begin{aligned} &= \frac{5 \times (2 - \sqrt{5})}{(2 + \sqrt{5})(2 - \sqrt{5})} \\ &= \frac{5(2 - \sqrt{5})}{4 - 5} \\ &= \frac{5(2 - \sqrt{5})}{-1} \\ &= 5(\sqrt{5} - 2) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 16

**Question:**

Rationalise the denominator:

$$\frac{5\sqrt{2}}{\sqrt{8} - \sqrt{7}}$$

**Solution:**

$$\begin{aligned} &= \frac{5\sqrt{2}(\sqrt{8} + \sqrt{7})}{(\sqrt{8} - \sqrt{7})(\sqrt{8} + \sqrt{7})} \\ &= \frac{5(\sqrt{8 \times 2} + \sqrt{2}\sqrt{7})}{8 - 7} \\ &= \frac{5(\sqrt{16} + \sqrt{14})}{1} \\ &= 5(4 + \sqrt{14}) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 17

**Question:**

Rationalise the denominator:

$$\frac{11}{3 + \sqrt{11}}$$

**Solution:**

$$\begin{aligned} &= \frac{11(3 - \sqrt{11})}{(3 + \sqrt{11})(3 - \sqrt{11})} \\ &= \frac{11(3 - \sqrt{11})}{9 - 11} \\ &= \frac{11(3 - \sqrt{11})}{-2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 18

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{3} - \sqrt{7}}{\sqrt{3} + \sqrt{7}}$$

**Solution:**

$$\begin{aligned} &= \frac{(\sqrt{3} - \sqrt{7})(\sqrt{3} - \sqrt{7})}{(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})} \\ &= \frac{3 - \sqrt{21} - \sqrt{21} + 7}{3 - 7} \\ &= \frac{10 - 2\sqrt{21}}{-4} \\ &= \frac{5 - \sqrt{21}}{-2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 19

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{17} - \sqrt{11}}{\sqrt{17} + \sqrt{11}}$$

**Solution:**

$$\begin{aligned} &= \frac{(\sqrt{17} - \sqrt{11})(\sqrt{17} - \sqrt{11})}{(\sqrt{17} + \sqrt{11})(\sqrt{17} - \sqrt{11})} \\ &= \frac{17 - \sqrt{187} - \sqrt{187} + 11}{17 - 11} \\ &= \frac{28 - 2\sqrt{187}}{6} \\ &= \frac{14 - \sqrt{187}}{3} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise H, Question 20

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{41} + \sqrt{29}}{\sqrt{41} - \sqrt{29}}$$

**Solution:**

$$\begin{aligned} &= \frac{(\sqrt{41} + \sqrt{29})(\sqrt{41} + \sqrt{29})}{(\sqrt{41} - \sqrt{29})(\sqrt{41} + \sqrt{29})} \\ &= \frac{41 + 2\sqrt{41}\sqrt{29} + 29}{41 - 29} \\ &= \frac{70 + 2\sqrt{1189}}{12} \\ &= \frac{35 + \sqrt{1189}}{6} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

**Algebra and functions**  
Exercise H, Question 21

**Question:**

Rationalise the denominator:

$$\frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}}$$

**Solution:**

$$\begin{aligned} &= \frac{(\sqrt{2} - \sqrt{3})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} \\ &= \frac{\sqrt{6} - 3 + 2 - \sqrt{6}}{3 - 2} \\ &= \frac{-1}{1} \\ &= -1 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 1

**Question:**

Simplify:

(a)  $y^3 \times y^5$

(b)  $3x^2 \times 2x^5$

(c)  $(4x^2)^3 \div 2x^5$

(d)  $4b^2 \times 3b^3 \times b^4$

**Solution:**

(a)  $= y^{3+5}$   
 $= y^8$

(b)  $= 3 \times 2 \times x^{2+5}$   
 $= 6x^7$

(c)  $= 4^3 x^{2 \times 3} \div 2x^5$   
 $= 64x^6 \div 2x^5$   
 $= 32x^{6-5}$   
 $= 32x$

(d)  $= 4 \times 3 \times b^{2+3+4}$   
 $= 12b^9$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 2

**Question:**

Expand the brackets:

(a)  $3(5y + 4)$

(b)  $5x^2(3 - 5x + 2x^2)$

(c)  $5x(2x + 3) - 2x(1 - 3x)$

(d)  $3x^2(1 + 3x) - 2x(3x - 2)$

**Solution:**

(a)  $= 15y + 12$

(b)  $= 15x^2 - 25x^3 + 10x^4$

(c)  $= 10x^2 + 15x - 2x + 6x^2$   
 $= 16x^2 + 13x$

(d)  $= 3x^2 + 9x^3 - 6x^2 + 4x$   
 $= 9x^3 - 3x^2 + 4x$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 3

**Question:**

Factorise these expressions completely:

(a)  $3x^2 + 4x$

(b)  $4y^2 + 10y$

(c)  $x^2 + xy + xy^2$

(d)  $8xy^2 + 10x^2y$

**Solution:**

(a)  $= x ( 3x + 4 )$

(b)  $= 2y ( 2y + 5 )$

(c)  $= x ( x + y + y^2 )$

(d)  $= 2xy ( 4y + 5x )$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 4

**Question:**

Factorise:

(a)  $x^2 + 3x + 2$

(b)  $3x^2 + 6x$

(c)  $x^2 - 2x - 35$

(d)  $2x^2 - x - 3$

(e)  $5x^2 - 13x - 6$

(f)  $6 - 5x - x^2$

**Solution:**

$$\begin{aligned} \text{(a)} &= x^2 + x + 2x + 2 \\ &= x(x + 1) + 2(x + 1) \\ &= (x + 1)(x + 2) \end{aligned}$$

$$\text{(b)} = 3x(x + 2)$$

$$\begin{aligned} \text{(c)} &= x^2 - 7x + 5x - 35 \\ &= x(x - 7) + 5(x - 7) \\ &= (x - 7)(x + 5) \end{aligned}$$

$$\begin{aligned} \text{(d)} &= 2x^2 - 3x + 2x - 3 \\ &= x(2x - 3) + (2x - 3) \\ &= (2x - 3)(x + 1) \end{aligned}$$

$$\begin{aligned} \text{(e)} &= 5x^2 + 2x - 15x - 6 \\ &= x(5x + 2) - 3(5x + 2) \\ &= (5x + 2)(x - 3) \end{aligned}$$

$$\begin{aligned} \text{(f)} &= 6 + x - 6x - x^2 \\ &= (6 + x) - x(6 + x) \\ &= (1 - x)(6 + x) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 5

#### Question:

Simplify:

$$(a) 9x^3 \div 3x^{-3}$$

$$(b) \left( 4^{\frac{3}{2}} \right)^{\frac{1}{3}}$$

$$(c) 3x^{-2} \times 2x^4$$

$$(d) 3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$$

#### Solution:

$$(a) = 3x^{3 - -3}$$

$$= 3x^6$$

$$(b) [ (\sqrt{4})^3 ]^{\frac{1}{3}}$$

$$= (\sqrt{4})^{3 \times \frac{1}{3}}$$

$$= \sqrt{4}$$

$$= \pm 2$$

$$(c) = 6x^{-2+4}$$

$$= 6x^2$$

$$(d) = \frac{1}{2}x^{\frac{1}{3} - \frac{2}{3}}$$

$$= \frac{1}{2}x^{-\frac{1}{3}} \text{ or}$$

$$= \frac{1}{2(\sqrt[3]{x})}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 6

**Question:**

Evaluate:

$$(a) \left( \frac{8}{27} \right)^{\frac{2}{3}}$$

$$(b) \left( \frac{225}{289} \right)^{\frac{3}{2}}$$

**Solution:**

$$\begin{aligned}(a) &= \left( \frac{\sqrt[3]{8}}{\sqrt[3]{27}} \right)^2 \\ &= \left( \frac{2}{3} \right)^2 \\ &= \frac{4}{9}\end{aligned}$$

$$\begin{aligned}(b) &= \left( \frac{\sqrt{225}}{\sqrt{289}} \right)^3 \\ &= \frac{15^3}{17^3} \\ &= \frac{3375}{4913}\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 7

#### Question:

Simplify:

(a)  $\frac{3}{\sqrt{63}}$

(b)  $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

#### Solution:

(a)  $= \frac{3}{\sqrt{9 \times 7}}$

$= \frac{3}{3\sqrt{7}}$

$= \frac{1}{\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$  (If you rationalise)

(b)  $= 2\sqrt{5} + 2 \times 3\sqrt{5} - 4\sqrt{5}$   
 $= 4\sqrt{5}$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise I, Question 8

#### Question:

Rationalise:

$$(a) \frac{1}{\sqrt{3}}$$

$$(b) \frac{1}{\sqrt{2}-1}$$

$$(c) \frac{3}{\sqrt{3}-2}$$

$$(d) \frac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$$

#### Solution:

$$(a) = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

$$(b) = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{\sqrt{2}+1}{2-1}$$

$$= \sqrt{2}+1$$

$$(c) = \frac{3(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}$$

$$= \frac{3\sqrt{3}+6}{3-4}$$

$$= -3\sqrt{3}-6$$

$$(d) = \frac{(\sqrt{23}-\sqrt{37})(\sqrt{23}-\sqrt{37})}{(\sqrt{23}+\sqrt{37})(\sqrt{23}-\sqrt{37})}$$

$$= \frac{23-2\sqrt{23}\sqrt{37}+37}{23-37}$$

$$= \frac{60-2\sqrt{851}}{-14}$$

$$= \frac{30-\sqrt{851}}{-7}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 1

#### Question:

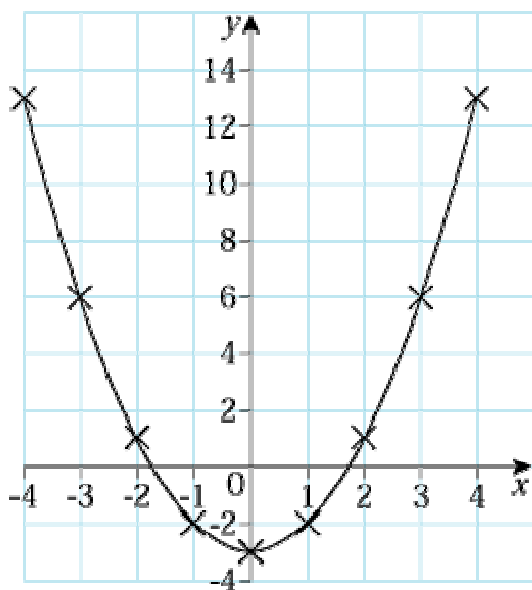
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = x^2 - 3$$

#### Solution:

$$y = x^2 - 3.$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$x^2 - 3$	$16 - 3$	$9 - 3$	$4 - 3$	$1 - 3$	$0 - 3$	$1 - 3$	$4 - 3$	$9 - 3$	$16 - 3$
$y$	$13$	$6$	$1$	$-2$	$-3$	$-2$	$1$	$6$	$13$



Equation of line of symmetry is  $x = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 2

#### Question:

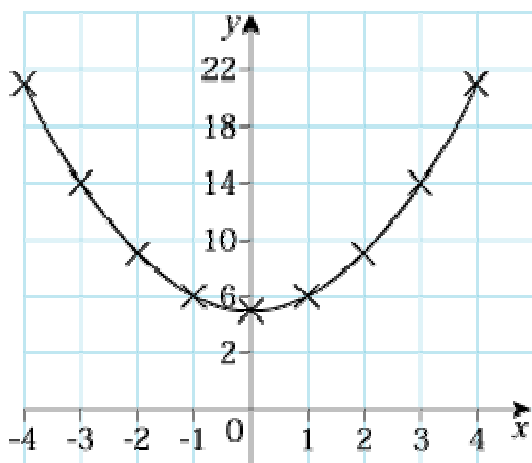
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = x^2 + 5$$

#### Solution:

$$y = x^2 + 5.$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$x^2 + 5$	$16 + 5$	$9 + 5$	$4 + 5$	$1 + 5$	$0 + 5$	$1 + 5$	$4 + 5$	$9 + 5$	$16 + 5$
$y$	$21$	$14$	$9$	$6$	$5$	$6$	$9$	$14$	$21$



Equation of line of symmetry is  $x = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 3

#### Question:

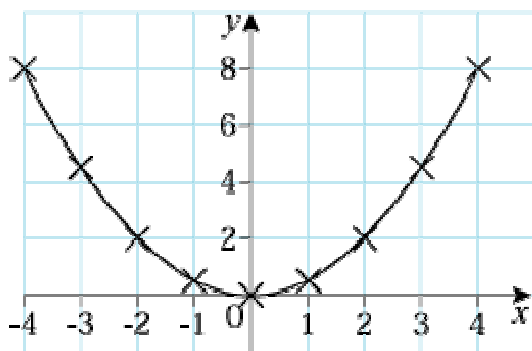
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = \frac{1}{2}x^2$$

#### Solution:

$$y = \frac{1}{2}x^2$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$\frac{1}{2}x^2$	$8$	$4\frac{1}{2}$	$2$	$\frac{1}{2}$	$0$	$\frac{1}{2}$	$2$	$4\frac{1}{2}$	$8$
$y$	$8$	$4\frac{1}{2}$	$2$	$\frac{1}{2}$	$0$	$\frac{1}{2}$	$2$	$4\frac{1}{2}$	$8$



Equation of line of symmetry is  $x = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 4

#### Question:

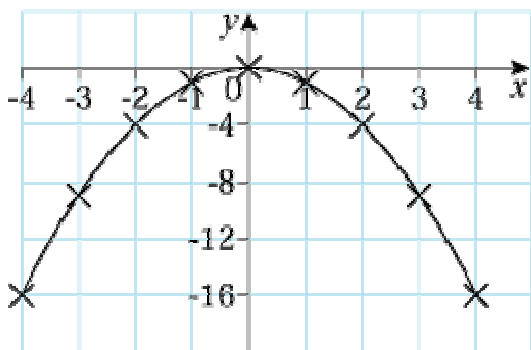
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = -x^2$$

#### Solution:

$$y = -x^2$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$-x^2$	$-16$	$-9$	$-4$	$-1$	$0$	$-1$	$-4$	$-9$	$-16$
$y$	$-16$	$-9$	$-4$	$-1$	$0$	$-1$	$-4$	$-9$	$-16$



Equation of line of symmetry is  $x = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 5

#### Question:

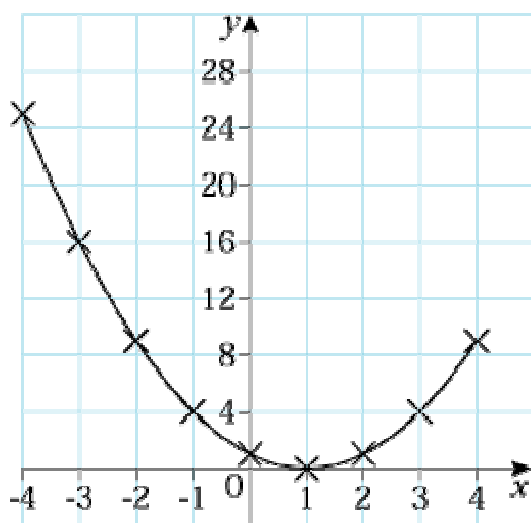
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = (x - 1)^2$$

#### Solution:

$$y = (x - 1)^2$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$(x - 1)^2$	$25$	$16$	$9$	$4$	$1$	$0$	$1$	$4$	$9$
$y$	$25$	$16$	$9$	$4$	$1$	$0$	$1$	$4$	$9$



Equation of line of symmetry is  $x = 1$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 6

#### Question:

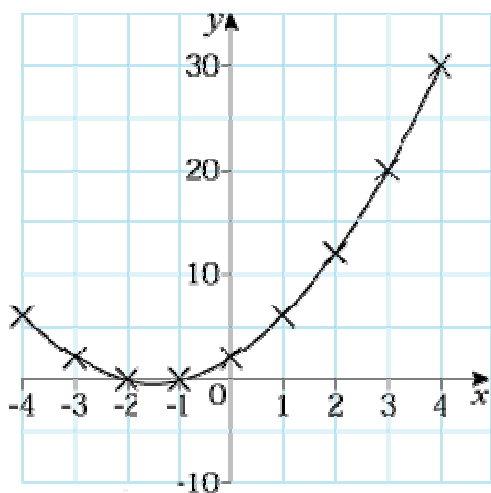
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = x^2 + 3x + 2$$

#### Solution:

$$y = x^2 + 3x + 2$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$x^2 + 3x + 2$	$16 - 12 + 2$	$9 - 9 + 2$	$4 - 6 + 2$	$1 - 3 + 2$	$0 + 0 + 2$	$1 + 3 + 2$	$4 + 6 + 2$	$9 + 9 + 2$	$16 + 12 + 2$
$y$	$6$	$2$	$0$	$0$	$2$	$6$	$12$	$20$	$30$



Equation of line of symmetry is  $x = -1\frac{1}{2}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 7

#### Question:

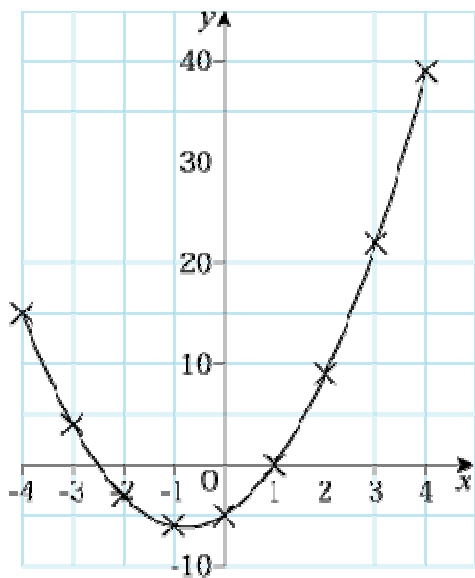
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = 2x^2 + 3x - 5$$

#### Solution:

$$y = 2x^2 + 3x - 5$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$2x^2 + 3x - 5$	$32 - 12 - 5$	$18 - 9 - 5$	$8 - 6 - 5$	$2 - 3 - 5$	$0 + 0 - 5$	$2 + 3 - 5$	$8 + 6 - 5$	$18 + 9 - 5$	$32 + 12 - 5$
$y$	$15$	$4$	$-3$	$-6$	$-5$	$0$	$9$	$22$	$39$



Equation of line of symmetry is  $x = -\frac{3}{4}$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 8

#### Question:

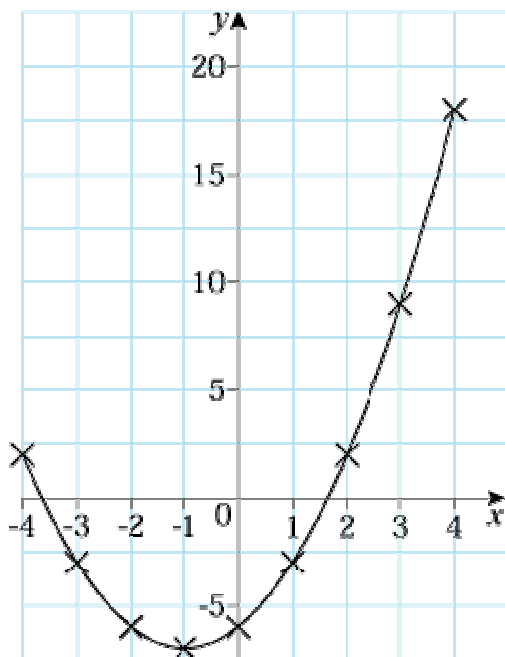
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = x^2 + 2x - 6$$

#### Solution:

$$y = x^2 + 2x - 6$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$x^2 + 2x - 6$	$16 - 8 - 6$	$9 - 6 - 6$	$4 - 4 - 6$	$1 - 2 - 6$	$0 + 0 - 6$	$1 + 2 - 6$	$4 + 4 - 6$	$9 + 6 - 6$	$16 + 8 - 6$
$y$	$2$	$-3$	$-6$	$-7$	$-6$	$-3$	$2$	$9$	$18$



Equation of line of symmetry is  $x = -1$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise A, Question 9

#### Question:

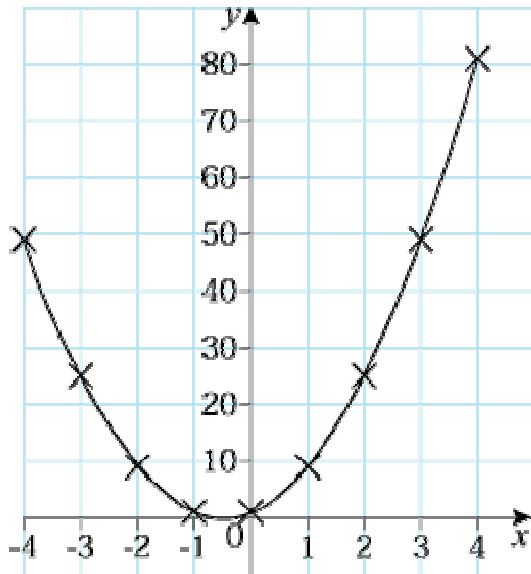
Draw a graph with the following equation, taking values of  $x$  from  $-4$  to  $+4$ . For each graph write down the equation of the line of symmetry.

$$y = (2x + 1)^2$$

#### Solution:

$$y = (2x + 1)^2$$

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$2x + 1$	$-8 + 1$	$-6 + 1$	$-4 + 1$	$-2 + 1$	$0 + 1$	$2 + 1$	$4 + 1$	$6 + 1$	$8 + 1$
$(2x + 1)^2$	$49$	$25$	$9$	$1$	$1$	$9$	$25$	$49$	$81$



Equation of line of symmetry is  $x = -\frac{1}{2}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 1

**Question:**

Solve the following equation:

$$x^2 = 4x$$

**Solution:**

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \text{ or } x - 4 = 0$$

$$\text{So } x = 0 \text{ or } x = 4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 2

**Question:**

Solve the following equation:

$$x^2 = 25x$$

**Solution:**

$$x^2 - 25x = 0$$

$$x(x - 25) = 0$$

$$x = 0 \text{ or } x - 25 = 0$$

$$\text{So } x = 0 \text{ or } x = 25$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 3

**Question:**

Solve the following equation:

$$3x^2 = 6x$$

**Solution:**

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ or } x - 2 = 0$$

$$\text{So } x = 0 \text{ or } x = 2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 4

**Question:**

Solve the following equation:

$$5x^2 = 30x$$

**Solution:**

$$5x^2 - 30x = 0$$

$$5x(x - 6) = 0$$

$$x = 0 \text{ or } x - 6 = 0$$

$$\text{So } x = 0 \text{ or } x = 6$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 5

**Question:**

Solve the following equation:

$$x^2 + 3x + 2 = 0$$

**Solution:**

$$(x + 1)(x + 2) = 0$$
$$x + 1 = 0 \text{ or } x + 2 = 0$$
$$\text{So } x = -1 \text{ or } x = -2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 6

**Question:**

Solve the following equation:

$$x^2 + 5x + 4 = 0$$

**Solution:**

$$(x + 1)(x + 4) = 0$$
$$x + 1 = 0 \text{ or } x + 4 = 0$$
$$\text{So } x = -1 \text{ or } x = -4$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 7

**Question:**

Solve the following equation:

$$x^2 + 7x + 10 = 0$$

**Solution:**

$$(x + 2)(x + 5) = 0$$
$$x + 2 = 0 \text{ or } x + 5 = 0$$
$$x = -2 \text{ or } x = -5$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 8

**Question:**

Solve the following equation:

$$x^2 - x - 6 = 0$$

**Solution:**

$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \text{ or } x + 2 = 0$$
$$\text{So } x = 3 \text{ or } x = -2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 9

**Question:**

Solve the following equation:

$$x^2 - 8x + 15 = 0$$

**Solution:**

$$(x - 3)(x - 5) = 0$$
$$x - 3 = 0 \text{ or } x - 5 = 0$$
$$\text{So } x = 3 \text{ or } x = 5$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 10

**Question:**

Solve the following equation:

$$x^2 - 9x + 20 = 0$$

**Solution:**

$$(x - 4)(x - 5) = 0$$
$$x - 4 = 0 \text{ or } x - 5 = 0$$
$$\text{So } x = 4 \text{ or } x = 5$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 11

**Question:**

Solve the following equation:

$$x^2 - 5x - 6 = 0$$

**Solution:**

$$(x - 6)(x + 1) = 0$$
$$x - 6 = 0 \text{ or } x + 1 = 0$$
$$\text{So } x = 6 \text{ or } x = -1$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 12

**Question:**

Solve the following equation:

$$x^2 - 4x - 12 = 0$$

**Solution:**

$$(x - 6)(x + 2) = 0$$
$$x - 6 = 0 \text{ or } x + 2 = 0$$
$$\text{So } x = 6 \text{ or } x = -2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 13

**Question:**

Solve the following equation:

$$2x^2 + 7x + 3 = 0$$

**Solution:**

$$(2x + 1)(x + 3) = 0$$

$$2x + 1 = 0 \text{ or } x + 3 = 0$$

$$2x = -1 \text{ or } x = -3$$

$$\text{So } x = -\frac{1}{2} \text{ or } x = -3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 14

**Question:**

Solve the following equation:

$$6x^2 - 7x - 3 = 0$$

**Solution:**

$$(3x + 1)(2x - 3) = 0$$
$$3x + 1 = 0 \text{ or } 2x - 3 = 0$$

$$\text{So } x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 15

**Question:**

Solve the following equation:

$$6x^2 - 5x - 6 = 0$$

**Solution:**

$$(3x + 2)(2x - 3) = 0$$
$$3x + 2 = 0 \text{ or } 2x - 3 = 0$$

$$\text{So } x = -\frac{2}{3} \text{ or } x = \frac{3}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 16

**Question:**

Solve the following equation:

$$4x^2 - 16x + 15 = 0$$

**Solution:**

$$(2x - 3)(2x - 5) = 0$$
$$2x - 3 = 0 \text{ or } 2x - 5 = 0$$

$$\text{So } x = \frac{3}{2} \text{ or } x = \frac{5}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 17

**Question:**

Solve the following equation:

$$3x^2 + 5x = 2$$

**Solution:**

$$\begin{aligned}3x^2 + 5x - 2 &= 0 \\(3x - 1)(x + 2) &= 0 \\3x - 1 = 0 \text{ or } x + 2 &= 0\end{aligned}$$

$$\text{So } x = \frac{1}{3} \text{ or } x = -2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 18

**Question:**

Solve the following equation:

$$(2x - 3)^2 = 9$$

**Solution:**

$$2x - 3 = \pm 3$$

$$2x = \pm 3 + 3$$

$$x = \frac{\pm 3 + 3}{2}$$

$$\text{So } x = 3 \text{ or } x = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 19

**Question:**

Solve the following equation:

$$(x - 7)^2 = 36$$

**Solution:**

$$x - 7 = \pm 6$$

$$x = \pm 6 + 7$$

$$\text{So } x = 1 \text{ or } x = 13$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 20

**Question:**

Solve the following equation:

$$2x^2 = 8$$

**Solution:**

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{So } x = 2 \text{ or } x = -2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 21

**Question:**

Solve the following equation:

$$3x^2 = 5$$

**Solution:**

$$x^2 = \frac{5}{3}$$

$$x = \pm \sqrt{\frac{5}{3}}$$

$$\text{So } x = \sqrt{\frac{5}{3}} \text{ or } x = -\sqrt{\frac{5}{3}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 22

**Question:**

Solve the following equation:

$$(x - 3)^2 = 13$$

**Solution:**

$$x - 3 = \pm \sqrt{13}$$

$$x = 3 \pm \sqrt{13}$$

$$\text{So } x = 3 + \sqrt{13} \text{ or } x = 3 - \sqrt{13}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 23

**Question:**

Solve the following equation:

$$(3x - 1)^2 = 11$$

**Solution:**

$$3x - 1 = \pm \sqrt{11}$$

$$3x = 1 \pm \sqrt{11}$$

$$x = \frac{1 \pm \sqrt{11}}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 24

**Question:**

Solve the following equation:

$$5x^2 - 10x^2 = -7 + x + x^2$$

**Solution:**

$$\begin{aligned} -6x^2 - x + 7 &= 0 \\ (1 - x)(7 + 6x) &= 0 \\ x = 1 \text{ or } 6x &= -7 \end{aligned}$$

$$\text{So } x = 1 \text{ or } x = -\frac{7}{6}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 25

**Question:**

Solve the following equation:

$$6x^2 - 7 = 11x$$

**Solution:**

$$\begin{aligned}6x^2 - 11x - 7 &= 0 \\(3x - 7)(2x + 1) &= 0 \\3x - 7 = 0 \text{ or } 2x + 1 &= 0\end{aligned}$$

$$\text{So } x = \frac{7}{3} \text{ or } x = -\frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise B, Question 26

**Question:**

Solve the following equation:

$$4x^2 + 17x = 6x - 2x^2$$

**Solution:**

$$6x^2 + 11x = 0$$

$$x(6x + 11) = 0$$

$$x = 0 \text{ or } 6x + 11 = 0$$

$$\text{So } x = 0 \text{ or } x = -\frac{11}{6}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 1

**Question:**

Complete the square for the expression:

$$x^2 + 4x$$

**Solution:**

$$= (x + 2)^2 - 4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 2

**Question:**

Complete the square for the expression:

$$x^2 - 6x$$

**Solution:**

$$= (x - 3)^2 - 9$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 3

**Question:**

Complete the square for the expression:

$$x^2 - 16x$$

**Solution:**

$$= (x - 8)^2 - 64$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 4

**Question:**

Complete the square for the expression:

$$x^2 + x$$

**Solution:**

$$= \left( x + \frac{1}{2} \right)^2 - \frac{1}{4}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 5

**Question:**

Complete the square for the expression:

$$x^2 - 14x$$

**Solution:**

$$= (x - 7)^2 - 49$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 6

**Question:**

Complete the square for the expression:

$$2x^2 + 16x$$

**Solution:**

$$\begin{aligned} &= 2 ( x^2 + 8x ) \\ &= 2 [ ( x + 4 )^2 - 16 ] \\ &= 2 ( x + 4 )^2 - 32 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 7

**Question:**

Complete the square for the expression:

$$3x^2 - 24x$$

**Solution:**

$$\begin{aligned} &= 3 ( x^2 - 8x ) \\ &= 3 [ ( x - 4 )^2 - 16 ] \\ &= 3 ( x - 4 )^2 - 48 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 8

**Question:**

Complete the square for the expression:

$$2x^2 - 4x$$

**Solution:**

$$\begin{aligned} &= 2 ( x^2 - 2x ) \\ &= 2 [ ( x - 1 )^2 - 1 ] \\ &= 2 ( x - 1 )^2 - 2 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 9

**Question:**

Complete the square for the expression:

$$5x^2 + 20x$$

**Solution:**

$$\begin{aligned} &= 5 ( x^2 + 4x ) \\ &= 5 [ ( x + 2 )^2 - 4 ] \\ &= 5 ( x + 2 )^2 - 20 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 10

**Question:**

Complete the square for the expression:

$$2x^2 - 5x$$

**Solution:**

$$\begin{aligned} &= 2 \left( x^2 - \frac{5}{2}x \right) \\ &= 2 \left[ \left( x - \frac{5}{4} \right)^2 - \frac{25}{16} \right] \\ &= 2 \left( x - \frac{5}{4} \right)^2 - \frac{25}{8} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 11

**Question:**

Complete the square for the expression:

$$3x^2 + 9x$$

**Solution:**

$$\begin{aligned} &= 3 (x^2 + 3x) \\ &= 3 \left[ \left( x + \frac{3}{2} \right)^2 - \frac{9}{4} \right] \\ &= 3 \left( x + \frac{3}{2} \right)^2 - \frac{27}{4} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise C, Question 12

**Question:**

Complete the square for the expression:

$$3x^2 - x$$

**Solution:**

$$\begin{aligned} &= 3 \left( x^2 - \frac{1}{3}x \right) \\ &= 3 \left[ \left( x - \frac{1}{6} \right)^2 - \frac{1}{36} \right] \\ &= 3 \left( x - \frac{1}{6} \right)^2 - \frac{3}{36} \\ &= 3 \left( x - \frac{1}{6} \right)^2 - \frac{1}{12} \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 1

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 6x + 1 = 0$$

**Solution:**

$$x^2 + 6x = -1$$

$$(x + 3)^2 - 9 = -1$$

$$(x + 3)^2 = -1 + 9$$

$$(x + 3)^2 = 8$$

$$x + 3 = \pm \sqrt{8}$$

$$x = -3 \pm \sqrt{8}$$

$$x = -3 \pm \sqrt{2} \sqrt{4}$$

$$x = -3 \pm 2\sqrt{2}$$

$$\text{So } x = -3 + 2\sqrt{2} \text{ or } x = -3 - 2\sqrt{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 2

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 12x + 3 = 0$$

**Solution:**

$$x^2 + 12x = -3$$

$$(x + 6)^2 - 36 = -3$$

$$(x + 6)^2 = 33$$

$$x + 6 = \pm \sqrt{33}$$

$$x = -6 \pm \sqrt{33}$$

$$\text{So } x = -6 + \sqrt{33} \text{ or } x = -6 - \sqrt{33}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 3

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 - 10x = 5$$

**Solution:**

$$(x - 5)^2 - 25 = 5$$

$$(x - 5)^2 = 5 + 25$$

$$(x - 5)^2 = 30$$

$$x - 5 = \pm \sqrt{30}$$

$$x = 5 \pm \sqrt{30}$$

$$\text{So } x = 5 + \sqrt{30} \text{ or } x = 5 - \sqrt{30}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 4

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 4x - 2 = 0$$

**Solution:**

$$x^2 + 4x = 2$$

$$(x + 2)^2 - 4 = 2$$

$$(x + 2)^2 = 6$$

$$x + 2 = \pm \sqrt{6}$$

$$\text{So } x = -2 + \sqrt{6} \text{ or } x = -2 - \sqrt{6}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 5

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 - 3x - 5 = 0$$

**Solution:**

$$x^2 - 3x = 5$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} = 5$$

$$\left(x - \frac{3}{2}\right)^2 = 5 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

$$x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

$$\text{So } x = \frac{3 + \sqrt{29}}{2} \text{ or } x = \frac{3 - \sqrt{29}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 6

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$2x^2 - 7 = 4x$$

**Solution:**

$$2x^2 - 4x = 7$$

$$x^2 - 2x = \frac{7}{2}$$

$$(x - 1)^2 - 1 = \frac{7}{2}$$

$$(x - 1)^2 = \frac{9}{2}$$

$$x - 1 = \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3\sqrt{2}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 7

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$4x^2 - x = 8$$

**Solution:**

$$x^2 - \frac{1}{4}x = 2$$

$$\left(x - \frac{1}{8}\right)^2 - \frac{1}{64} = 2$$

$$\left(x - \frac{1}{8}\right)^2 = 2 + \frac{1}{64}$$

$$\left(x - \frac{1}{8}\right)^2 = \frac{129}{64}$$

$$x - \frac{1}{8} = \pm \frac{\sqrt{129}}{8}$$

$$x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$$

$$\text{So } x = \frac{1 + \sqrt{129}}{8} \text{ or } x = \frac{1 - \sqrt{129}}{8}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 8

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$10 = 3x - x^2$$

**Solution:**

$$x^2 - 3x = -10$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} = -10$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{31}{4}$$

No real roots as RHS is negative.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 9

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$15 - 6x - 2x^2 = 0$$

**Solution:**

$$2x^2 + 6x = 15$$

$$x^2 + 3x = \frac{15}{2}$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} = \frac{15}{2}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{39}{4}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{39}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

$$\text{So } x = -\frac{3}{2} + \frac{\sqrt{39}}{2} \text{ or } x = -\frac{3}{2} - \frac{\sqrt{39}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise D, Question 10

**Question:**

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$5x^2 + 8x - 2 = 0$$

**Solution:**

$$x^2 + \frac{8}{5}x = \frac{2}{5}$$

$$\left(x + \frac{4}{5}\right)^2 - \frac{16}{25} = \frac{2}{5}$$

$$\left(x + \frac{4}{5}\right)^2 = \frac{26}{25}$$

$$x + \frac{4}{5} = \pm \frac{\sqrt{26}}{5}$$

$$x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$$

$$\text{So } x = \frac{-4 + \sqrt{26}}{5} \text{ or } x = \frac{-4 - \sqrt{26}}{5}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 1

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 + 3x + 1 = 0$$

**Solution:**

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

$$\text{Then } x = \frac{-3 + \sqrt{5}}{2} \text{ or } x = \frac{-3 - \sqrt{5}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 2

#### Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 - 3x - 2 = 0$$

#### Solution:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+3 \pm \sqrt{9+8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$$\text{Then } x = \frac{3 + \sqrt{17}}{2} \text{ or } x = \frac{3 - \sqrt{17}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 3

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 + 6x + 6 = 0$$

**Solution:**

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2 \times 1}$$

$$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = -3 \pm \sqrt{3}$$

Then  $x = -3 + \sqrt{3}$  or  $x = -3 - \sqrt{3}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 4

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 - 5x - 2 = 0$$

**Solution:**

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{5 \pm \sqrt{33}}{2}$$

$$\text{Then } x = \frac{5 + \sqrt{33}}{2} \text{ or } x = \frac{5 - \sqrt{33}}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 5

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$3x^2 + 10x - 2 = 0$$

**Solution:**

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-2)}}{2 \times 3}$$

$$x = \frac{-10 \pm \sqrt{100 + 24}}{6}$$

$$x = \frac{-10 \pm \sqrt{124}}{6}$$

$$x = \frac{-10 \pm 2\sqrt{31}}{6}$$

$$\text{Then } x = \frac{-5 + \sqrt{31}}{3} \text{ or } x = \frac{-5 - \sqrt{31}}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 6

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$4x^2 - 4x - 1 = 0$$

**Solution:**

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$

$$x = \frac{+4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4\sqrt{2}}{8}$$

$$\text{Then } x = \frac{1 + \sqrt{2}}{2} \text{ or } x = \frac{1 - \sqrt{2}}{2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 7

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$7x^2 + 9x + 1 = 0$$

**Solution:**

$$x = \frac{-9 \pm \sqrt{9^2 - 4(7)(1)}}{2 \times 7}$$

$$x = \frac{-9 \pm \sqrt{81 - 28}}{14}$$

$$x = \frac{-9 \pm \sqrt{53}}{14}$$

$$\text{Then } x = \frac{-9 + \sqrt{53}}{14} \text{ or } x = \frac{-9 - \sqrt{53}}{14}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 8

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$5x^2 + 4x - 3 = 0$$

**Solution:**

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-3)}}{2 \times 5}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{10}$$

$$x = \frac{-4 \pm \sqrt{76}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{19}}{10}$$

$$\text{Then } x = \frac{-2 + \sqrt{19}}{5} \text{ or } x = \frac{-2 - \sqrt{19}}{5}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 9

**Question:**

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$4x^2 - 7x = 2$$

**Solution:**

$$4x^2 - 7x - 2 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-2)}}{2 \times 4}$$

$$x = \frac{+7 \pm \sqrt{49 + 32}}{8}$$

$$x = \frac{7 \pm \sqrt{81}}{8}$$

$$x = \frac{7 \pm 9}{8}$$

Then  $x = 2$  or  $x = -\frac{1}{4}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise E, Question 10

#### Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$11x^2 + 2x - 7 = 0$$

#### Solution:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-7)}}{2 \times 11}$$

$$x = \frac{-2 \pm \sqrt{4 + 308}}{22}$$

$$x = \frac{-2 \pm \sqrt{312}}{22}$$

$$x = \frac{-2 \pm 2\sqrt{78}}{22}$$

$$x = \frac{-1 \pm \sqrt{78}}{11}$$

$$\text{Then } x = \frac{-1 + \sqrt{78}}{11} \text{ or } x = \frac{-1 - \sqrt{78}}{11}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise F, Question 1

#### Question:

Sketch the graphs of the following equations:

(a)  $y = x^2 + 3x + 2$

(b)  $y = x^2 - 3x + 10$

(c)  $y = x^2 + 2x - 15$

(d)  $y = 2x^2 + 7x + 3$

(e)  $y = 2x^2 + x - 3$

(f)  $y = 6x^2 - 19x + 10$

(g)  $y = 3x^2 - 2x - 5$

(h)  $y = 3x^2 - 13x$

(i)  $y = -x^2 + 6x + 7$

(j)  $y = 4 - 7x - 2x^2$

#### Solution:

(a)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 9, 4ac = 8$$

$b^2 > 4ac$ , so there are two different roots of the equation  $y = 0$ .

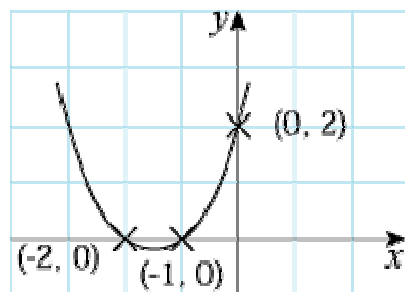
When  $y = 0$ ,

$$(x + 2)(x + 1) = 0$$

$$x = -2 \text{ or } x = -1$$

So crossing points are  $(-2, 0)$  and  $(-1, 0)$ .

When  $x = 0, y = 2$ , so  $(0, 2)$  is a crossing point.



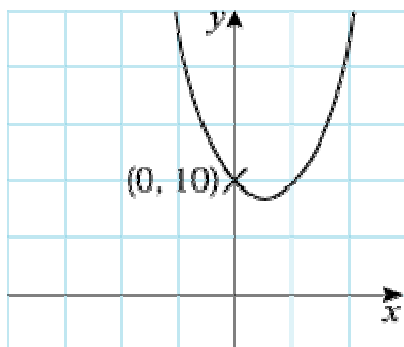
(b)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 9, 4ac = 40$$

$b^2 < 4ac$ , so there are no real roots of the equation  $y = 0$ .

So there are no crossing points at  $y = 0$ .

When  $x = 0, y = 10$ , so crossing point is  $(0, 10)$ .



(c)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 4, 4ac = -60$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

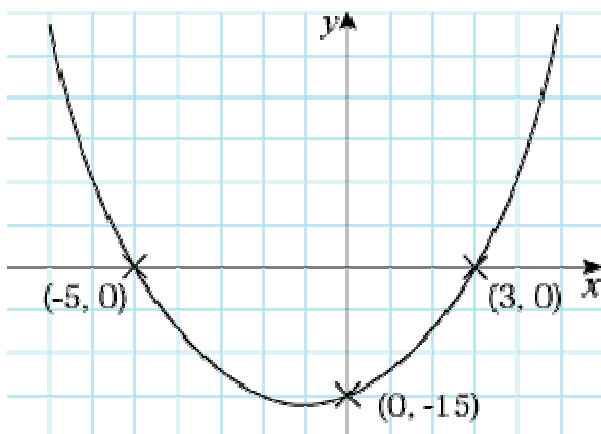
When  $y = 0$ ,

$$0 = (x + 5)(x - 3)$$

$$x = -5 \text{ or } x = 3$$

So crossing points are  $(-5, 0)$  and  $(3, 0)$ .

When  $x = 0, y = -15$ , so crossing point is  $(0, -15)$ .



(d)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 49, 4ac = 24$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

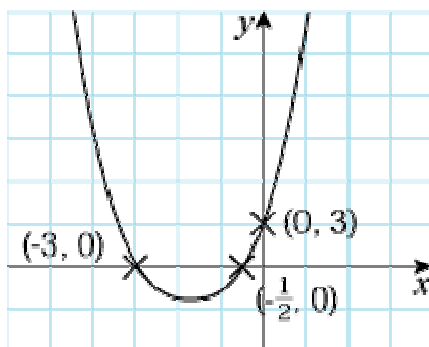
When  $y = 0$ ,

$$0 = (2x + 1)(x + 3)$$

$$x = -\frac{1}{2} \text{ or } x = -3$$

So crossing points are  $\left(-\frac{1}{2}, 0\right)$  and  $(-3, 0)$ .

When  $x = 0, y = 3$ , so crossing point is  $(0, 3)$ .



(e)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 1, 4ac = -24$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

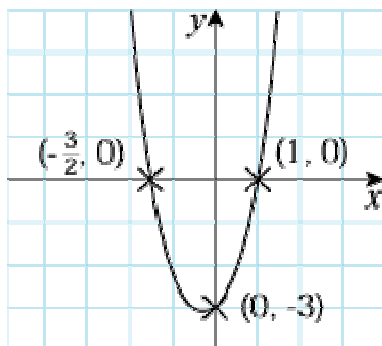
When  $y = 0$ ,

$$0 = (2x + 3)(x - 1)$$

$$x = -\frac{3}{2} \text{ or } x = 1$$

So crossing points are  $\left(-\frac{3}{2}, 0\right)$  and  $(1, 0)$ .

When  $x = 0, y = -3$ , so crossing point is  $(0, -3)$ .



(f)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 361, 4ac = 240$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

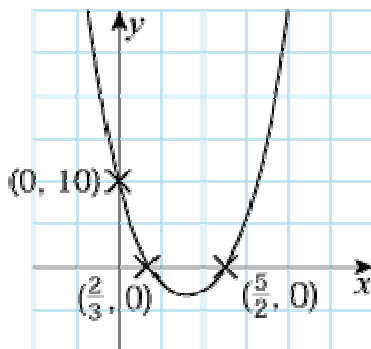
When  $y = 0$ ,

$$0 = (3x - 2)(2x - 5)$$

$$x = \frac{2}{3} \text{ or } x = \frac{5}{2}$$

So crossing points are  $\left(\frac{2}{3}, 0\right)$  and  $\left(\frac{5}{2}, 0\right)$ .

When  $x = 0, y = 10$ , so crossing point is  $(0, 10)$ .



(g)  $a >$  so graph is a  $\cup$  shape.

$$b^2 = 4, 4ac = -60$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

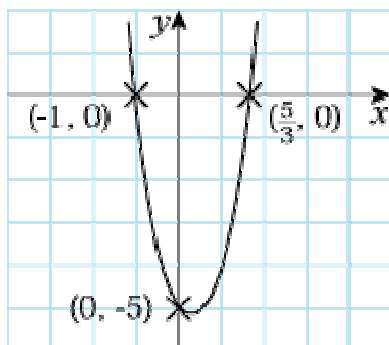
When  $y = 0$ ,

$$0 = (3x - 5)(x + 1)$$

$$x = \frac{5}{3} \text{ or } x = -1$$

So crossing points are  $\left(\frac{5}{3}, 0\right)$  and  $(-1, 0)$ .

When  $x = 0$ ,  $y = -5$ , so crossing point is  $(0, -5)$ .



(h)  $a > 0$  so graph is a  $\cup$  shape.

$$b^2 = 169, 4ac = 0$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

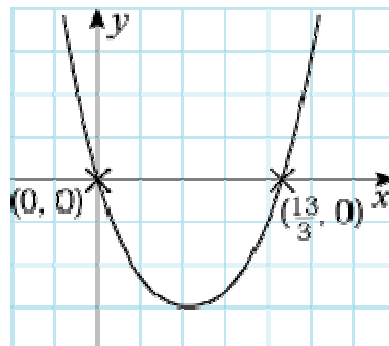
When  $y = 0$ ,

$$0 = x(3x - 13)$$

$$x = 0 \text{ or } x = \frac{13}{3}$$

So crossing points are  $(0, 0)$  and  $\left(\frac{13}{3}, 0\right)$ .

When  $x = 0$ ,  $y = 0$ , so crossing point is  $(0, 0)$ .



(i)  $a < 0$  so graph is a  $\cap$  shape.

$$b^2 = 36, 4ac = -28$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

When  $y = 0$ ,

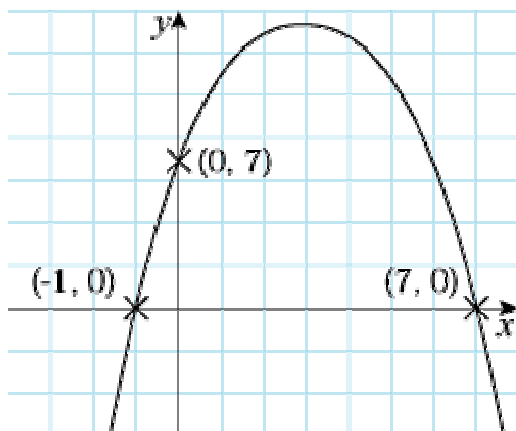
$$0 = (7 - x)(1 + x)$$

$$x = 7 \text{ or } x = -1$$

So crossing points are  $(7, 0)$  and  $(-1, 0)$ .

When  $x = 0$ ,  $y = 7$ , so crossing point is  $(0, 7)$ .





(j)  $a < 0$  so graph is a  $\cap$  shape.

$$b^2 = 49, 4ac = -32$$

$b^2 > 4ac$ , so two different roots of  $y = 0$ .

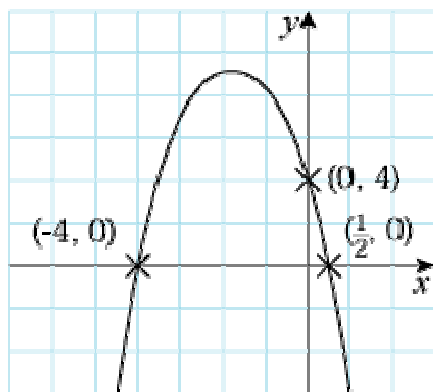
When  $y = 0$ ,

$$0 = (1 - 2x)(4 + x)$$

$$x = \frac{1}{2} \text{ or } x = -4$$

So crossing points are  $\left(\frac{1}{2}, 0\right)$  and  $(-4, 0)$ .

When  $x = 0$ ,  $y = 4$ , so crossing point is  $(0, 4)$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise F, Question 2

**Question:**

Find the values of  $k$  for which  $x^2 + kx + 4 = 0$  has equal roots.

**Solution:**

$x^2 + kx + 4 = 0$  has equal roots if

$$b^2 = 4ac$$

i.e.

$$k^2 = 4 \times 1 \times 4 = 16 \quad \Rightarrow \quad k = \pm 4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise F, Question 3

**Question:**

Find the values of  $k$  for which  $kx^2 + 8x + k = 0$  has equal roots.

**Solution:**

$kx^2 + 8x + k = 0$  has equal roots if

$$b^2 = 4ac$$

i.e.

$$8^2 = 4 \times k \times k = 4k^2$$

$$\text{So } k^2 = \frac{64}{4} = 16 \quad \Rightarrow \quad k = \pm 4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 1

#### Question:

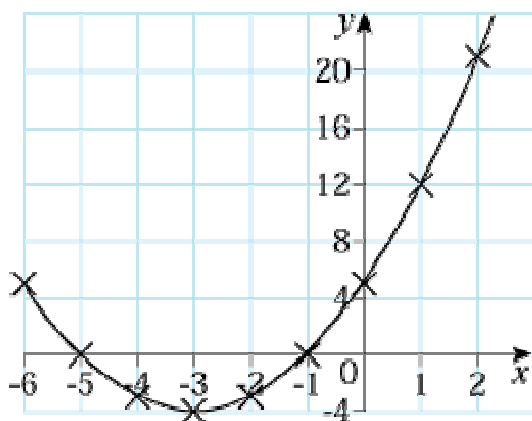
Draw the graphs with the following equations, choosing appropriate values for  $x$ . For each graph write down the equation of the line of symmetry.

(a)  $y = x^2 + 6x + 5$

(b)  $y = 2x^2 - 3x - 4$

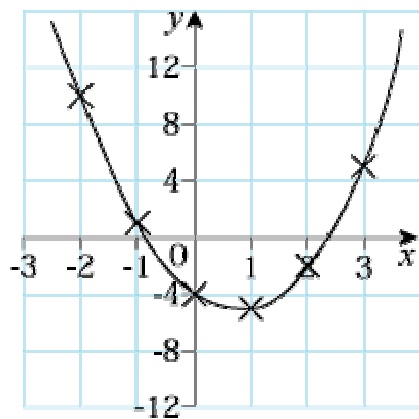
#### Solution:

$x$	-6	-5	-4	-3	-2	-1	0	1	2
$x^2$	36	25	16	9	4	1	0	1	4
(a) $+6x$	-36	-30	-24	-18	-12	-6	0	+6	+12
	+5	+5	+5	+5	+5	+5	+5	+5	+5
$y$	5	0	-3	-4	-3	0	5	12	21



$x = -3$  is line of symmetry.

$x$	-2	-1	0	1	2	3
$2x^2$	8	2	0	2	8	18
(b) $-3x$	+6	+3	0	-3	-6	-9
	-4	-4	-4	-4	-4	-4
$y$	10	1	-4	-5	-2	5



$x = \frac{3}{4}$  is line of symmetry.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 2

**Question:**

Solve the following equations:

(a)  $y^2 + 3y + 2 = 0$

(b)  $3x^2 + 13x - 10 = 0$

(c)  $5x^2 - 10x = 4x + 3$

(d)  $(2x - 5)^2 = 7$

**Solution:**

(a)  $(y + 1)(y + 2) = 0$   
 $y = -1$  or  $y = -2$

(b)  $(3x - 2)(x + 5) = 0$   
 $x = \frac{2}{3}$  or  $x = -5$

(c)  $5x^2 - 14x - 3 = 0$   
 $(5x + 1)(x - 3) = 0$   
 $x = -\frac{1}{5}$  or  $x = 3$

(d)  $2x - 5 = \pm \sqrt{7}$   
 $2x = \pm \sqrt{7} + 5$   
 $x = \frac{5 \pm \sqrt{7}}{2}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 3

#### Question:

Solve the following equations by:

(i) Completing the square.

(ii) Using the formula.

(a)  $x^2 + 5x + 2 = 0$

(b)  $x^2 - 4x - 3 = 0$

(c)  $5x^2 + 3x - 1 = 0$

(d)  $3x^2 - 5x = 4$

#### Solution:

(a) (i)  $x^2 + 5x = -2$

$$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} = -2$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(ii)  $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2}$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(b)(i)  $x^2 - 4x = 3$

$$(x - 2)^2 - 4 = 3$$

$$(x - 2)^2 = 7$$

$$x - 2 = \pm \sqrt{7}$$

$$x = 2 \pm \sqrt{7}$$

(ii)  $x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-3)}}{2}$

$$x = \frac{+4 \pm \sqrt{16 + 12}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 7}}{2}$$

$$x = \frac{4 \pm 2\sqrt{7}}{2}$$

$$x = 2 \pm \sqrt{7}$$

$$(c) (i) 5x^2 + 3x = 1$$

$$5 \left( x^2 + \frac{3}{5}x \right) = 1$$

$$5 \left[ \left( x + \frac{3}{10} \right)^2 - \frac{9}{100} \right] = 1$$

$$\left( x + \frac{3}{10} \right)^2 - \frac{9}{100} = \frac{1}{5}$$

$$\left( x + \frac{3}{10} \right)^2 = \frac{29}{100}$$

$$x + \frac{3}{10} = \pm \frac{\sqrt{29}}{10}$$

$$x = \frac{-3 \pm \sqrt{29}}{10}$$

$$(ii) x = \frac{-3 \pm \sqrt{9 - 4(5)(-1)}}{10}$$

$$x = \frac{-3 \pm \sqrt{29}}{10}$$

$$(d)(i) 3 \left( x^2 - \frac{5}{3}x \right) = 4$$

$$3 \left[ \left( x - \frac{5}{6} \right)^2 - \frac{25}{36} \right] = 4$$

$$\left( x - \frac{5}{6} \right)^2 - \frac{25}{36} = \frac{4}{3}$$

$$\left( x - \frac{5}{6} \right)^2 = \frac{73}{36}$$

$$x - \frac{5}{6} = \pm \frac{\sqrt{73}}{6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$

$$(ii) x = \frac{-(-5) \pm \sqrt{25 - 4(3)(-4)}}{6}$$

$$x = \frac{+5 \pm \sqrt{25 + 48}}{6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 4

#### Question:

Sketch graphs of the following equations:

(a)  $y = x^2 + 5x + 4$

(b)  $y = 2x^2 + x - 3$

(c)  $y = 6 - 10x - 4x^2$

(d)  $y = 15x - 2x^2$

#### Solution:

(a)  $a > 0$  so  $\cup$  shape

$$b^2 = 25, 4ac = 16$$

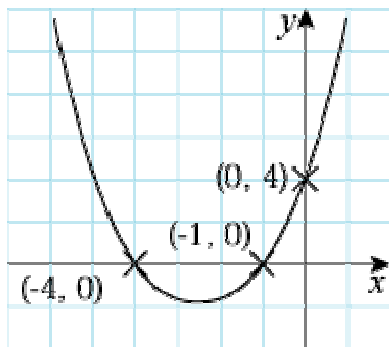
$b^2 > 4ac$ , so two different roots of  $y = 0$ .

$$y = 0 \Rightarrow 0 = (x + 1)(x + 4)$$

$$x = -1 \text{ or } x = -4$$

So  $x$ -axis crossing points are  $(-1, 0)$  and  $(-4, 0)$ .

$$x = 0 \Rightarrow y = 4 \text{ So } y\text{-axis crossing point is } (0, 4).$$



(b)  $a > 0$  So  $\cup$  shape

$$b^2 = 1, 4ac = -24$$

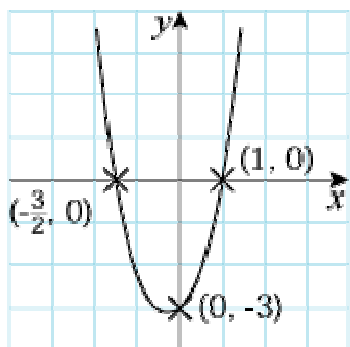
$b^2 > 4ac$ , so two different roots of  $y = 0$ .

$$y = 0 \Rightarrow 0 = (2x + 3)(x - 1)$$

$$x = -\frac{3}{2} \text{ or } x = 1$$

So  $x$ -axis crossing points are  $\left(-\frac{3}{2}, 0\right)$  and  $(1, 0)$ .

$$x = 0 \Rightarrow y = -3 \text{ so } y\text{-axis crossing point in } (0, -3).$$



(c)  $a < 0$  So  $\cap$  shape

$$b^2 = 100, 4ac = -96$$

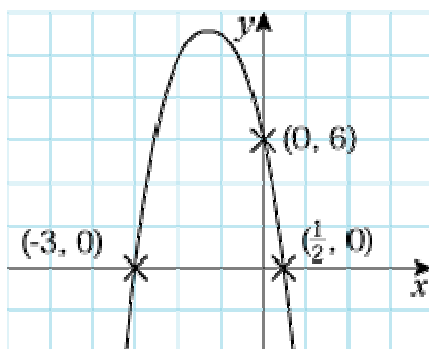
$b^2 > 4ac$ , so two different roots of  $y = 0$ .

$$y = 0 \Rightarrow 0 = (1 - 2x)(6 + 2x)$$

$$x = \frac{1}{2} \text{ or } x = -3$$

So  $x$ -axis crossing points are  $\left(\frac{1}{2}, 0\right)$  and  $(-3, 0)$ .

$x = 0 \Rightarrow y = 6$  so  $y$ -axis crossing point is  $(0, 6)$ .



(d)  $a < 0$  so  $\cap$  shape

$$b^2 = 225, 4ac = 0$$

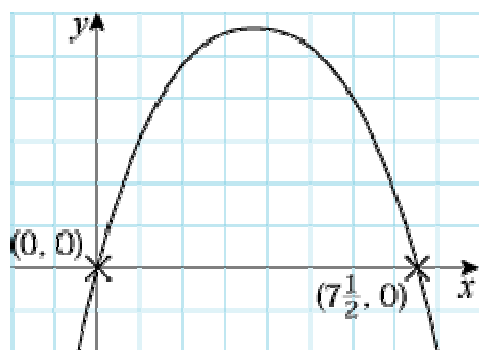
$b^2 > 4ac$ , so two different roots of  $y = 0$ .

$$y = 0 \Rightarrow 0 = x(15 - 2x)$$

$$x = 0 \text{ or } x = 7\frac{1}{2}$$

So  $x$ -axis crossing points are  $(0, 0)$  and  $\left(7\frac{1}{2}, 0\right)$ .

$x = 0 \Rightarrow y = 0$  So  $y$ -axis crossing point is  $(0, 0)$ .





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 5

#### Question:

Given that for all values of  $x$  :

$$3x^2 + 12x + 5 = p(x + q)^2 + r$$

(a) Find the values of  $p$ ,  $q$  and  $r$ .

(b) Solve the equation  $3x^2 + 12x + 5 = 0$ . **[E]**

#### Solution:

$$(a) 3x^2 + 12x + 5 = p(x^2 + 2qx + q^2) + r$$

$$3x^2 + 12x + 5 = px^2 + 2pqx + pq^2 + r$$

$$\text{Comparing } x^2 : p = 3 \text{ ①}$$

$$\text{Comparing } x : 2pq = 12 \text{ ②}$$

$$\text{Comparing constants : } pq^2 + r = 5 \text{ ③}$$

Substitute ① into ②:

$$2 \times 3q = 12$$

$$q = 2$$

Substitute  $p = 3$  and  $q = 2$  into ③:

$$3 \times 2^2 + r = 5$$

$$12 + r = 5$$

$$r = -7$$

$$\text{So } p = 3, q = 2, r = -7$$

$$(b) 3x^2 + 12x + 5 = 0$$

$$\Rightarrow 3(x + 2)^2 - 7 = 0$$

$$\Rightarrow 3(x + 2)^2 = 7$$

$$\Rightarrow (x + 2)^2 = \frac{7}{3}$$

$$\Rightarrow x + 2 = \pm \sqrt{\frac{7}{3}}$$

$$\text{So } x = -2 \pm \sqrt{\frac{7}{3}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 6

#### Question:

Find, as surds, the roots of the equation

$$2(x + 1)(x - 4) - (x - 2)^2 = 0$$

#### Solution:

$$2(x^2 - 3x - 4) - (x^2 - 4x + 4) = 0$$

$$2x^2 - 6x - 8 - x^2 + 4x - 4 = 0$$

$$x^2 - 2x - 12 = 0$$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(-12)}}{2}$$

$$x = \frac{+2 \pm \sqrt{52}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 13}}{2}$$

$$x = \frac{2 \pm 2\sqrt{13}}{2}$$

$$x = 1 \pm \sqrt{13}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Quadratic Equations

#### Exercise G, Question 7

**Question:**

Use algebra to solve  $(x - 1)(x + 2) = 18$ . [E]

**Solution:**

$$\begin{aligned}x^2 + x - 2 &= 18 \\x^2 + x - 20 &= 0 \\(x + 5)(x - 4) &= 0 \\x &= -5 \text{ or } x = 4\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 1

**Question:**

Solve these simultaneous equations by elimination:

$$\begin{aligned}2x - y &= 6 \\4x + 3y &= 22\end{aligned}$$

**Solution:**

$$\begin{aligned}6x - 3y &= 18 \\4x + 3y &= 22\end{aligned}$$

Add:

$$10x = 40$$

$$x = 4$$

Substitute into  $2x - y = 6$ :

$$8 - y = 6$$

$$y = 2$$

So solution is  $x = 4, y = 2$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 2

**Question:**

Solve these simultaneous equations by elimination:

$$7x + 3y = 16$$

$$2x + 9y = 29$$

**Solution:**

$$21x + 9y = 48$$

$$2x + 9y = 29$$

Subtract:

$$19x = 19$$

$$x = 1$$

Substitute into  $7x + 3y = 16$ :

$$7 + 3y = 16$$

$$3y = 9$$

$$y = 3$$

So solution is  $x = 1, y = 3$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 3

**Question:**

Solve these simultaneous equations by elimination:

$$5x + 2y = 6$$

$$3x - 10y = 26$$

**Solution:**

$$25x + 10y = 30$$

$$3x - 10y = 26$$

Add:

$$28x = 56$$

$$x = 2$$

Substitute into  $5x + 2y = 6$ :

$$10 + 2y = 6$$

$$2y = -4$$

$$y = -2$$

So solution is  $x = 2, y = -2$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 4

**Question:**

Solve these simultaneous equations by elimination:

$$\begin{aligned}2x - y &= 12 \\6x + 2y &= 21\end{aligned}$$

**Solution:**

$$\begin{aligned}4x - 2y &= 24 \\6x + 2y &= 21\end{aligned}$$

Add:

$$10x = 45$$

$$x = 4\frac{1}{2}$$

Substitute into  $2x - y = 12$ :

$$9 - y = 12$$

$$-y = 3$$

$$y = -3$$

So solution is  $x = 4\frac{1}{2}$ ,  $y = -3$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 5

**Question:**

Solve these simultaneous equations by elimination:

$$\begin{aligned}3x - 2y &= -6 \\6x + 3y &= 2\end{aligned}$$

**Solution:**

$$\begin{aligned}6x - 4y &= -12 \\6x + 3y &= 2\end{aligned}$$

Subtract:

$$\begin{aligned}-7y &= -14 \\y &= 2\end{aligned}$$

Substitute into  $3x - 2y = -6$ :

$$\begin{aligned}3x - 4 &= -6 \\3x &= -2\end{aligned}$$

$$x = -\frac{2}{3}$$

So solution is  $x = -\frac{2}{3}, y = 2$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise A, Question 6

**Question:**

Solve these simultaneous equations by elimination:

$$3x + 8y = 33$$

$$6x = 3 + 5y$$

**Solution:**

$$6x + 16y = 66$$

$$6x = 3 + 5y$$

$$6x + 16y = 66$$

$$6x - 5y = 3$$

Subtract:

$$21y = 63$$

$$y = 3$$

Substitute into  $3x + 8y = 33$ :

$$3x + 24 = 33$$

$$3x = 9$$

$$x = 3$$

So solution is  $x = 3, y = 3$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise B, Question 1

**Question:**

Solve these simultaneous equations by substitution:

$$\begin{aligned}x + 3y &= 11 \\4x - 7y &= 6\end{aligned}$$

**Solution:**

$$\begin{aligned}x &= 11 - 3y \\ \text{Substitute into } 4x - 7y &= 6: \\ 4(11 - 3y) - 7y &= 6 \\ 44 - 12y - 7y &= 6 \\ -19y &= -38 \\ y &= 2 \\ \text{Substitute into } x = 11 - 3y: \\ x &= 11 - 6 \\ x &= 5 \\ \text{So solution is } x = 5, y = 2\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise B, Question 2

#### Question:

Solve these simultaneous equations by substitution:

$$4x - 3y = 40$$

$$2x + y = 5$$

#### Solution:

$$y = 5 - 2x$$

Substitute into  $4x - 3y = 40$ :

$$4x - 3(5 - 2x) = 40$$

$$4x - 15 + 6x = 40$$

$$10x = 55$$

$$x = 5 \frac{1}{2}$$

Substitute into  $y = 5 - 2x$ :

$$y = 5 - 11$$

$$y = -6$$

So solution is  $x = 5 \frac{1}{2}$ ,  $y = -6$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise B, Question 3

**Question:**

Solve these simultaneous equations by substitution:

$$\begin{aligned}3x - y &= 7 \\10x + 3y &= -2\end{aligned}$$

**Solution:**

$$\begin{aligned}-y &= 7 - 3x \\y &= 3x - 7\end{aligned}$$

Substitute into  $10x + 3y = -2$ :

$$\begin{aligned}10x + 3(3x - 7) &= -2 \\10x + 9x - 21 &= -2 \\19x &= 19 \\x &= 1\end{aligned}$$

Substitute into  $y = 3x - 7$ :

$$\begin{aligned}y &= 3 - 7 \\y &= -4\end{aligned}$$

So solution is  $x = 1, y = -4$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise B, Question 4

**Question:**

Solve these simultaneous equations by substitution:

$$2y = 2x - 3$$

$$3y = x - 1$$

**Solution:**

$$x = 3y + 1$$

Substitute into  $2y = 2x - 3$ :

$$2y = 2(3y + 1) - 3$$

$$2y = 6y + 2 - 3$$

$$-4y = -1$$

$$y = \frac{1}{4}$$

Substitute into  $x = 3y + 1$ :

$$x = \frac{3}{4} + 1$$

$$x = 1\frac{3}{4}$$

So solution is  $x = 1\frac{3}{4}$ ,  $y = \frac{1}{4}$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise C, Question 1

#### Question:

Solve the simultaneous equations:

(a)  $x + y = 11$   
 $xy = 30$

(b)  $2x + y = 1$   
 $x^2 + y^2 = 1$

(c)  $y = 3x$   
 $2y^2 - xy = 15$

(d)  $x + y = 9$   
 $x^2 - 3xy + 2y^2 = 0$

(e)  $3a + b = 8$   
 $3a^2 + b^2 = 28$

(f)  $2u + v = 7$   
 $uv = 6$

#### Solution:

(a)  $y = 11 - x$   
 Substitute into  $xy = 30$ :  
 $x(11 - x) = 30$   
 $11x - x^2 = 30$   
 $0 = x^2 - 11x + 30$   
 $0 = (x - 5)(x - 6)$   
 $x = 5$  or  $x = 6$   
 Substitute into  $y = 11 - x$ :  
 when  $x = 5$ ,  $y = 11 - 5 = 6$   
 when  $x = 6$ ,  $y = 11 - 6 = 5$   
 Solutions are  $x = 5$ ,  $y = 6$  and  $x = 6$ ,  $y = 5$

(b)  $y = 1 - 2x$   
 Substitute into  $x^2 + y^2 = 1$ :  
 $x^2 + (1 - 2x)^2 = 1$   
 $x^2 + 1 - 4x + 4x^2 = 1$   
 $5x^2 - 4x = 0$   
 $x(5x - 4) = 0$   
 $x = 0$  or  $x = \frac{4}{5}$

Substitute into  $y = 1 - 2x$ :  
 when  $x = 0$ ,  $y = 1$   
 when  $x = \frac{4}{5}$ ,  $y = 1 - \frac{8}{5} = -\frac{3}{5}$

Solutions are  $x = 0$ ,  $y = 1$  and  $x = \frac{4}{5}$ ,  $y = -\frac{3}{5}$

(c)  $y = 3x$   
 Substitute into  $2y^2 - xy = 15$ :

$$2(3x)^2 - x(3x) = 15$$

$$18x^2 - 3x^2 = 15$$

$$15x^2 = 15$$

$$x^2 = 1$$

$$x = -1 \text{ or } x = 1$$

Substitute into  $y = 3x$ :

$$\text{when } x = -1, y = -3$$

$$\text{when } x = 1, y = 3$$

Solutions are  $x = -1, y = -3$  and  $x = 1, y = 3$

(d)  $x = 9 - y$

Substitute into  $x^2 - 3xy + 2y^2 = 0$ :

$$(9 - y)^2 - 3y(9 - y) + 2y^2 = 0$$

$$81 - 18y + y^2 - 27y + 3y^2 + 2y^2 = 0$$

$$6y^2 - 45y + 81 = 0$$

Divide by 3:

$$2y^2 - 15y + 27 = 0$$

$$(2y - 9)(y - 3) = 0$$

$$y = \frac{9}{2} \text{ or } y = 3$$

Substitute into  $x = 9 - y$ :

$$\text{when } y = \frac{9}{2}, x = 9 - \frac{9}{2} = \frac{9}{2}$$

$$\text{when } y = 3, x = 9 - 3 = 6$$

Solutions are  $x = 4\frac{1}{2}, y = 4\frac{1}{2}$  and  $x = 6, y = 3$

(e)  $b = 8 - 3a$

Substitute into  $3a^2 + b^2 = 28$ :

$$3a^2 + (8 - 3a)^2 = 28$$

$$3a^2 + 64 - 48a + 9a^2 = 28$$

$$12a^2 - 48a + 36 = 0$$

Divide by 12:

$$a^2 - 4a + 3 = 0$$

$$(a - 1)(a - 3) = 0$$

$$a = 1 \text{ or } a = 3$$

Substitute into  $b = 8 - 3a$ :

$$\text{when } a = 1, b = 8 - 3 = 5$$

$$\text{when } a = 3, b = 8 - 9 = -1$$

Solutions are  $a = 1, b = 5$  and  $a = 3, b = -1$

(f)  $v = 7 - 2u$

Substitute into  $uv = 6$ :

$$u(7 - 2u) = 6$$

$$7u - 2u^2 = 6$$

$$0 = 2u^2 - 7u + 6$$

$$0 = (2u - 3)(u - 2)$$

$$u = \frac{3}{2} \text{ or } u = 2$$

Substitute into  $v = 7 - 2u$ :

$$\text{when } u = \frac{3}{2}, v = 7 - 3 = 4$$

$$\text{when } u = 2, v = 7 - 4 = 3$$

Solutions are  $u = \frac{3}{2}, v = 4$  and  $u = 2, v = 3$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise C, Question 2

#### Question:

Find the coordinates of the points at which the line with equation  $y = x - 4$  intersects the curve with equation  $y^2 = 2x^2 - 17$ .

#### Solution:

$$y = x - 4$$

Substitute into  $y^2 = 2x^2 - 17$ :

$$(x - 4)^2 = 2x^2 - 17$$

$$x^2 - 8x + 16 = 2x^2 - 17$$

$$0 = x^2 + 8x - 33$$

$$0 = (x + 11)(x - 3)$$

$$x = -11 \text{ or } x = 3$$

Substitute into  $y = x - 4$ :

$$\text{when } x = -11, y = -11 - 4 = -15$$

$$\text{when } x = 3, y = 3 - 4 = -1$$

Intersection points:  $(-11, -15)$  and  $(3, -1)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise C, Question 3

**Question:**

Find the coordinates of the points at which the line with equation  $y = 3x - 1$  intersects the curve with equation  $y^2 - xy = 15$ .

**Solution:**

$$y = 3x - 1$$

Substitute into  $y^2 - xy = 15$ :

$$(3x - 1)^2 - x(3x - 1) = 15$$

$$9x^2 - 6x + 1 - 3x^2 + x = 15$$

$$6x^2 - 5x - 14 = 0$$

$$(6x + 7)(x - 2) = 0$$

$$x = -\frac{7}{6} \text{ or } x = 2$$

Substitute into  $y = 3x - 1$ :

$$\text{when } x = -\frac{7}{6}, y = -\frac{21}{6} - 1 = -\frac{9}{2}$$

$$\text{when } x = 2, y = 6 - 1 = 5$$

Intersection points:  $\left(-1\frac{1}{6}, -4\frac{1}{2}\right)$  and  $(2, 5)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise C, Question 4

#### Question:

Solve the simultaneous equations:

$$(a) \begin{aligned} 3x + 2y &= 7 \\ x^2 + y &= 8 \end{aligned}$$

$$(b) \begin{aligned} 2x + 2y &= 7 \\ x^2 - 4y^2 &= 8 \end{aligned}$$

#### Solution:

$$(a) 2y = 7 - 3x$$

$$y = \frac{1}{2}(7 - 3x)$$

Substitute into  $x^2 + y = 8$ :

$$x^2 + \frac{1}{2}(7 - 3x) = 8$$

Multiply by 2:

$$2x^2 + (7 - 3x) = 16$$

$$2x^2 - 3x - 9 = 0$$

$$(2x + 3)(x - 3) = 0$$

$$x = -\frac{3}{2} \text{ or } x = 3$$

$$\text{Substitute into } y = \frac{1}{2} \left( 7 - 3x \right) :$$

$$\text{when } x = -\frac{3}{2}, y = \frac{1}{2} \left( 7 + \frac{9}{2} \right) = \frac{23}{4}$$

$$\text{when } x = 3, y = \frac{1}{2} \left( 7 - 9 \right) = -1$$

Solutions are  $x = -1\frac{1}{2}, y = 5\frac{3}{4}$  and  $x = 3, y = -1$

$$(b) 2x = 7 - 2y$$

$$x = \frac{1}{2} \left( 7 - 2y \right)$$

Substitute into  $x^2 - 4y^2 = 8$ :

$$\left[ \frac{1}{2} \left( 7 - 2y \right) \right]^2 - 4y^2 = 8$$

$$\frac{1}{4} (7 - 2y)^2 - 4y^2 = 8$$

Multiply by 4:

$$(7 - 2y)^2 - 16y^2 = 32$$

$$49 - 28y + 4y^2 - 16y^2 = 32$$

$$0 = 12y^2 + 28y - 17$$

$$0 = (6y + 17)(2y - 1)$$

$$y = -\frac{17}{6} \text{ or } y = \frac{1}{2}$$

Substitute into  $x = \frac{1}{2} (7 - 2y)$  :

$$\text{when } y = -\frac{17}{6}, x = \frac{1}{2} \left( 7 + \frac{17}{3} \right) = \frac{19}{3}$$

$$\text{when } y = \frac{1}{2}, x = \frac{1}{2} (7 - 1) = 3$$

Solutions are  $x = 6\frac{1}{3}, y = -2\frac{5}{6}$  and  $x = 3, y = \frac{1}{2}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise C, Question 5

#### Question:

Solve the simultaneous equations, giving your answers in their simplest surd form:

(a)  $x - y = 6$   
 $xy = 4$

(b)  $2x + 3y = 13$   
 $x^2 + y^2 = 78$

#### Solution:

(a)  $x = 6 + y$   
 Substitute into  $xy = 4$ :  
 $y(6 + y) = 4$   
 $6y + y^2 = 4$   
 $y^2 + 6y - 4 = 0$   
 $a = 1, b = 6, c = -4$   

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{-6 \pm \sqrt{(36 + 16)}}{2} = \frac{-6 \pm \sqrt{52}}{2}$$

$$\sqrt{52} = \sqrt{(4 \times 13)} = \sqrt{4} \sqrt{13} = 2\sqrt{13}$$

$$y = \frac{-6 \pm 2\sqrt{13}}{2} = -3 \pm \sqrt{13}$$

Substitute into  $x = 6 + y$ :  
 when  $y = -3 - \sqrt{13}$ ,  $x = 6 - 3 - \sqrt{13} = 3 - \sqrt{13}$   
 when  $y = -3 + \sqrt{13}$ ,  $x = 6 - 3 + \sqrt{13} = 3 + \sqrt{13}$   
 Solutions are  $x = 3 - \sqrt{13}$ ,  $y = -3 - \sqrt{13}$  and  $x = 3 + \sqrt{13}$ ,  $y = -3 + \sqrt{13}$

(b)  $2x = 13 - 3y$

$$x = \frac{1}{2} (13 - 3y)$$

Substitute into  $x^2 + y^2 = 78$ :

$$\left[ \frac{1}{2} (13 - 3y) \right]^2 + y^2 = 78$$

$$\frac{1}{4} (13 - 3y)^2 + y^2 = 78$$

Multiply by 4:

$$(13 - 3y)^2 + 4y^2 = 312$$

$$169 - 78y + 9y^2 + 4y^2 = 312$$

$$13y^2 - 78y - 143 = 0$$

Divide by 13:

$$y^2 - 6y - 11 = 0$$

$$a = 1, b = -6, c = -11$$

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{6 \pm \sqrt{(36 + 44)}}{2} = \frac{6 \pm \sqrt{80}}{2}$$

$$\sqrt{80} = \sqrt{(16 \times 5)} = \sqrt{16} \sqrt{5} = 4\sqrt{5}$$

$$y = \frac{6 \pm 4\sqrt{5}}{2} = 3 \pm 2\sqrt{5}$$



Substitute into  $x = \frac{1}{2} (13 - 3y)$  :

$$\text{when } y = 3 - 2\sqrt{5}, x = \frac{1}{2} \left[ 13 - 3(3 - 2\sqrt{5}) \right] = \frac{1}{2} \left[ 13 - 9 + 6\sqrt{5} \right] = 2 + 3\sqrt{5}$$

$$\text{when } y = 3 + 2\sqrt{5}, x = \frac{1}{2} \left[ 13 - 3(3 + 2\sqrt{5}) \right] = \frac{1}{2} \left[ 13 - 9 - 6\sqrt{5} \right] = 2 - 3\sqrt{5}$$

Solutions are  $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$  and  $x = 2 + 3\sqrt{5}, y = 3 - 2\sqrt{5}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise D, Question 1

#### Question:

Find the set of values of  $x$  for which:

(a)  $2x - 3 < 5$

(b)  $5x + 4 \geq 39$

(c)  $6x - 3 > 2x + 7$

(d)  $5x + 6 \leq -12 - x$

(e)  $15 - x > 4$

(f)  $21 - 2x > 8 + 3x$

(g)  $1 + x < 25 + 3x$

(h)  $7x - 7 < 7 - 7x$

(i)  $5 - 0.5x \geq 1$

(j)  $5x + 4 > 12 - 2x$

#### Solution:

(a)  $2x < 5 + 3$

$$2x < 8$$

$$x < 4$$

(b)  $5x \geq 39 - 4$

$$5x \geq 35$$

$$x \geq 7$$

(c)  $6x - 2x > 7 + 3$

$$4x > 10$$

$$x > 2\frac{1}{2}$$

(d)  $5x + x \leq -12 - 6$

$$6x \leq -18$$

$$x \leq -3$$

(e)  $-x > 4 - 15$

$$-x > -11$$

$$x < 11$$

(f)  $21 - 8 > 3x + 2x$

$$13 > 5x$$

$$5x < 13$$

$$x < 2\frac{3}{5}$$

$$\begin{aligned} \text{(g)} \quad x - 3x &< 25 - 1 \\ -2x &< 24 \\ x &> -12 \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad 7x + 7x &< 7 + 7 \\ 14x &< 14 \\ x &< 1 \end{aligned}$$

$$\begin{aligned} \text{(i)} \quad -0.5x &\geq 1 - 5 \\ -0.5x &\geq -4 \\ x &\leq 8 \end{aligned}$$

$$\begin{aligned} \text{(j)} \quad 5x + 2x &> 12 - 4 \\ 7x &> 8 \\ x &> 1 \frac{1}{7} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise D, Question 2

#### Question:

Find the set of values of  $x$  for which:

(a)  $2(x - 3) \geq 0$

(b)  $8(1 - x) > x - 1$

(c)  $3(x + 7) \leq 8 - x$

(d)  $2(x - 3) - (x + 12) < 0$

(e)  $1 + 11(2 - x) < 10(x - 4)$

(f)  $2(x - 5) \geq 3(4 - x)$

(g)  $12x - 3(x - 3) < 45$

(h)  $x - 2(5 + 2x) < 11$

(i)  $x(x - 4) \geq x^2 + 2$

(j)  $x(5 - x) \geq 3 + x - x^2$

#### Solution:

(a)  $2x - 6 \geq 0$

$$2x \geq 6$$

$$x \geq 3$$

(b)  $8 - 8x > x - 1$

$$8 + 1 > x + 8x$$

$$9 > 9x$$

$$1 > x$$

$$x < 1$$

(c)  $3x + 21 \leq 8 - x$

$$3x + x \leq 8 - 21$$

$$4x \leq -13$$

$$x \leq -3\frac{1}{4}$$

(d)  $2x - 6 - x - 12 < 0$

$$2x - x < 6 + 12$$

$$x < 18$$

(e)  $1 + 22 - 11x < 10x - 40$

$$1 + 22 + 40 < 10x + 11x$$

$$63 < 21x$$

$$3 < x$$

$$x > 3$$

(f)  $2x - 10 \geq 12 - 3x$

$$2x + 3x \geq 12 + 10$$

$$5x \geq 22$$

$$x \geq 4 \frac{2}{5}$$

$$(g) 12x - 3x + 9 < 45$$

$$12x - 3x < 45 - 9$$

$$9x < 36$$

$$x < 4$$

$$(h) x - 10 - 4x < 11$$

$$x - 4x < 11 + 10$$

$$-3x < 21$$

$$x > -7$$

$$(i) x^2 - 4x \geq x^2 + 2$$

$$x^2 - x^2 - 4x \geq 2$$

$$-4x \geq 2$$

$$x \leq -\frac{1}{2}$$

$$(j) 5x - x^2 \geq 3 + x - x^2$$

$$5x - x - x^2 + x^2 \geq 3$$

$$4x \geq 3$$

$$x \geq \frac{3}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise D, Question 3

#### Question:

Find the set of values of  $x$  for which:

(a)  $3(x - 2) > x - 4$  and  $4x + 12 > 2x + 17$

(b)  $2x - 5 < x - 1$  and  $7(x + 1) > 23 - x$

(c)  $2x - 3 > 2$  and  $3(x + 2) < 12 + x$

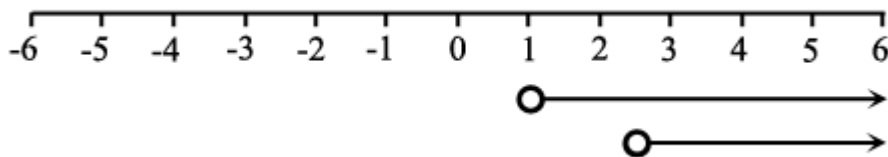
(d)  $15 - x < 2(11 - x)$  and  $5(3x - 1) > 12x + 19$

(e)  $3x + 8 \leq 20$  and  $2(3x - 7) \geq x + 6$

#### Solution:

(a)  $3x - 6 > x - 4$   
 $3x - x > -4 + 6$   
 $2x > 2$   
 $x > 1$

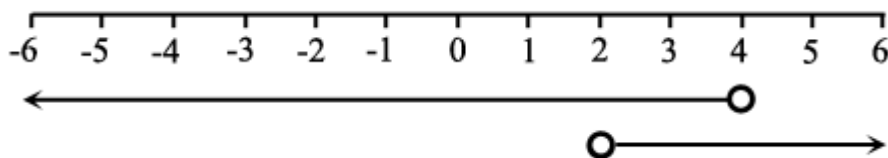
$4x + 12 > 2x + 17$   
 $4x - 2x > 17 - 12$   
 $2x > 5$   
 $x > 2\frac{1}{2}$



So the required set of values is  $x > 2\frac{1}{2}$

(b)  $2x - x < -1 + 5$   
 $x < 4$

$7(x + 1) > 23 - x$   
 $7x + 7 > 23 - x$   
 $7x + x > 23 - 7$   
 $8x > 16$   
 $x > 2$



So the required set of values is  $2 < x < 4$

$$(c) 2x > 2 + 3$$

$$2x > 5$$

$$x > 2\frac{1}{2}$$

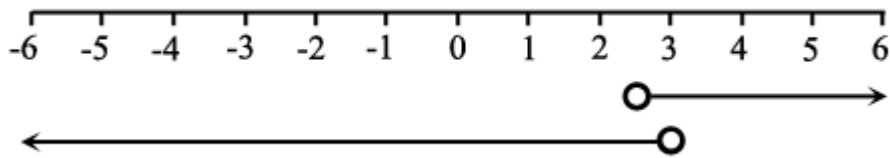
$$3(x + 2) < 12 + x$$

$$3x + 6 < 12 + x$$

$$3x - x < 12 - 6$$

$$2x < 6$$

$$x < 3$$



So the required set of values is  $2\frac{1}{2} < x < 3$

$$(d) 15 - x < 22 - 2x$$

$$-x + 2x < 22 - 15$$

$$x < 7$$

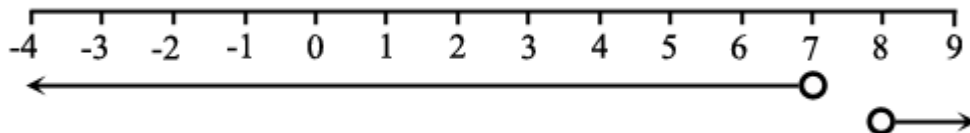
$$5(3x - 1) > 12x + 19$$

$$15x - 5 > 12x + 19$$

$$15x - 12x > 19 + 5$$

$$3x > 24$$

$$x > 8$$



There are no values satisfying both inequalities.

$$(e) 3x \leq 20 - 8$$

$$3x \leq 12$$

$$x \leq 4$$

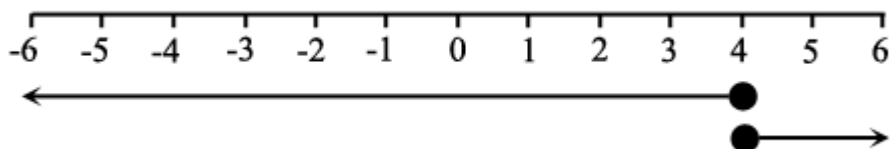
$$2(3x - 7) \geq x + 6$$

$$6x - 14 \geq x + 6$$

$$6x - x \geq 6 + 14$$

$$5x \geq 20$$

$$x \geq 4$$



There is just one value,  $x = 4$ .

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise E, Question 1

**Question:**

Find the set of values of  $x$  for which:

(a)  $x^2 - 11x + 24 < 0$

(b)  $12 - x - x^2 > 0$

(c)  $x^2 - 3x - 10 > 0$

(d)  $x^2 + 7x + 12 \geq 0$

(e)  $7 + 13x - 2x^2 > 0$

(f)  $10 + x - 2x^2 < 0$

(g)  $4x^2 - 8x + 3 \leq 0$

(h)  $-2 + 7x - 3x^2 < 0$

(i)  $x^2 - 9 < 0$

(j)  $6x^2 + 11x - 10 > 0$

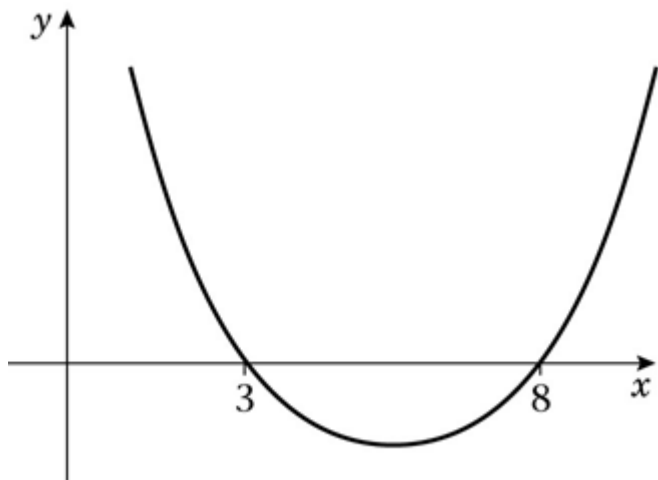
(k)  $x^2 - 5x > 0$

(l)  $2x^2 + 3x \leq 0$

**Solution:**

$$(a) \quad x^2 - 11x + 24 = 0$$
$$(x - 3)(x - 8) = 0$$
$$x = 3, x = 8$$

Sketch of  $y = x^2 - 11x + 24$ :



$$x^2 - 11x + 24 < 0 \text{ when } 3 < x < 8$$

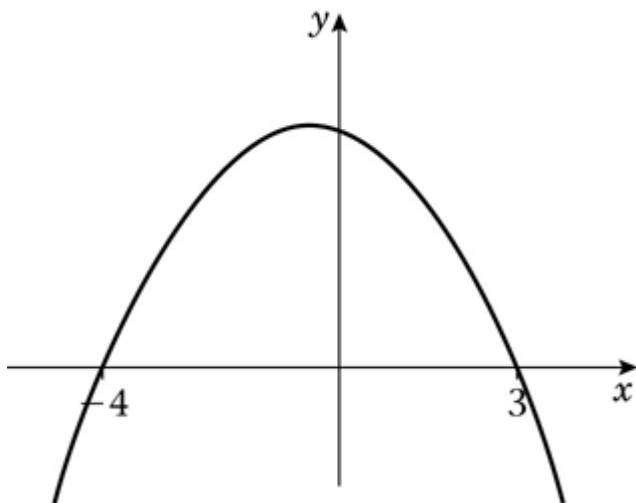
$$(b) 12 - x - x^2 = 0$$

$$0 = x^2 + x - 12$$

$$0 = (x + 4)(x - 3)$$

$$x = -4, x = 3$$

Sketch of  $y = 12 - x - x^2$ :



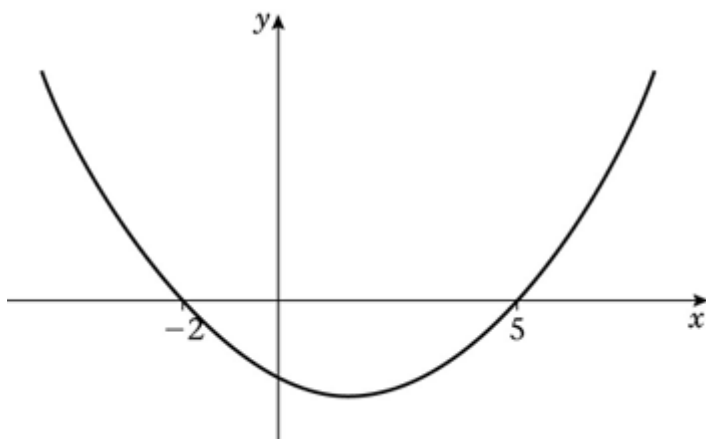
$$12 - x - x^2 > 0 \text{ when } -4 < x < 3$$

$$(c) x^2 - 3x - 10 = 0$$

$$(x + 2)(x - 5) = 0$$

$$x = -2, x = 5$$

Sketch of  $y = x^2 - 3x - 10$ :



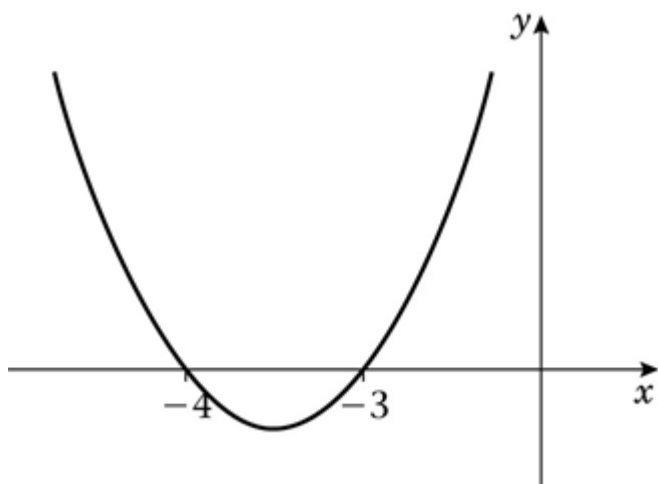
$$x^2 - 3x - 10 > 0 \text{ when } x < -2 \text{ or } x > 5$$

$$(d) x^2 + 7x + 12 = 0$$

$$(x + 4)(x + 3) = 0$$

$$x = -4, x = -3$$

Sketch of  $y = x^2 + 7x + 12$ :



$$x^2 + 7x + 12 \geq 0 \text{ when } x \leq -4 \text{ or } x \geq -3$$

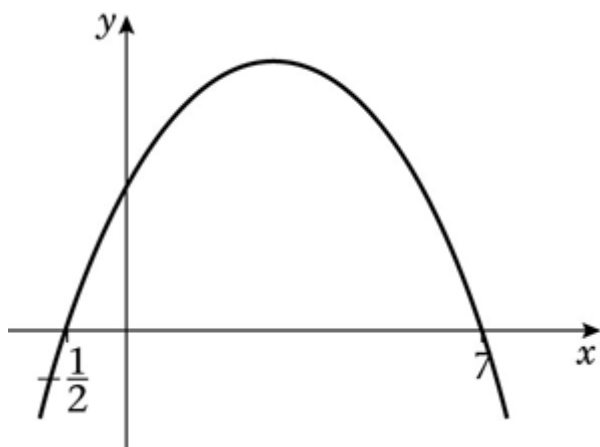
$$(e) 7 + 13x - 2x^2 = 0$$

$$2x^2 - 13x - 7 = 0$$

$$(2x + 1)(x - 7) = 0$$

$$x = -\frac{1}{2}, x = 7$$

Sketch of  $y = 7 + 13x - 2x^2$ :



$$7 + 13x - 2x^2 > 0 \text{ when } -\frac{1}{2} < x < 7$$

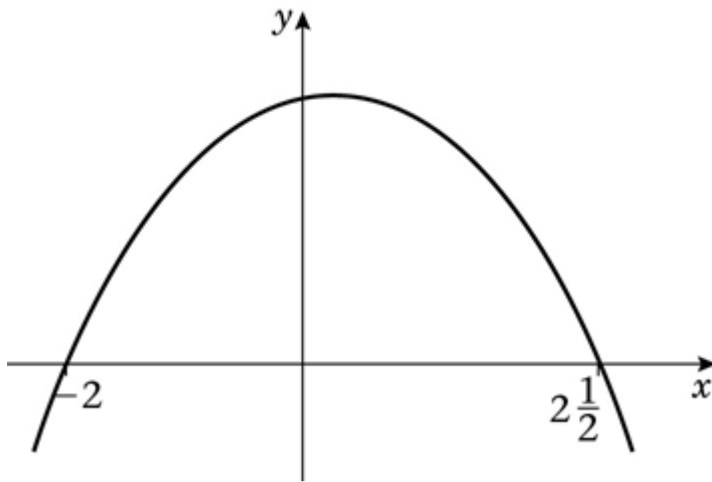
$$(f) 10 + x - 2x^2 = 0$$

$$2x^2 - x - 10 = 0$$

$$(2x - 5)(x + 2) = 0$$

$$x = 2\frac{1}{2}, x = -2$$

Sketch of  $y = 10 + x - 2x^2$ :

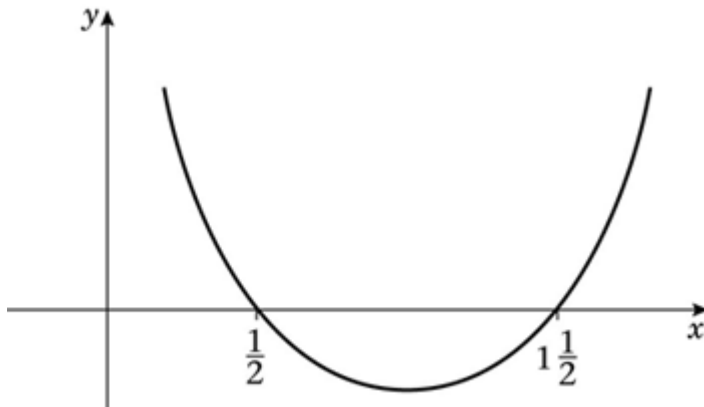


$$10 + x - 2x^2 < 0 \text{ when } x < -2 \text{ or } x > 2 \frac{1}{2}$$

$$\begin{aligned} \text{(g) } 4x^2 - 8x + 3 &= 0 \\ (2x - 1)(2x - 3) &= 0 \end{aligned}$$

$$x = \frac{1}{2}, x = 1 \frac{1}{2}$$

Sketch of  $y = 4x^2 - 8x + 3$ :

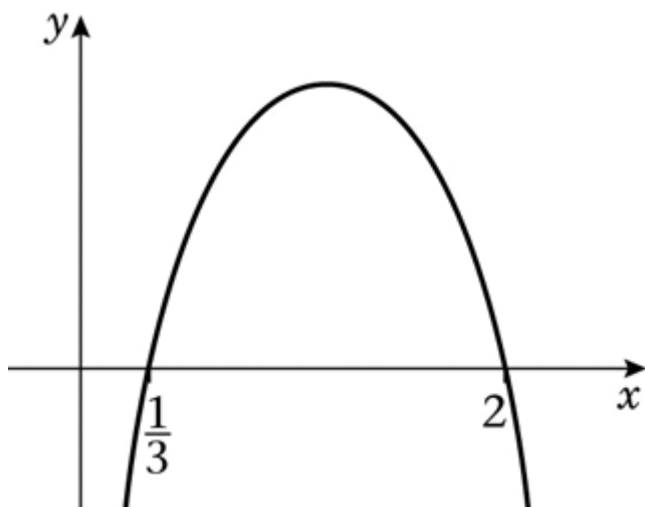


$$4x^2 - 8x + 3 \leq 0 \text{ when } \frac{1}{2} \leq x \leq 1 \frac{1}{2}$$

$$\begin{aligned} \text{(h) } -2 + 7x - 3x^2 &= 0 \\ 3x^2 - 7x + 2 &= 0 \\ (3x - 1)(x - 2) &= 0 \end{aligned}$$

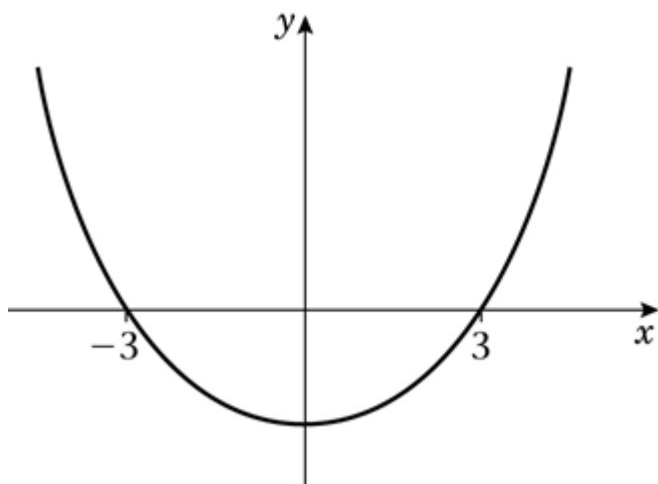
$$x = \frac{1}{3}, x = 2$$

Sketch of  $y = -2 + 7x - 3x^2$ :



$$-2 + 7x - 3x^2 < 0 \text{ when } x < \frac{1}{3} \text{ or } x > 2$$

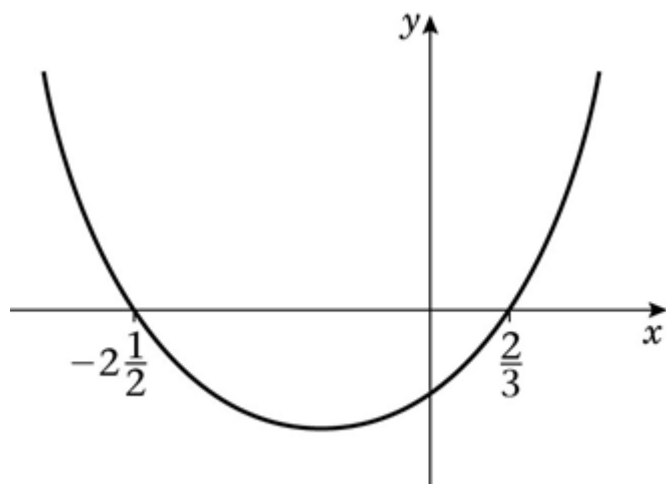
(i)  $x^2 - 9 = 0$   
 $(x + 3)(x - 3) = 0$   
 $x = -3, x = 3$   
 Sketch of  $y = x^2 - 9$ :



$$x^2 - 9 < 0 \text{ when } -3 < x < 3$$

(j)  $6x^2 + 11x - 10 = 0$   
 $(3x - 2)(2x + 5) = 0$   
 $x = \frac{2}{3}, x = -2\frac{1}{2}$

Sketch of  $y = 6x^2 + 11x - 10$ :



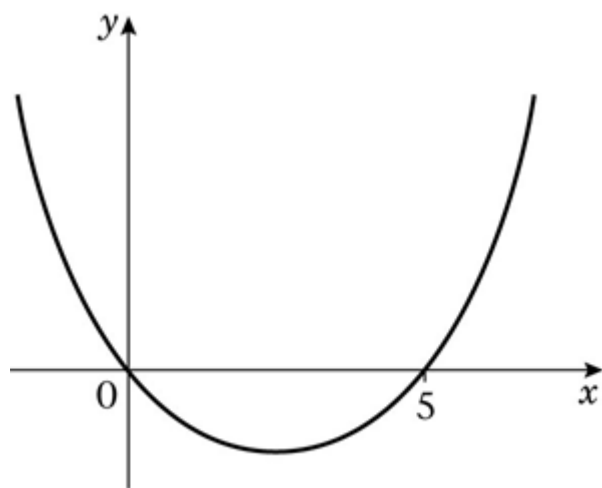
$$6x^2 + 11x - 10 > 0 \text{ when } x < -2\frac{1}{2} \text{ or } x > \frac{2}{3}$$

$$(k) x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$x = 0, x = 5$$

Sketch of  $y = x^2 - 5x$ :



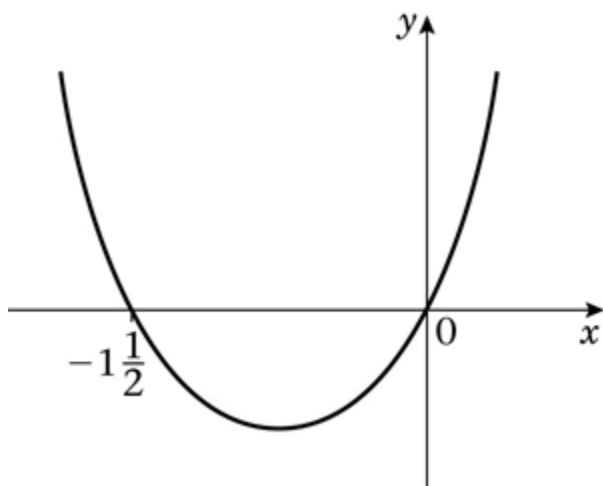
$$x^2 - 5x > 0 \text{ when } x < 0 \text{ or } x > 5$$

$$(l) 2x^2 + 3x = 0$$

$$x(2x + 3) = 0$$

$$x = 0, x = -1\frac{1}{2}$$

Sketch of  $y = 2x^2 + 3x$ :



$$2x^2 + 3x \leq 0 \text{ when } -1\frac{1}{2} \leq x \leq 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise E, Question 2

#### Question:

Find the set of values of  $x$  for which:

(a)  $x^2 < 10 - 3x$

(b)  $11 < x^2 + 10$

(c)  $x(3 - 2x) > 1$

(d)  $x(x + 11) < 3(1 - x^2)$

#### Solution:

(a)  $x^2 = 10 - 3x$

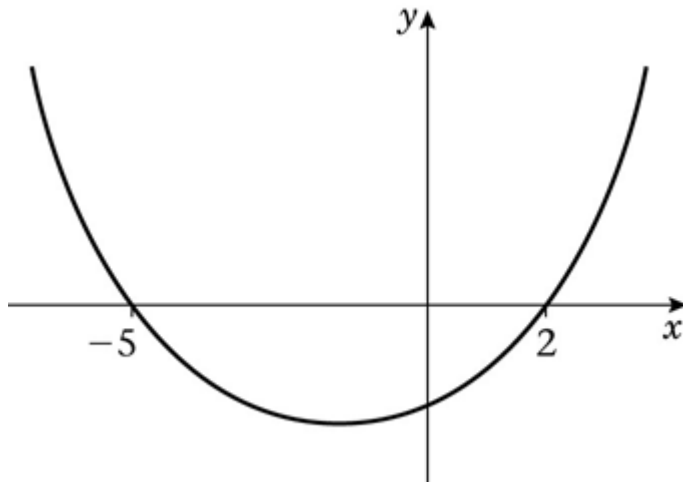
$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$$x = -5, x = 2$$

$$x^2 < 10 - 3x \Rightarrow x^2 + 3x - 10 < 0$$

Sketch of  $y = x^2 + 3x - 10$ :



$$x^2 + 3x - 10 < 0 \text{ when } -5 < x < 2$$

(b)  $11 = x^2 + 10$

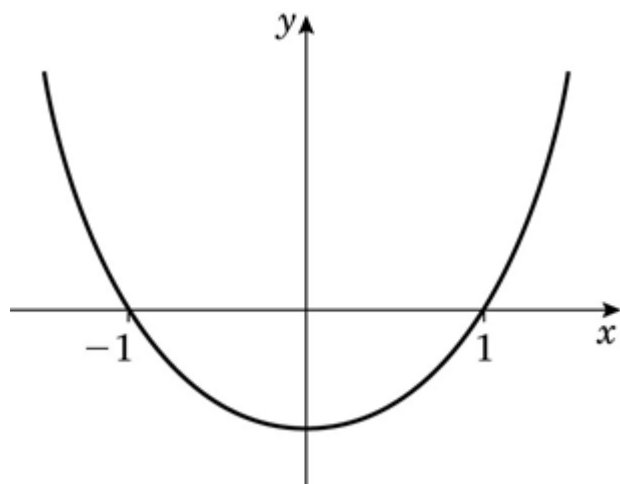
$$x^2 = 1$$

$$x = -1, x = 1$$

$$11 < x^2 + 10 \Rightarrow 0 < x^2 + 10 - 11 \Rightarrow x^2 - 1 > 0$$

Sketch of  $y = x^2 - 1$ :





$x^2 - 1 > 0$  when  $x < -1$  or  $x > 1$

$$(c) x(3 - 2x) = 1$$

$$3x - 2x^2 = 1$$

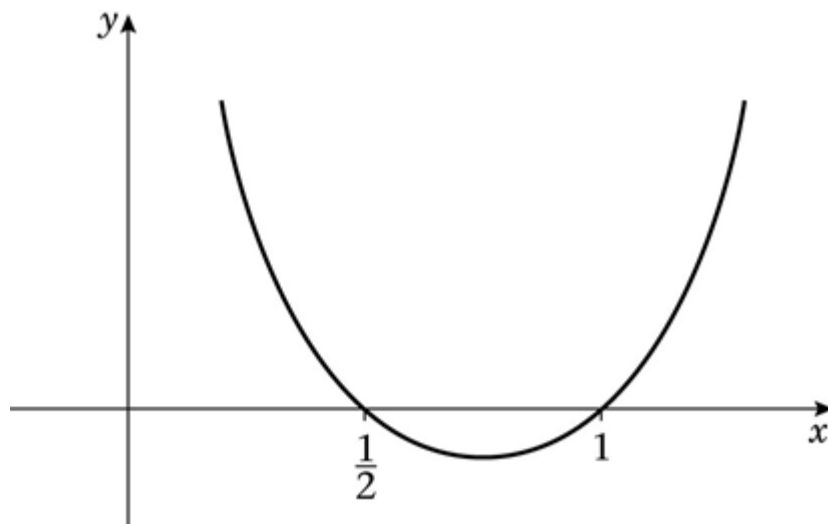
$$0 = 2x^2 - 3x + 1$$

$$0 = (2x - 1)(x - 1)$$

$$x = \frac{1}{2}, x = 1$$

$$x(3 - 2x) > 1 \Rightarrow -2x^2 + 3x - 1 > 0 \Rightarrow 2x^2 - 3x + 1 < 0$$

Sketch of  $y = 2x^2 - 3x + 1$ :



$2x^2 - 3x + 1 < 0$  when  $\frac{1}{2} < x < 1$

$$(d) x(x + 11) = 3(1 - x^2)$$

$$x^2 + 11x = 3 - 3x^2$$

$$x^2 + 3x^2 + 11x - 3 = 0$$

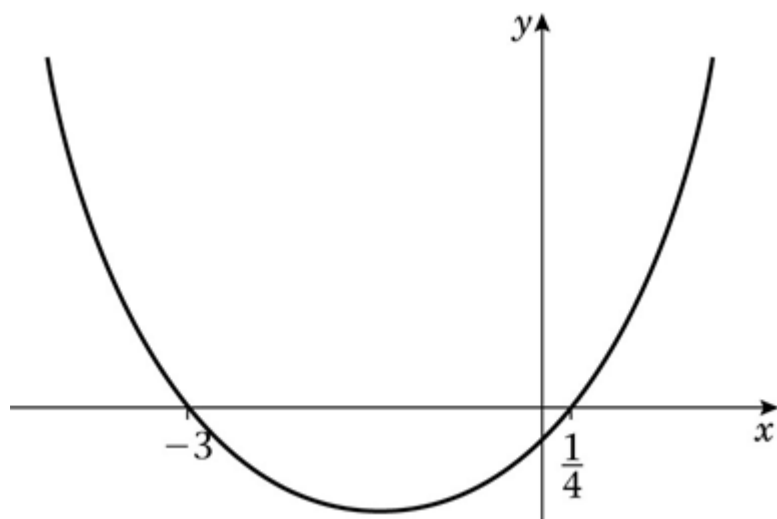
$$4x^2 + 11x - 3 = 0$$

$$(4x - 1)(x + 3) = 0$$

$$x = \frac{1}{4}, x = -3$$

$$x(x + 11) < 3(1 - x^2) \Rightarrow 4x^2 + 11x - 3 < 0$$

Sketch of  $y = 4x^2 + 11x - 3$ :



$$4x^2 + 11x - 3 < 0 \text{ when } -3 < x < \frac{1}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise E, Question 3

#### Question:

Find the set of values of  $x$  for which:

(a)  $x^2 - 7x + 10 < 0$  and  $3x + 5 < 17$

(b)  $x^2 - x - 6 > 0$  and  $10 - 2x < 5$

(c)  $4x^2 - 3x - 1 < 0$  and  $4(x + 2) < 15 - (x + 7)$

(d)  $2x^2 - x - 1 < 0$  and  $14 < 3x - 2$

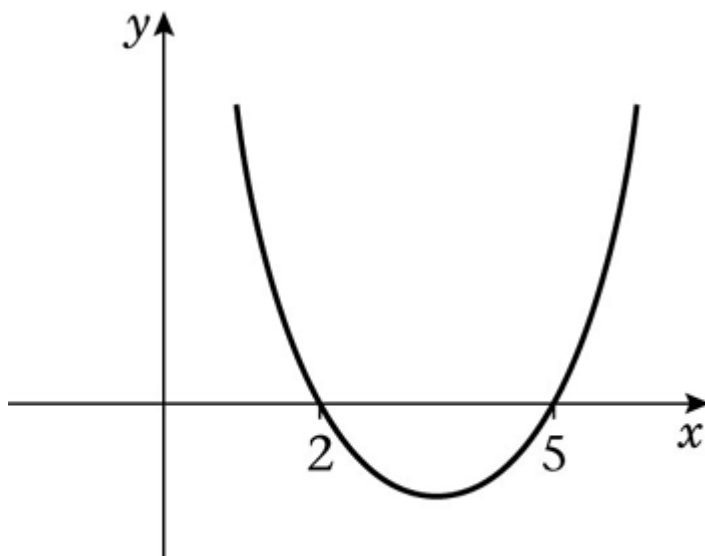
(e)  $x^2 - x - 12 > 0$  and  $3x + 17 > 2$

(f)  $x^2 - 2x - 3 < 0$  and  $x^2 - 3x + 2 > 0$

#### Solution:

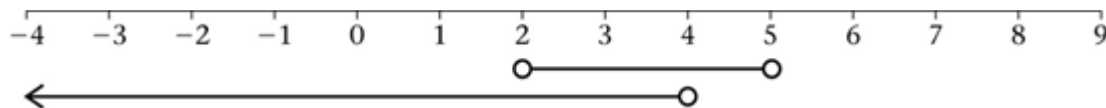
(a)  $x^2 - 7x + 10 = 0$   
 $(x - 2)(x - 5) = 0$   
 $x = 2, x = 5$

Sketch of  $y = x^2 - 7x + 10$ :



$x^2 - 7x + 10 < 0$  when  $2 < x < 5$ .

$3x + 5 < 17$   
 $3x < 17 - 5$   
 $3x < 12$   
 $x < 4$



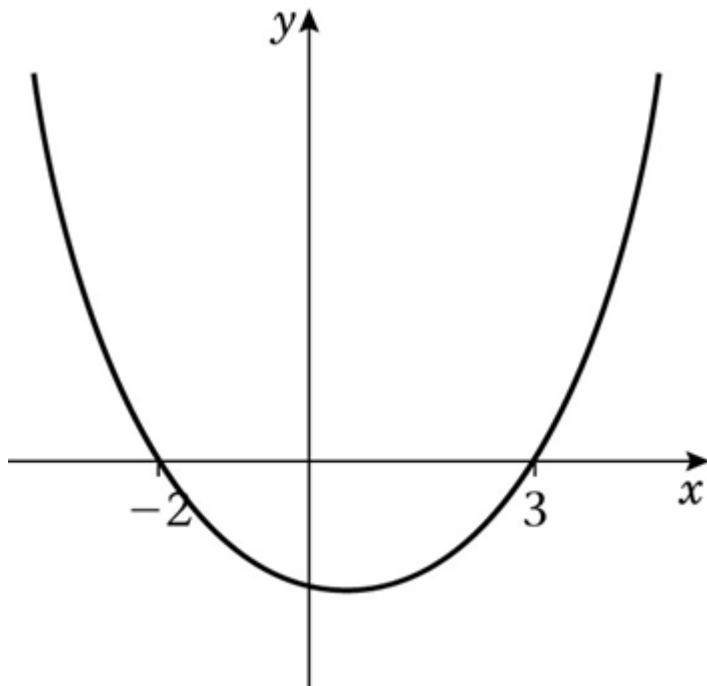
Intersection is  $2 < x < 4$ .

$$(b) x^2 - x - 6 = 0$$

$$(x + 2)(x - 3) = 0$$

$$x = -2, x = 3$$

Sketch of  $y = x^2 - x - 6$ :



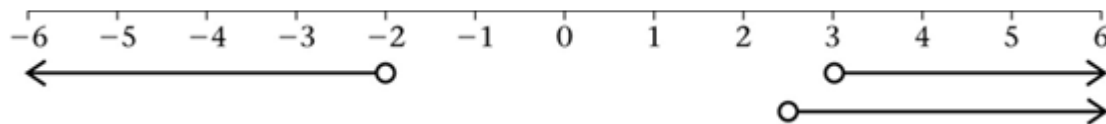
$$x^2 - x - 6 > 0 \text{ when } x < -2 \text{ or } x > 3$$

$$10 - 2x < 5$$

$$-2x < 5 - 10$$

$$-2x < -5$$

$$x > 2\frac{1}{2}$$



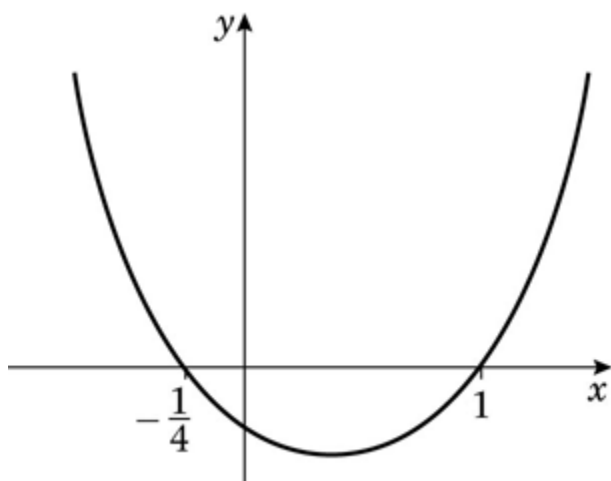
Intersection is  $x > 3$ .

$$(c) 4x^2 - 3x - 1 = 0$$

$$(4x + 1)(x - 1) = 0$$

$$x = -\frac{1}{4}, x = 1$$

Sketch of  $y = 4x^2 - 3x - 1$ :



$$4x^2 - 3x - 1 < 0 \text{ when } -\frac{1}{4} < x < 1$$

$$4(x+2) < 15 - (x+7)$$

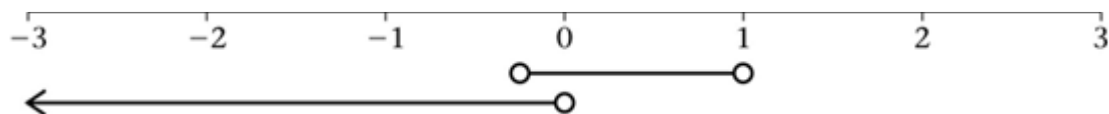
$$4x + 8 < 15 - x - 7$$

$$4x + 8 < 8 - x$$

$$4x + x < 8 - 8$$

$$5x < 0$$

$$x < 0$$



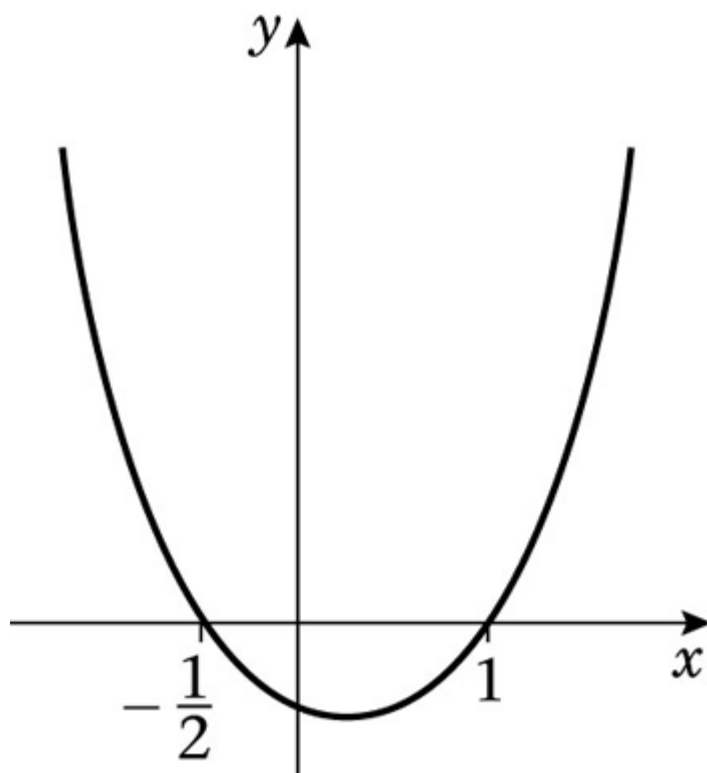
$$\text{Intersection is } -\frac{1}{4} < x < 0$$

$$(d) 2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, x = 1$$

Sketch of  $y = 2x^2 - x - 1$ :



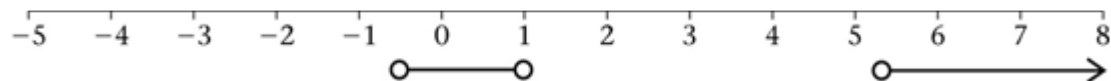
$$2x^2 - x - 1 < 0 \text{ when } -\frac{1}{2} < x < 1$$

$$14 < 3x - 2$$

$$14 + 2 < 3x$$

$$3x > 16$$

$$x > 5\frac{1}{3}$$



No intersection.

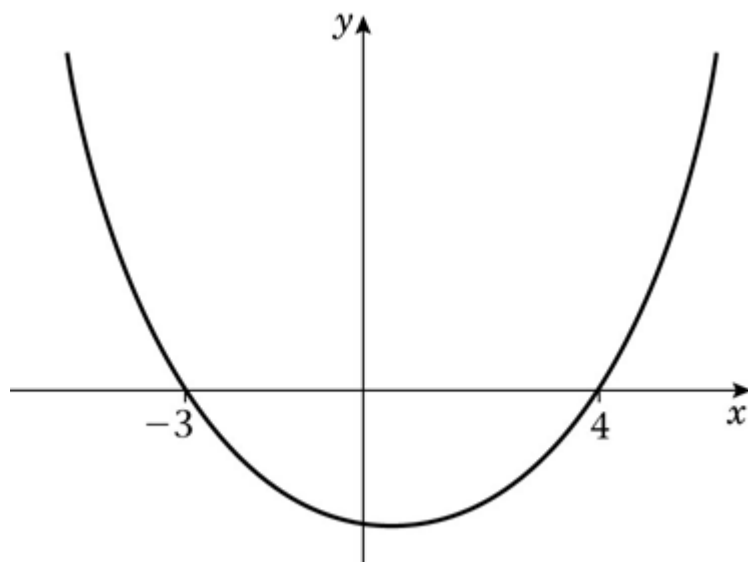
There are no values of  $x$  for which both inequalities are true.

$$(e) x^2 - x - 12 = 0$$

$$(x + 3)(x - 4) = 0$$

$$x = -3, x = 4$$

Sketch of  $y = x^2 - x - 12$ :



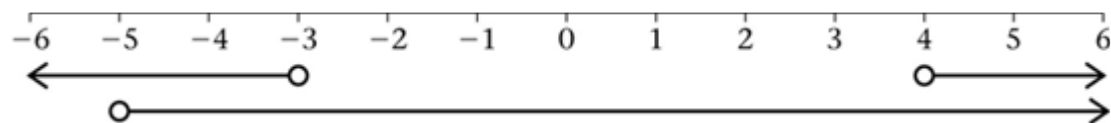
$$x^2 - x - 12 > 0 \text{ when } x < -3 \text{ or } x > 4$$

$$3x + 17 > 2$$

$$3x > 2 - 17$$

$$3x > -15$$

$$x > -5$$



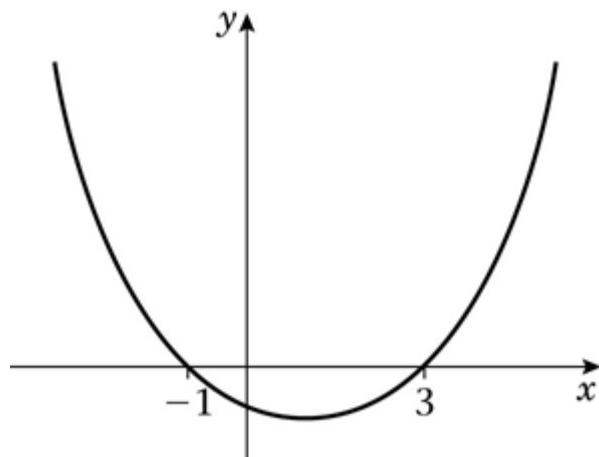
Intersection is  $-5 < x < -3, x > 4$ .

$$(f) x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1, x = 3$$

Sketch of  $y = x^2 - 2x - 3$ :



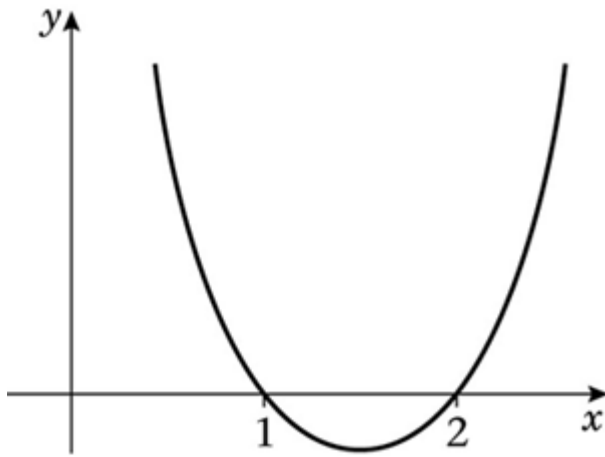
$$x^2 - 2x - 3 < 0 \text{ when } -1 < x < 3$$

$$x^2 - 3x + 2 = 0$$

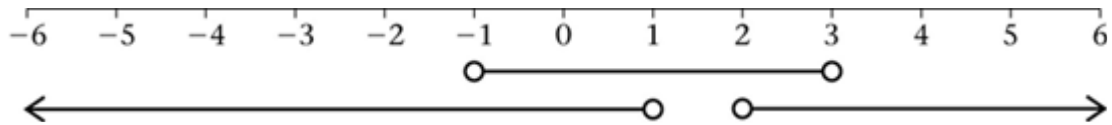
$$(x - 1)(x - 2) = 0$$

$$x = 1, x = 2$$

Sketch of  $y = x^2 - 3x + 2$ :



$$x^2 - 3x + 2 > 0 \text{ when } x < 1 \text{ or } x > 2$$



Intersection is  $-1 < x < 1, 2 < x < 3$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise E, Question 4

**Question:**

(a) Find the range of values of  $k$  for which the equation  $x^2 - kx + (k + 3) = 0$  has no real roots.

(b) Find the range of values of  $p$  for which the roots of the equation  $px^2 + px - 2 = 0$  are real.

**Solution:**

(a)  $a = 1, b = -k, c = k + 3$

$b^2 - 4ac < 0$  for no real roots, so

$$k^2 - 4(k + 3) < 0$$

$$k^2 - 4k - 12 < 0$$

$$(k - 6)(k + 2) < 0$$

$$-2 < k < 6$$

(b)  $a = p, b = p, c = -2$

$b^2 - 4ac < 0$  for no real roots, so

$$p^2 + 8p < 0$$

$$p(p + 8) < 0$$

$$-8 < p < 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 1

**Question:**

Solve the simultaneous equations:

$$\begin{aligned}x + 2y &= 3 \\x^2 - 4y^2 &= -33 \quad \text{[E]}\end{aligned}$$

**Solution:**

$$\begin{aligned}x &= 3 - 2y \\ \text{Substitute into } x^2 - 4y^2 &= -33: \\ (3 - 2y)^2 - 4y^2 &= -33 \\ 9 - 12y + 4y^2 - 4y^2 &= -33 \\ -12y &= -33 - 9 \\ -12y &= -42 \\ y &= 3 \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{Substitute into } x &= 3 - 2y: \\ x &= 3 - 7 = -4\end{aligned}$$

$$\text{So solution is } x = -4, y = 3 \frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 2

#### Question:

Show that the elimination of  $x$  from the simultaneous equations:

$$x - 2y = 1$$

$$3xy - y^2 = 8$$

produces the equation

$$5y^2 + 3y - 8 = 0.$$

Solve this quadratic equation and hence find the pairs  $(x, y)$  for which the simultaneous equations are satisfied. **[E]**

#### Solution:

$$x = 1 + 2y$$

Substitute into  $3xy - y^2 = 8$ :

$$3y(1 + 2y) - y^2 = 8$$

$$3y + 6y^2 - y^2 = 8$$

$$5y^2 + 3y - 8 = 0$$

$$(5y + 8)(y - 1) = 0$$

$$y = -\frac{8}{5} \text{ or } y = 1$$

Substitute into  $x = 1 + 2y$ :

$$\text{when } y = -\frac{8}{5}, x = 1 - \frac{16}{5} = -\frac{11}{5}$$

$$\text{when } y = 1, x = 1 + 2 = 3$$

Solutions are  $\left(-2\frac{1}{5}, -1\frac{3}{5}\right)$  and  $(3, 1)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 3

#### Question:

(a) Given that  $3^x = 9^{y-1}$ , show that  $x = 2y - 2$ .

(b) Solve the simultaneous equations:

$$x = 2y - 2$$

$$x^2 = y^2 + 7 \quad \text{[E]}$$

#### Solution:

(a)  $9 = 3^2$ , so  $3^x = (3^2)^{y-1} \Rightarrow 3^x = 3^{2(y-1)}$

Equate powers:  $x = 2(y-1) \Rightarrow x = 2y - 2$

(b)  $x = 2y - 2$

Substitute into  $x^2 = y^2 + 7$ :

$$(2y - 2)^2 = y^2 + 7$$

$$4y^2 - 8y + 4 = y^2 + 7$$

$$4y^2 - y^2 - 8y + 4 - 7 = 0$$

$$3y^2 - 8y - 3 = 0$$

$$(3y + 1)(y - 3) = 0$$

$$y = -\frac{1}{3} \text{ or } y = 3$$

Substitute into  $x = 2y - 2$ :

when  $y = -\frac{1}{3}$ ,  $x = -\frac{2}{3} - 2 = -2\frac{2}{3}$

when  $y = 3$ ,  $x = 6 - 2 = 4$

Solutions are  $x = -2\frac{2}{3}$ ,  $y = -\frac{1}{3}$  and  $x = 4$ ,  $y = 3$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 4

**Question:**

Solve the simultaneous equations:

$$\begin{aligned}x + 2y &= 3 \\x^2 - 2y + 4y^2 &= 18 \quad \text{[E]}\end{aligned}$$

**Solution:**

$$\begin{aligned}x &= 3 - 2y \\ \text{Substitute into } x^2 - 2y + 4y^2 &= 18: \\ (3 - 2y)^2 - 2y + 4y^2 &= 18 \\ 9 - 12y + 4y^2 - 2y + 4y^2 &= 18 \\ 8y^2 - 14y + 9 - 18 &= 0 \\ 8y^2 - 14y - 9 &= 0 \\ (4y - 9)(2y + 1) &= 0 \\ y &= \frac{9}{4} \text{ or } y = -\frac{1}{2}\end{aligned}$$

Substitute into  $x = 3 - 2y$ :

$$\text{when } y = \frac{9}{4}, x = 3 - \frac{9}{2} = -\frac{3}{2}$$

$$\text{when } y = -\frac{1}{2}, x = 3 + 1 = 4$$

Solutions are  $x = -1\frac{1}{2}, y = 2\frac{1}{4}$  and  $x = 4, y = -\frac{1}{2}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 5

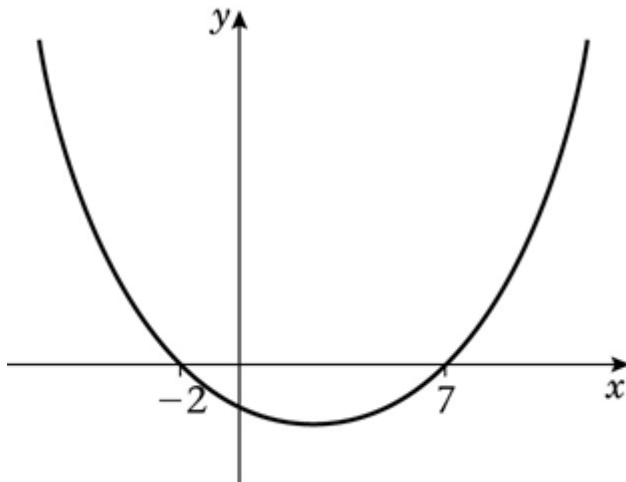
#### Question:

- (a) Solve the inequality  $3x - 8 > x + 13$ .
- (b) Solve the inequality  $x^2 - 5x - 14 > 0$ . **[E]**

#### Solution:

(a)  $3x - x > 13 + 8$   
 $2x > 21$   
 $x > 10 \frac{1}{2}$

(b)  $x^2 - 5x - 14 = 0$   
 $(x + 2)(x - 7) = 0$   
 $x = -2$  or  $x = 7$   
Sketch of  $y = x^2 - 5x - 14$ :



$x^2 - 5x - 14 > 0$  when  $x < -2$  or  $x > 7$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 6

#### Question:

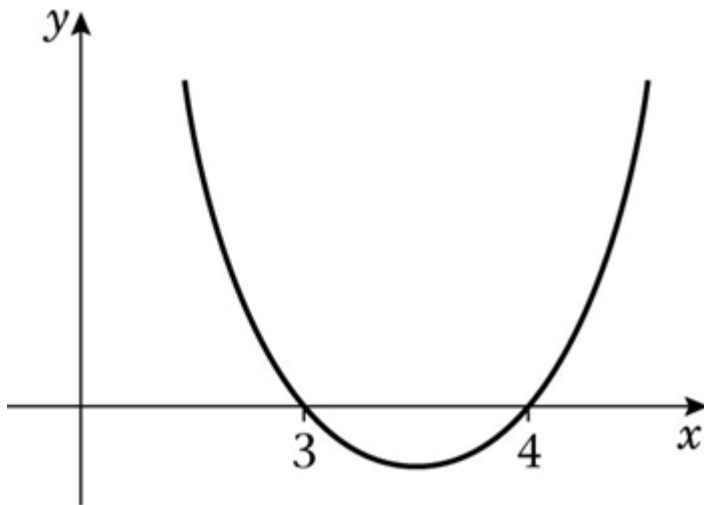
Find the set of values of  $x$  for which  $(x - 1)(x - 4) < 2(x - 4)$ . [E]

#### Solution:

$$\begin{aligned}x^2 - 5x + 4 &< 2x - 8 \\x^2 - 5x - 2x + 4 + 8 &< 0 \\x^2 - 7x + 12 &< 0\end{aligned}$$

$$\begin{aligned}x^2 - 7x + 12 &= 0 \\(x - 3)(x - 4) &= 0 \\x = 3 \text{ or } x = 4\end{aligned}$$

Sketch of  $y = x^2 - 7x + 12$ :



$$x^2 - 7x + 12 < 0 \text{ when } 3 < x < 4.$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 7

#### Question:

(a) Use algebra to solve  $(x - 1)(x + 2) = 18$ .

(b) Hence, or otherwise, find the set of values of  $x$  for which  $(x - 1)(x + 2) > 18$ . **[E]**

#### Solution:

(a)  $x^2 + x - 2 = 18$

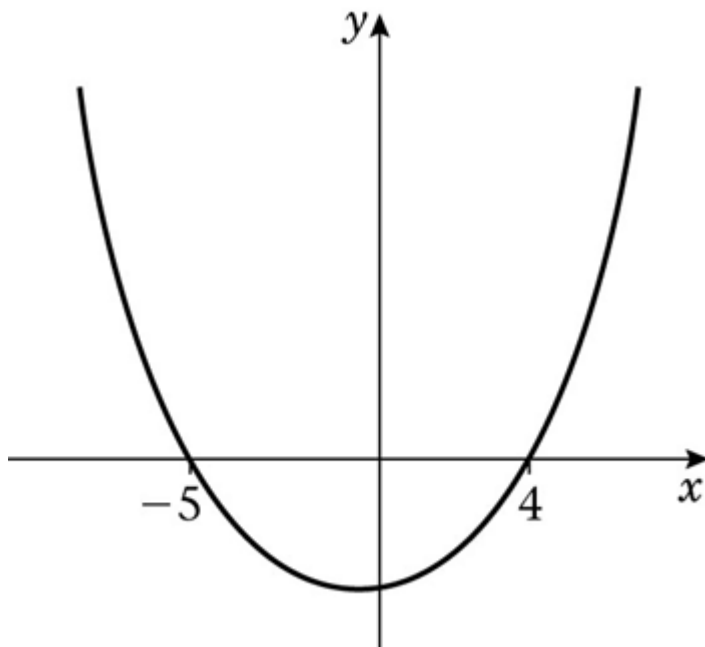
$$x^2 + x - 20 = 0$$

$$(x + 5)(x - 4) = 0$$

$$x = -5 \text{ or } x = 4$$

(b)  $(x - 1)(x + 2) > 18 \Rightarrow x^2 + x - 20 > 0$

Sketch of  $y = x^2 + x - 20$ :



$$x^2 + x - 20 > 0 \text{ when } x < -5 \text{ or } x > 4$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 8

#### Question:

Find the set of values of  $x$  for which:

(a)  $6x - 7 < 2x + 3$

(b)  $2x^2 - 11x + 5 < 0$

(c) both  $6x - 7 < 2x + 3$  and  $2x^2 - 11x + 5 < 0$ . **[E]**

#### Solution:

(a)  $6x - 2x < 3 + 7$

$$4x < 10$$

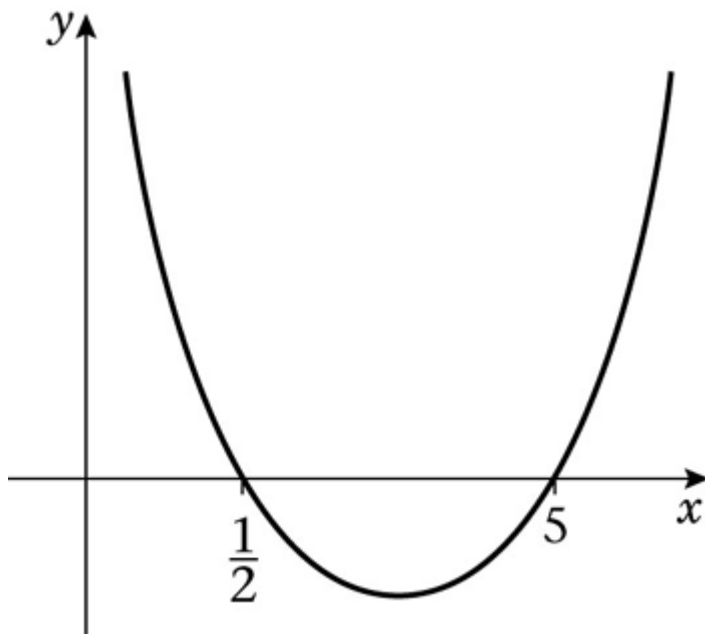
$$x < 2\frac{1}{2}$$

(b)  $2x^2 - 11x + 5 = 0$

$$(2x - 1)(x - 5) = 0$$

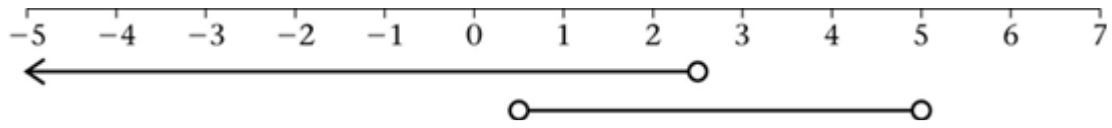
$$x = \frac{1}{2} \text{ or } x = 5$$

Sketch of  $y = 2x^2 - 11x + 5$ :



$$2x^2 - 11x + 5 < 0 \text{ when } \frac{1}{2} < x < 5$$

(c)



Intersection is  $\frac{1}{2} < x < 2\frac{1}{2}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 9

**Question:**

Find the values of  $k$  for which  $kx^2 + 8x + 5 = 0$  has real roots.

**Solution:**

$$a = k, b = 8, c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac \geq 0 \text{ for real roots. So}$$

$$8^2 - 4k \times 5 \geq 0$$

$$64 - 20k \geq 0$$

$$64 \geq 20k$$

$$\frac{64}{20} \geq k$$

$$k \leq 3 \frac{1}{5}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 10

#### Question:

Find algebraically the set of values of  $x$  for which  $(2x - 3)(x + 2) > 3(x - 2)$ . [E]

#### Solution:

$$2x^2 + x - 6 > 3x - 6$$

$$2x^2 + x - 3x - 6 + 6 > 0$$

$$2x^2 - 2x > 0$$

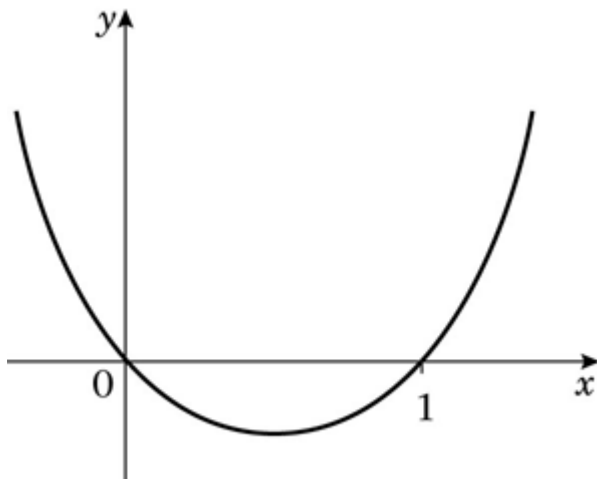
$$2x(x - 1) > 0$$

Solve the equation:

$$2x(x - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

Sketch of  $y = 2x^2 - 2x$ :



$$2x^2 - 2x > 0 \text{ when } x < 0 \text{ or } x > 1$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 11

#### Question:

- (a) Find, as surds, the roots of the equation  $2(x+1)(x-4) - (x-2)^2 = 0$ .
- (b) Hence find the set of values of  $x$  for which  $2(x+1)(x-4) - (x-2)^2 > 0$ . **[E]**

#### Solution:

$$(a) 2(x^2 - 3x - 4) - (x^2 - 4x + 4) = 0$$

$$2x^2 - 6x - 8 - x^2 + 4x - 4 = 0$$

$$x^2 - 2x - 12 = 0$$

$$a = 1, b = -2, c = -12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} :$$

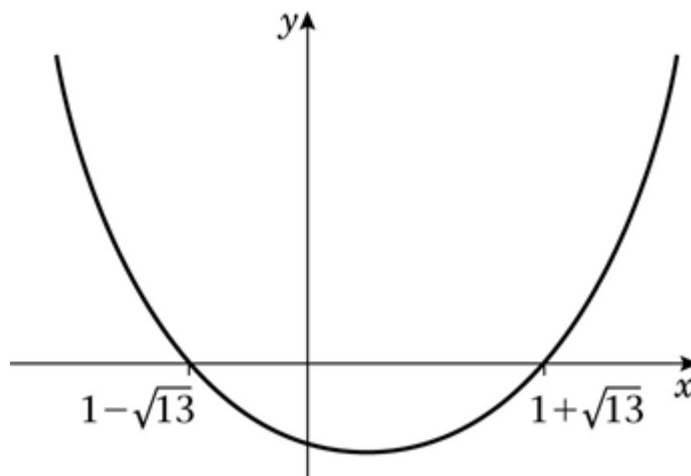
$$x = \frac{2 \pm \sqrt{(-2)^2 + 48}}{2} = \frac{2 \pm \sqrt{52}}{2}$$

$$\sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$$

$$x = 1 + \sqrt{13} \text{ or } x = 1 - \sqrt{13}$$

$$(b) 2(x+1)(x-4) - (x-2)^2 > 0 \Rightarrow x^2 - 2x - 12 > 0$$

Sketch of  $y = x^2 - 2x - 12$ :



$$x^2 - 2x - 12 > 0 \text{ when } x < 1 - \sqrt{13} \text{ or } x > 1 + \sqrt{13}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 12

#### Question:

- (a) Use algebra to find the set of values of  $x$  for which  $x(x - 5) > 36$ .
- (b) Using your answer to part (a), find the set of values of  $y$  for which  $y^2(y^2 - 5) > 36$ .

#### Solution:

(a)  $x^2 - 5x > 36$

$$x^2 - 5x - 36 > 0$$

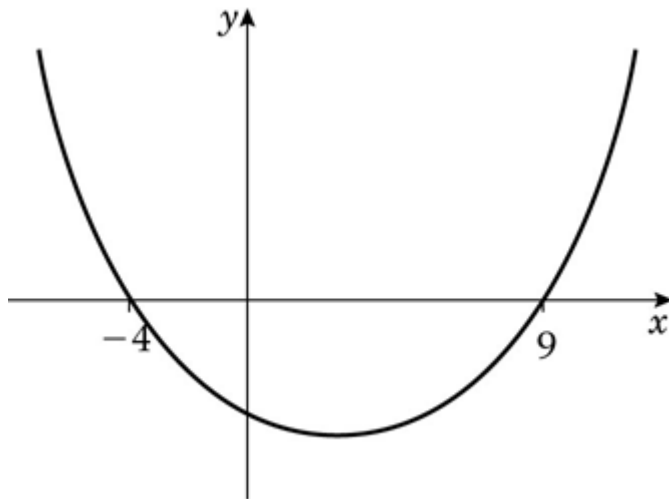
Solve the equation:

$$x^2 - 5x - 36 = 0$$

$$(x + 4)(x - 9) = 0$$

$$x = -4 \text{ or } x = 9$$

Sketch of  $y = x^2 - 5x - 36$ :



$$x^2 - 5x - 36 > 0 \text{ when } x < -4 \text{ or } x > 9$$

(b) Either  $y^2 < -4$  or  $y^2 > 9$

$y^2 < -4$  is not possible. No values.

$$y^2 > 9 \Rightarrow y > 3 \text{ or } y < -3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Equations and inequalities

#### Exercise F, Question 13

#### Question:

The specification for a rectangular car park states that the length  $x$  m is to be 5 m more than the breadth. The perimeter of the car park is to be greater than 32 m.

(a) Form a linear inequality in  $x$ .

The area of the car park is to be less than  $104\text{m}^2$ .

(b) Form a quadratic inequality in  $x$ .

(c) By solving your inequalities, determine the set of possible values of  $x$ . **[E]**

#### Solution:

(a) Length is  $x$  metres, breadth is  $(x - 5)$  metres.

Perimeter is  $x + x + (x - 5) + (x - 5) = (4x - 10)$  metres

So  $4x - 10 > 32$

(b) Area is  $x(x - 5)$   $\text{m}^2$ .

So  $x(x - 5) < 104$

(c) Linear:

$$4x - 10 > 32$$

$$4x > 32 + 10$$

$$4x > 42$$

$$x > 10 \frac{1}{2}$$

Quadratic:

$$x^2 - 5x < 104$$

$$x^2 - 5x - 104 < 0$$

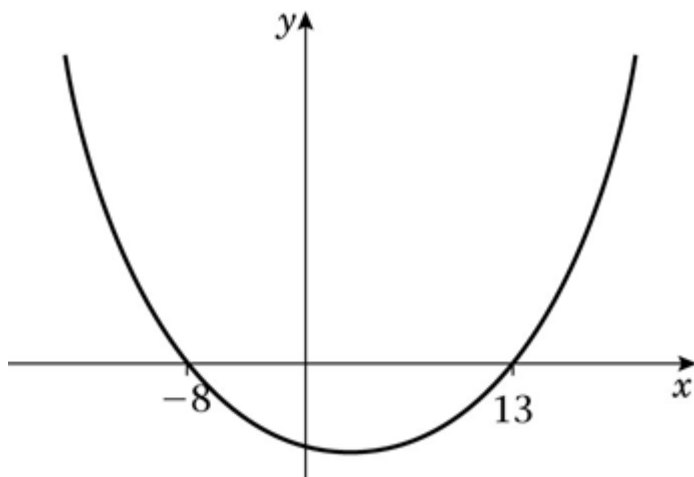
Solve the equation:

$$x^2 - 5x - 104 = 0$$

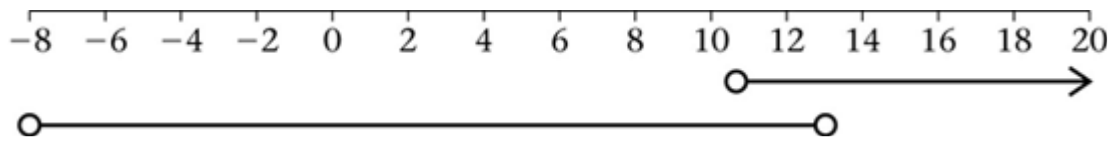
$$(x + 8)(x - 13) = 0$$

$$x = -8 \text{ or } x = 13$$

Sketch of  $y = x^2 - 5x - 104$ :



$$x^2 - 5x - 104 < 0 \text{ when } -8 < x < 13$$



Intersection is  $10\frac{1}{2} < x < 13$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise A, Question 1

#### Question:

Sketch the following curves and indicate clearly the points of intersection with the axes:

(a)  $y = (x - 3)(x - 2)(x + 1)$

(b)  $y = (x - 1)(x + 2)(x + 3)$

(c)  $y = (x + 1)(x + 2)(x + 3)$

(d)  $y = (x + 1)(1 - x)(x + 3)$

(e)  $y = (x - 2)(x - 3)(4 - x)$

(f)  $y = x(x - 2)(x + 1)$

(g)  $y = x(x + 1)(x - 1)$

(h)  $y = x(x + 1)(1 - x)$

(i)  $y = (x - 2)(2x - 1)(2x + 1)$

(j)  $y = x(2x - 1)(x + 3)$

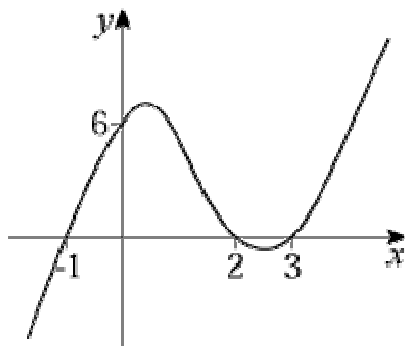
#### Solution:

(a)  $y = 0 \Rightarrow x = -1, 2, 3$

$x = 0 \Rightarrow y = 6$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$

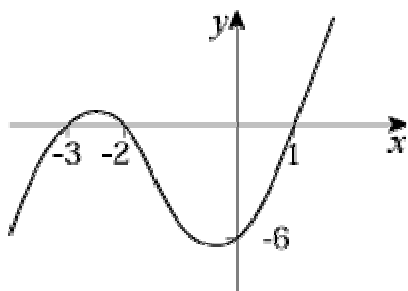


(b)  $y = 0 \Rightarrow x = 1, -2, -3$

$x = 0 \Rightarrow y = -6$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$

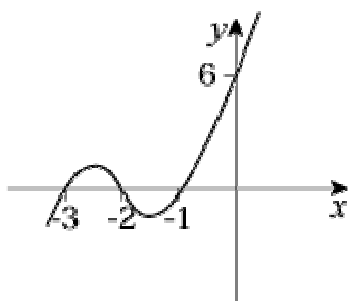


$$(c) y = 0 \Rightarrow x = -1, -2, -3$$

$$x = 0 \Rightarrow y = 6$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

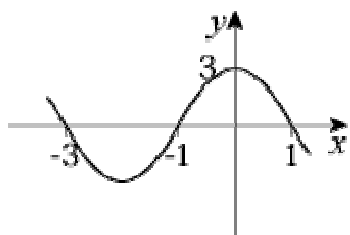


$$(d) y = 0 \Rightarrow x = -1, 1, -3$$

$$x = 0 \Rightarrow y = 3$$

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$

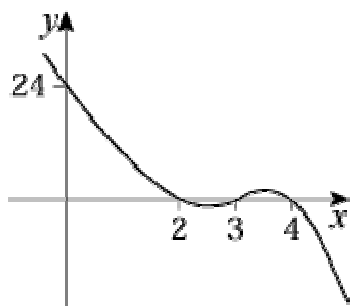


$$(e) y = 0 \Rightarrow x = 2, 3, 4$$

$$x = 0 \Rightarrow y = 24$$

$$x \rightarrow \infty, y \rightarrow -\infty$$

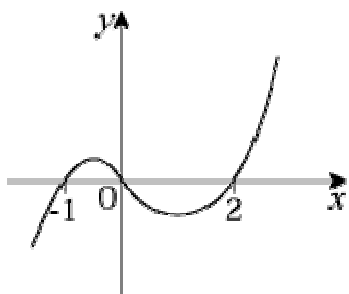
$$x \rightarrow -\infty, y \rightarrow \infty$$



$$(f) y = 0 \Rightarrow x = 0, -1, 2$$

$$x \rightarrow \infty, y \rightarrow \infty$$

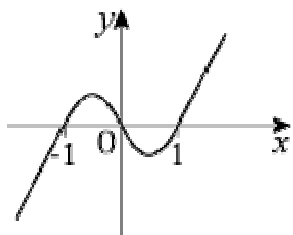
$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$(g) y = 0 \Rightarrow x = 0, -1, 1$$

$$x \rightarrow \infty, y \rightarrow \infty$$

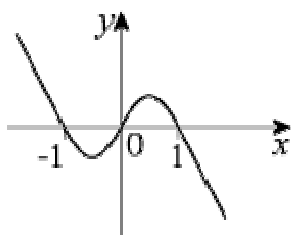
$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$(h) y = 0 \Rightarrow x = 0, -1, 1$$

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$

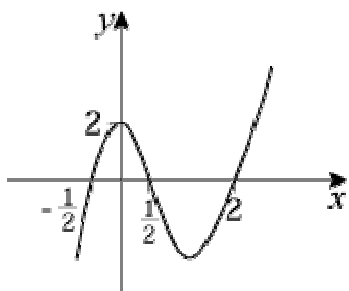


$$(i) y = 0 \Rightarrow x = 2, \frac{1}{2}, -\frac{1}{2}$$

$$x = 0 \Rightarrow y = 2$$

$$x \rightarrow \infty, y \rightarrow \infty$$

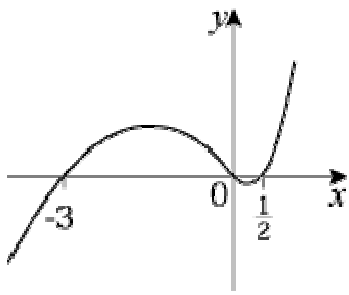
$$x \rightarrow -\infty, y \rightarrow -\infty$$



$$(j) y = 0 \Rightarrow x = 0, \frac{1}{2}, -3$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise A, Question 2

#### Question:

Sketch the curves with the following equations:

(a)  $y = (x + 1)^2(x - 1)$

(b)  $y = (x + 2)(x - 1)^2$

(c)  $y = (2 - x)(x + 1)^2$

(d)  $y = (x - 2)(x + 1)^2$

(e)  $y = x^2(x + 2)$

(f)  $y = (x - 1)^2x$

(g)  $y = (1 - x)^2(3 + x)$

(h)  $y = (x - 1)^2(3 - x)$

(i)  $y = x^2(2 - x)$

(j)  $y = x^2(x - 2)$

#### Solution:

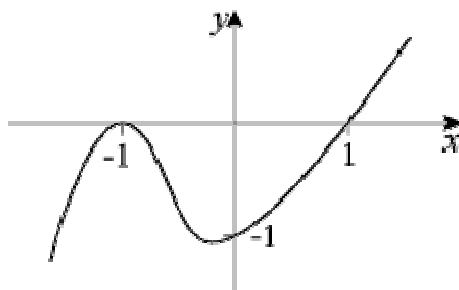
(a)  $y = 0 \Rightarrow x = -1$  (twice),  $1$

$x = 0 \Rightarrow y = -1$

Turning point at  $(-1, 0)$ .

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$



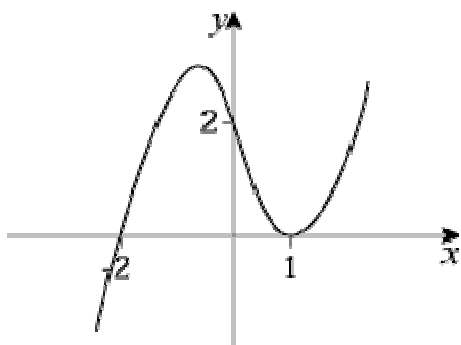
(b)  $y = 0 \Rightarrow x = -2, 1$  (twice)

$x = 0 \Rightarrow y = 2$

Turning point at  $(1, 0)$ .

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$



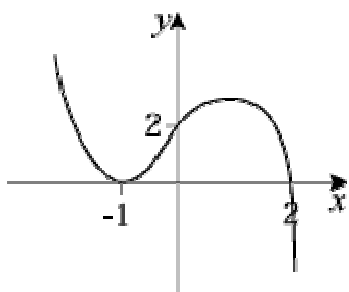
$$(c) y = 0 \Rightarrow x = 2, -1 \text{ (twice)}$$

$$x = 0 \Rightarrow y = 2$$

Turning point at  $(-1, 0)$ .

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$



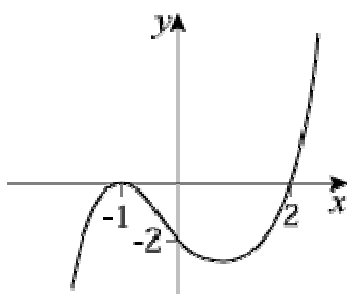
$$(d) y = 0 \Rightarrow x = 2, -1 \text{ (twice)}$$

$$x = 0 \Rightarrow y = -2$$

Turning point at  $(-1, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

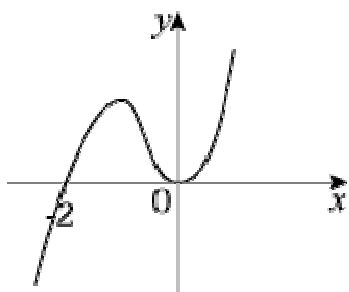


$$(e) y = 0 \Rightarrow x = 0 \text{ (twice)}, -2$$

Turning point at  $(0, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

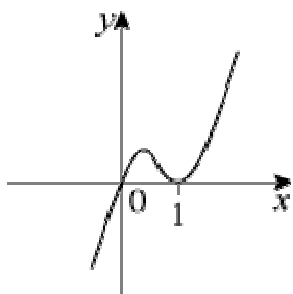


$$(f) y = 0 \Rightarrow x = 0, 1 \text{ (twice)}$$

Turning point at  $(1, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



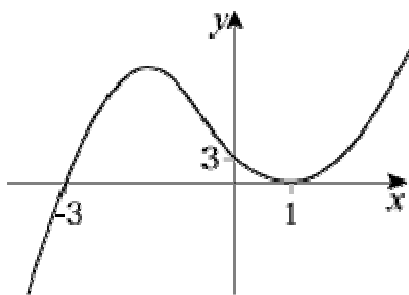
$$(g) y = 0 \Rightarrow x = 1 \text{ (twice)}, -3$$

$$x = 0 \Rightarrow y = 3$$

Turning point at  $(1, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



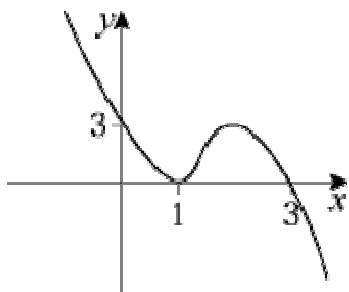
$$(h) y = 0 \Rightarrow x = 1 \text{ (twice)}, 3$$

$$x = 0 \Rightarrow y = 3$$

Turning point at  $(1, 0)$ .

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$

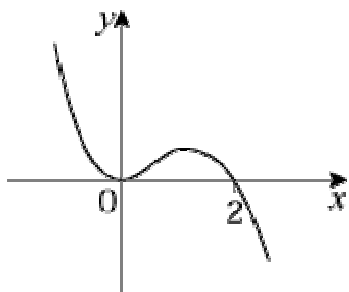


$$(i) y = 0 \Rightarrow x = 0 \text{ (twice)}, 2$$

Turning point at  $(0, 0)$ .

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$

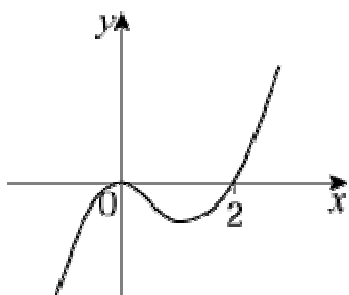


(j)  $y = 0 \Rightarrow x = 0$  (twice), 2

Turning point at (0, 0).

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise A, Question 3

#### Question:

Factorise the following equations and then sketch the curves:

(a)  $y = x^3 + x^2 - 2x$

(b)  $y = x^3 + 5x^2 + 4x$

(c)  $y = x^3 + 2x^2 + x$

(d)  $y = 3x + 2x^2 - x^3$

(e)  $y = x^3 - x^2$

(f)  $y = x - x^3$

(g)  $y = 12x^3 - 3x$

(h)  $y = x^3 - x^2 - 2x$

(i)  $y = x^3 - 9x$

(j)  $y = x^3 - 9x^2$

#### Solution:

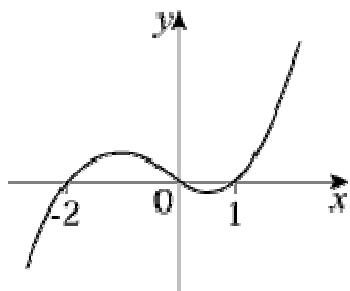
(a)  $y = x^3 + x^2 - 2x = x(x^2 + x - 2)$

So  $y = x(x + 2)(x - 1)$

$y = 0 \Rightarrow x = 0, 1, -2$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$



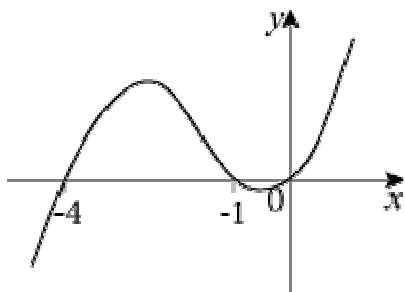
(b)  $y = x^3 + 5x^2 + 4x = x(x^2 + 5x + 4)$

So  $y = x(x + 4)(x + 1)$

$y = 0 \Rightarrow x = 0, -4, -1$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$



$$(c) y = x^3 + 2x^2 + x = x(x^2 + 2x + 1)$$

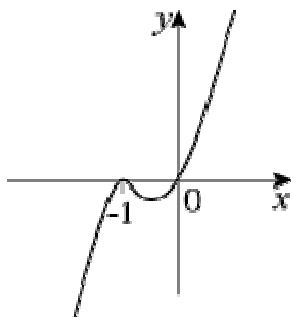
$$\text{So } y = x(x + 1)^2$$

$$y = 0 \Rightarrow x = 0, -1 \text{ (twice)}$$

Turning point at  $(-1, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



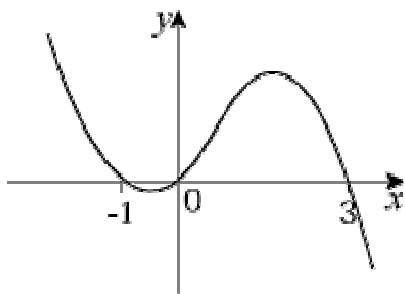
$$(d) y = 3x + 2x^2 - x^3 = x(3 + 2x - x^2)$$

$$\text{So } y = x(3 - x)(1 + x)$$

$$y = 0 \Rightarrow x = 0, 3, -1$$

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$



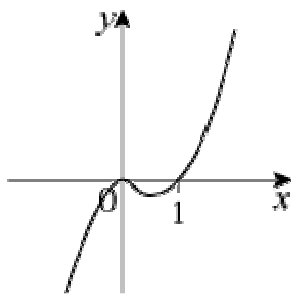
$$(e) y = x^3 - x^2 = x^2(x - 1)$$

$$y = 0 \Rightarrow x = 0 \text{ (twice), } 1$$

Turning point at  $(0, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



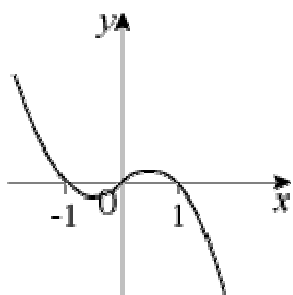
$$(f) y = x - x^3 = x(1 - x^2)$$

$$\text{So } y = x(1 - x)(1 + x)$$

$$y = 0 \Rightarrow x = 0, 1, -1$$

$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow \infty$$



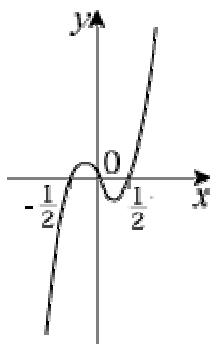
$$(g) y = 12x^3 - 3x = 3x(4x^2 - 1)$$

$$\text{So } y = 3x(2x - 1)(2x + 1)$$

$$y = 0 \Rightarrow x = 0, \frac{1}{2}, -\frac{1}{2}$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



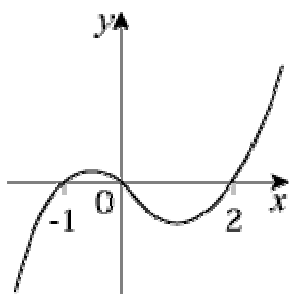
$$(h) y = x^3 - x^2 - 2x = x(x^2 - x - 2)$$

$$\text{So } y = x(x + 1)(x - 2)$$

$$y = 0 \Rightarrow x = 0, -1, 2$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



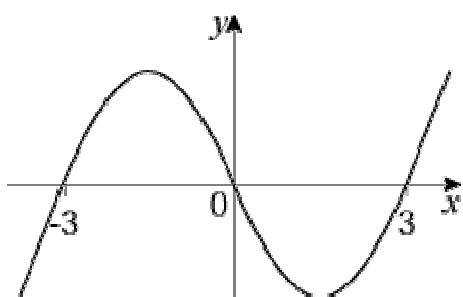
$$(i) y = x^3 - 9x = x(x^2 - 9)$$

$$\text{So } y = x(x - 3)(x + 3)$$

$$y = 0 \Rightarrow x = 0, 3, -3$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



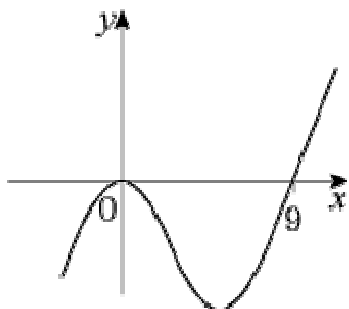
$$(j) y = x^3 - 9x^2 = x^2(x - 9)$$

$$y = 0 \Rightarrow x = 0 \text{ (twice), } 9$$

Turning point at (0,0).

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise B, Question 1

#### Question:

Sketch the following curves and show their positions relative to the curve  $y = x^3$ :

(a)  $y = (x - 2)^3$

(b)  $y = (2 - x)^3$

(c)  $y = (x - 1)^3$

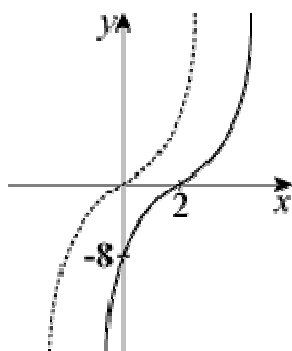
(d)  $y = (x + 2)^3$

(e)  $y = -(x + 2)^3$

#### Solution:

(a)  $y = 0 \Rightarrow x = 2$ , so curve crosses  $x$ -axis at  $(2, 0)$

$x = 0 \Rightarrow y = -8$ , so curve crosses  $y$ -axis at  $(0, -8)$

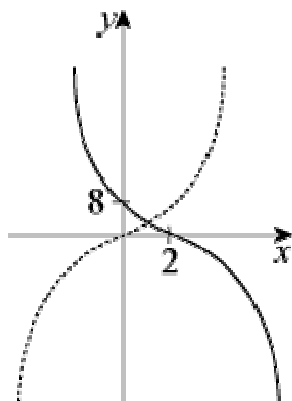


Curve is a translation of  $+2$  in  $x$  direction of the curve  $y = x^3$ .

(b)  $y = 0 \Rightarrow x = 2$ , so curve crosses  $x$ -axis at  $(2, 0)$

$x = 0 \Rightarrow y = 8$ , so curve crosses  $y$ -axis at  $(0, 8)$

$y = (2 - x)^3 = -(x - 2)^3$ , so shape is like  $y = -x^3$

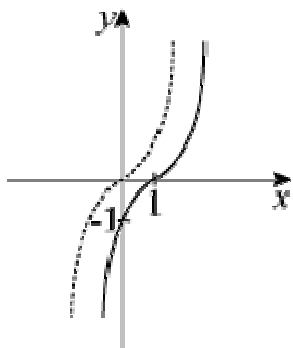


This is a horizontal translation of + 2 of the curve  $y = -x^3$ .

(c)  $y = 0 \Rightarrow x = 1$ , so curve crosses  $x$ -axis at  $(1, 0)$

$x = 0 \Rightarrow y = -1$ , so curve crosses  $y$ -axis at  $(0, -1)$

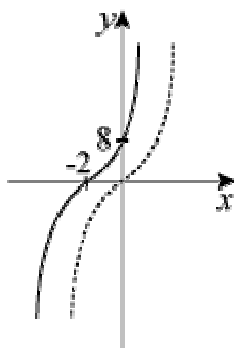
$y = (x - 1)^3$  is a horizontal translation of + 1 of  $y = x^3$ .



(d)  $y = 0 \Rightarrow x = -2$ , so curve crosses  $x$ -axis at  $(-2, 0)$

$x = 0 \Rightarrow y = 8$ , so curve crosses  $y$ -axis at  $(0, 8)$

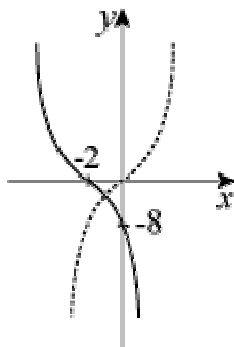
$y = (x + 2)^3$  is same shape as  $y = x^3$  but translated horizontally by - 2.



(e)  $y = 0 \Rightarrow x = -2$ , so curve crosses  $x$ -axis at  $(-2, 0)$

$x = 0 \Rightarrow y = -8$ , so curve crosses  $y$ -axis at  $(0, -8)$

$y = -(x + 2)^3$  is a reflection in  $x$ -axis of  $y = (x + 2)^3$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise B, Question 2

#### Question:

Sketch the following and indicate the coordinates of the points where the curves cross the axes:

(a)  $y = (x + 3)^3$

(b)  $y = (x - 3)^3$

(c)  $y = (1 - x)^3$

(d)  $y = -(x - 2)^3$

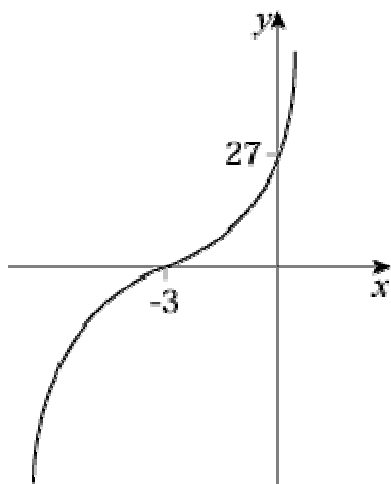
(e)  $y = -(x - \frac{1}{2})^3$

#### Solution:

(a)  $y = 0 \Rightarrow x = -3$ , so curve crosses  $x$ -axis at  $(-3, 0)$

$x = 0 \Rightarrow y = 27$ , so curve crosses  $y$ -axis at  $(0, 27)$

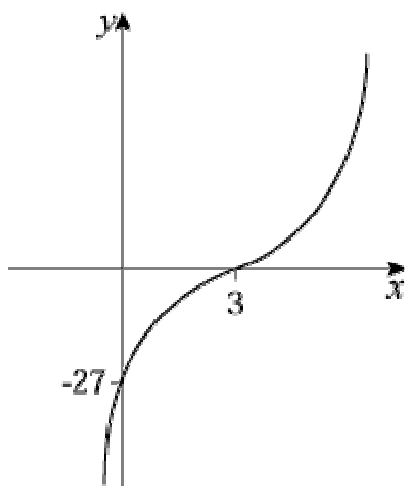
$y = (x + 3)^3$  is a translation of  $-3$  in  $x$ -direction of  $y = x^3$ .



(b)  $y = 0 \Rightarrow x = 3$ , so curve crosses  $x$ -axis at  $(3, 0)$

$x = 0 \Rightarrow y = -27$ , so curve crosses  $y$ -axis at  $(0, -27)$

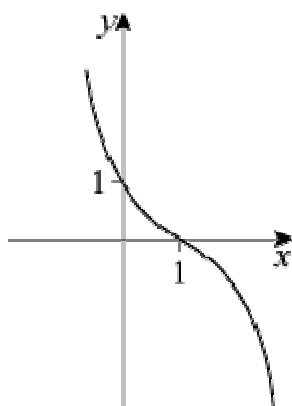
$y = (x - 3)^3$  is a horizontal translation of  $+3$  of  $y = x^3$ .



(c)  $y = 0 \Rightarrow x = 1$ , so curve crosses  $x$ -axis at  $(1, 0)$

$x = 0 \Rightarrow y = 1$ , so curve crosses  $y$ -axis at  $(0, 1)$

$y = (1 - x)^3$  is a horizontal translation of  $y = -x^3$ .

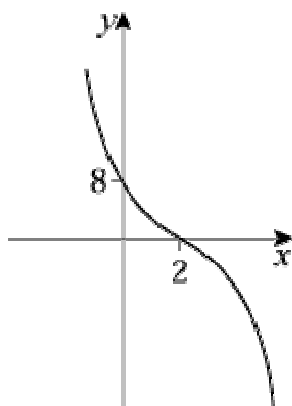


Horizontal translation  $+1$  of  $y = -x^3$ .

(d)  $y = 0 \Rightarrow x = 2$ , so curve crosses  $x$ -axis at  $(2, 0)$

$x = 0 \Rightarrow y = 8$ , so curve crosses  $y$ -axis at  $(0, 8)$

$y = -(x - 2)^3$  is a translation ( $+2$  in  $x$ -direction) of  $y = -x^3$ .

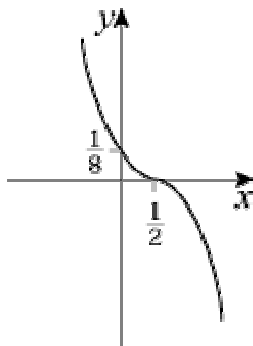


(e)  $y = 0 \Rightarrow x = \frac{1}{2}$ , so curve crosses  $x$ -axis at  $(\frac{1}{2}, 0)$



$x = 0 \Rightarrow y = \frac{1}{8}$ , so curve crosses  $y$ -axis at  $(0, \frac{1}{8})$

$y = -(x - \frac{1}{2})^3$  is a horizontal translation  $(+ \frac{1}{2})$  of  $y = -x^3$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

Exercise C, Question 1

#### Question:

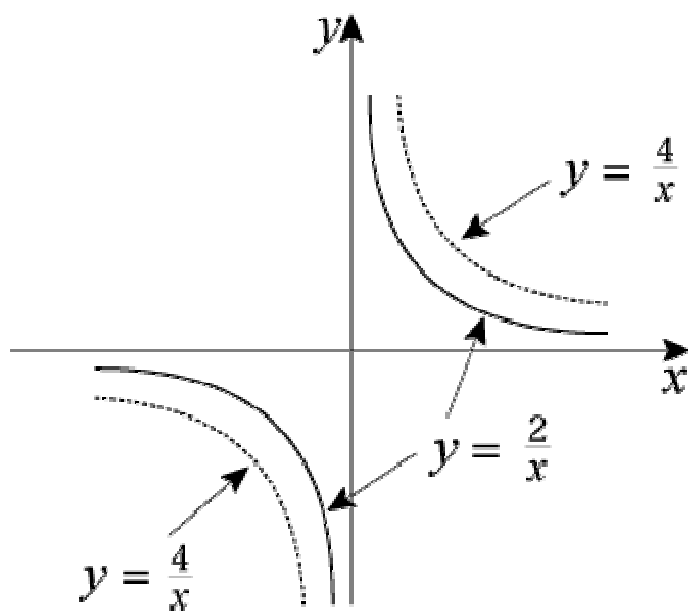
Sketch on the same diagram

$$y = \frac{2}{x} \text{ and } y = \frac{4}{x}$$

#### Solution:

For  $x > 0$ ,  $\frac{4}{x} > \frac{2}{x}$  (since  $4 > 2$ )

So  $\frac{4}{x}$  is 'on top' of  $\frac{2}{x}$  in 1st quadrant and 'below' in 3rd quadrant



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise C, Question 2

#### Question:

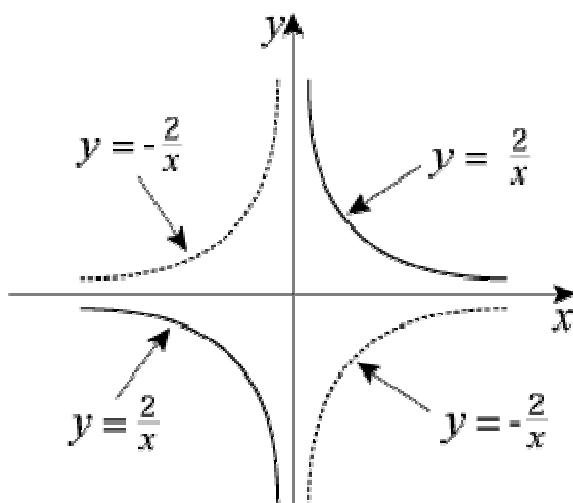
Sketch on the same diagram

$$y = \frac{2}{x} \text{ and } y = -\frac{2}{x}$$

#### Solution:

$$y = \frac{2}{x} > 0 \text{ for } x > 0$$

$$y = -\frac{2}{x} < 0 \text{ for } x > 0$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise C, Question 3

#### Question:

Sketch on the same diagram

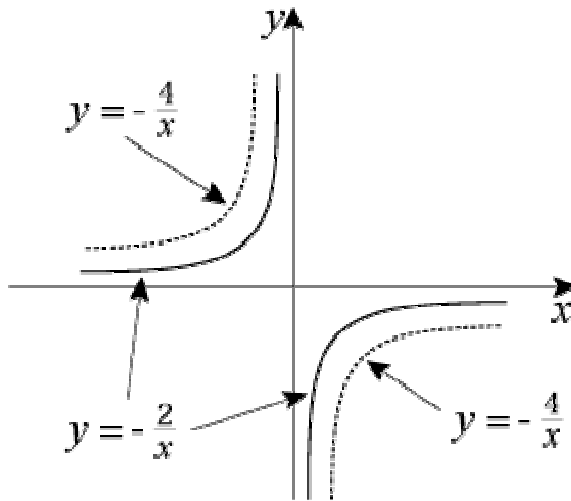
$$y = -\frac{4}{x} \text{ and } y = -\frac{2}{x}$$

#### Solution:

Graphs are like  $y = -\frac{1}{x}$  and so exist in 2nd and 4th quadrants.

$$\text{For } x < 0, -\frac{4}{x} > -\frac{2}{x}$$

So  $-\frac{4}{x}$  is 'on top' of  $-\frac{2}{x}$  in 2nd quadrant and 'below' in 4th quadrant.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise C, Question 4

#### Question:

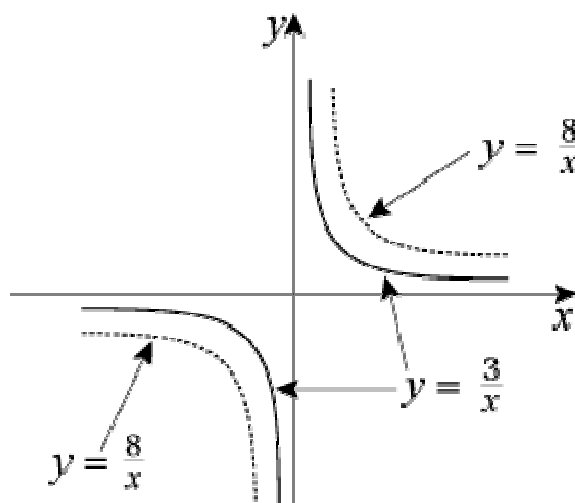
Sketch on the same diagram

$$y = \frac{3}{x} \text{ and } y = \frac{8}{x}$$

#### Solution:

For  $x > 0$ ,  $\frac{8}{x} > \frac{3}{x}$

So  $y = \frac{8}{x}$  is 'on top' of  $y = \frac{3}{x}$  in 1st quadrant and 'below' in 3rd quadrant.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise C, Question 5

#### Question:

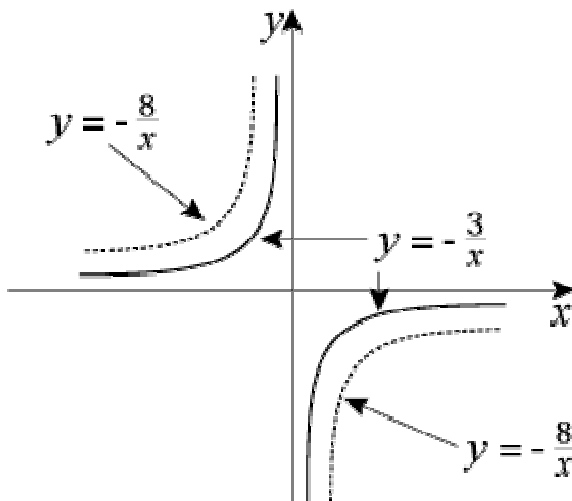
Sketch on the same diagram

$$y = -\frac{3}{x} \text{ and } y = -\frac{8}{x}$$

#### Solution:

$$\text{For } x < 0, -\frac{8}{x} > -\frac{3}{x}$$

So  $y = -\frac{8}{x}$  is 'on top' of  $y = -\frac{3}{x}$  in 2nd quadrant and 'below' in 4th quadrant.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 1

#### Question:

In each case:

(i) sketch the two curves on the same axes

(ii) state the number of points of intersection

(iii) write down a suitable equation which would give the  $x$ -coordinates of these points. (You are not required to solve this equation.)

(a)  $y = x^2, y = x(x^2 - 1)$

(b)  $y = x(x + 2), y = -\frac{3}{x}$

(c)  $y = x^2, y = (x + 1)(x - 1)^2$

(d)  $y = x^2(1 - x), y = -\frac{2}{x}$

(e)  $y = x(x - 4), y = \frac{1}{x}$

(f)  $y = x(x - 4), y = -\frac{1}{x}$

(g)  $y = x(x - 4), y = (x - 2)^3$

(h)  $y = -x^3, y = -\frac{2}{x}$

(i)  $y = -x^3, y = x^2$

(j)  $y = -x^3, y = -x(x + 2)$

#### Solution:

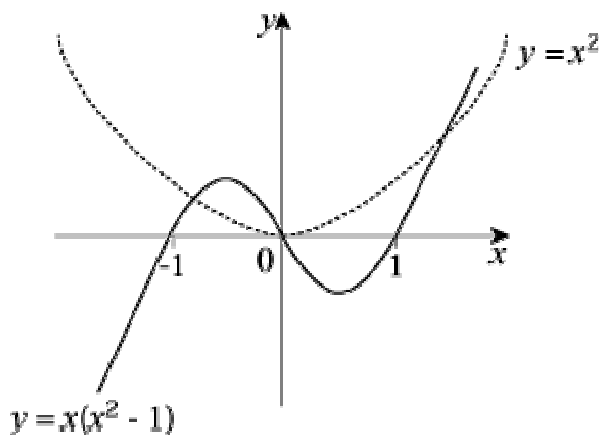
(a) (i)  $y = x^2$  is standard

$$y = x(x^2 - 1) = x(x - 1)(x + 1)$$

$$y = 0 \Rightarrow x = 0, 1, -1$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



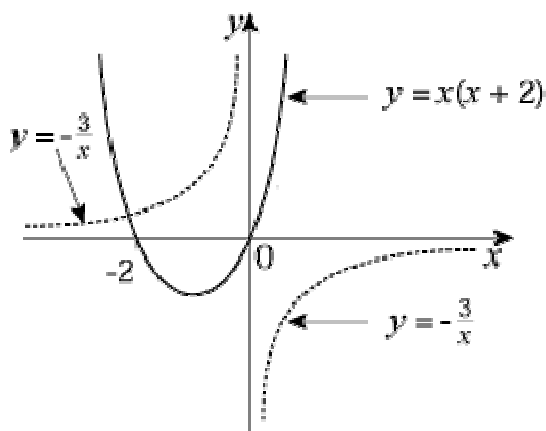
(ii)  $y = x^2$  cuts  $y = x(x^2 - 1)$  in 3 places.

(iii) Solutions given by  $x^2 = x(x^2 - 1)$

(b) (i)  $y = x(x + 2)$  is a  $\cup$ -shaped curve

$$y = 0 \Rightarrow x = 0, -2$$

$$y = -\frac{3}{x} \text{ is like } y = -\frac{1}{x}$$



(ii) Curves cross at only 1 point.

(iii) Equation:  $-\frac{3}{x} = x(x + 2)$

(c) (i)  $y = x^2$  is standard

$$y = (x + 1)(x - 1)^2$$

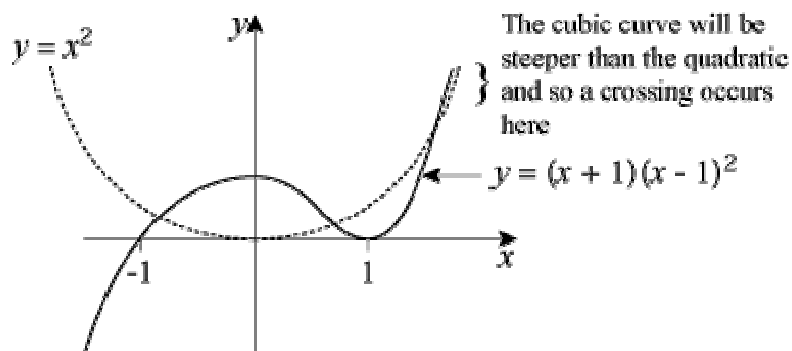
$$y = 0 \Rightarrow x = -1, 1 \text{ (twice)}$$

Turning point at  $(1, 0)$

$$x \rightarrow \infty, y \rightarrow +\infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$





(ii) 3 points of intersection

(iii) Equation:  $x^2 = (x + 1)(x - 1)^2$

(d) (i)  $y = x^2(1 - x)$

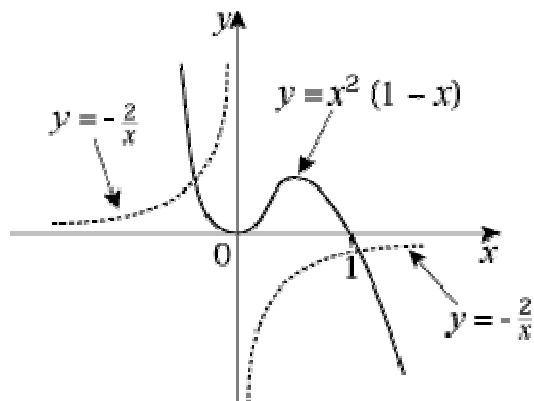
$y = 0 \Rightarrow x = 0$  (twice), 1

Turning point at  $(0, 0)$

$x \rightarrow \infty, y \rightarrow -\infty$

$x \rightarrow -\infty, y \rightarrow \infty$

$y = -\frac{2}{x}$  is like  $y = -\frac{1}{x}$  and in 2nd and 4th quadrants



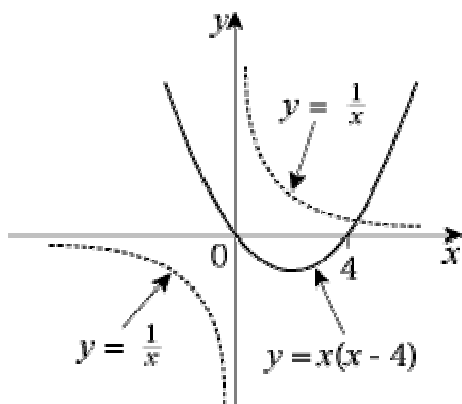
(ii) 2 points of intersection

(iii) Equation:  $-\frac{2}{x} = x^2(1 - x)$

(e) (i)  $y = x(x - 4)$  is a  $\cup$ -shaped curve

$y = 0 \Rightarrow x = 0, 4$

$y = \frac{1}{x}$  is standard



(ii) 1 point of intersection

(iii) Equation:  $\frac{1}{x} = x(x - 4)$

(f) (i)  $y = x(x - 4)$  is a  $\cup$ -shaped curve

$y = 0 \Rightarrow x = 0, 4$

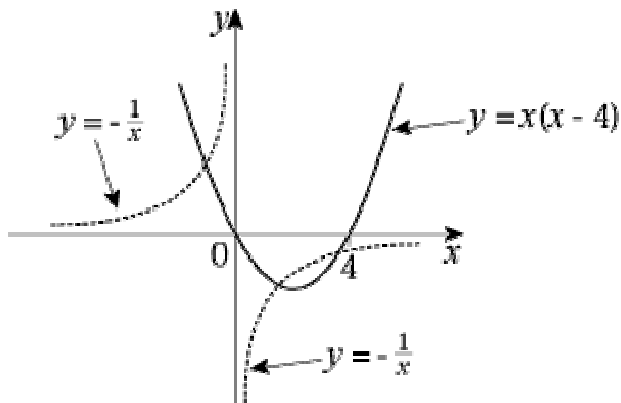
$y = -\frac{1}{x}$  is standard and in 2nd and 4th quadrants

At  $x = 2$ ,

$y = -\frac{1}{x}$  gives  $y = -\frac{1}{2}$

$y = x(x - 4)$  gives  $y = 2(-2) = -4$

So  $y = -\frac{1}{x}$  cuts  $y = x(x - 4)$  in 4th quadrant.



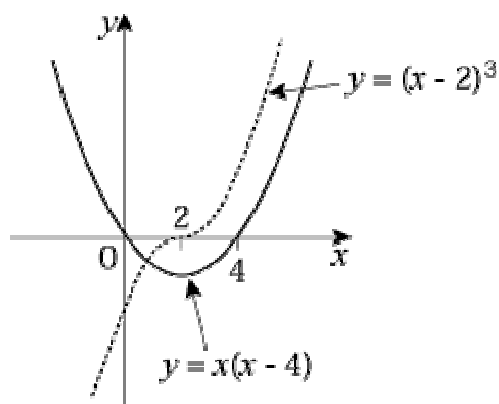
(ii) 3 points of intersection

(iii) Equation:  $-\frac{1}{x} = x(x - 4)$

(g) (i)  $y = x(x - 4)$  is a  $\cup$ -shaped curve

$y = 0 \Rightarrow x = 0, 4$

$y = (x - 2)^3$  is a translation of  $+2$  in the  $x$ -direction of  $y = x^3$ .

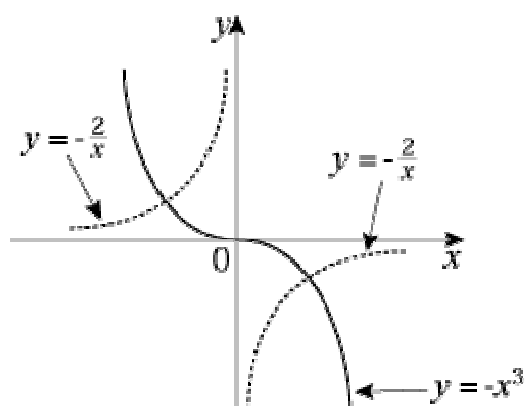


(ii) 1 point of intersection

(iii)  $x(x - 4) = (x - 2)^3$

(h) (i)  $y = -x^3$  is standard

$y = -\frac{2}{x}$  is like  $y = -\frac{1}{x}$  and in 2nd and 4th quadrants.

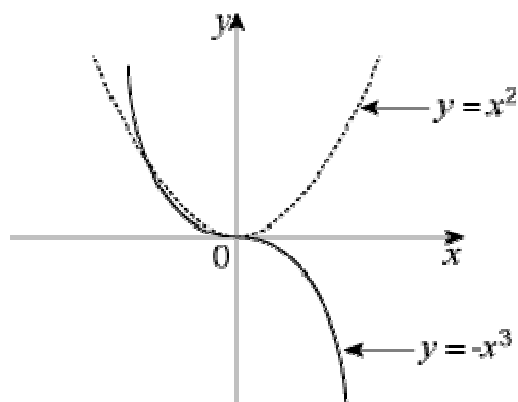


(ii) 2 points of intersection

(iii)  $-x^3 = -\frac{2}{x}$  or  $x^3 = \frac{2}{x}$

(i) (i)  $y = -x^3$  is standard

$y = x^2$  is standard



(ii) 2 points of intersection

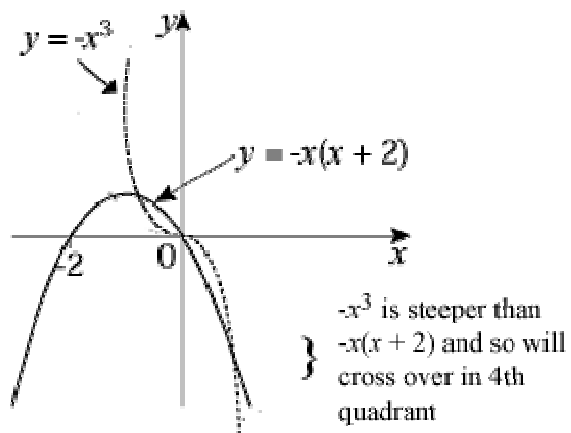
[At (0,0) the curves actually touch. They intersect in the second quadrant.]

(iii)  $x^2 = -x^3$

(j) (i)  $y = -x^3$  is standard

$y = -x(x+2)$  is  $\cap$  shaped

$y = 0 \Rightarrow x = 0, -2$



(ii) 3 points of intersection

(iii)  $-x^3 = -x(x+2)$  or  $x^3 = x(x+2)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 2

#### Question:

- (a) On the same axes sketch the curves given by  $y = x^2(x - 4)$  and  $y = x(4 - x)$ .
- (b) Find the coordinates of the points of intersection.

#### Solution:

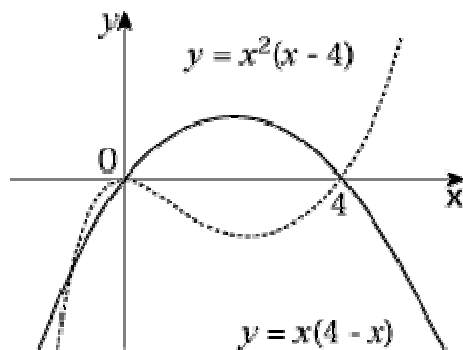
(a)  $y = x^2(x - 4)$

$y = 0 \Rightarrow x = 0$  (twice), 4

Turning point at  $(0, 0)$

$y = x(4 - x)$  is  $\cap$  shaped

$y = 0 \Rightarrow x = 0, 4$



(b)  $x(4 - x) = x^2(x - 4)$

$\Rightarrow 0 = x^2(x - 4) - x(4 - x)$

Factorise:  $0 = x(x - 4)(x + 1)$

So intersections at  $x = 0, -1, 4$

So points are [using  $y = x(4 - x)$ ]  $(0, 0)$ ;  $(-1, -5)$ ;  $(4, 0)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves Exercise D, Question 3

#### Question:

- (a) On the same axes sketch the curves given by  $y = x(2x + 5)$  and  $y = x(1 + x)^2$
- (b) Find the coordinates of the points of intersection.

#### Solution:

(a)  $y = x(2x + 5)$  is  $\cup$  shaped

$$y = 0 \Rightarrow x = 0, -\frac{5}{2}$$

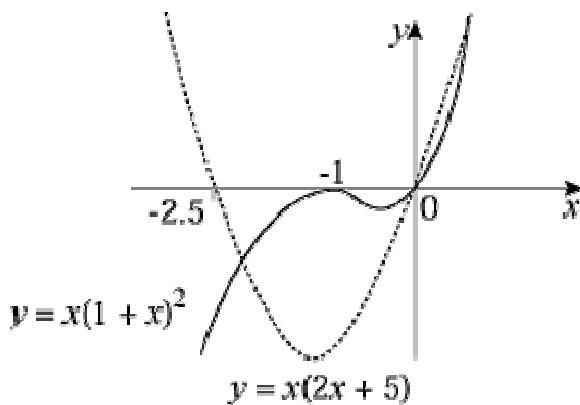
$$y = x(1 + x)^2$$

$$y = 0 \Rightarrow x = 0, -1 \text{ (twice)}$$

Turning point at  $(-1, 0)$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



(b)  $x(1 + x)^2 = x(2x + 5)$

$$\Rightarrow x [ x^2 + 2x + 1 - (2x + 5) ] = 0$$

$$\Rightarrow x(x^2 - 4) = 0$$

$$\Rightarrow x(x - 2)(x + 2) = 0$$

$$\Rightarrow x = 0, 2, -2$$

So points are [using  $y = x(2x + 5)$ ]:  $(0, 0)$ ;  $(2, 18)$ ;  $(-2, -2)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

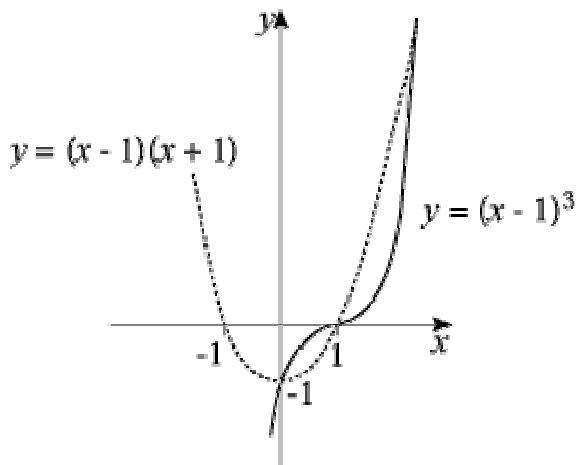
#### Exercise D, Question 4

#### Question:

- (a) On the same axes sketch the curves given by  $y = (x - 1)^3$  and  $y = (x - 1)(1 + x)$ .
- (b) Find the coordinates of the points of intersection.

#### Solution:

- (a)  $y = (x - 1)^3$  is like  $y = x^3$  with crossing points at  $(1, 0)$  and  $(0, -1)$   
 $y = (x - 1)(1 + x)$  is a  $\cup$ -shaped curve.  
 $y = 0 \Rightarrow x = 1, -1$



- (b) Intersect when  $(x - 1)^3 = (x - 1)(x + 1)$   
 i.e.  $(x - 1)^3 - (x - 1)(x + 1) = 0$   
 $\Rightarrow (x - 1) [ x^2 - 2x + 1 - (x + 1) ] = 0$   
 $\Rightarrow (x - 1)(x^2 - 3x) = 0$   
 $\Rightarrow (x - 1)(x - 3)x = 0$   
 $\Rightarrow x = 0, 1, 3$

So intersections at  $(0, -1)$ ;  $(1, 0)$ ;  $(3, 8)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 5

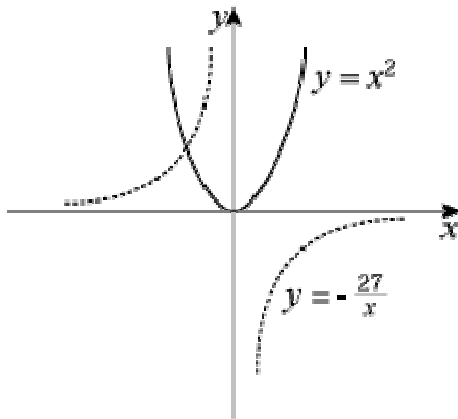
#### Question:

- (a) On the same axes sketch the curves given by  $y = x^2$  and  $y = -\frac{27}{x}$ .
- (b) Find the coordinates of the point of intersection.

#### Solution:

- (a)  $y = -\frac{27}{x}$  is like  $y = -\frac{1}{x}$  and in 2nd and 4th quadrants.

$y = x^2$  is standard



- (b)  $-\frac{27}{x} = x^2$   
 $\Rightarrow -27 = x^3$   
 $\Rightarrow x = -3$

Substitute in  $y = -\frac{27}{x} \Rightarrow y = 9$

So intersection at  $(-3, 9)$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 6

#### Question:

- (a) On the same axes sketch the curves given by  $y = x^2 - 2x$  and  $y = x(x - 2)(x - 3)$ .
- (b) Find the coordinates of the point of intersection.

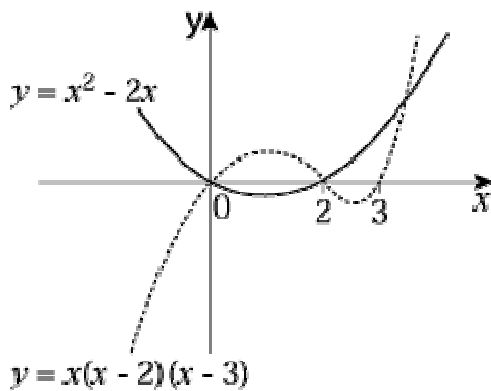
#### Solution:

$$(a) y = x(x - 2)(x - 3)$$

$$y = 0 \Rightarrow x = 0, 2, 3$$

$$y = x^2 - 2x = x(x - 2) \text{ is } \cup \text{ shaped}$$

$$y = 0 \Rightarrow x = 0, 2$$



$$(b) x(x - 2) = x(x - 2)(x - 3)$$

$$\Rightarrow 0 = x(x - 2)(x - 3 - 1)$$

$$\Rightarrow 0 = x(x - 2)(x - 4)$$

$$\Rightarrow x = 0, 2, 4$$

$$\text{Substitute in } y = x(x - 2) \Rightarrow y = 0, 0, 8$$

So intersections at  $(0, 0)$ ;  $(2, 0)$ ;  $(4, 8)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

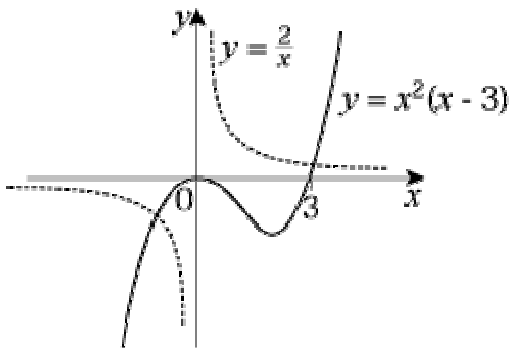
#### Exercise D, Question 7

#### Question:

- (a) On the same axes sketch the curves given by  $y = x^2(x - 3)$  and  $y = \frac{2}{x}$ .
- (b) Explain how your sketch shows that there are only two solutions to the equation  $x^3(x - 3) = 2$ .

#### Solution:

- (a)  $y = x^2(x - 3)$   
 $y = 0 \Rightarrow x = 0$  (twice), 3  
 Turning point at  $(0, 0)$   
 $y = \frac{2}{x}$  is like  $y = \frac{1}{x}$



- (b) Curves only cross at two points. So two solutions to  
 $\frac{2}{x} = x^2(x - 3)$   
 $2 = x^3(x - 3)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

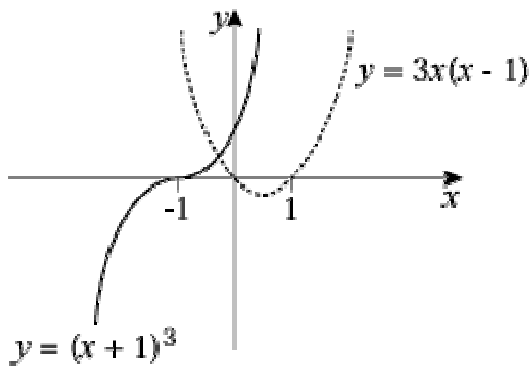
#### Exercise D, Question 8

#### Question:

- (a) On the same axes sketch the curves given by  $y = (x + 1)^3$  and  $y = 3x(x - 1)$ .
- (b) Explain how your sketch shows that there is only one solution to the equation  $x^3 + 6x + 1 = 0$ .

#### Solution:

- (a)  $y = (x + 1)^3$  is like  $y = x^3$  and crosses at  $(-1, 0)$  and  $(0, 1)$ .  
 $y = 3x(x - 1)$  is  $\cup$  shaped  
 $y = 0 \Rightarrow x = 0, 1$



- (b) Curves only cross once. So only one solution to  
 $(x + 1)^3 = 3x(x - 1)$   
 $x^3 + \cancel{3x^2} + 3x + 1 = \cancel{3x^2} - 3x$   
 $x^3 + 6x + 1 = 0$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

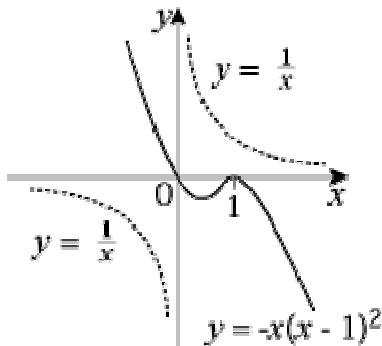
#### Exercise D, Question 9

#### Question:

- (a) On the same axes sketch the curves given by  $y = \frac{1}{x}$  and  $y = -x(x-1)^2$ .
- (b) Explain how your sketch shows that there are no solutions to the equation  $1 + x^2(x-1)^2 = 0$ .

#### Solution:

- (a)  $y = -x(x-1)^2$   
 $y = 0 \Rightarrow x = 0, 1$  (twice)  
 Turning point at  $(1, 0)$   
 $x \rightarrow \infty, y \rightarrow -\infty$   
 $x \rightarrow -\infty, y \rightarrow \infty$



- (b) Curves do not cross, so no solutions to

$$\frac{1}{x} = -x(x-1)^2$$

$$1 = -x^2(x-1)^2$$

$$1 + x^2(x-1)^2 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 10

#### Question:

- (a) On the same axes sketch the curves given by  $y = 1 - 4x^2$  and  $y = x(x - 2)^2$ .
- (b) State, with a reason, the number of solutions to the equation  $x^3 + 4x - 1 = 0$ .

#### Solution:

(a)  $y = x(x - 2)^2$

$y = 0 \Rightarrow x = 0, 2$  (twice)

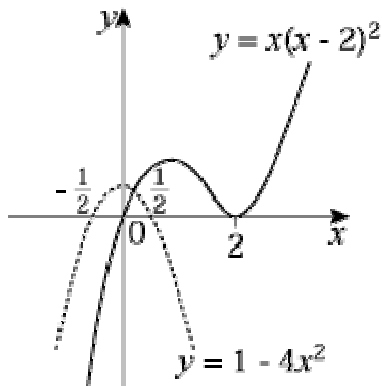
Turning point at  $(2, 0)$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$

$y = 1 - 4x^2 = (1 - 2x)(1 + 2x)$  is  $\cap$  shaped

$y = 0 \Rightarrow x = \frac{1}{2}, -\frac{1}{2}$



- (b) Curves cross once. So one solution to

$$1 - 4x^2 = x(x - 2)^2$$

$$1 - 4x^2 = x(x^2 - 4x + 4)$$

$$1 - 4x^2 = x^3 - 4x^2 + 4x$$

$$0 = x^3 + 4x - 1$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 11

#### Question:

- (a) On the same axes sketch the curve  $y = x^3 - 3x^2 - 4x$  and the line  $y = 6x$ .
- (b) Find the coordinates of the points of intersection.

#### Solution:

$$(a) y = x^3 - 3x^2 - 4x = x(x^2 - 3x - 4)$$

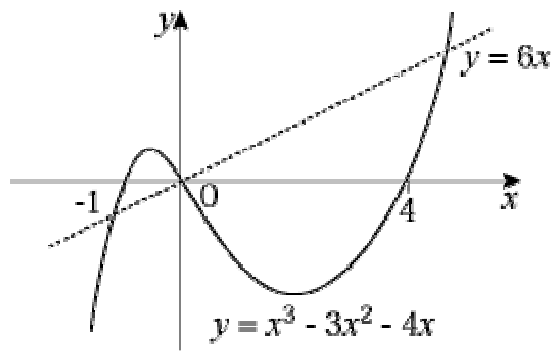
$$\text{So } y = x(x - 4)(x + 1)$$

$$y = 0 \Rightarrow x = 0, -1, 4$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$$y = 6x \text{ is a straight line through } (0, 0)$$



$$(b) x^3 - 3x^2 - 4x = 6x$$

$$\Rightarrow x^3 - 3x^2 - 10x = 0$$

$$\Rightarrow x(x^2 - 3x - 10) = 0$$

$$\Rightarrow x(x - 5)(x + 2) = 0$$

$$\Rightarrow x = 0, 5, -2$$

So (using  $y = 6x$ ) the points of intersection are:  $(0, 0)$ ;  $(5, 30)$ ;  $(-2, -12)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 12

#### Question:

- (a) On the same axes sketch the curve  $y = (x^2 - 1)(x - 2)$  and the line  $y = 14x + 2$ .
- (b) Find the coordinates of the points of intersection.

#### Solution:

$$(a) y = (x^2 - 1)(x - 2) = (x - 1)(x + 1)(x - 2)$$

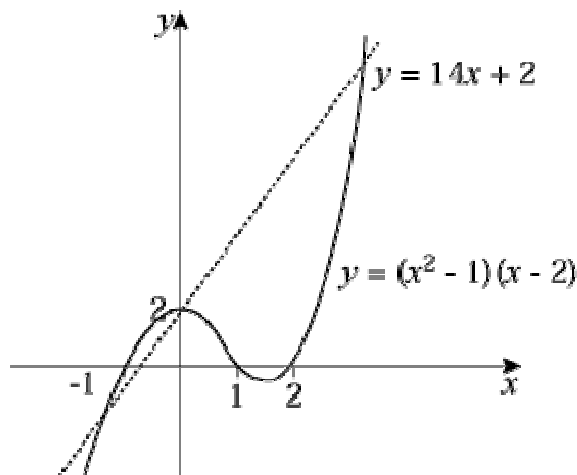
$$y = 0 \Rightarrow x = 1, -1, 2$$

$$x = 0 \Rightarrow y = 2$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$y = 14x + 2$  is a straight line passing through  $(0, 2)$  and  $(-\frac{1}{7}, 0)$ .



$$(b) \text{ Intersection when } 14x + 2 = (x^2 - 1)(x - 2)$$

$$\Rightarrow 14x + 2 = x^3 - 2x^2 - x + 2$$

$$\Rightarrow 0 = x^3 - 2x^2 - 15x$$

$$\Rightarrow 0 = x(x^2 - 2x - 15)$$

$$\Rightarrow 0 = x(x - 5)(x + 3)$$

$$\Rightarrow x = 0, 5, -3$$

So (using  $y = 14x + 2$ ) the points of intersection are:  $(0, 2)$ ;  $(5, 72)$ ;  $(-3, -40)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise D, Question 13

#### Question:

- (a) On the same axes sketch the curves with equations  $y = (x - 2)(x + 2)^2$  and  $y = -x^2 - 8$ .
- (b) Find the coordinates of the points of intersection.

#### Solution:

(a)  $y = (x - 2)(x + 2)^2$

$$y = 0 \Rightarrow x = -2 \text{ (twice), } 2$$

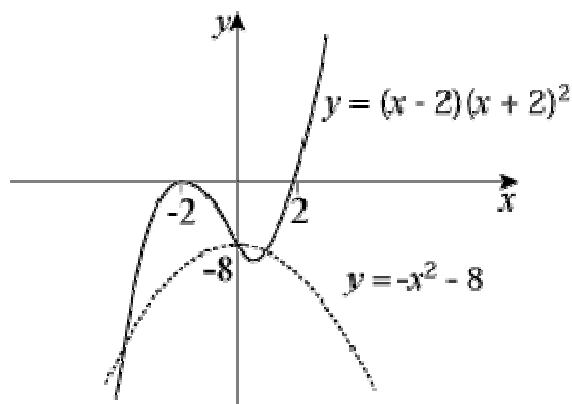
$$x = 0 \Rightarrow y = -8$$

Turning point at  $(-2, 0)$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$y = -x^2 - 8$  is  $\cap$  shaped with a maximum at  $(0, -8)$



- (b) Intersections when  $-x^2 - 8 = (x + 2)^2(x - 2)$
- $$\Rightarrow -x^2 - 8 = (x^2 + 4x + 4)(x - 2)$$
- $$\Rightarrow -x^2 - 8 = x^3 + 4x^2 + 4x - 2x^2 - 8x - 8$$
- $$\Rightarrow 0 = x^3 + 3x^2 - 4x$$
- $$\Rightarrow 0 = x(x^2 + 3x - 4)$$
- $$\Rightarrow 0 = x(x + 4)(x - 1)$$
- $$\Rightarrow x = 0, 1, -4$$

So (using  $y = -x^2 - 8$ ) points of intersection are:  $(0, -8)$ ;  $(1, -9)$ ;  $(-4, -24)$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise E, Question 1

#### Question:

Apply the following transformations to the curves with equations  $y = f(x)$  where:

(i)  $f(x) = x^2$

(ii)  $f(x) = x^3$

(iii)  $f(x) = \frac{1}{x}$

In each case state the coordinates of points where the curves cross the axes and in (iii) state the equations of any asymptotes.

(a)  $f(x + 2)$

(b)  $f(x) + 2$

(c)  $f(x - 1)$

(d)  $f(x) - 1$

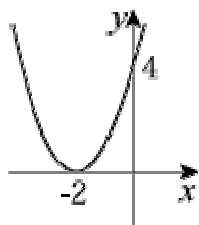
(e)  $f(x) - 3$

(f)  $f(x - 3)$

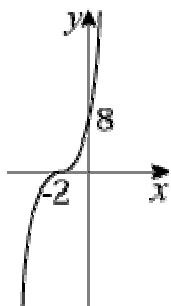
#### Solution:

(a)  $f(x + 2)$  is a horizontal translation of  $-2$ .

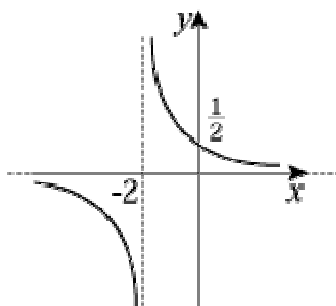
(i)  $y = x^2 \rightarrow y = (x + 2)^2$



(ii)  $y = x^3 \rightarrow y = (x + 2)^3$



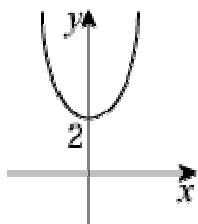
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x+2}$



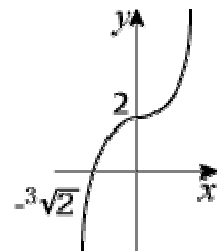
Asymptotes:  $x = -2$  and  $y = 0$

(b)  $f(x) + 2$  is a vertical translation of  $f(x)$ .

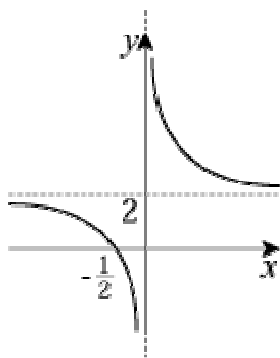
(i)  $y = x^2 \rightarrow y = x^2 + 2$



(ii)  $y = x^3 \rightarrow y = x^3 + 2$



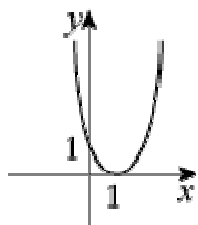
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x} + 2$



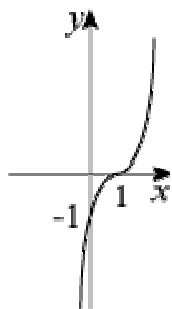
Asymptotes:  $x = 0$  and  $y = 2$

(c)  $f(x - 1)$  is a horizontal translation of  $+ 1$ .

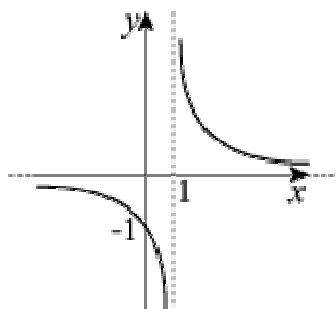
(i)  $y = x^2 \rightarrow y = (x - 1)^2$



(ii)  $y = x^3 \rightarrow y = (x - 1)^3$



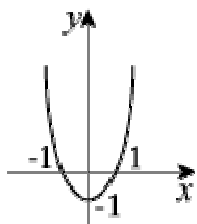
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x - 1}$



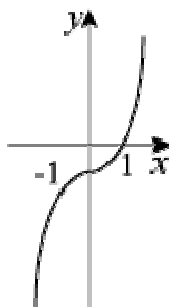
Asymptotes:  $x = 1, y = 0$

(d)  $f(x) - 1$  is a vertical translation of  $- 1$ .

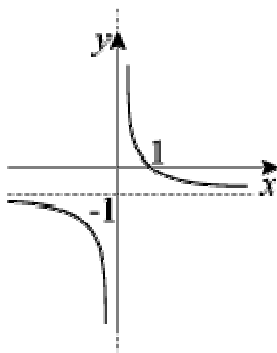
(i)  $y = x^2 \rightarrow y = x^2 - 1$



(ii)  $y = x^3 \rightarrow y = x^3 - 1$



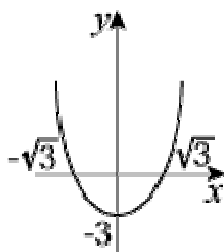
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x} - 1$



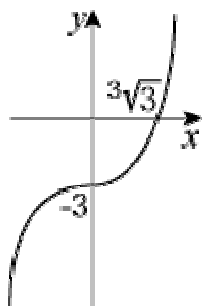
Asymptotes:  $x = 0, y = -1$

(e)  $f(x) - 3$  is a vertical translation of  $-3$ .

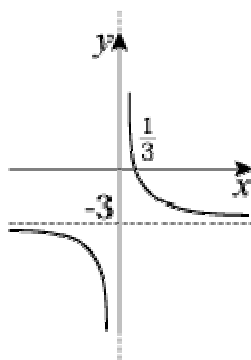
(i)  $y = x^2 \rightarrow y = x^2 - 3$



(ii)  $y = x^3 \rightarrow y = x^3 - 3$



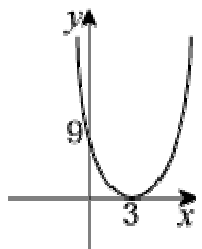
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x} - 3$



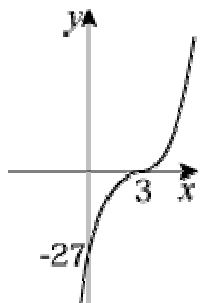
Asymptotes:  $x = 0, y = -3$

(f)  $f(x - 3)$  is a horizontal translation of  $f(x)$ .

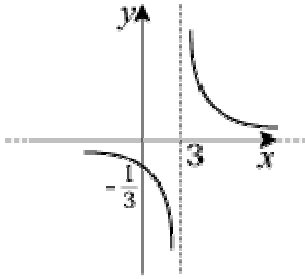
(i)  $y = x^2 \rightarrow y = (x - 3)^2$



(ii)  $y = x^3 \rightarrow y = (x - 3)^3$



(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{x - 3}$



Asymptotes:  $x = 3$ ,  $y = 0$

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# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

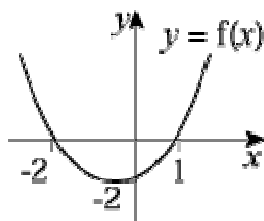
#### Exercise E, Question 2

#### Question:

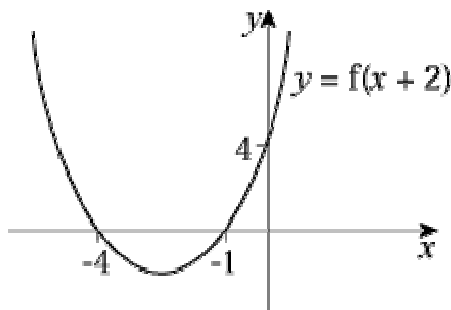
- (a) Sketch the curve  $y = f(x)$  where  $f(x) = (x - 1)(x + 2)$ .
- (b) On separate diagrams sketch the graphs of (i)  $y = f(x + 2)$  (ii)  $y = f(x) + 2$ .
- (c) Find the equations of the curves  $y = f(x + 2)$  and  $y = f(x) + 2$ , in terms of  $x$ , and use these equations to find the coordinates of the points where your graphs in part (b) cross the  $y$ -axis.

#### Solution:

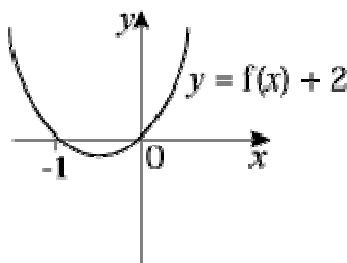
(a)  $f(x) = 0 \Rightarrow x = 1, -2$



- (b)(i)  $f(x + 2)$  is a horizontal translation of  $-2$ .



- (ii)  $f(x) + 2$  is a vertical translation of  $+2$



Since axis of symmetry of  $f(x)$  is at  $x = -\frac{1}{2}$ , the same axis of symmetry applies to  $f(x) + 2$ . Since one root is at  $x = 0$ , the other must be symmetric at  $x = -1$ .

(c)  $y = f(x + 2)$  is  $y = (x + 1)(x + 4)$ . So  $x = 0 \Rightarrow y = 4$

$y = f(x) + 2$  is  $y = x^2 + x = x(x + 1)$ . So  $x = 0 \Rightarrow y = 0$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

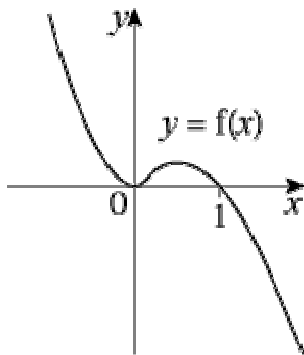
#### Exercise E, Question 3

#### Question:

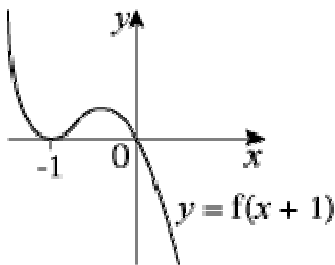
- (a) Sketch the graph of  $y = f(x)$  where  $f(x) = x^2(1 - x)$ .
- (b) Sketch the curve with equation  $y = f(x + 1)$ .
- (c) By finding the equation  $f(x + 1)$  in terms of  $x$ , find the coordinates of the point in part (b) where the curve crosses the  $y$ -axis.

#### Solution:

- (a)  $y = x^2(1 - x)$   
 $y = 0 \Rightarrow x = 0$  (twice),  $1$   
 Turning point at  $(0, 0)$   
 $x \rightarrow \infty, y \rightarrow -\infty$   
 $x \rightarrow -\infty, y \rightarrow \infty$



- (b)  $f(x + 1)$  is a horizontal translation of  $-1$ .



- (c)  $f(x + 1) = (x + 1)^2 [ 1 - (x + 1) ] = -(x + 1)^2 x$   
 So  $y = 0 \Rightarrow x = 0$ , i.e. curve passes through  $(0, 0)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

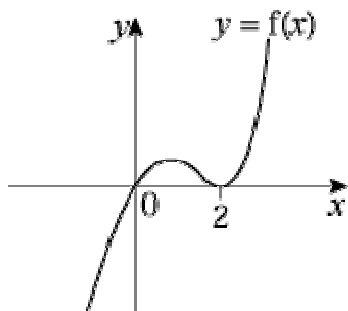
#### Exercise E, Question 4

#### Question:

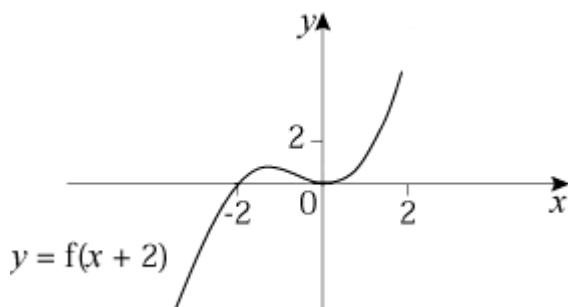
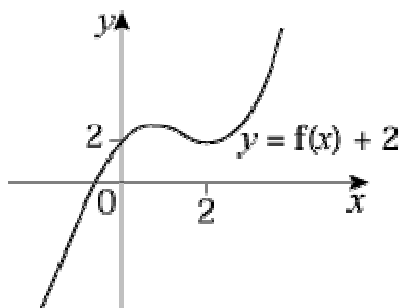
- (a) Sketch the graph of  $y = f(x)$  where  $f(x) = x(x - 2)^2$ .
- (b) Sketch the curves with equations  $y = f(x) + 2$  and  $y = f(x + 2)$ .
- (c) Find the coordinates of the points where the graph of  $y = f(x + 2)$  crosses the axes.

#### Solution:

- (a)  $y = x(x - 2)^2$   
 $y = 0 \Rightarrow x = 0, 2$  (twice)  
 Turning point at  $(2, 0)$   
 $x \rightarrow \infty, y \rightarrow \infty$   
 $x \rightarrow -\infty, y \rightarrow -\infty$



(b)



- (c)  $f(x + 2) = 0$  at points where  $(x + 2) [(x + 2) - 2]^2 = 0$

$$\Rightarrow (x + 2)(x)^2 = 0$$

$$\Rightarrow x = 0 \text{ and } x = -2$$

So graph crosses axes at  $(0, 0)$  ;  $(-2, 0)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

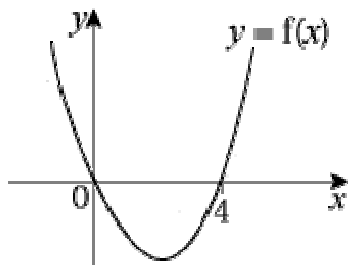
#### Exercise E, Question 5

#### Question:

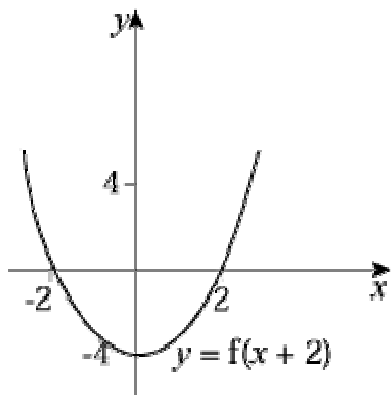
- (a) Sketch the graph of  $y = f(x)$  where  $f(x) = x(x - 4)$ .
- (b) Sketch the curves with equations  $y = f(x + 2)$  and  $y = f(x) + 4$ .
- (c) Find the equations of the curves in part (b) in terms of  $x$  and hence find the coordinates of the points where the curves cross the axes.

#### Solution:

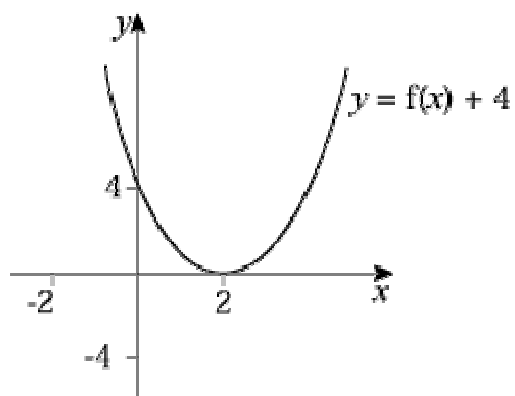
- (a)  $y = x(x - 4)$  is  $\cup$  shaped and passes through  $(0, 0)$  and  $(4, 0)$ .



- (b)  $f(x + 2)$  is a horizontal translation of  $-2$ .



- $f(x) + 4$  is a vertical translation of  $+4$ .



$$(c) f(x + 2) = (x + 2) [(x + 2) - 4] = (x + 2)(x - 2)$$

$$y = 0 \Rightarrow x = -2, 2$$

$$f(x) + 4 = x(x - 4) + 4 = x^2 - 4x + 4 = (x - 2)^2$$

$$y = 0 \Rightarrow x = 2$$

The minimum point on  $y = f(x)$  is when  $x = 2$  (by symmetry) and then  $f(2) = -4$ .

So  $y = f(x + 2)$  crosses  $y$ -axis at  $(0, -4)$

and  $y = f(x) + 4$  touches  $x$ -axis at  $(2, 0)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise F, Question 1

#### Question:

Apply the following transformations to the curves with equations  $y = f(x)$  where:

(i)  $f(x) = x^2$

(ii)  $f(x) = x^3$

(iii)  $f(x) = \frac{1}{x}$

In each case show both  $f(x)$  and the transformation on the same diagram.

(a)  $f(2x)$

(b)  $f(-x)$

(c)  $f\left(\frac{1}{2}x\right)$

(d)  $f(4x)$

(e)  $f\left(\frac{1}{4}x\right)$

(f)  $2f(x)$

(g)  $-f(x)$

(h)  $4f(x)$

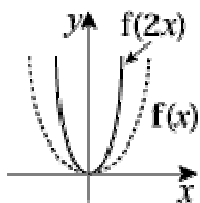
(i)  $\frac{1}{2}f(x)$

(j)  $\frac{1}{4}f(x)$

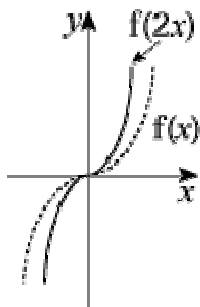
#### Solution:

(a)  $f(2x)$  means multiply  $x$ -coordinates by  $\frac{1}{2}$ .

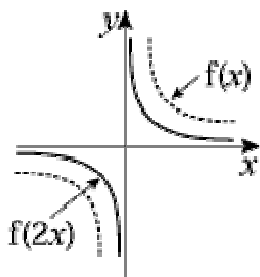
(i)  $y = x^2 \rightarrow y = (2x)^2 = 4x^2$



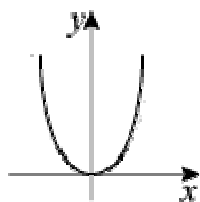
(ii)  $y = x^3 \rightarrow y = (2x)^3 = 8x^3$



(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{2x} = \frac{1}{2} \times \frac{1}{x}$

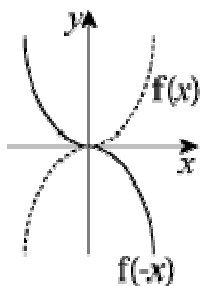


(b) (i)  $y = x^2 \rightarrow y = (-x)^2 = x^2$

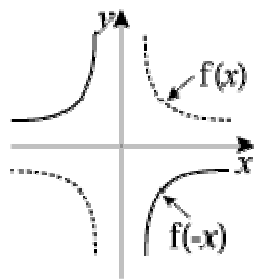


$f(x) = f(-x)$

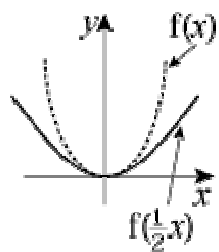
(ii)  $y = x^3 \rightarrow y = (-x)^3 = -x^3$



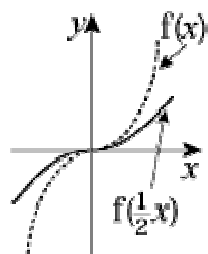
$$(iii) y = \frac{1}{x} \rightarrow y = \frac{1}{-x} = -\frac{1}{x}$$



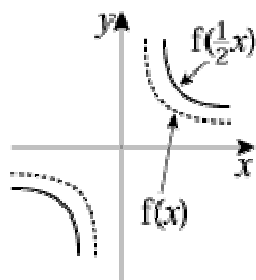
$$(c) (i) y = x^3 \rightarrow y = \left(\frac{1}{2}x\right)^2 = \frac{x^2}{4}$$



$$(ii) y = x^3 \rightarrow y = \left(\frac{1}{2}x\right)^3 = \frac{x^3}{8}$$

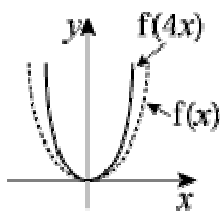


$$(iii) y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{2}x} = \frac{2}{x}$$

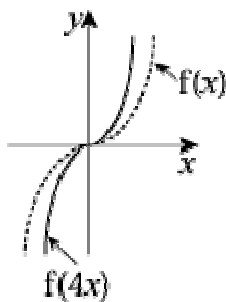


$$(d) (i) y = x^2 \rightarrow y = (4x)^2 = 16x^2$$

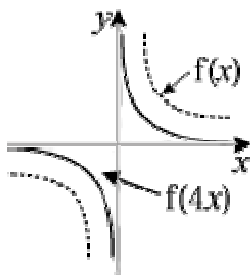




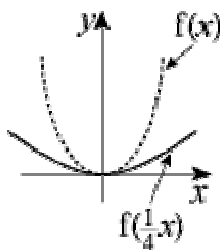
$$(ii) y = x^3 \rightarrow y = (4x)^3 = 64x^3$$



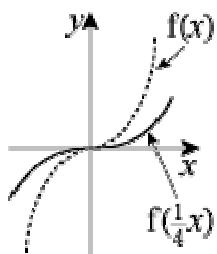
$$(iii) y = \frac{1}{x} \rightarrow y = \frac{1}{4x} = \frac{1}{4} \times \frac{1}{x}$$



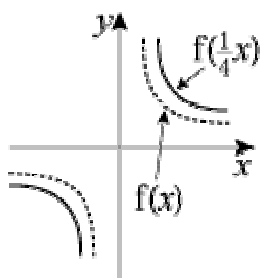
$$(e) (i) y = x^2 \rightarrow y = \left(\frac{1}{4}x\right)^2 = \frac{x^2}{16}$$



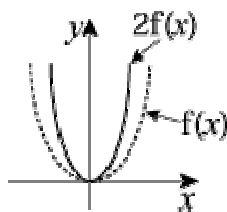
$$(ii) y = x^3 \rightarrow y = \left(\frac{1}{4}x\right)^3 = \frac{x^3}{64}$$



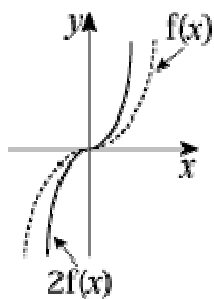
$$(iii) y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{4x}} = \frac{4}{x}$$



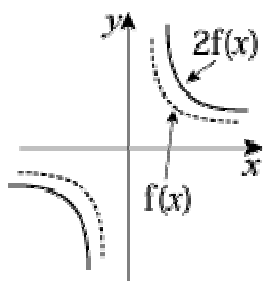
$$(f) (i) y = x^2 \rightarrow y = 2x^2$$



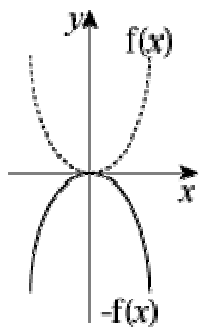
$$(ii) y = x^3 \rightarrow y = 2x^3$$



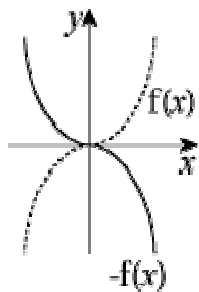
$$(iii) y = \frac{1}{x} \rightarrow y = 2 \times \frac{1}{x} = \frac{2}{x}$$



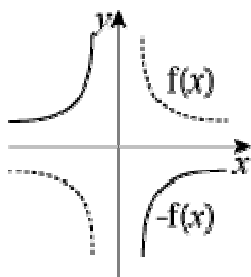
(g) (i)  $y = x^2 \rightarrow y = -x^2$



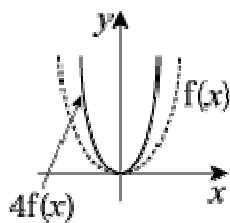
(ii)  $y = x^3 \rightarrow y = -x^3$



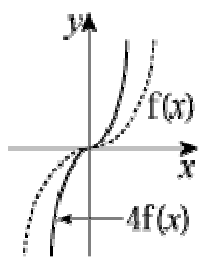
(iii)  $y = \frac{1}{x} \rightarrow y = -\frac{1}{x}$



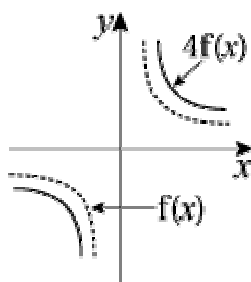
(h) (i)  $y = x^2 \rightarrow y = 4x^2$



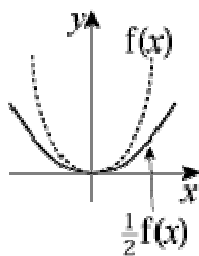
(ii)  $y = x^3 \rightarrow y = 4x^3$



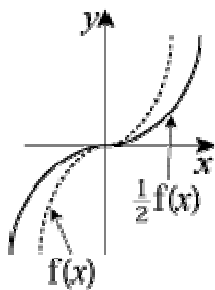
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{4}{x}$



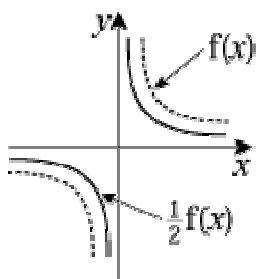
(i)  $y = x^2 \rightarrow y = \frac{1}{2}x^2$



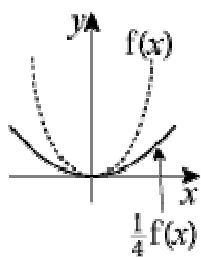
(ii)  $y = x^3 \rightarrow y = \frac{1}{2}x^3$



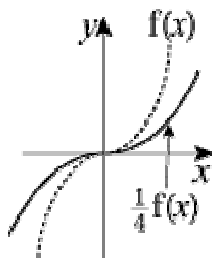
(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{2} \times \frac{1}{x}$



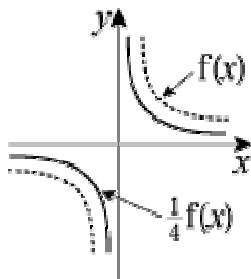
(j) (i)  $y = x^2 \rightarrow y = \frac{1}{4}x^2$



(ii)  $y = x^3 \rightarrow y = \frac{1}{4}x^3$



(iii)  $y = \frac{1}{x} \rightarrow y = \frac{1}{4} \times \frac{1}{x}$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

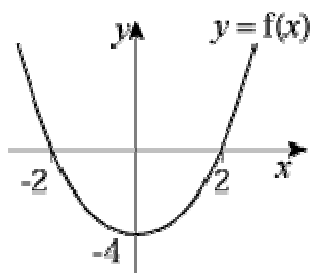
#### Exercise F, Question 2

#### Question:

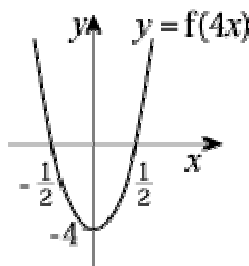
- (a) Sketch the curve with equation  $y = f(x)$  where  $f(x) = x^2 - 4$ .
- (b) Sketch the graphs of  $y = f(4x)$ ,  $y = 3f(x)$ ,  $y = f(-x)$  and  $y = -f(x)$ .

#### Solution:

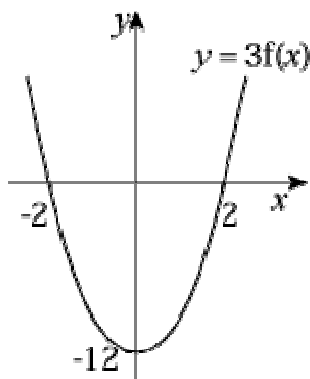
- (a)  $y = x^2 - 4 = (x - 2)(x + 2)$  and is  $\cup$  shaped  
 $y = 0 \Rightarrow x = 2, -2$



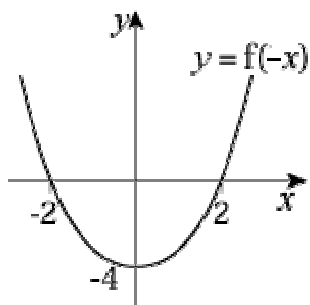
- (b)  $f(4x)$  is a stretch  $\times \frac{1}{4}$  horizontally



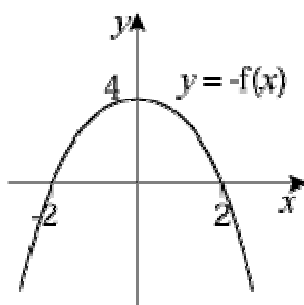
- $3f(x)$  is a stretch  $\times 3$  vertically



- $f(-x)$  is a reflection in y-axis



–  $f(x)$  is a reflection in  $x$ -axis



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise F, Question 3

#### Question:

(a) Sketch the curve with equation  $y = f(x)$  where  $f(x) = (x - 2)(x + 2)x$ .

(b) Sketch the graphs of  $y = f\left(\frac{1}{2}x\right)$ ,  $y = f(2x)$  and  $y = -f(x)$ .

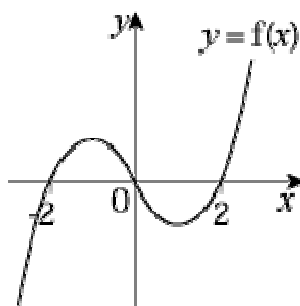
#### Solution:

(a)  $y = (x - 2)(x + 2)x$

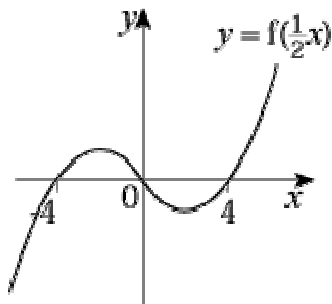
$y = 0 \Rightarrow x = 2, -2, 0$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$

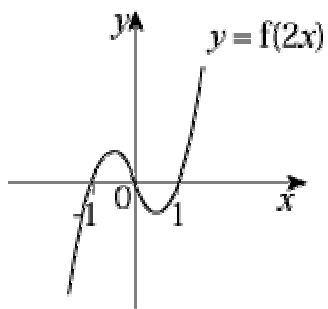


(b)  $f\left(\frac{1}{2}x\right)$  is a stretch  $\times 2$  horizontally

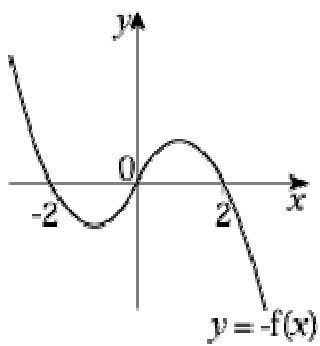


$f(2x)$  is a stretch  $\times \frac{1}{2}$  horizontally





–  $f(x)$  is a reflection in  $x$ -axis



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

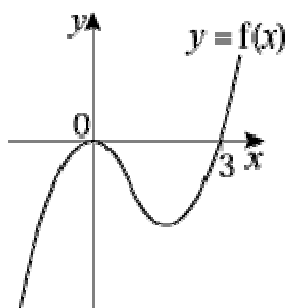
#### Exercise F, Question 4

#### Question:

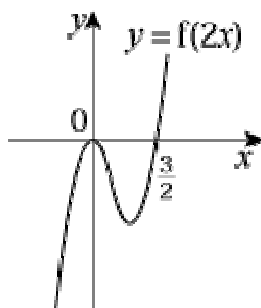
- (a) Sketch the curve with equation  $y = f(x)$  where  $f(x) = x^2(x - 3)$ .
- (b) Sketch the curves with equations  $y = f(2x)$ ,  $y = -f(x)$  and  $y = f(-x)$ .

#### Solution:

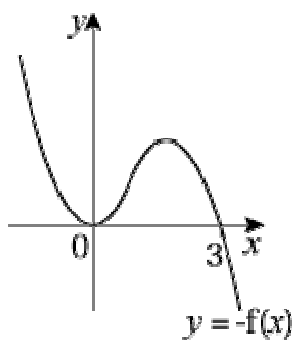
- (a)  $y = x^2(x - 3)$   
 $y = 0 \Rightarrow x = 0$  (twice),  $3$   
 Turning point at  $(0, 0)$   
 $x \rightarrow \infty, y \rightarrow \infty$   
 $x \rightarrow -\infty, y \rightarrow -\infty$



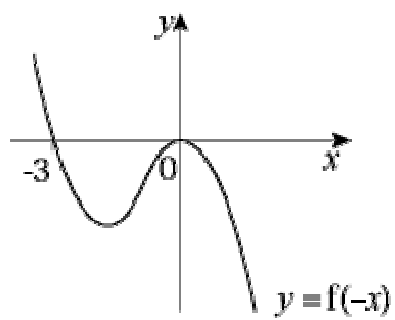
- (b)  $f(2x)$  is a stretch  $\times \frac{1}{2}$  horizontally



- $-f(x)$  is a reflection in  $x$ -axis



$f(-x)$  is a reflection in  $y$ -axis



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise F, Question 5

#### Question:

(a) Sketch the curve with equation  $y = f(x)$  where  $f(x) = (x - 2)(x - 1)(x + 2)$ .

(b) Sketch the curves with equations  $y = f(2x)$  and  $f(\frac{1}{2}x)$ .

#### Solution:

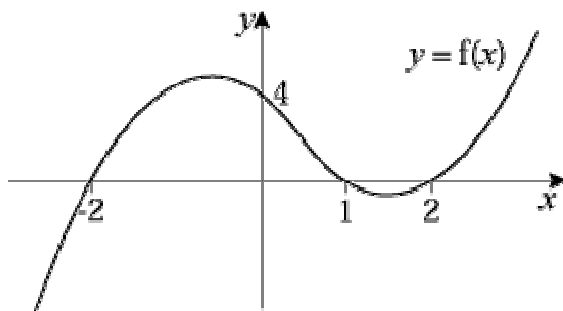
(a)  $y = (x - 2)(x - 1)(x + 2)$

$y = 0 \Rightarrow x = 2, 1, -2$

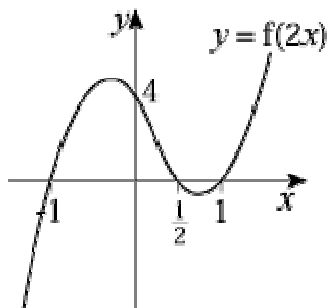
$x = 0 \Rightarrow y = 4$

$x \rightarrow \infty, y \rightarrow \infty$

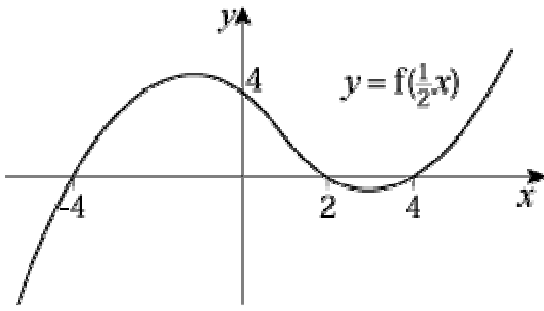
$x \rightarrow -\infty, y \rightarrow -\infty$



(b)  $f(2x)$  is a stretch  $\times \frac{1}{2}$  horizontally



$f(\frac{1}{2}x)$  is a stretch  $\times 2$  horizontally



# Solutionbank C1

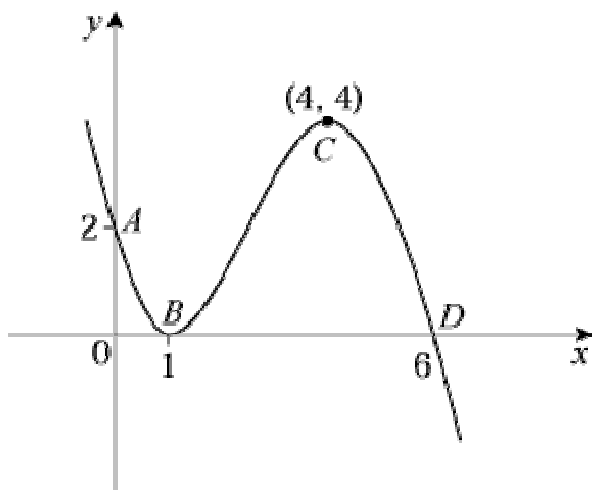
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise G, Question 1

#### Question:

The following diagram shows a sketch of the curve with equation  $y = f(x)$ . The points  $A(0, 2)$ ,  $B(1, 0)$ ,  $C(4, 4)$  and  $D(6, 0)$  lie on the curve.



Sketch the following graphs and give the coordinates of the points  $A$ ,  $B$ ,  $C$  and  $D$  after each transformation:

(a)  $f(x + 1)$

(b)  $f(x) - 4$

(c)  $f(x + 4)$

(d)  $f(2x)$

(e)  $3f(x)$

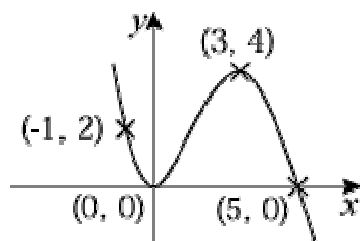
(f)  $f\left(\frac{1}{2}x\right)$

(g)  $\frac{1}{2}f(x)$

(h)  $f(-x)$

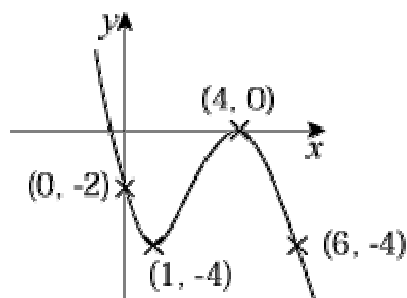
#### Solution:

(a)  $f(x + 1)$  is a translation of  $-1$  horizontally.



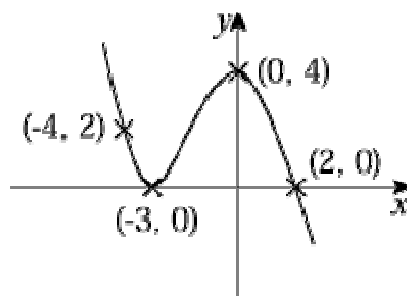
$A(-1, 2)$ ;  $B(0, 0)$ ;  $C(3, 4)$ ;  $D(5, 0)$

(b)  $f(x) - 4$  is a vertical translation of  $-4$ .



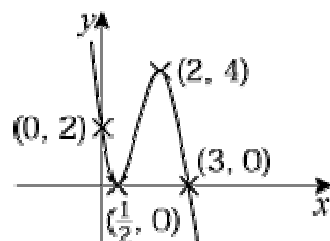
$A(0, -2)$ ;  $B(1, -4)$ ;  $C(4, 0)$ ;  $D(6, -4)$

(c)  $f(x + 4)$  is a translation of  $-4$  horizontally.



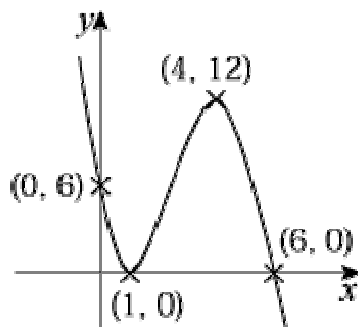
$A(-4, 2)$ ;  $B(-3, 0)$ ;  $C(0, 4)$ ;  $D(2, 0)$

(d)  $f(2x)$  is a stretch of  $\frac{1}{2}$  horizontally.



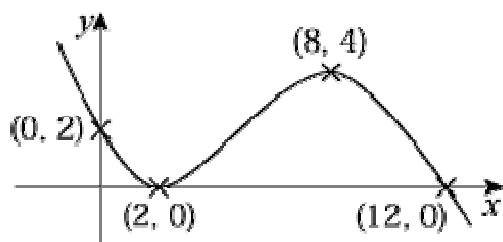
$A(0, 2)$ ;  $B(\frac{1}{2}, 0)$ ;  $C(2, 4)$ ;  $D(3, 0)$

(e)  $3f(x)$  is a stretch of 3 vertically.



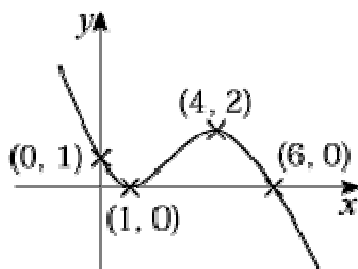
$A(0, 6)$ ;  $B(1, 0)$ ;  $C(4, 12)$ ;  $D(6, 0)$

(f)  $f(\frac{1}{2}x)$  is a stretch of 2 horizontally.



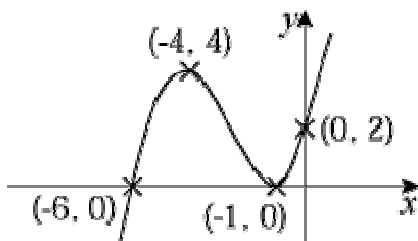
$A(0, 2)$ ;  $B(2, 0)$ ;  $C(8, 4)$ ;  $D(12, 0)$

(g)  $\frac{1}{2}f(x)$  is a stretch of  $\frac{1}{2}$  vertically.



$A(0, 1)$ ;  $B(1, 0)$ ;  $C(4, 2)$ ;  $D(6, 0)$

(h)  $f(-x)$  is a reflection in the y-axis.



$A(0, 2)$ ;  $B(-1, 0)$ ;  $C(-4, 4)$ ;  $D(-6, 0)$





# Solutionbank C1

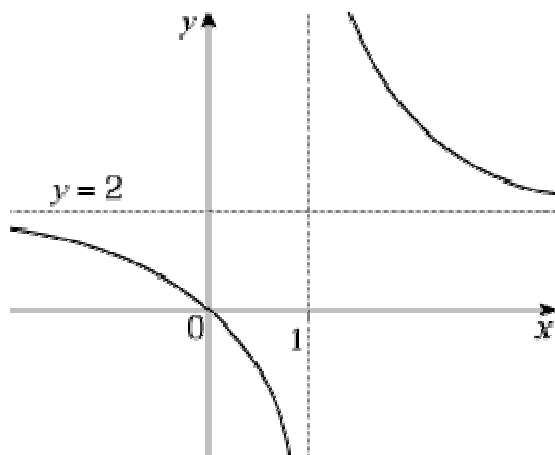
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise G, Question 2

#### Question:

The curve  $y = f(x)$  passes through the origin and has horizontal asymptote  $y = 2$  and vertical asymptote  $x = 1$ , as shown in the diagram.



Sketch the following graphs and give the equations of any asymptotes and, for all graphs except (a), give coordinates of intersections with the axes after each transformation.

(a)  $f(x) + 2$

(b)  $f(x + 1)$

(c)  $2f(x)$

(d)  $f(x) - 2$

(e)  $f(2x)$

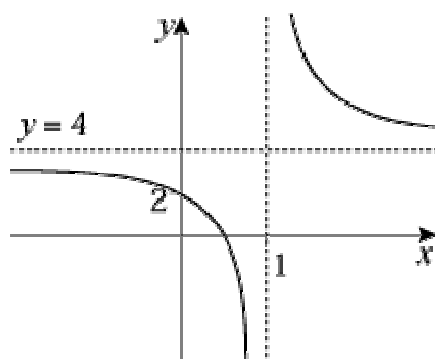
(f)  $f\left(\frac{1}{2}x\right)$

(g)  $\frac{1}{2}f(x)$

(h)  $-f(x)$

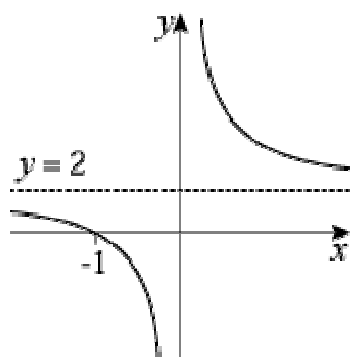
#### Solution:

(a)  $f(x) + 2$  is a translation of  $f(x)$  + 2 in a vertical direction.



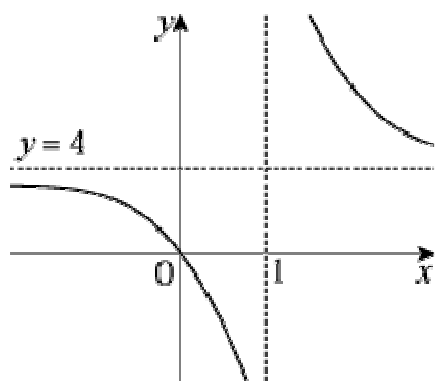
Asymptotes:  $x = 1, y = 4$ . Intersections:  $(0, 2)$  and  $(a, 0)$ , where  $0 < a < 1$ .

(b)  $f(x + 1)$  is a horizontal translation of  $-1$ .



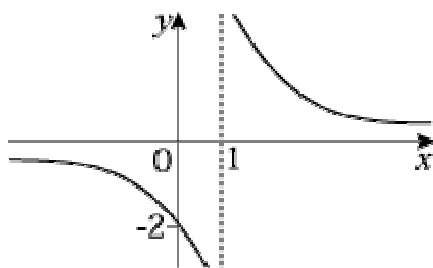
Asymptotes:  $x = 0, y = 2$ . Intersections:  $(-1, 0)$

(c)  $2f(x)$  is a stretch of 2 in a vertical direction.



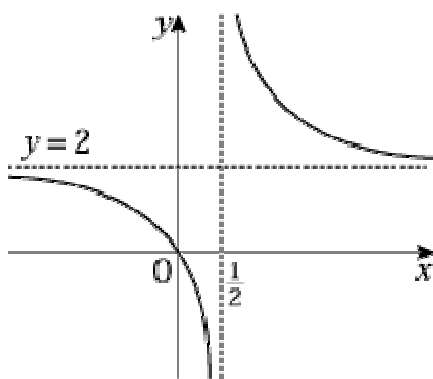
Asymptotes:  $x = 1, y = 4$ . Intersections:  $(0, 0)$

(d)  $f(x) - 2$  is a vertical translation of  $-2$ .



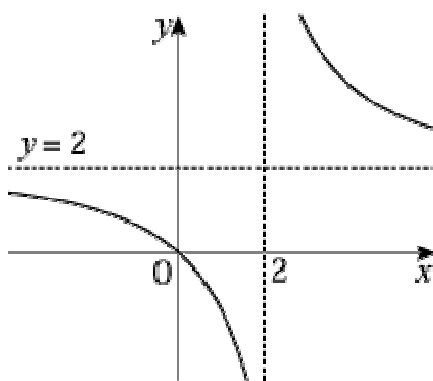
Asymptotes:  $x = 1, y = 0$ . Intersections:  $(0, -2)$

(e)  $f(2x)$  is a stretch of  $\frac{1}{2}$  in a horizontal direction.



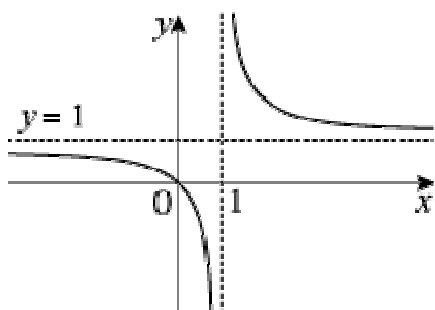
Asymptotes:  $x = \frac{1}{2}, y = 2$ . Intersections:  $(0, 0)$

(f)  $f(\frac{1}{2}x)$  is a stretch of 2 in a horizontal direction.



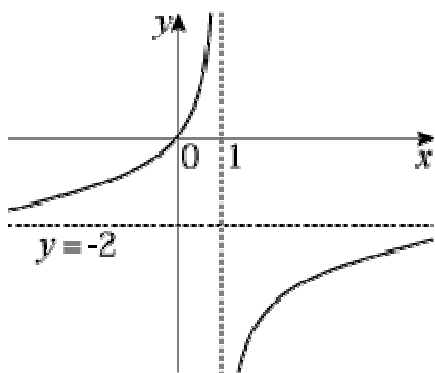
Asymptotes:  $x = 2, y = 2$ . Intersections:  $(0, 0)$

(g)  $\frac{1}{2}f(x)$  is a stretch of  $\frac{1}{2}$  in a vertical direction.



Asymptotes:  $x = 1, y = 1$ . Intesections:  $(0, 0)$

(h)  $-f(x)$  is a reflection in the  $x$ -axis.



Asymptotes:  $x = 1, y = -2$ . Intersections:  $(0, 0)$

# Solutionbank C1

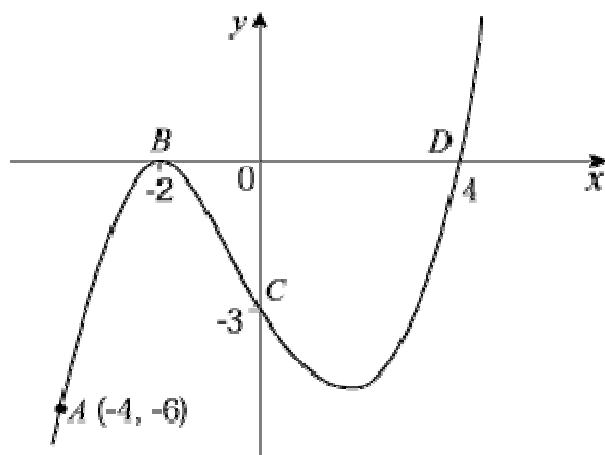
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise G, Question 3

#### Question:

The curve with equation  $y = f(x)$  passes through the points  $A(-4, -6)$ ,  $B(-2, 0)$ ,  $C(0, -3)$  and  $D(4, 0)$  as shown in the diagram.



Sketch the following and give the coordinates of the points  $A$ ,  $B$ ,  $C$  and  $D$  after each transformation.

(a)  $f(x - 2)$

(b)  $f(x) + 6$

(c)  $f(2x)$

(d)  $f(x + 4)$

(e)  $f(x) + 3$

(f)  $3f(x)$

(g)  $\frac{1}{3}f(x)$

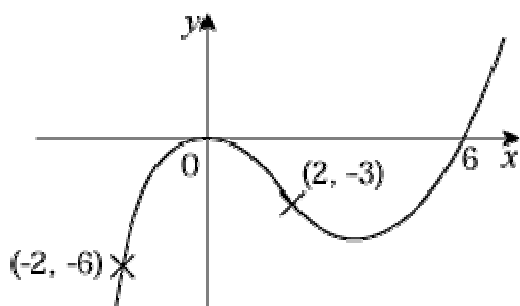
(h)  $f\left(\frac{1}{4}x\right)$

(i)  $-f(x)$

(j)  $f(-x)$

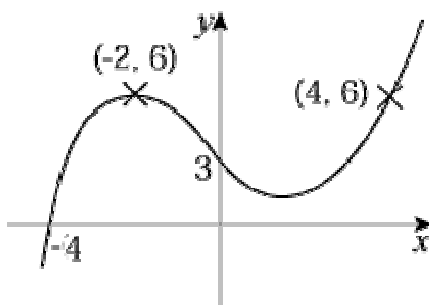
#### Solution:

(a)  $f(x - 2)$  is a horizontal translation of  $+ 2$ .



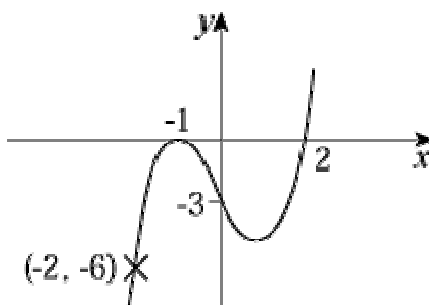
$A(-2, -6)$ ;  $B(0, 0)$ ;  $C(2, -3)$ ;  $D(6, 0)$

(b)  $f(x) + 6$  is a vertical translation of  $+6$ .



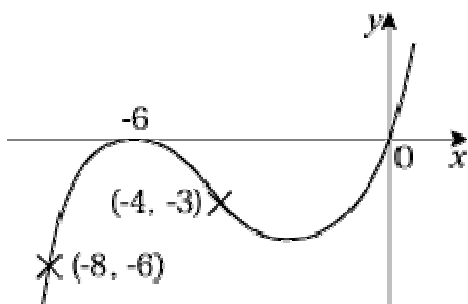
$A(-4, 0)$ ;  $B(-2, 6)$ ;  $C(0, 3)$ ;  $D(4, 6)$

(c)  $f(2x)$  is a horizontal stretch of  $\frac{1}{2}$ .



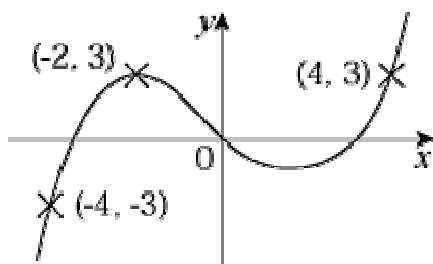
$A(-2, -6)$ ;  $B(-1, 0)$ ;  $C(0, -3)$ ;  $D(2, 0)$

(d)  $f(x + 4)$  is a horizontal translation of  $-4$ .



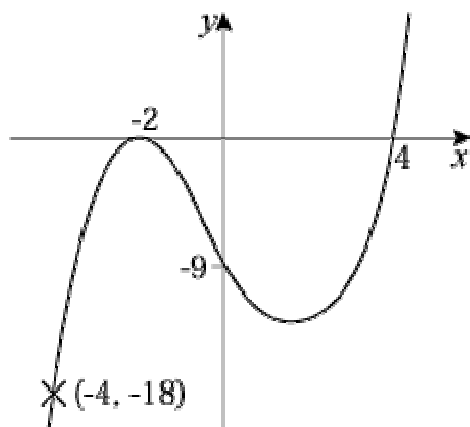
$A(-8, -6)$ ;  $B(-6, 0)$ ;  $C(-4, -3)$ ;  $D(0, 0)$

(e)  $f(x) + 3$  is a vertical translation of  $+3$ .



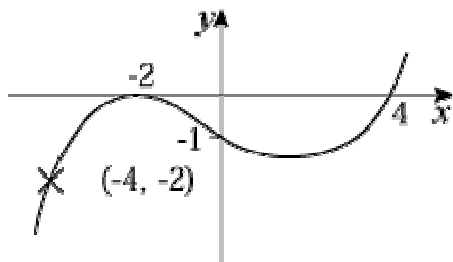
$A(-4, -3)$ ;  $B(-2, 3)$ ;  $C(0, 0)$ ;  $D(4, 3)$

(f)  $3f(x)$  is a vertical stretch of 3.



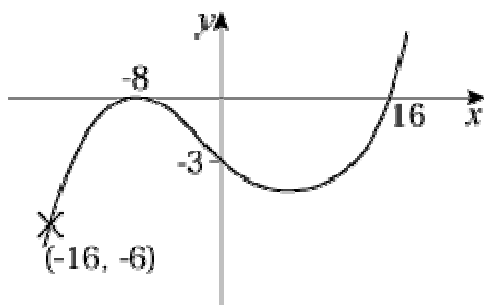
$A(-4, -18)$ ;  $B(-2, 0)$ ;  $C(0, -9)$ ;  $D(4, 0)$

(g)  $\frac{1}{3}f(x)$  is a vertical stretch of  $\frac{1}{3}$ .



$A(-4, -2)$ ;  $B(-2, 0)$ ;  $C(0, -1)$ ;  $D(4, 0)$

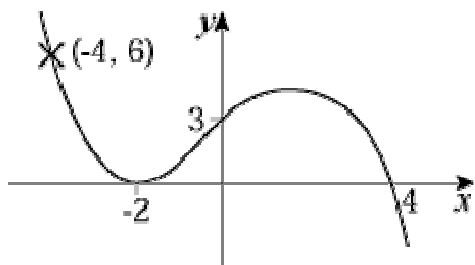
(h)  $f(\frac{1}{4}x)$  is a horizontal stretch of 4.





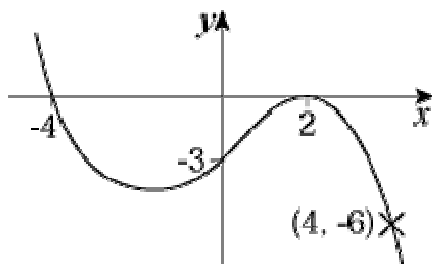
$A'(-16, -6)$ ;  $B'(-8, 0)$ ;  $C'(0, -3)$ ;  $D'(16, 0)$

(i)  $-f(x)$  is a reflection in the  $x$ -axis.



$A'(-4, 6)$ ;  $B'(-2, 0)$ ;  $C'(0, 3)$ ;  $D'(4, 0)$

(j)  $f(-x)$  is a reflection in the  $y$ -axis.



$A'(4, -6)$ ;  $B'(2, 0)$ ;  $C'(0, -3)$ ;  $D'(-4, 0)$

# Solutionbank C1

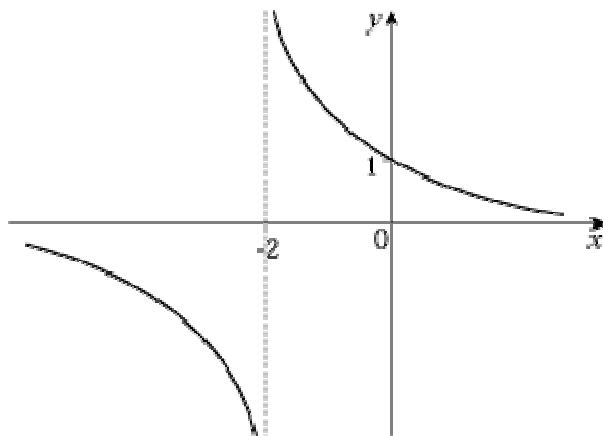
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise G, Question 4

#### Question:

A sketch of the curve  $y = f(x)$  is shown in the diagram. The curve has vertical asymptote  $x = -2$  and a horizontal asymptote with equation  $y = 0$ . The curve crosses the  $y$ -axis at  $(0, 1)$ .



(a) Sketch, on separate diagrams, the graphs of:

(i)  $2f(x)$

(ii)  $f(2x)$

(iii)  $f(x - 2)$

(iv)  $f(x) - 1$

(v)  $f(-x)$

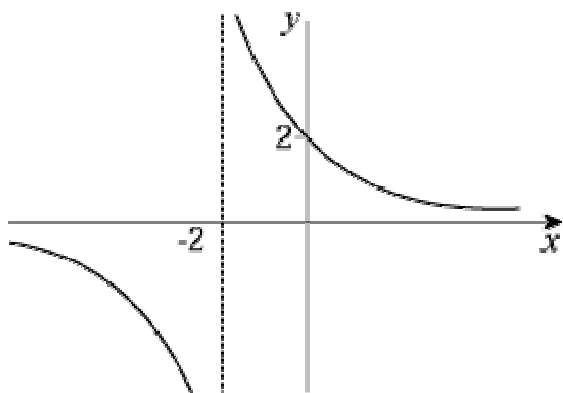
(vi)  $-f(x)$

In each case state the equations of any asymptotes and, if possible, points where the curve cuts the axes.

(b) Suggest a possible equation for  $f(x)$ .

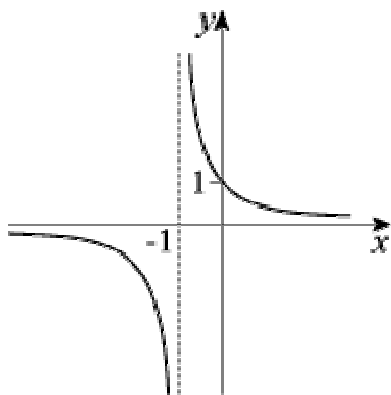
#### Solution:

(a) (i)  $2f(x)$  is a vertical stretch of 2.



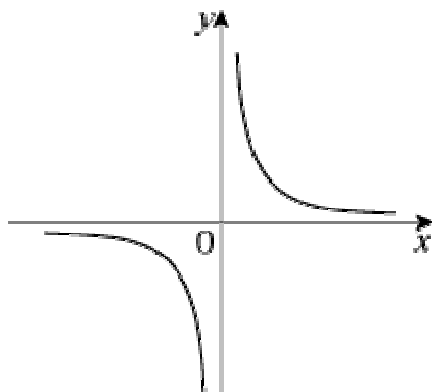
Asymptotes:  $x = -2, y = 0$ . Intersections:  $(0, 2)$

(ii)  $f(2x)$  is a horizontal stretch of  $\frac{1}{2}$ .



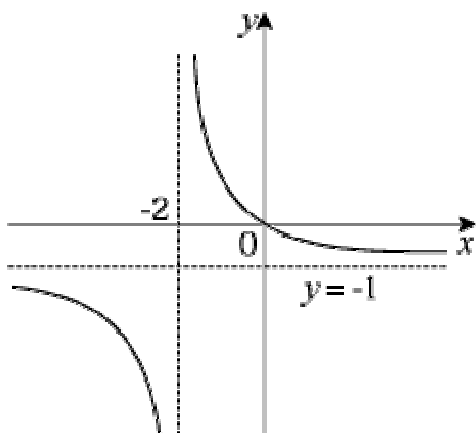
Asymptotes:  $x = -1, y = 0$ . Intersections:  $(0, 1)$

(iii)  $f(x - 2)$  is a translation of  $+2$  in the  $x$ -direction.



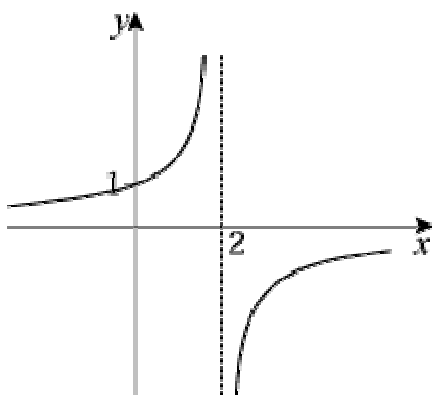
Asymptotes:  $x = 0, y = 0$ . No intersections with axes.

(iv)  $f(x) - 1$  is a translation of  $-1$  in the  $y$ -direction.



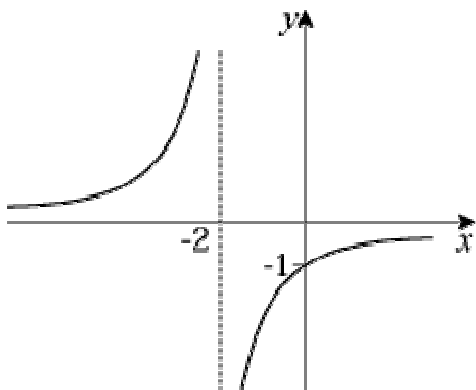
Asymptotes:  $x = -2, y = -1$ . Intersections:  $(0, 0)$

(v)  $f(-x)$  is a reflection in the  $y$ -axis.



Asymptotes:  $x = 2, y = 0$ . Intersections:  $(0, 1)$

(vi)  $-f(x)$  is a reflection in the  $x$ -axis.



Asymptotes:  $x = -2, y = 0$ . Intersections:  $(0, -1)$

(b) The shape of the curve is like  $y = \frac{k}{x}, k > 0$ .

$x = -2$  asymptote suggests denominator is zero when  $x = -2$ , so denominator is  $x + 2$ .  
Also,  $f(0) = 1$  means 2 required on numerator.

$$f(x) = \frac{2}{x+2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 1

#### Question:

- (a) On the same axes sketch the graphs of  $y = x^2(x - 2)$  and  $y = 2x - x^2$ .
- (b) By solving a suitable equation find the points of intersection of the two graphs.

#### Solution:

(a)  $y = x^2(x - 2)$

$$y = 0 \Rightarrow x = 0 \text{ (twice), } 2$$

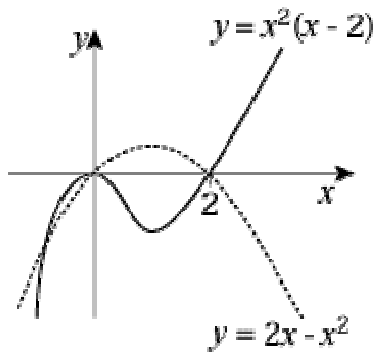
Turning point at  $(0, 0)$ .

$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$$y = 2x - x^2 = x(2 - x) \text{ is } \cap \text{ shaped}$$

$$y = 0 \Rightarrow x = 0, 2$$



(b)  $x^2(x - 2) = x(2 - x)$

$$\Rightarrow x^2(x - 2) - x(2 - x) = 0$$

$$\Rightarrow x(x - 2)(x + 1) = 0$$

$$\Rightarrow x = 0, 2, -1$$

Using  $y = x(2 - x)$  the points of intersection are:

$(0, 0)$ ;  $(2, 0)$ ;  $(-1, -3)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

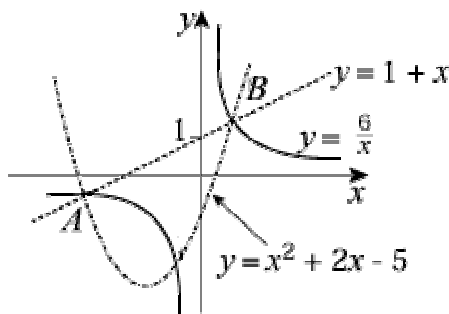
#### Exercise H, Question 2

#### Question:

- (a) On the same axes sketch the curves with equations  $y = \frac{6}{x}$  and  $y = 1 + x$ .
- (b) The curves intersect at the points  $A$  and  $B$ . Find the coordinates of  $A$  and  $B$ .
- (c) The curve  $C$  with equation  $y = x^2 + px + q$ , where  $p$  and  $q$  are integers, passes through  $A$  and  $B$ . Find the values of  $p$  and  $q$ .
- (d) Add  $C$  to your sketch.

#### Solution:

- (a)  $y = \frac{6}{x}$  is like  $y = \frac{1}{x}$  and  $y = 1 + x$  is a straight line.



- (b)  $\frac{6}{x} = 1 + x$
- $$\Rightarrow 6 = x + x^2$$
- $$\Rightarrow 0 = x^2 + x - 6$$
- $$\Rightarrow 0 = (x + 3)(x - 2)$$
- $$\Rightarrow x = 2, -3$$

So  $A$  is  $(-3, -2)$ ;  $B$  is  $(2, 3)$

- (c) Substitute the points  $A$  and  $B$  into  $y = x^2 + px + q$ :

$$A \Rightarrow -2 = 9 - 3p + q \quad \textcircled{1}$$

$$B \Rightarrow 3 = 4 + 2p + q \quad \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}: -5 = 5 - 5p$$

$$\Rightarrow p = 2$$

$$\Rightarrow q = -5$$

- (d)  $y = x^2 + 2x - 5 = (x + 1)^2 - 6 \Rightarrow$  minimum at  $(-1, -6)$

# Solutionbank C1

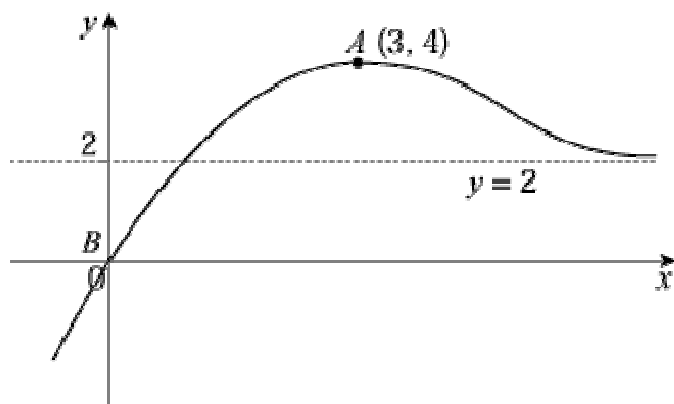
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 3

#### Question:

The diagram shows a sketch of the curve  $y = f(x)$ . The point  $B(0, 0)$  lies on the curve and the point  $A(3, 4)$  is a maximum point. The line  $y = 2$  is an asymptote.

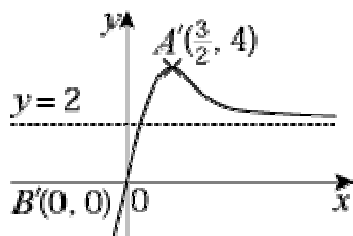


Sketch the following and in each case give the coordinates of the new positions of  $A$  and  $B$  and state the equation of the asymptote:

- (a)  $f(2x)$
- (b)  $\frac{1}{2}f(x)$
- (c)  $f(x) - 2$
- (d)  $f(x + 3)$
- (e)  $f(x - 3)$
- (f)  $f(x) + 1$

#### Solution:

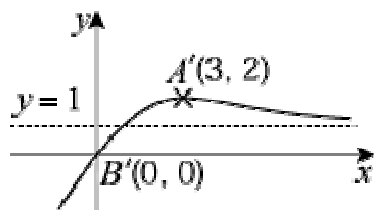
- (a)  $f(2x)$  is a horizontal stretch of  $\frac{1}{2}$ .



$A'(\frac{3}{2}, 4)$ ;  $B'(0, 0)$ . Asymptote:  $y = 2$ .

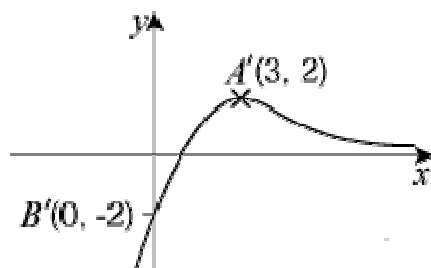


(b)  $\frac{1}{2}f(x)$  is a vertical stretch of  $\frac{1}{2}$ .



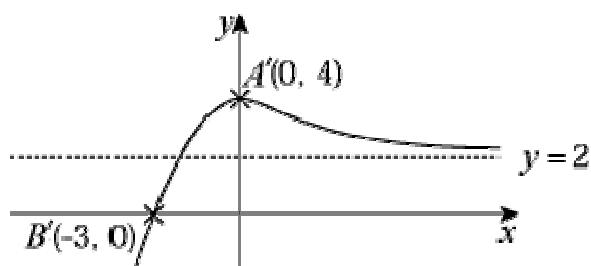
$A'(3, 2)$ ;  $B'(0, 0)$ . Asymptote:  $y = 1$ .

(c)  $f(x) - 2$  is a vertical translation of  $-2$ .



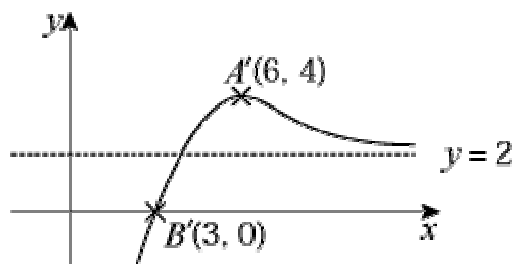
$A'(3, 2)$ ;  $B'(0, -2)$ . Asymptote:  $y = 0$ .

(d)  $f(x + 3)$  is a horizontal translation of  $-3$ .



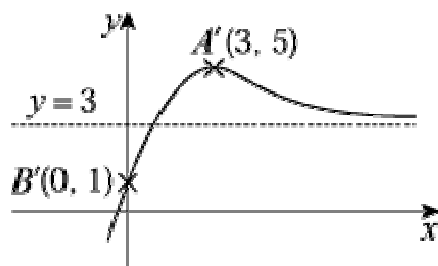
$A'(0, 4)$ ;  $B'(-3, 0)$ . Asymptote:  $y = 2$ .

(e)  $f(x - 3)$  is a horizontal translation of  $+3$ .



$A'(6, 4)$ ;  $B'(3, 0)$ . Asymptote:  $y = 2$ .

(f)  $f(x) + 1$  is a vertical translation of  $+1$ .



$A(3, 5)$ ;  $B(0, 1)$ . Asymptote:  $y = 3$ .

# Solutionbank C1

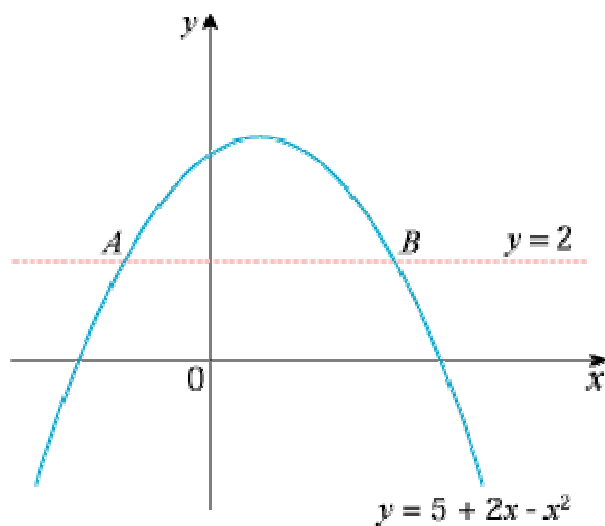
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 4

#### Question:

The diagram shows the curve with equation  $y = 5 + 2x - x^2$  and the line with equation  $y = 2$ . The curve and the line intersect at the points  $A$  and  $B$ .



Find the  $x$ -coordinates of  $A$  and  $B$ . **[E]**

#### Solution:

$$\begin{aligned}2 &= 5 + 2x - x^2 \\x^2 - 2x - 3 &= 0 \\(x - 3)(x + 1) &= 0 \\x &= -1, 3\end{aligned}$$

# Solutionbank C1

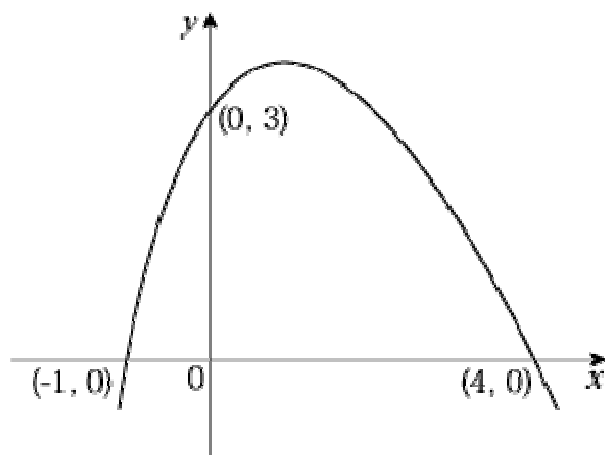
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 5

#### Question:

The curve with equation  $y = f(x)$  meets the coordinate axes at the points  $(-1, 0)$ ,  $(4, 0)$  and  $(0, 3)$ , as shown in the diagram.



Using a separate diagram for each, sketch the curve with equation

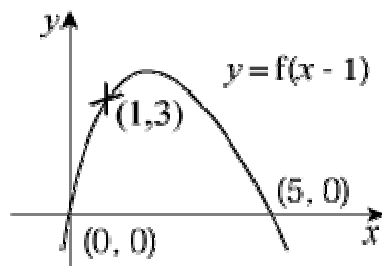
(a)  $y = f(x - 1)$

(b)  $y = -f(x)$

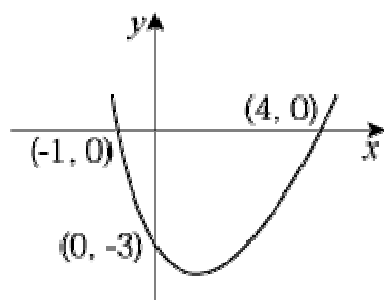
On each sketch, write in the coordinates of the points at which the curve meets the coordinate axes. **[E]**

#### Solution:

(a)  $f(x - 1)$  is a translation of  $f(x)$  by  $+1$  in the  $x$ -direction.



(b)  $-f(x)$  is a reflection in the  $x$ -axis.



# Solutionbank C1

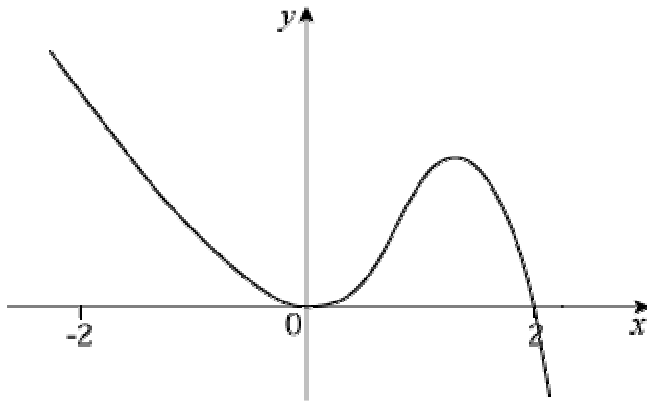
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 6

#### Question:

The figure shows a sketch of the curve with equation  $y = f(x)$ .



In separate diagrams show, for  $-2 \leq x \leq 2$ , sketches of the curves with equation:

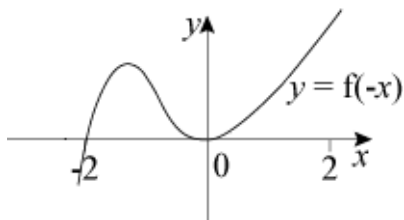
(a)  $y = f(-x)$

(b)  $y = -f(x)$

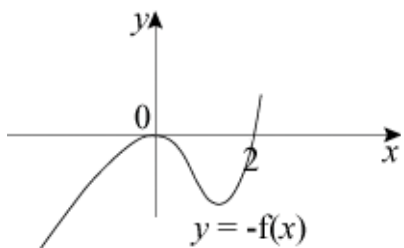
Mark on each sketch the  $x$ -coordinate of any point, or points, where a curve touches or crosses the  $x$ -axis. **[E]**

#### Solution:

(a)  $f(-x)$  is a reflection in the  $y$ -axis.



(b)  $-f(x)$  is a reflection in the  $x$ -axis.



# Solutionbank C1

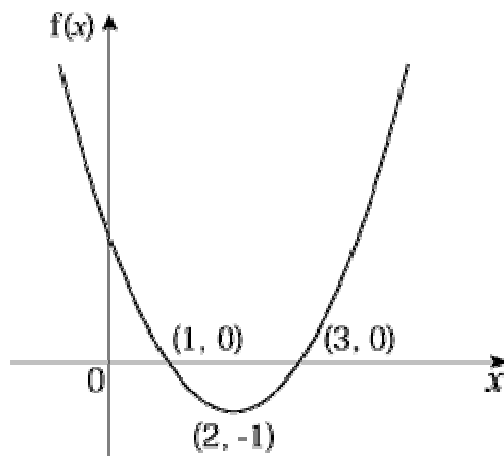
## Edexcel Modular Mathematics for AS and A-Level

### Sketching curves

#### Exercise H, Question 7

#### Question:

The diagram shows the graph of the quadratic function  $f$ . The graph meets the  $x$ -axis at  $(1, 0)$  and  $(3, 0)$  and the minimum point is  $(2, -1)$ .

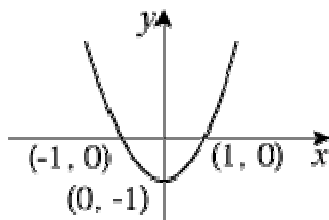


- (a) Find the equation of the graph in the form  $y = f(x)$ .
- (b) On separate axes, sketch the graphs of
- (i)  $y = f(x + 2)$
- (ii)  $y = f(2x)$
- (c) On each graph write in the coordinates of the points at which the graph meets the  $x$ -axis and write in the coordinates of the minimum point. **[E]**

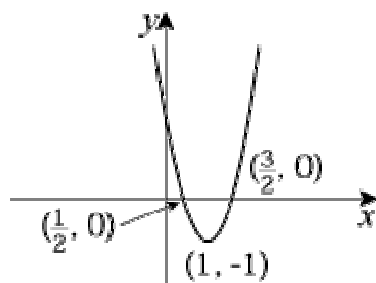
#### Solution:

(a) Let  $y = a(x - p)(x - q)$   
 Since  $(1, 0)$  and  $(3, 0)$  are on the curve then  $p = 1, q = 3$   
 So  $y = a(x - 1)(x - 3)$   
 Using  $(2, -1) \Rightarrow -1 = a(1)(-1) \Rightarrow a = 1$   
 So  $y = (x - 1)(x - 3) = x^2 - 4x + 3$

(b) (i)  $f(x + 2) = (x + 1)(x - 1)$ , or translation of  $-2$  in the  $x$ -direction.



(ii)  $f(2x) = (2x - 1)(2x - 3)$ , or horizontal stretch of  $\frac{1}{2}$ .





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 1

**Question:**

Work out the gradients of these lines:

(a)  $y = -2x + 5$

(b)  $y = -x + 7$

(c)  $y = 4 + 3x$

(d)  $y = \frac{1}{3}x - 2$

(e)  $y = -\frac{2}{3}x$

(f)  $y = \frac{5}{4}x + \frac{2}{3}$

(g)  $2x - 4y + 5 = 0$

(h)  $10x - 5y + 1 = 0$

(i)  $-x + 2y - 4 = 0$

(j)  $-3x + 6y + 7 = 0$

(k)  $4x + 2y - 9 = 0$

(l)  $9x + 6y + 2 = 0$

**Solution:**

(a) Gradient =  $-2$

(b) Gradient =  $-1$

(c) Gradient =  $3$

(d) Gradient =  $\frac{1}{3}$

(e) Gradient =  $-\frac{2}{3}$

(f) Gradient =  $\frac{5}{4}$

(g)  $2x - 4y + 5 = 0$   
 $2x + 5 = 4y$

$$4y = 2x + 5$$

$$y = \frac{2}{4}x + \frac{5}{4}$$

$$y = \frac{1}{2}x + \frac{5}{4}$$

$$\text{Gradient} = \frac{1}{2}$$

$$(h) 10x - 5y + 1 = 0$$

$$10x + 1 = 5y$$

$$5y = 10x + 1$$

$$y = \frac{10}{5}x + \frac{1}{5}$$

$$y = 2x + \frac{1}{5}$$

$$\text{Gradient} = 2$$

$$(i) -x + 2y - 4 = 0$$

$$2y - 4 = x$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

$$\text{Gradient} = \frac{1}{2}$$

$$(j) -3x + 6y + 7 = 0$$

$$6y + 7 = 3x$$

$$6y = 3x - 7$$

$$y = \frac{3}{6}x - \frac{7}{6}$$

$$y = \frac{1}{2}x - \frac{7}{6}$$

$$\text{Gradient} = \frac{1}{2}$$

$$(k) 4x + 2y - 9 = 0$$

$$2y - 9 = -4x$$

$$2y = -4x + 9$$

$$y = -\frac{4}{2}x + \frac{9}{2}$$

$$y = -2x + \frac{9}{2}$$

$$\text{Gradient} = -2$$

$$(l) 9x + 6y + 2 = 0$$

$$6y + 2 = -9x$$

$$6y = -9x - 2$$

$$y = -\frac{9}{6}x - \frac{2}{6}$$

$$y = -\frac{3}{2}x - \frac{1}{3}$$

$$\text{Gradient} = -\frac{3}{2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 2

**Question:**

These lines intercept the y-axis at  $(0, c)$ . Work out the value of  $c$  in each case.

(a)  $y = -x + 4$

(b)  $y = 2x - 5$

(c)  $y = \frac{1}{2}x - \frac{2}{3}$

(d)  $y = -3x$

(e)  $y = \frac{6}{7}x + \frac{7}{5}$

(f)  $y = 2 - 7x$

(g)  $3x - 4y + 8 = 0$

(h)  $4x - 5y - 10 = 0$

(i)  $-2x + y - 9 = 0$

(j)  $7x + 4y + 12 = 0$

(k)  $7x - 2y + 3 = 0$

(l)  $-5x + 4y + 2 = 0$

**Solution:**

(a)  $c = 4$

(b)  $c = -5$

(c)  $c = -\frac{2}{3}$

(d)  $y = -3x$   
 $y = -3x + 0$   
 $c = 0$

(e)  $c = \frac{7}{5}$

(f)  $y = 2 - 7x$   
 $y = -7x + 2$   
 $c = 2$

(g)  $3x - 4y + 8 = 0$

$$3x + 8 = 4y$$

$$4y = 3x + 8$$

$$y = \frac{3}{4}x + \frac{8}{4}$$

$$y = \frac{3}{4}x + 2$$

$$c = 2$$

$$(h) 4x - 5y - 10 = 0$$

$$4x - 10 = 5y$$

$$5y = 4x - 10$$

$$y = \frac{4}{5}x - \frac{10}{5}$$

$$y = \frac{4}{5}x - 2$$

$$c = -2$$

$$(i) -2x + y - 9 = 0$$

$$y - 9 = 2x$$

$$y = 2x + 9$$

$$c = 9$$

$$(j) 7x + 4y + 12 = 0$$

$$4y + 12 = -7x$$

$$4y = -7x - 12$$

$$y = -\frac{7}{4}x - \frac{12}{4}$$

$$y = -\frac{7}{4}x - 3$$

$$c = -3$$

$$(k) 7x - 2y + 3 = 0$$

$$7x + 3 = 2y$$

$$2y = 7x + 3$$

$$y = \frac{7}{2}x + \frac{3}{2}$$

$$c = \frac{3}{2}$$

$$(l) -5x + 4y + 2 = 0$$

$$4y + 2 = 5x$$

$$4y = 5x - 2$$

$$y = \frac{5}{4}x - \frac{2}{4}$$

$$y = \frac{5}{4}x - \frac{1}{2}$$

$$c = -\frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 3

**Question:**

Write these lines in the form  $ax + by + c = 0$ .

(a)  $y = 4x + 3$

(b)  $y = 3x - 2$

(c)  $y = -6x + 7$

(d)  $y = \frac{4}{5}x - 6$

(e)  $y = \frac{5}{3}x + 2$

(f)  $y = \frac{7}{3}x$

(g)  $y = 2x - \frac{4}{7}$

(h)  $y = -3x + \frac{2}{9}$

(i)  $y = -6x - \frac{2}{3}$

(j)  $y = -\frac{1}{3}x + \frac{1}{2}$

(k)  $y = \frac{2}{3}x + \frac{5}{6}$

(l)  $y = \frac{3}{5}x + \frac{1}{2}$

**Solution:**

(a)  $y = 4x + 3$

$0 = 4x + 3 - y$

$4x + 3 - y = 0$

$4x - y + 3 = 0$

(b)  $y = 3x - 2$

$0 = 3x - 2 - y$

$3x - 2 - y = 0$

$3x - y - 2 = 0$

$$\begin{aligned} \text{(c) } y &= -6x + 7 \\ 6x + y &= 7 \\ 6x + y - 7 &= 0 \end{aligned}$$

$$\text{(d) } y = \frac{4}{5}x - 6$$

Multiply each term by 5:

$$\begin{aligned} 5y &= 4x - 30 \\ 0 &= 4x - 30 - 5y \\ 4x - 30 - 5y &= 0 \\ 4x - 5y - 30 &= 0 \end{aligned}$$

$$\text{(e) } y = \frac{5}{3}x + 2$$

Multiply each term by 3:

$$\begin{aligned} 3y &= 5x + 6 \\ 0 &= 5x + 6 - 3y \\ 5x + 6 - 3y &= 0 \\ 5x - 3y + 6 &= 0 \end{aligned}$$

$$\text{(f) } y = \frac{7}{3}x$$

Multiply each term by 3:

$$\begin{aligned} 3y &= 7x \\ 0 &= 7x - 3y \\ 7x - 3y &= 0 \end{aligned}$$

$$\text{(g) } y = 2x - \frac{4}{7}$$

Multiply each term by 7:

$$\begin{aligned} 7y &= 14x - 4 \\ 0 &= 14x - 4 - 7y \\ 14x - 4 - 7y &= 0 \\ 14x - 7y - 4 &= 0 \end{aligned}$$

$$\text{(h) } y = -3x + \frac{2}{9}$$

Multiply each term by 9:

$$\begin{aligned} 9y &= -27x + 2 \\ 27x + 9y &= 2 \\ 27x + 9y - 2 &= 0 \end{aligned}$$

$$\text{(i) } y = -6x - \frac{2}{3}$$

Multiply each term by 3:

$$\begin{aligned} 3y &= -18x - 2 \\ 18x + 3y &= -2 \\ 18x + 3y + 2 &= 0 \end{aligned}$$

$$\text{(j) } y = -\frac{1}{3}x + \frac{1}{2}$$

Multiply each term by 6 (6 is divisible by both 3 and 2):

$$\begin{aligned} 6y &= -2x + 3 \\ 2x + 6y &= 3 \\ 2x + 6y - 3 &= 0 \end{aligned}$$

$$\text{(k) } y = \frac{2}{3}x + \frac{5}{6}$$

Multiply each term by 6 (6 is divisible by both 3 and 6):

$$6y = 4x + 5$$

$$0 = 4x + 5 - 6y$$

$$4x + 5 - 6y = 0$$

$$4x - 6y + 5 = 0$$

$$(1) y = \frac{3}{5}x + \frac{1}{2}$$

Multiply each term by 10 (10 is divisible by both 5 and 2):

$$10y = 6x + 5$$

$$0 = 6x + 5 - 10y$$

$$6x + 5 - 10y = 0$$

$$6x - 10y + 5 = 0$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the  $(x, y)$  plane  
Exercise A, Question 4

**Question:**

A line is parallel to the line  $y = 5x + 8$  and its intercept on the  $y$ -axis is  $(0, 3)$ . Write down the equation of the line.

**Solution:**

The line is parallel to  $y = 5x + 8$ , so  $m = 5$ .

The line intercepts the  $y$ -axis at  $(0, 3)$ , so  $c = 3$ .

Using  $y = mx + c$ , the equation of the line is  $y = 5x + 3$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 5

**Question:**

A line is parallel to the line  $y = -\frac{2}{5}x + 1$  and its intercept on the y-axis is  $(0, -4)$ . Work out the equation of the line. Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**Solution:**

The line is parallel to  $y = -\frac{2}{5}x + 1$ , so  $m = -\frac{2}{5}$ .

The line intercepts the y-axis at  $(0, -4)$ , so  $c = -4$ .  
Using  $y = mx + c$ , the equation of the line is

$$y = -\frac{2}{5}x - 4$$

Multiply each term by 5:

$$5y = -2x - 20$$

$$2x + 5y = -20$$

$$2x + 5y + 20 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 6

**Question:**

A line is parallel to the line  $3x + 6y + 11 = 0$  and its intercept on the y-axis is  $(0, 7)$ . Write down the equation of the line.

**Solution:**

$$3x + 6y + 11 = 0$$

$$6y + 11 = -3x$$

$$6y = -3x - 11$$

$$y = -\frac{3}{6}x - \frac{11}{6}$$

$$y = -\frac{1}{2}x - \frac{11}{6}$$

The line is parallel to  $y = -\frac{1}{2}x - \frac{11}{6}$ , so  $m = -\frac{1}{2}$ .

The line intercepts the y-axis at  $(0, 7)$ , so  $c = 7$ .

Using  $y = mx + c$ , the equation of the line is  $y = -\frac{1}{2}x + 7$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 7

**Question:**

A line is parallel to the line  $2x - 3y - 1 = 0$  and it passes through the point  $(0, 0)$ . Write down the equation of the line.

**Solution:**

$$2x - 3y - 1 = 0$$

$$2x - 1 = 3y$$

$$3y = 2x - 1$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

The line is parallel to  $y = \frac{2}{3}x - \frac{1}{3}$ , so  $m = \frac{2}{3}$ .

The intercept on the y-axis is  $(0, 0)$ , so  $c = 0$ .

Using  $y = mx + c$ :

$$y = \frac{2}{3}x + 0$$

$$y = \frac{2}{3}x$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the  $(x, y)$  plane  
Exercise A, Question 8

**Question:**

The line  $y = 6x - 18$  meets the  $x$ -axis at the point  $P$ . Work out the coordinates of  $P$ .

**Solution:**

$$y = 6x - 18$$

Substitute  $y = 0$ :

$$6x - 18 = 0$$

$$6x = 18$$

$$x = 3$$

The line meets the  $x$ -axis at  $P ( 3 , 0 )$  .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 9

**Question:**

The line  $3x + 2y - 5 = 0$  meets the  $x$ -axis at the point  $R$ . Work out the coordinates of  $R$ .

**Solution:**

$$3x + 2y - 5 = 0$$

Substitute  $y = 0$ :

$$3x + 2(0) - 5 = 0$$

$$3x - 5 = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

The line meets the  $x$ -axis at  $R \left( \frac{5}{3}, 0 \right)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise A, Question 10

**Question:**

The line  $5x - 4y + 20 = 0$  meets the  $y$ -axis at the point  $A$  and the  $x$ -axis at the point  $B$ . Work out the coordinates of the points  $A$  and  $B$ .

**Solution:**

$$5x - 4y + 20 = 0$$

Substitute  $x = 0$ :

$$5(0) - 4y + 20 = 0$$

$$-4y + 20 = 0$$

$$20 = 4y$$

$$4y = 20$$

$$y = 5$$

The line meets the  $y$ -axis at  $A(0, 5)$ .

Substitute  $y = 0$ :

$$5x - 4(0) + 20 = 0$$

$$5x + 20 = 0$$

$$5x = -20$$

$$x = -4$$

The line meets the  $x$ -axis at  $B(-4, 0)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 1

**Question:**

Work out the gradient of the line joining these pairs of points:

(a)  $(4, 2), (6, 3)$

(b)  $(-1, 3), (5, 4)$

(c)  $(-4, 5), (1, 2)$

(d)  $(2, -3), (6, 5)$

(e)  $(-3, 4), (7, -6)$

(f)  $(-12, 3), (-2, 8)$

(g)  $(-2, -4), (10, 2)$

(h)  $\left(\frac{1}{2}, 2\right), \left(\frac{3}{4}, 4\right)$

(i)  $\left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{2}{3}\right)$

(j)  $(-2.4, 9.6), (0, 0)$

(k)  $(1.3, -2.2), (8.8, -4.7)$

(l)  $(0, 5a), (10a, 0)$

(m)  $(3b, -2b), (7b, 2b)$

(n)  $(p, p^2), (q, q^2)$

**Solution:**

(a)  $(x_1, y_1) = (4, 2), (x_2, y_2) = (6, 3)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{6 - 4} = \frac{1}{2}$$

(b)  $(x_1, y_1) = (-1, 3), (x_2, y_2) = (5, 4)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{5 - (-1)} = \frac{1}{6}$$

(c)  $(x_1, y_1) = (-4, 5), (x_2, y_2) = (1, 2)$



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{1 - (-4)} = -\frac{3}{5}$$

(d)  $(x_1, y_1) = (2, -3)$ ,  $(x_2, y_2) = (6, 5)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{6 - 2} = \frac{8}{4} = 2$$

(e)  $(x_1, y_1) = (-3, 4)$ ,  $(x_2, y_2) = (7, -6)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 4}{7 - (-3)} = -\frac{10}{10} = -1$$

(f)  $(x_1, y_1) = (-12, 3)$ ,  $(x_2, y_2) = (-2, 8)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{-2 - (-12)} = \frac{5}{-2 + 12} = \frac{5}{10} = \frac{1}{2}$$

(g)  $(x_1, y_1) = (-2, -4)$ ,  $(x_2, y_2) = (10, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{10 - (-2)} = \frac{6}{12} = \frac{1}{2}$$

(h)  $\left( x_1, y_1 \right) = \left( \frac{1}{2}, 2 \right)$ ,  $\left( x_2, y_2 \right) = \left( \frac{3}{4}, 4 \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{\frac{3}{4} - \frac{1}{2}} = \frac{2}{\frac{1}{4}} = 8$$

(i)  $\left( x_1, y_1 \right) = \left( \frac{1}{4}, \frac{1}{2} \right)$ ,  $\left( x_2, y_2 \right) = \left( \frac{1}{2}, \frac{2}{3} \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{4}} = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$$

(j)  $(x_1, y_1) = (-2.4, 9.6)$ ,  $(x_2, y_2) = (0, 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 9.6}{0 - (-2.4)} = \frac{-9.6}{2.4} = -4$$

(k)  $(x_1, y_1) = (1.3, -2.2)$ ,  $(x_2, y_2) = (8.8, -4.7)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4.7 - (-2.2)}{8.8 - 1.3} = \frac{-2.5}{7.5} = -\frac{1}{3}$$

(l)  $(x_1, y_1) = (0, 5a)$ ,  $(x_2, y_2) = (10a, 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5a}{10a - 0} = \frac{-5a}{10a} = \frac{-5}{10} = -\frac{1}{2}$$

(m)  $(x_1, y_1) = (3b, -2b)$ ,  $(x_2, y_2) = (7b, 2b)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - (-2b)}{7b - 3b} = \frac{4b}{4b} = 1$$

$$\text{(ii)} \quad (x_1, y_1) = (p, p^2), \quad (x_2, y_2) = (q, q^2)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{q^2 - p^2}{q - p} = \frac{(q - p)(q + p)}{q - p} = q + p$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 2

**Question:**

The line joining  $(3, -5)$  to  $(6, a)$  has gradient 4. Work out the value of  $a$ .

**Solution:**

$$(x_1, y_1) = (3, -5), (x_2, y_2) = (6, a)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = 4$$

$$\text{so } \frac{a - (-5)}{6 - 3} = 4$$

$$\Rightarrow \frac{a + 5}{3} = 4$$

$$\Rightarrow a + 5 = 12$$

$$\Rightarrow a = 7$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane  
Exercise B, Question 3

**Question:**

The line joining  $(5, b)$  to  $(8, 3)$  has gradient  $-3$ . Work out the value of  $b$ .

**Solution:**

$$(x_1, y_1) = (5, b), (x_2, y_2) = (8, 3)$$

$$\frac{3-b}{8-5} = -3$$

$$\frac{3-b}{3} = -3$$

$$3-b = -9$$

$$b = 12$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 4

**Question:**

The line joining  $(c, 4)$  to  $(7, 6)$  has gradient  $\frac{3}{4}$ . Work out the value of  $c$ .

**Solution:**

$$(x_1, y_1) = (c, 4), (x_2, y_2) = (7, 6)$$

$$\frac{6-4}{7-c} = \frac{3}{4}$$

$$\frac{2}{7-c} = \frac{3}{4}$$

$$2 = \frac{3}{4} (7 - c)$$

$$8 = 3(7 - c)$$

$$8 = 21 - 3c$$

$$-13 = -3c$$

$$c = \frac{-13}{-3} = \frac{13}{3} = 4\frac{1}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 5

**Question:**

The line joining  $(-1, 2b)$  to  $(1, 4)$  has gradient  $-\frac{1}{4}$ . Work out the value of  $b$ .

**Solution:**

$$(x_1, y_1) = (-1, 2b), (x_2, y_2) = (1, 4)$$

$$\frac{4 - 2b}{1 - (-1)} = -\frac{1}{4}$$

$$\frac{4 - 2b}{2} = -\frac{1}{4}$$

$$2 - b = -\frac{1}{4}$$

$$2\frac{1}{4} - b = 0$$

$$b = 2\frac{1}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane  
Exercise B, Question 6

**Question:**

The line joining  $(-3, -2)$  to  $(2e, 5)$  has gradient 2. Work out the value of  $e$ .

**Solution:**

$$(x_1, y_1) = (-3, -2), (x_2, y_2) = (2e, 5)$$

$$\frac{5 - (-2)}{2e - (-3)} = 2$$

$$\frac{7}{2e + 3} = 2$$

$$7 = 2(2e + 3)$$

$$7 = 4e + 6$$

$$4e = 1$$

$$e = \frac{1}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane  
Exercise B, Question 7

**Question:**

The line joining  $(7, 2)$  to  $(f, 3f)$  has gradient 4. Work out the value of  $f$ .

**Solution:**

$$(x_1, y_1) = (7, 2), (x_2, y_2) = (f, 3f)$$

$$\frac{3f-2}{f-7} = 4$$

$$3f - 2 = 4(f - 7)$$

$$3f - 2 = 4f - 28$$

$$-2 = f - 28$$

$$28 - 2 = f$$

$$f = 26$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 8

**Question:**

The line joining  $(3, -4)$  to  $(-g, 2g)$  has gradient  $-3$ . Work out the value of  $g$ .

**Solution:**

$$(x_1, y_1) = (3, -4), (x_2, y_2) = (-g, 2g)$$

$$\frac{2g - (-4)}{-g - 3} = -3$$

$$\frac{2g + 4}{-g - 3} = -3$$

$$2g + 4 = -3(-g - 3)$$

$$2g + 4 = 3g + 9$$

$$4 = g + 9$$

$$g = -5$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 9

#### Question:

Show that the points  $A(2, 3)$ ,  $B(4, 4)$ ,  $C(10, 7)$  can be joined by a straight line. (Hint: Find the gradient of the lines joining the points: **i**  $A$  and  $B$  and **ii**  $A$  and  $C$ .)

#### Solution:

The gradient of  $AB$  is  $\frac{4-3}{4-2} = \frac{1}{2}$

The gradient of  $AC$  is  $\frac{7-3}{10-2} = \frac{4}{8} = \frac{1}{2}$

The gradients are equal so the points can be joined by a straight line.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise B, Question 10

**Question:**

Show that the points  $(-2a, 5a)$ ,  $(0, 4a)$ ,  $(6a, a)$  are collinear (i.e. on the same straight line).

**Solution:**

The gradient of the line joining  $(-2a, 5a)$  and  $(0, 4a)$  is

$$\frac{4a - 5a}{0 - (-2a)} = \frac{-a}{2a} = \frac{-1}{2}$$

The gradient of the line joining  $(-2a, 5a)$  and  $(6a, a)$  is

$$\frac{a - 5a}{6a - (-2a)} = \frac{-4a}{8a} = \frac{-4}{8} = \frac{-1}{2}$$

The gradients are equal so the points can be joined by a straight line (i.e. they are collinear).

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 1

#### Question:

Find the equation of the line with gradient  $m$  that passes through the point  $(x_1, y_1)$  when:

(a)  $m = 2$  and  $(x_1, y_1) = (2, 5)$

(b)  $m = 3$  and  $(x_1, y_1) = (-2, 1)$

(c)  $m = -1$  and  $(x_1, y_1) = (3, -6)$

(d)  $m = -4$  and  $(x_1, y_1) = (-2, -3)$

(e)  $m = \frac{1}{2}$  and  $(x_1, y_1) = (-4, 10)$

(f)  $m = -\frac{2}{3}$  and  $(x_1, y_1) = (-6, -1)$

(g)  $m = 2$  and  $(x_1, y_1) = (a, 2a)$

(h)  $m = -\frac{1}{2}$  and  $(x_1, y_1) = (-2b, 3b)$

#### Solution:

(a)  $y - y_1 = m(x - x_1)$

$$y - 5 = 2(x - 2)$$

$$y - 5 = 2x - 4$$

$$y = 2x + 1$$

(b)  $y - y_1 = m(x - x_1)$

$$y - 1 = 3[x - (-2)]$$

$$y - 1 = 3(x + 2)$$

$$y - 1 = 3x + 6$$

$$y = 3x + 7$$

(c)  $y - y_1 = m(x - x_1)$

$$y - (-6) = -1(x - 3)$$

$$y + 6 = -x + 3$$

$$y = -x - 3$$

(d)  $y - y_1 = m(x - x_1)$

$$y - (-3) = -4[x - (-2)]$$

$$y + 3 = -4(x + 2)$$

$$y + 3 = -4x - 8$$

$$y = -4x - 11$$

(e)  $y - y_1 = m(x - x_1)$

$$y - 10 = \frac{1}{2} \left[ x - \left( -4 \right) \right]$$

$$y - 10 = \frac{1}{2} \left( x + 4 \right)$$

$$y - 10 = \frac{1}{2}x + 2$$

$$y = \frac{1}{2}x + 12$$

$$(f) y - y_1 = m ( x - x_1 )$$

$$y - \left( -1 \right) = -\frac{2}{3} \left[ x - \left( -6 \right) \right]$$

$$y + 1 = -\frac{2}{3} \left( x + 6 \right)$$

$$y + 1 = -\frac{2}{3}x - 4$$

$$y = -\frac{2}{3}x - 5$$

$$(g) y - y_1 = m ( x - x_1 )$$

$$y - 2a = 2 ( x - a )$$

$$y - 2a = 2x - 2a$$

$$y = 2x$$

$$(h) y - y_1 = m ( x - x_1 )$$

$$y - 3b = -\frac{1}{2} \left[ x - \left( -2b \right) \right]$$

$$y - 3b = -\frac{1}{2} \left( x + 2b \right)$$

$$y - 3b = -\frac{1}{2}x - b$$

$$y = -\frac{1}{2}x - b + 3b$$

$$y = -\frac{1}{2}x + 2b$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 2

**Question:**

The line  $y = 4x - 8$  meets the  $x$ -axis at the point  $A$ . Find the equation of the line with gradient 3 that passes through the point  $A$ .

**Solution:**

$$y = 4x - 8$$

Substitute  $y = 0$ :

$$4x - 8 = 0$$

$$4x = 8$$

$$x = 2$$

So  $A$  has coordinates  $(2, 0)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 3(x - 2)$$

$$y = 3x - 6$$

The equation of the line is  $y = 3x - 6$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 3

**Question:**

The line  $y = -2x + 8$  meets the y-axis at the point  $B$ . Find the equation of the line with gradient 2 that passes through the point  $B$ .

**Solution:**

$$y = -2x + 8$$

Substitute  $x = 0$ :

$$y = -2(0) + 8$$

$$y = 8$$

So  $B$  has coordinates  $(0, 8)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 2(x - 0)$$

$$y - 8 = 2x$$

$$y = 2x + 8$$

The equation of the line is  $y = 2x + 8$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 4

**Question:**

The line  $y = \frac{1}{2}x + 6$  meets the  $x$ -axis at the point  $C$ . Find the equation of the line with gradient  $\frac{2}{3}$  that passes through the point  $C$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**Solution:**

$$y = \frac{1}{2}x + 6$$

Substitute  $y = 0$ :

$$\frac{1}{2}x + 6 = 0$$

$$\frac{1}{2}x = -6$$

$$x = -12$$

So  $C$  has coordinates  $(-12, 0)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{2}{3} \left[ x - \left( -12 \right) \right]$$

$$y = \frac{2}{3} \left( x + 12 \right)$$

$$y = \frac{2}{3}x + 8$$

Multiply each term by 3:

$$3y = 2x + 24$$

$$0 = 2x + 24 - 3y$$

$$2x - 3y + 24 = 0$$

The equation of the line is  $2x - 3y + 24 = 0$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 5

**Question:**

The line  $y = \frac{1}{4}x + 2$  meets the y-axis at the point  $B$ . The point  $C$  has coordinates  $(-5, 3)$ . Find the gradient of the line joining the points  $B$  and  $C$ .

**Solution:**

$$y = \frac{1}{4}x + 2$$

Substitute  $x = 0$ :

$$y = \frac{1}{4} \left( 0 \right) + 2$$

$$y = 2$$

So  $B$  has coordinates  $(0, 2)$ .

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-5 - 0} = \frac{1}{-5} = -\frac{1}{5}$$

The gradient of the line joining  $B$  and  $C$  is  $-\frac{1}{5}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 6

**Question:**

The lines  $y = x$  and  $y = 2x - 5$  intersect at the point  $A$ . Find the equation of the line with gradient  $\frac{2}{5}$  that passes through the point  $A$ . (Hint: Solve  $y = x$  and  $y = 2x - 5$  simultaneously.)

**Solution:**

Substitute  $y = x$ :

$$x = 2x - 5$$

$$0 = x - 5$$

$$x = 5$$

$$y = x$$

Substitute  $x = 5$ :

$$y = 5$$

The coordinates of  $A$  are  $(5, 5)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{2}{5} \left( x - 5 \right)$$

$$y - 5 = \frac{2}{5}x - 2$$

$$y = \frac{2}{5}x + 3$$

The equation of the line is  $y = \frac{2}{5}x + 3$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 7

#### Question:

The lines  $y = 4x - 10$  and  $y = x - 1$  intersect at the point  $T$ . Find the equation of the line with gradient  $-\frac{2}{3}$  that passes through the point  $T$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

Substitute  $y = x - 1$ :

$$x - 1 = 4x - 10$$

$$-1 = 3x - 10$$

$$9 = 3x$$

$$x = 3$$

$$y = x - 1$$

Substitute  $x = 3$ :

$$y = 3 - 1 = 2$$

The coordinates of  $T$  are  $(3, 2)$ .

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 3)$$

$$y - 2 = -\frac{2}{3}x + 2$$

$$\frac{2}{3}x + y - 2 = 2$$

$$\frac{2}{3}x + y - 4 = 0$$

$$2x + 3y - 12 = 0$$

The equation of the line is  $2x + 3y - 12 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 8

#### Question:

The line  $p$  has gradient  $\frac{2}{3}$  and passes through the point  $(6, -12)$ . The line  $q$  has gradient  $-1$  and passes through the point  $(5, 5)$ . The line  $p$  meets the  $y$ -axis at  $A$  and the line  $q$  meets the  $x$ -axis at  $B$ . Work out the gradient of the line joining the points  $A$  and  $B$ .

#### Solution:

The equation of  $p$  is

$$y - \begin{pmatrix} -12 \end{pmatrix} = \frac{2}{3} \begin{pmatrix} x - 6 \end{pmatrix}$$

$$y + 12 = \frac{2}{3}x - 4$$

$$y = \frac{2}{3}x - 16$$

The equation of  $q$  is

$$y - 5 = -1(x - 5)$$

$$y - 5 = -x + 5$$

$$y = -x + 10$$

For the coordinates of  $A$  substitute  $x = 0$  into

$$y = \frac{2}{3}x - 16$$

$$y = \frac{2}{3} \begin{pmatrix} 0 \end{pmatrix} - 16$$

$$y = -16$$

Coordinates are  $A(0, -16)$

For the coordinates of  $B$  substitute  $y = 0$  into

$$y = -x + 10$$

$$0 = -x + 10$$

$$x = 10$$

Coordinates are  $B(10, 0)$

Gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-16 - 0}{0 - 10} = \frac{-16}{-10} = \frac{8}{5}$$

The gradient of the line joining  $A$  and  $B$  is  $\frac{8}{5}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 9

#### Question:

The line  $y = -2x + 6$  meets the  $x$ -axis at the point  $P$ . The line  $y = \frac{3}{2}x - 4$  meets the  $y$ -axis at the point  $Q$ . Find the equation of the line joining the points  $P$  and  $Q$ . (Hint: First work out the gradient of the line joining the points  $P$  and  $Q$ .)

#### Solution:

$$y = -2x + 6$$

Substitute  $y = 0$ :

$$0 = -2x + 6$$

$$2x = 6$$

$$x = 3$$

$P$  has coordinates  $(3, 0)$ .

$$y = \frac{3}{2}x - 4$$

Substitute  $x = 0$ :

$$y = \frac{3}{2} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 4$$

$$y = -4$$

$Q$  has coordinates  $(0, -4)$

Gradient of  $PQ$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{3 - 0} = \frac{4}{3}$$

Equation of  $PQ$  is

$$y - y_1 = m(x - x_1)$$

Substitute  $(3, 0)$ :

$$y - 0 = \frac{4}{3} \begin{pmatrix} x - 3 \end{pmatrix}$$

$$y = \frac{4}{3}x - 4$$

The equation of the line through  $P$  and  $Q$  is  $y = \frac{4}{3}x - 4$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise C, Question 10

#### Question:

The line  $y = 3x - 5$  meets the  $x$ -axis at the point  $M$ . The line  $y = -\frac{2}{3}x + \frac{2}{3}$  meets the  $y$ -axis at the point  $N$ . Find the equation of the line joining the points  $M$  and  $N$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

$$y = 3x - 5$$

Substitute  $y = 0$ :

$$3x - 5 = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

$M$  has coordinates  $\left(\frac{5}{3}, 0\right)$ .

$$y = -\frac{2}{3}x + \frac{2}{3}$$

Substitute  $x = 0$ :

$$y = -\frac{2}{3}\left(0\right) + \frac{2}{3} = \frac{2}{3}$$

$N$  has coordinates  $\left(0, \frac{2}{3}\right)$ .

Gradient of  $MN$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \frac{2}{3}}{\frac{5}{3} - 0} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{2}{5}$$

Equation of  $MN$  is

$$y - y_1 = m(x - x_1)$$

Substitute  $\left(\frac{5}{3}, 0\right)$ :

$$y - 0 = -\frac{2}{5}\left(x - \frac{5}{3}\right)$$

$$y = -\frac{2}{5}x + \frac{2}{3}$$

Multiply each term by 15:

$$15y = -6x + 10$$

$$6x + 15y = 10$$

$$6x + 15y - 10 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 1

#### Question:

Find the equation of the line that passes through these pairs of points:

(a)  $(2, 4)$  and  $(3, 8)$

(b)  $(0, 2)$  and  $(3, 5)$

(c)  $(-2, 0)$  and  $(2, 8)$

(d)  $(5, -3)$  and  $(7, 5)$

(e)  $(3, -1)$  and  $(7, 3)$

(f)  $(-4, -1)$  and  $(6, 4)$

(g)  $(-1, -5)$  and  $(-3, 3)$

(h)  $(-4, -1)$  and  $(-3, -9)$

(i)  $\left(\frac{1}{3}, \frac{2}{5}\right)$  and  $\left(\frac{2}{3}, \frac{4}{5}\right)$

(j)  $\left(-\frac{3}{4}, \frac{1}{7}\right)$  and  $\left(\frac{1}{4}, \frac{3}{7}\right)$

#### Solution:

(a)  $(x_1, y_1) = (2, 4)$ ,  $(x_2, y_2) = (3, 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 4}{8 - 4} = \frac{x - 2}{3 - 2}$$

$$\frac{y - 4}{4} = \frac{x - 2}{1}$$

$$\frac{y - 4}{4} = x - 2$$

Multiply each side by 4:

$$4 \times \frac{y - 4}{4} = 4 \left( x - 2 \right)$$

$$y - 4 = 4(x - 2)$$

$$y - 4 = 4x - 8$$

$$y = 4x - 4$$

(b)  $(x_1, y_1) = (0, 2)$ ,  $(x_2, y_2) = (3, 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{5 - 2} = \frac{x - 0}{3 - 0}$$

$$\frac{y - 2}{3} = \frac{x}{3}$$

Multiply each side by 3:

$$3 \times \frac{y - 2}{3} = 3 \times \frac{x}{3}$$

$$y - 2 = x$$

$$y = x + 2$$

(c)  $(x_1, y_1) = (-2, 0)$ ,  $(x_2, y_2) = (2, 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{8 - 0} = \frac{x - (-2)}{2 - (-2)}$$

$$\frac{y}{8} = \frac{x + 2}{4}$$

Multiply each side by 8:

$$8 \times \frac{y}{8} = 8 \times \frac{x + 2}{4}$$

$$y = 2(x + 2)$$

$$y = 2x + 4$$

(d)  $(x_1, y_1) = (5, -3)$ ,  $(x_2, y_2) = (7, 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-3)}{5 - (-3)} = \frac{x - 5}{7 - 5}$$

$$\frac{y + 3}{8} = \frac{x - 5}{2}$$

Multiply each side by 8:

$$8 \times \frac{y + 3}{8} = 8 \times \frac{x - 5}{2}$$

$$y + 3 = 4(x - 5)$$

$$y + 3 = 4x - 20$$

$$y = 4x - 23$$

(e)  $(x_1, y_1) = (3, -1)$ ,  $(x_2, y_2) = (7, 3)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{3 - (-1)} = \frac{x - 3}{7 - 3}$$

$$\frac{y + 1}{4} = \frac{x - 3}{4}$$

Multiply each side by 4:

$$y + 1 = x - 3$$

$$y = x - 4$$

(f)  $(x_1, y_1) = (-4, -1)$ ,  $(x_2, y_2) = (6, 4)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



$$\frac{y - (-1)}{4 - (-1)} = \frac{x - (-4)}{6 - (-4)}$$

$$\frac{y+1}{5} = \frac{x+4}{10}$$

Multiply each side by 10:

$$2(y+1) = x+4$$

$$2y+2 = x+4$$

$$2y = x+2$$

Divide each term by 2:

$$y = \frac{1}{2}x + 1$$

(g)  $(x_1, y_1) = (-1, -5)$ ,  $(x_2, y_2) = (-3, 3)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - (-5)}{3 - (-5)} = \frac{x - (-1)}{-3 - (-1)}$$

$$\frac{y+5}{8} = \frac{x+1}{-2}$$

Multiply each side by 8:

$$y+5 = -4(x+1) \quad (\text{Note: } \frac{8}{-2} = -4)$$

$$y+5 = -4x-4$$

$$y = -4x-9$$

(h)  $(x_1, y_1) = (-4, -1)$ ,  $(x_2, y_2) = (-3, -9)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - (-1)}{-9 - (-1)} = \frac{x - (-4)}{-3 - (-4)}$$

$$\frac{y+1}{-8} = \frac{x+4}{1}$$

Multiply each side by  $-8$ :

$$y+1 = -8(x+4)$$

$$y+1 = -8x-32$$

$$y = -8x-33$$

(i)  $\left(x_1, y_1\right) = \left(\frac{1}{3}, \frac{2}{5}\right)$ ,  $\left(x_2, y_2\right) = \left(\frac{2}{3}, \frac{4}{5}\right)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - \frac{2}{5}}{\frac{4}{5} - \frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{2}{3} - \frac{1}{3}}$$

$$\frac{y - \frac{2}{5}}{\frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{1}{3}}$$

$$\frac{5}{2} \left( y - \frac{2}{5} \right) = 3 \left( x - \frac{1}{3} \right) \quad (\text{Note: } \frac{1}{\frac{2}{5}} = \frac{5}{2} \text{ and } \frac{1}{\frac{1}{3}} = 3)$$

$$\frac{5}{2}y - 1 = 3x - 1$$

$$\frac{5}{2}y = 3x$$

$$5y = 6x$$

$$y = \frac{6}{5}x$$

$$(j) \left( x_1, y_1 \right) = \left( \frac{-3}{4}, \frac{1}{7} \right), \left( x_2, y_2 \right) = \left( \frac{1}{4}, \frac{3}{7} \right)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - \frac{1}{7}}{\frac{3}{7} - \frac{1}{7}} = \frac{x - \left( -\frac{3}{4} \right)}{\frac{1}{4} - \left( -\frac{3}{4} \right)}$$

$$\frac{y - \frac{1}{7}}{\frac{2}{7}} = \frac{x + \frac{3}{4}}{1}$$

Multiply each side by  $\frac{2}{7}$ :

$$y - \frac{1}{7} = \frac{2}{7} \left( x + \frac{3}{4} \right)$$

$$y - \frac{1}{7} = \frac{2}{7}x + \frac{3}{14}$$

$$y = \frac{2}{7}x + \frac{3}{14} + \frac{1}{7}$$

$$y = \frac{2}{7}x + \frac{5}{14}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 2

#### Question:

The line that passes through the points  $(2, -5)$  and  $(-7, 4)$  meets the  $x$ -axis at the point  $P$ . Work out the coordinates of the point  $P$ .

#### Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{4 - (-5)} = \frac{x - 2}{-7 - 2}$$

$$\frac{y + 5}{9} = \frac{x - 2}{-9}$$

Multiply each side by 9:

$$y + 5 = -1(x - 2) \quad (\text{Note: } \frac{9}{-9} = -1)$$

$$y + 5 = -x + 2$$

$$y = -x - 3$$

Substitute  $y = 0$ :

$$0 = -x - 3$$

$$x = -3$$

So the line meets the  $x$ -axis at  $P(-3, 0)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 3

**Question:**

The line that passes through the points  $(-3, -5)$  and  $(4, 9)$  meets the y-axis at the point  $G$ . Work out the coordinates of the point  $G$ .

**Solution:**

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{9 - (-5)} = \frac{x - (-3)}{4 - (-3)}$$

$$\frac{y + 5}{14} = \frac{x + 3}{7}$$

Multiply each side by 14:

$$y + 5 = 2(x + 3)$$

$$y + 5 = 2x + 6$$

$$y = 2x + 1$$

Substitute  $x = 0$ :

$$y = 2(0) + 1 = 1$$

The coordinates of  $G$  are  $(0, 1)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 4

#### Question:

The line that passes through the points  $\left( 3, 2\frac{1}{2} \right)$  and  $\left( -1\frac{1}{2}, 4 \right)$  meets the y-axis at the point  $J$ . Work out the coordinates of the point  $J$ .

#### Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2\frac{1}{2}}{4 - 2\frac{1}{2}} = \frac{x - 3}{-1\frac{1}{2} - 3}$$

$$\frac{y - 2\frac{1}{2}}{1\frac{1}{2}} = \frac{x - 3}{-4\frac{1}{2}}$$

Multiply top and bottom of each fraction by 2:

$$\frac{2y - 5}{3} = \frac{2x - 6}{-9}$$

Multiply each side by 9:

$$3(2y - 5) = -1(2x - 6) \quad (\text{Note: } \frac{9}{-9} = -1)$$

$$6y - 15 = -2x + 6$$

$$6y = -2x + 21$$

$$y = -\frac{2}{6}x + \frac{21}{6}$$

$$y = -\frac{1}{3}x + \frac{7}{2}$$

Substitute  $x = 0$ :

$$y = -\frac{1}{3} \left( 0 \right) + \frac{7}{2} = \frac{7}{2}$$

The coordinates of  $J$  are  $\left( 0, \frac{7}{2} \right)$  or  $\left( 0, 3\frac{1}{2} \right)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 5

#### Question:

The line  $y = 2x - 10$  meets the  $x$ -axis at the point  $A$ . The line  $y = -2x + 4$  meets the  $y$ -axis at the point  $B$ . Find the equation of the line joining the points  $A$  and  $B$ . (Hint: First work out the coordinates of the points  $A$  and  $B$ .)

#### Solution:

$$y = 2x - 10$$

Substitute  $y = 0$ :

$$2x - 10 = 0$$

$$2x = 10$$

$$x = 5$$

The coordinates of  $A$  are  $(5, 0)$ .

$$y = -2x + 4$$

Substitute  $x = 0$ :

$$y = -2(0) + 4 = 4$$

The coordinates of  $B$  are  $(0, 4)$ .

Equation of  $AB$ :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{4 - 0} = \frac{x - 5}{0 - 5}$$

$$\frac{y}{4} = \frac{x - 5}{-5}$$

Multiply each side by 4:

$$y = 4 \frac{(x - 5)}{-5} = \frac{4}{-5} (x - 5) = -\frac{4}{5} (x - 5) = -\frac{4}{5}x + 4$$

The equation of the line is  $y = -\frac{4}{5}x + 4$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 6

**Question:**

The line  $y = 4x + 5$  meets the  $y$ -axis at the point  $C$ . The line  $y = -3x - 15$  meets the  $x$ -axis at the point  $D$ . Find the equation of the line joining the points  $C$  and  $D$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**Solution:**

$$y = 4x + 5$$

Substitute  $x = 0$ :

$$y = 4(0) + 5 = 5$$

The coordinates of  $C$  are  $(0, 5)$ .

$$y = -3x - 15$$

Substitute  $y = 0$ :

$$0 = -3x - 15$$

$$3x = -15$$

$$x = -5$$

The coordinates of  $D$  are  $(-5, 0)$ .

Equation of  $CD$ :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{0 - 5} = \frac{x - 0}{-5 - 0}$$

$$\frac{y - 5}{-5} = \frac{x}{-5}$$

Multiply each side by  $-5$ :

$$y - 5 = x$$

$$-5 = x - y$$

$$0 = x - y + 5$$

The equation of the line is  $x - y + 5 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 7

#### Question:

The lines  $y = x - 5$  and  $y = 3x - 13$  intersect at the point  $S$ . The point  $T$  has coordinates  $(-4, 2)$ . Find the equation of the line that passes through the points  $S$  and  $T$ .

#### Solution:

$$y = 3x - 13$$

$$y = x - 5$$

$$\text{So } 3x - 13 = x - 5$$

$$\Rightarrow 3x = x + 8$$

$$\Rightarrow 2x = 8$$

$$\Rightarrow x = 4$$

when  $x = 4$ ,  $y = 4 - 5 = -1$

The coordinates of  $S$  are  $(4, -1)$ .

Equation of  $ST$ :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{2 - (-1)} = \frac{x - 4}{-4 - 4}$$

$$\frac{y + 1}{3} = \frac{x - 4}{-8}$$

Multiply each side by 3:

$$y + 1 = 3 \times \frac{(x - 4)}{-8}$$

$$y + 1 = \frac{3}{-8} \times (x - 4)$$

$$y + 1 = -\frac{3}{8} (x - 4)$$

$$y + 1 = -\frac{3}{8}x + \frac{3}{2}$$

$$y = -\frac{3}{8}x + \frac{1}{2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 8

#### Question:

The lines  $y = -2x + 1$  and  $y = x + 7$  intersect at the point  $L$ . The point  $M$  has coordinates  $(-3, 1)$ . Find the equation of the line that passes through the points  $L$  and  $M$ .

#### Solution:

$$y = x + 7$$

$$y = -2x + 1$$

$$\text{So } x + 7 = -2x + 1$$

$$\Rightarrow 3x + 7 = 1$$

$$\Rightarrow 3x = -6$$

$$\Rightarrow x = -2$$

$$\text{when } x = -2, y = (-2) + 7 = 5$$

The coordinates of  $L$  are  $(-2, 5)$ .

Equation of  $LM$ :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{1 - 5} = \frac{x - (-2)}{-3 - (-2)}$$

$$\frac{y - 5}{-4} = \frac{x + 2}{-1}$$

Multiply each side by  $-4$ :

$$y - 5 = 4(x + 2) \quad (\text{Note: } \frac{-4}{-1} = 4)$$

$$y - 5 = 4x + 8$$

$$y = 4x + 13$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 9

#### Question:

The vertices of the triangle  $ABC$  have coordinates  $A(3, 5)$ ,  $B(-2, 0)$  and  $C(4, -1)$ . Find the equations of the sides of the triangle.

#### Solution:

(1) Equation of  $AB$ :

$$(x_1, y_1) = (3, 5), (x_2, y_2) = (-2, 0)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{0 - 5} = \frac{x - 3}{-2 - 3}$$

$$\frac{y - 5}{-5} = \frac{x - 3}{-5}$$

Multiply each side by  $-5$ :

$$y - 5 = x - 3$$

$$y = x + 2$$

(2) Equation of  $AC$ :

$$(x_1, y_1) = (3, 5), (x_2, y_2) = (4, -1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-1 - 5} = \frac{x - 3}{4 - 3}$$

$$\frac{y - 5}{-6} = \frac{x - 3}{1}$$

Multiply each side by  $-6$ :

$$y - 5 = -6(x - 3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 23$$

(3) Equation of  $BC$ :

$$(x_1, y_1) = (-2, 0), (x_2, y_2) = (4, -1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{-1 - 0} = \frac{x - (-2)}{4 - (-2)}$$

$$\frac{y}{-1} = \frac{x + 2}{6}$$

Multiply each side by  $-1$ :

$$y = -1 \frac{(x + 2)}{6}$$

$$y = -\frac{1}{6} \left( x + 2 \right)$$

$$y = -\frac{1}{6}x - \frac{1}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise D, Question 10

#### Question:

The line  $V$  passes through the points  $(-5, 3)$  and  $(7, -3)$  and the line  $W$  passes through the points  $(2, -4)$  and  $(4, 2)$ . The lines  $V$  and  $W$  intersect at the point  $A$ . Work out the coordinates of the point  $A$ .

#### Solution:

(1) The equation of  $V$ :

$$(x_1, y_1) = (-5, 3), (x_2, y_2) = (7, -3)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-3 - 3} = \frac{x - (-5)}{7 - (-5)}$$

$$\frac{y - 3}{-6} = \frac{x + 5}{12}$$

Multiply each side by  $-6$ :

$$y - 3 = -\frac{1}{2} \left( x + 5 \right) \quad (\text{Note: } \frac{-6}{12} = -\frac{1}{2})$$

$$y - 3 = -\frac{1}{2}x - \frac{5}{2}$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

(2) The equation of  $W$ :

$$(x_1, y_1) = (2, -4), (x_2, y_2) = (4, 2)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-4)}{2 - (-4)} = \frac{x - 2}{4 - 2}$$

$$\frac{y + 4}{6} = \frac{x - 2}{2}$$

Multiply each side by 6:

$$y + 4 = 3(x - 2) \quad (\text{Note: } \frac{6}{2} = 3)$$

$$y + 4 = 3x - 6$$

$$y = 3x - 10$$

Solving simultaneously:

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$y = 3x - 10$$

$$\text{So } 3x - 10 = -\frac{1}{2}x + \frac{1}{2}$$

$$\Rightarrow \frac{7}{2}x - 10 = \frac{1}{2}$$

$$\Rightarrow \frac{7}{2}x = \frac{21}{2}$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = 3$$

When  $x = 3$ ,  $y = 3(3) - 10 = 9 - 10 = -1$

The lines intersect at  $A(3, -1)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 1

#### Question:

Work out if these pairs of lines are parallel, perpendicular or neither:

(a)  $y = 4x + 2$

$$y = -\frac{1}{4}x - 7$$

(b)  $y = \frac{2}{3}x - 1$

$$y = \frac{2}{3}x - 11$$

(c)  $y = \frac{1}{5}x + 9$

$$y = 5x + 9$$

(d)  $y = -3x + 2$

$$y = \frac{1}{3}x - 7$$

(e)  $y = \frac{3}{5}x + 4$

$$y = -\frac{5}{3}x - 1$$

(f)  $y = \frac{5}{7}x$

$$y = \frac{5}{7}x - 3$$

(g)  $y = 5x - 3$

$$5x - y + 4 = 0$$

(h)  $5x - y - 1 = 0$

$$y = -\frac{1}{5}x$$

(i)  $y = -\frac{3}{2}x + 8$

$$2x - 3y - 9 = 0$$

(j)  $4x - 5y + 1 = 0$

$$8x - 10y - 2 = 0$$

(k)  $3x + 2y - 12 = 0$

$$2x + 3y - 6 = 0$$

(l)  $5x - y + 2 = 0$

$$2x + 10y - 4 = 0$$

**Solution:**

(a) The gradients of the lines are 4 and  $-\frac{1}{4}$ .

$$4 \times -\frac{1}{4} = -1$$

The lines are **perpendicular**.

(b) The gradients of the lines are  $\frac{2}{3}$  and  $\frac{2}{3}$ , i.e. they have the same gradient.

The lines are **parallel**.

(c) The gradients of the lines are  $\frac{1}{5}$  and 5.

$$\frac{1}{5} \times 5 = 1$$

The lines are **neither** perpendicular nor parallel.

(d) The gradients of the lines are  $-3$  and  $\frac{1}{3}$ .

$$-3 \times \frac{1}{3} = -1$$

The lines are **perpendicular**.

(e) The gradients of the lines are  $\frac{3}{5}$  and  $-\frac{5}{3}$ .

$$\frac{3}{5} \times -\frac{5}{3} = -1$$

The lines are **perpendicular**.

(f) The gradients of the lines are  $\frac{5}{7}$  and  $\frac{5}{7}$ , i.e. they have the same gradient.

The lines are **parallel**.

(g) The gradient of  $y = 5x - 3$  is 5.

$$5x - y + 4 = 0$$

$$5x + 4 = y$$

$$y = 5x + 4$$

The gradient of  $5x - y + 4 = 0$  is 5.

The lines have the same gradient.

The lines are **parallel**.

(h)  $5x - y - 1 = 0$

$$5x - 1 = y$$

$$y = 5x - 1$$

The gradient of  $5x - y - 1 = 0$  is 5.

The gradient of  $y = -\frac{1}{5}x$  is  $-\frac{1}{5}$ .

The product of the gradients is  $5 \times -\frac{1}{5} = -1$

So the lines are **perpendicular**.

(i) The gradient of  $y = -\frac{3}{2}x + 8$  is  $-\frac{3}{2}$ .

$$2x - 3y - 9 = 0$$

$$2x - 9 = 3y$$

$$3y = 2x - 9$$

$$y = \frac{2}{3}x - 3$$

The gradient of  $2x - 3y - 9 = 0$  is  $\frac{2}{3}$ .

The product of the gradients is  $\frac{2}{3} \times -\frac{3}{2} = -1$

So the lines are **perpendicular**.

$$(j) 4x - 5y + 1 = 0$$

$$4x + 1 = 5y$$

$$5y = 4x + 1$$

$$y = \frac{4}{5}x + \frac{1}{5}$$

The gradient of  $4x - 5y + 1 = 0$  is  $\frac{4}{5}$ .

$$8x - 10y - 2 = 0$$

$$8x - 2 = 10y$$

$$10y = 8x - 2$$

$$y = \frac{8}{10}x - \frac{2}{10}$$

$$y = \frac{4}{5}x - \frac{1}{5}$$

The gradient of  $8x - 10y - 2 = 0$  is  $\frac{4}{5}$ .

The lines have the same gradient, they are **parallel**.

$$(k) 3x + 2y - 12 = 0$$

$$3x + 2y = 12$$

$$2y = -3x + 12$$

$$y = -\frac{3}{2}x + 6$$

The gradient of  $3x + 2y - 12 = 0$  is  $-\frac{3}{2}$ .

$$2x + 3y - 6 = 0$$

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

The gradient of  $2x + 3y - 6 = 0$  is  $-\frac{2}{3}$ .

The product of the gradient is

$$-\frac{3}{2} \times -\frac{2}{3} = 1$$

So the lines are **neither** parallel nor perpendicular.

$$(l) 5x - y + 2 = 0$$

$$5x + 2 = y$$

$$y = 5x + 2$$

The gradient of  $5x - y + 2 = 0$  is 5.

$$2x + 10y - 4 = 0$$

$$2x + 10y = 4$$

$$10y = -2x + 4$$

$$y = -\frac{2}{10}x + \frac{4}{10}$$

$$y = -\frac{1}{5}x + \frac{2}{5}$$

The gradient of  $2x + 10y - 4 = 0$  is  $-\frac{1}{5}$ .

The product of the gradients is

$$5 \times -\frac{1}{5} = -1$$

So the lines are **perpendicular**.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 2

**Question:**

Find an equation of the line that passes through the point  $(6, -2)$  and is perpendicular to the line  $y = 3x + 5$ .

**Solution:**

The gradient of  $y = 3x + 5$  is 3.

The gradient of a line perpendicular to  $y = 3x + 5$  is  $-\frac{1}{3}$ .

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -2 \end{pmatrix} = -\frac{1}{3} \begin{pmatrix} x - 6 \end{pmatrix}$$

$$y + 2 = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x$$

The equation of the line is  $y = -\frac{1}{3}x$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 3

**Question:**

Find an equation of the line that passes through the point  $(-2, 7)$  and is parallel to the line  $y = 4x + 1$ . Write your answer in the form  $ax + by + c = 0$ .

**Solution:**

The gradient of a line parallel to  $y = 4x + 1$  is 4.

$$y - y_1 = m (x - x_1)$$

$$y - 7 = 4 [x - (-2)]$$

$$y - 7 = 4(x + 2)$$

$$y - 7 = 4x + 8$$

$$y = 4x + 15$$

$$0 = 4x + 15 - y$$

$$4x - y + 15 = 0$$

The equation of the line is  $4x - y + 15 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 4

#### Question:

Find an equation of the line:

(a) parallel to the line  $y = -2x - 5$ , passing through  $\left(-\frac{1}{2}, \frac{3}{2}\right)$ .

(b) parallel to the line  $x - 2y - 1 = 0$ , passing through  $(0, 0)$ .

(c) perpendicular to the line  $y = x - 4$ , passing through  $(-1, -2)$ .

(d) perpendicular to the line  $2x + y - 9 = 0$ , passing through  $(4, -6)$ .

#### Solution:

(a) The gradient of a line parallel to  $y = -2x - 5$  is  $-2$ .

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = -2 \left[ x - \left(-\frac{1}{2}\right) \right]$$

$$y - \frac{3}{2} = -2 \left( x + \frac{1}{2} \right)$$

$$y - \frac{3}{2} = -2x - 1$$

$$y = -2x + \frac{1}{2}$$

(b)  $x - 2y - 1 = 0$

$$x - 1 = 2y$$

$$2y = x - 1$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

The gradient of  $x - 2y - 1 = 0$  is  $\frac{1}{2}$ .

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{2} \left( x - 0 \right)$$

$$y = \frac{1}{2}x$$

(c) The gradient of  $y = x - 4$  is 1.

The gradient of a line perpendicular to  $y = x - 4$  is  $-\frac{1}{1} = -1$ .

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -1 [ x - (-1) ]$$

$$y + 2 = -1(x + 1)$$

$$y + 2 = -x - 1$$

$$y = -x - 3$$

$$(d) 2x + y - 9 = 0$$

$$2x + y = 9$$

$$y = -2x + 9$$

The gradient of  $2x + y - 9 = 0$  is  $-2$ .

The gradient of a line perpendicular to  $2x + y - 9 = 0$  is  $-\frac{1}{-2} = \frac{1}{2}$ .

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x - 4 \end{pmatrix}$$

$$y + 6 = \frac{1}{2} \begin{pmatrix} x - 4 \end{pmatrix}$$

$$y + 6 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x - 8$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 5

#### Question:

Find an equation of the line:

- (a) parallel to the line  $y = 3x + 6$ , passing through  $(-2, 5)$ .
- (b) perpendicular to the line  $y = 3x + 6$ , passing through  $(-2, 5)$ .
- (c) parallel to the line  $4x - 6y + 7 = 0$ , passing through  $(3, 4)$ .
- (d) perpendicular to the line  $4x - 6y + 7 = 0$ , passing through  $(3, 4)$ .

#### Solution:

- (a) The gradient of a line parallel to  $y = 3x + 6$  is 3.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3[x - (-2)]$$

$$y - 5 = 3(x + 2)$$

$$y - 5 = 3x + 6$$

$$y = 3x + 11$$

- (b) The gradient of a line perpendicular to  $y = 3x + 6$  is  $-\frac{1}{3}$ .

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{3}\left[x - (-2)\right]$$

$$y - 5 = -\frac{1}{3}(x + 2)$$

$$y - 5 = -\frac{1}{3}x - \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{13}{3}$$

- (c)  $4x - 6y + 7 = 0$

$$4x + 7 = 6y$$

$$6y = 4x + 7$$

$$y = \frac{4}{6}x + \frac{7}{6}$$

$$y = \frac{2}{3}x + \frac{7}{6}$$

The gradient of a line parallel to  $4x - 6y + 7 = 0$  is  $\frac{2}{3}$ .

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x - 3)$$

$$y - 4 = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x + 2$$

(d) The gradient of the line  $4x - 6y + 7 = 0$  is  $\frac{2}{3}$  [see part (c)].

The gradient of a line perpendicular to  $4x - 6y + 7 = 0$  is  $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$ .

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{3}{2}(x - 3)$$

$$y - 4 = -\frac{3}{2}x + \frac{9}{2}$$

$$y = -\frac{3}{2}x + \frac{17}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 6

#### Question:

Find an equation of the line that passes through the point  $(5, -5)$  and is perpendicular to the line  $y = \frac{2}{3}x + 5$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

The gradient of a line perpendicular to  $y = \frac{2}{3}x + 5$  is  $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$ .

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -5 \end{pmatrix} = -\frac{3}{2} \begin{pmatrix} x - 5 \end{pmatrix}$$

$$y + 5 = -\frac{3}{2} \begin{pmatrix} x - 5 \end{pmatrix}$$

Multiply each term by 2:

$$2y + 10 = -3(x - 5)$$

$$2y + 10 = -3x + 15$$

$$3x + 2y + 10 = 15$$

$$3x + 2y - 5 = 0$$

The equation of the line is  $3x + 2y - 5 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 7

#### Question:

Find an equation of the line that passes through the point  $(-2, -3)$  and is perpendicular to the line  $y = -\frac{4}{7}x + 5$ .

Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

The gradient of a line perpendicular to  $y = -\frac{4}{7}x + 5$  is  $-\frac{1}{-\frac{4}{7}} = \frac{7}{4}$ .

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -3 \end{pmatrix} = \frac{7}{4} \left[ x - \begin{pmatrix} -2 \end{pmatrix} \right]$$

$$y + 3 = \frac{7}{4} \begin{pmatrix} x + 2 \end{pmatrix}$$

Multiply each term by 4:

$$4y + 12 = 7(x + 2)$$

$$4y + 12 = 7x + 14$$

$$4y = 7x + 2$$

$$0 = 7x + 2 - 4y$$

$$7x - 4y + 2 = 0$$

The equation of the line is  $7x - 4y + 2 = 0$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 8

#### Question:

The line  $r$  passes through the points  $(1, 4)$  and  $(6, 8)$  and the line  $s$  passes through the points  $(5, -3)$  and  $(20, 9)$ . Show that the lines  $r$  and  $s$  are parallel.

#### Solution:

The gradient of  $r$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{6 - 1} = \frac{4}{5}$$

The gradient of  $s$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{20 - 5} = \frac{12}{15} = \frac{4}{5}$$

The gradients are equal, so the lines are **parallel**.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 9

#### Question:

The line  $l$  passes through the points  $(-3, 0)$  and  $(3, -2)$  and the line  $n$  passes through the points  $(1, 8)$  and  $(-1, 2)$ . Show that the lines  $l$  and  $n$  are perpendicular.

#### Solution:

The gradient of  $l$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - (-3)} = -\frac{2}{6} = -\frac{1}{3}$$

The gradient of  $n$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-1 - 1} = \frac{-6}{-2} = 3$$

The product of the gradients is

$$-\frac{1}{3} \times 3 = -1$$

So the lines are **perpendicular**.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

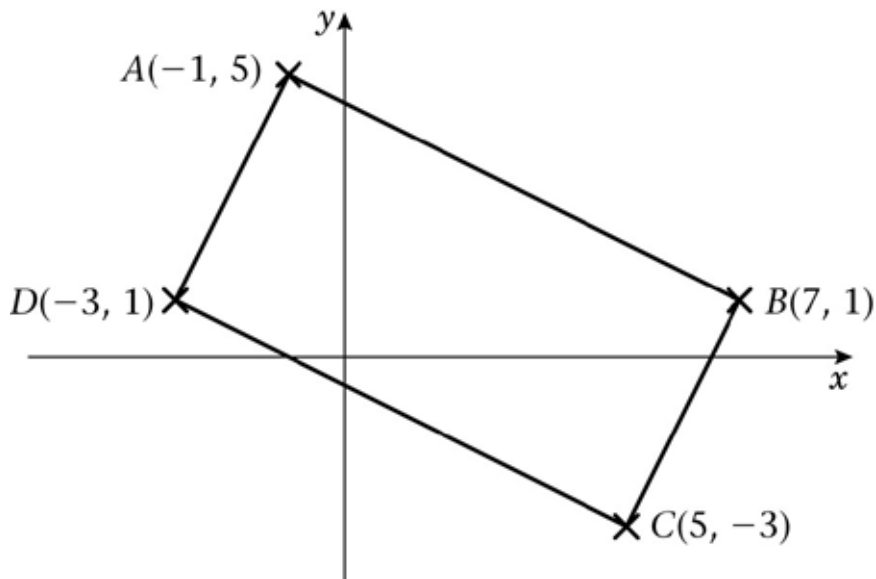
### Coordinate geometry in the (x, y) plane

#### Exercise E, Question 10

#### Question:

The vertices of a quadrilateral  $ABCD$  has coordinates  $A(-1, 5)$ ,  $B(7, 1)$ ,  $C(5, -3)$ ,  $D(-3, 1)$ . Show that the quadrilateral is a rectangle.

#### Solution:



(1) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - 7} = \frac{4}{-8} = -\frac{1}{2}$$

(2) The gradient of  $DC$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - (-3)} = -\frac{4}{8} = -\frac{1}{2}$$

The gradient of  $AB$  is the same as the gradient of  $DC$ , so the lines are parallel.

(3) The gradient of  $AD$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - (-3)} = \frac{4}{-1 + 3} = \frac{4}{2} = 2$$

(4) The gradient of  $BC$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - 7} = \frac{-4}{-2} = 2$$

The gradient of  $AD$  is the same as the gradient of  $BC$ , so the lines are parallel.

The line  $AD$  is perpendicular to the line  $AB$  as

$$2 \times -\frac{1}{2} = -1$$

So  $ABCD$  is a rectangle.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 1

#### Question:

The points  $A$  and  $B$  have coordinates  $(-4, 6)$  and  $(2, 8)$  respectively. A line  $p$  is drawn through  $B$  perpendicular to  $AB$  to meet the  $y$ -axis at the point  $C$ .

- (a) Find an equation of the line  $p$ .
- (b) Determine the coordinates of  $C$ . **[E]**

#### Solution:

- (a) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{2 - (-4)} = \frac{2}{6} = \frac{1}{3}$$

The gradient of a line perpendicular to  $AB$  is

$$-\frac{1}{\frac{1}{3}} = -3$$

The equation of  $p$  is

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -3(x - 2)$$

$$y - 8 = -3x + 6$$

$$y = -3x + 14$$

- (b) Substitute  $x = 0$ :

$$y = -3(0) + 14 = 14$$

The coordinates of  $C$  are  $(0, 14)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 2

#### Question:

The line  $l$  has equation  $2x - y - 1 = 0$ .

The line  $m$  passes through the point  $A(0, 4)$  and is perpendicular to the line  $l$ .

(a) Find an equation of  $m$  and show that the lines  $l$  and  $m$  intersect at the point  $P(2, 3)$ .  
The line  $n$  passes through the point  $B(3, 0)$  and is parallel to the line  $m$ .

(b) Find an equation of  $n$  and hence find the coordinates of the point  $Q$  where the lines  $l$  and  $n$  intersect. **[E]**

#### Solution:

$$(a) 2x - y - 1 = 0$$

$$2x - 1 = y$$

$$y = 2x - 1$$

The gradient of  $2x - y - 1 = 0$  is 2.

The gradient of a line perpendicular to  $2x - y - 1 = 0$  is  $-\frac{1}{2}$ .

The equation of the line  $m$  is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2}(x - 0)$$

$$y - 4 = -\frac{1}{2}x$$

$$y = -\frac{1}{2}x + 4$$

To find  $P$  solve  $y = -\frac{1}{2}x + 4$  and  $2x - y - 1 = 0$  simultaneously.

Substitute:

$$2x - \left(-\frac{1}{2}x + 4\right) - 1 = 0$$

$$2x + \frac{1}{2}x - 4 - 1 = 0$$

$$\frac{5}{2}x - 5 = 0$$

$$\frac{5}{2}x = 5$$

$$5x = 10$$

$$x = 2$$

Substitute  $x = 2$  into  $y = -\frac{1}{2}x + 4$ :

$$y = -\frac{1}{2}\left(2\right) + 4 = -1 + 4 = 3$$

The lines intersect at  $P(2, 3)$ , as required.

(b) A line parallel to the line  $m$  has gradient  $-\frac{1}{2}$ .

The equation of the line  $n$  is

$$y - y_1 = m (x - x_1)$$

$$y - 0 = -\frac{1}{2} \left( x - 3 \right)$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

To find  $Q$  solve  $2x - y - 1 = 0$  and  $y = -\frac{1}{2}x + \frac{3}{2}$  simultaneously.

Substitute:

$$2x - \left( -\frac{1}{2}x + \frac{3}{2} \right) - 1 = 0$$

$$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$$

$$\frac{5}{2}x - \frac{5}{2} = 0$$

$$\frac{5}{2}x = \frac{5}{2}$$

$$x = 1$$

Substitute  $x = 1$  into  $y = -\frac{1}{2}x + \frac{3}{2}$ :

$$y = -\frac{1}{2} \left( 1 \right) + \frac{3}{2} = -\frac{1}{2} + \frac{3}{2} = 1$$

The lines intersect at  $Q ( 1 , 1 )$  .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 3

#### Question:

The line  $L_1$  has gradient  $\frac{1}{7}$  and passes through the point  $A(2, 2)$ . The line  $L_2$  has gradient  $-1$  and passes through the point  $B(4, 8)$ . The lines  $L_1$  and  $L_2$  intersect at the point  $C$ .

(a) Find an equation for  $L_1$  and an equation for  $L_2$ .

(b) Determine the coordinates of  $C$ . **[E]**

#### Solution:

(a) The equation of  $L_1$  is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{7} \left( x - 2 \right)$$

$$y - 2 = \frac{1}{7}x - \frac{2}{7}$$

$$y = \frac{1}{7}x + \frac{12}{7}$$

The equation of  $L_2$  is

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -1(x - 4)$$

$$y - 8 = -x + 4$$

$$y = -x + 12$$

(b) Solve  $y = \frac{1}{7}x + \frac{12}{7}$  and  $y = -x + 12$  simultaneously.

Substitute:

$$-x + 12 = \frac{1}{7}x + \frac{12}{7}$$

$$12 = \frac{8}{7}x + \frac{12}{7}$$

$$10 \frac{2}{7} = \frac{8}{7}x$$

$$x = \frac{10 \frac{2}{7}}{\frac{8}{7}} = 9$$

Substitute  $x = 9$  into  $y = -x + 12$ :

$$y = -9 + 12 = 3$$

The lines intersect at  $C(9, 3)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 4

#### Question:

The straight line passing through the point  $P(2, 1)$  and the point  $Q(k, 11)$  has gradient  $-\frac{5}{12}$ .

(a) Find the equation of the line in terms of  $x$  and  $y$  only.

(b) Determine the value of  $k$ . **[E]**

#### Solution:

$$(a) m = -\frac{5}{12}, (x_1, y_1) = (2, 1)$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{5}{12} \left( x - 2 \right)$$

$$y - 1 = -\frac{5}{12}x + \frac{5}{6}$$

$$y = -\frac{5}{12}x + \frac{11}{6}$$

(b) Substitute  $(k, 11)$  into  $y = -\frac{5}{12}x + \frac{11}{6}$ :

$$11 = -\frac{5}{12}k + \frac{11}{6}$$

$$11 - \frac{11}{6} = -\frac{5}{12}k$$

$$\frac{55}{6} = -\frac{5}{12}k$$

Multiply each side by 12:

$$110 = -5k$$

$$k = -22$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 5

#### Question:

(a) Find an equation of the line  $l$  which passes through the points  $A(1, 0)$  and  $B(5, 6)$ .  
The line  $m$  with equation  $2x + 3y = 15$  meets  $l$  at the point  $C$ .

(b) Determine the coordinates of the point  $C$ . **[E]**

#### Solution:

(a) The equation of  $l$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{6 - 0} = \frac{x - 1}{5 - 1}$$

$$\frac{y}{6} = \frac{x - 1}{4}$$

Multiply each side by 6:

$$y = 6 \frac{(x - 1)}{4}$$

$$y = \frac{3}{2} (x - 1)$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

(b) Solve  $2x + 3y = 15$  and  $y = \frac{3}{2}x - \frac{3}{2}$  simultaneously.

Substitute:

$$2x + 3 \left( \frac{3}{2}x - \frac{3}{2} \right) = 15$$

$$2x + \frac{9}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x = \frac{39}{2}$$

$$13x = 39$$

$$x = 3$$

Substitute  $x = 3$  into  $y = \frac{3}{2}x - \frac{3}{2}$ :

$$y = \frac{3}{2} \left( 3 \right) - \frac{3}{2} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3$$

The coordinates of  $C$  are  $(3, 3)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 6

**Question:**

The line  $L$  passes through the points  $A ( 1 , 3 )$  and  $B ( - 19 , - 19 )$  .

Find an equation of  $L$  in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. **[E]**

**Solution:**

$$(x_1, y_1) = (1, 3), (x_2, y_2) = (-19, -19)$$

The equation of  $L$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-19 - 3} = \frac{x - 1}{-19 - 1}$$

$$\frac{y - 3}{-22} = \frac{x - 1}{-20}$$

Multiply each side by  $-22$ :

$$y - 3 = \frac{-22}{-20} (x - 1)$$

$$y - 3 = \frac{11}{10} (x - 1)$$

Multiply each term by 10:

$$10y - 30 = 11(x - 1)$$

$$10y - 30 = 11x - 11$$

$$10y = 11x + 19$$

$$0 = 11x - 10y + 19$$

The equation of  $L$  is  $11x - 10y + 19 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 7

#### Question:

The straight line  $l_1$  passes through the points  $A$  and  $B$  with coordinates  $(2, 2)$  and  $(6, 0)$  respectively.

(a) Find an equation of  $l_1$ .

The straight line  $l_2$  passes through the point  $C$  with coordinates  $(-9, 0)$  and has gradient  $\frac{1}{4}$ .

(b) Find an equation of  $l_2$ . **[E]**

#### Solution:

(a) The equation of  $l_1$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{0 - 2} = \frac{x - 2}{6 - 2}$$

$$\frac{y - 2}{-2} = \frac{x - 2}{4}$$

Multiply each side by  $-2$ :

$$y - 2 = -\frac{1}{2} \left( x - 2 \right) \quad (\text{Note: } -\frac{2}{4} = -\frac{1}{2})$$

$$y - 2 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 3$$

(b) The equation of  $l_2$  is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{4} \left[ x - \left( -9 \right) \right]$$

$$y = \frac{1}{4} \left( x + 9 \right)$$

$$y = \frac{1}{4}x + \frac{9}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 8

#### Question:

The straight line  $l_1$  passes through the points  $A$  and  $B$  with coordinates  $(0, -2)$  and  $(6, 7)$  respectively.

(a) Find the equation of  $l_1$  in the form  $y = mx + c$ .

The straight line  $l_2$  with equation  $x + y = 8$  cuts the  $y$ -axis at the point  $C$ . The lines  $l_1$  and  $l_2$  intersect at the point  $D$ .

(b) Calculate the coordinates of the point  $D$ .

(c) Calculate the area of  $\triangle ACD$ . **[E]**

#### Solution:

(a) The equation of  $l_1$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-2)}{7 - (-2)} = \frac{x - 0}{6 - 0}$$

$$\frac{y + 2}{9} = \frac{x}{6}$$

Multiply each term by 9:

$$y + 2 = \frac{9}{6}x$$

$$y + 2 = \frac{3}{2}x$$

$$y = \frac{3}{2}x - 2$$

(b) Solve  $x + y = 8$  and  $y = \frac{3}{2}x - 2$  simultaneously.

Substitute:

$$x + \left( \frac{3}{2}x - 2 \right) = 8$$

$$x + \frac{3}{2}x - 2 = 8$$

$$\frac{5}{2}x - 2 = 8$$

$$\frac{5}{2}x = 10$$

$$5x = 20$$

$$x = 4$$

Substitute  $x = 4$  into  $x + y = 8$ :

$$(4) + y = 8$$

$$y = 4$$

The coordinates of  $D$  are  $(4, 4)$ .

(c)  $x + y = 8$  cuts the  $y$ -axis when  $x = 0$ .

Substitute  $x = 0$ :

$$0 + y = 8$$

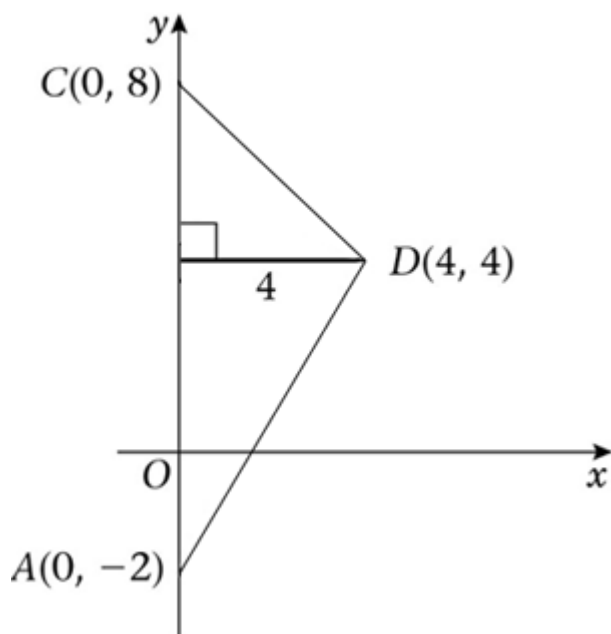
$$y = 8$$

The coordinates of  $C$  are  $(0, 8)$

$$AC = 10$$

$$h = 4$$

$$\text{Area} = \frac{1}{2} \times 10 \times 4 = 20$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 9

#### Question:

The points  $A$  and  $B$  have coordinates  $(2, 16)$  and  $(12, -4)$  respectively. A straight line  $l_1$  passes through  $A$  and  $B$ .

(a) Find an equation for  $l_1$  in the form  $ax + by = c$ .

The line  $l_2$  passes through the point  $C$  with coordinates  $(-1, 1)$  and has gradient  $\frac{1}{3}$ .

(b) Find an equation for  $l_2$ . **[E]**

#### Solution:

(a) The equation of  $l_1$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}$$

$$\frac{y - 16}{-20} = \frac{x - 2}{10}$$

Multiply each side by  $-20$ :

$$y - 16 = -2(x - 2) \quad (\text{Note: } -\frac{20}{10} = -2)$$

$$y - 16 = -2x + 4$$

$$y = -2x + 20$$

$$2x + y = 20$$

(b) The equation of  $l_2$  is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3} \left[ x - \left( -1 \right) \right]$$

$$y - 1 = \frac{1}{3} (x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 10

#### Question:

The points  $A(-1, -2)$ ,  $B(7, 2)$  and  $C(k, 4)$ , where  $k$  is a constant, are the vertices of  $\triangle ABC$ . Angle  $ABC$  is a right angle.

(a) Find the gradient of  $AB$ .

(b) Calculate the value of  $k$ .

(c) Find an equation of the straight line passing through  $B$  and  $C$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. **[E]**

#### Solution:

(a) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{7 - (-1)} = \frac{4}{8} = \frac{1}{2}$$

(b) The gradient of  $BC$  is

$$\frac{-1}{\frac{1}{2}} = -2$$

$$\text{So } \frac{y_2 - y_1}{x_2 - x_1} = -2$$

$$\Rightarrow \frac{4 - 2}{k - 7} = -2$$

$$\Rightarrow \frac{2}{k - 7} = -2$$

Multiply each side by  $(k - 7)$  :

$$2 = -2(k - 7)$$

$$2 = -2k + 14$$

$$-12 = -2k$$

$$k = 6$$

(c) The equation of the line passing through  $B$  and  $C$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{4 - 2} = \frac{x - 7}{6 - 7}$$

$$\frac{y - 2}{2} = \frac{x - 7}{-1}$$

Multiply each side by 2:

$$y - 2 = -2(x - 7) \quad (\text{Note: } \frac{2}{-1} = -2)$$

$$y - 2 = -2x + 14$$

$$y = -2x + 16$$

$$2x + y = 16$$

$$2x + y - 16 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 11

#### Question:

The straight line  $l$  passes through  $A ( 1 , 3 \sqrt{3} )$  and  $B ( 2 + \sqrt{3} , 3 + 4 \sqrt{3} )$ .

- (a) Calculate the gradient of  $l$  giving your answer as a surd in its simplest form.
- (b) Give the equation of  $l$  in the form  $y = mx + c$ , where constants  $m$  and  $c$  are surds given in their simplest form.
- (c) Show that  $l$  meets the  $x$ -axis at the point  $C ( -2 , 0 )$ . **[E]**

#### Solution:

(a) The gradient of  $l$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(3 + 4\sqrt{3}) - 3\sqrt{3}}{(2 + \sqrt{3}) - 1} = \frac{3 + \sqrt{3}}{1 + \sqrt{3}}$$

Rationalise the denominator:

$$\frac{3 + \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{3 - 3\sqrt{3} + \sqrt{3} - 3}{1 - 3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

(b) The equation of  $l$  is

$$\begin{aligned} y - y_1 &= m ( x - x_1 ) \\ y - 3\sqrt{3} &= \sqrt{3} ( x - 1 ) \\ y - 3\sqrt{3} &= \sqrt{3}x - \sqrt{3} \\ y &= \sqrt{3}x + 2\sqrt{3} \end{aligned}$$

(c) Substitute  $y = 0$ :

$$\begin{aligned} 0 &= \sqrt{3}x + 2\sqrt{3} \\ \sqrt{3}x &= -2\sqrt{3} \\ x &= \frac{-2\sqrt{3}}{\sqrt{3}} = -2 \end{aligned}$$

The coordinates of  $C$  are  $( -2 , 0 )$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 12

#### Question:

(a) Find an equation of the straight line passing through the points with coordinates  $(-1, 5)$  and  $(4, -2)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers.  
The line crosses the  $x$ -axis at the point  $A$  and the  $y$ -axis at the point  $B$ , and  $O$  is the origin.

(b) Find the area of  $\triangle OAB$ . **[E]**

#### Solution:

(a) The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-2 - 5} = \frac{x - (-1)}{4 - (-1)}$$

$$\frac{y - 5}{-7} = \frac{x + 1}{5}$$

Multiply each side by  $-35$ :

$$5(y - 5) = -7(x + 1) \quad (\text{Note: } \frac{-35}{-7} = 5 \text{ and } \frac{-35}{5} = -7)$$

$$5y - 25 = -7x - 7$$

$$7x + 5y - 25 = -7$$

$$7x + 5y - 18 = 0$$

(b) For the coordinates of  $A$  substitute  $y = 0$ :

$$7x + 5(0) - 18 = 0$$

$$7x - 18 = 0$$

$$7x = 18$$

$$x = \frac{18}{7}$$

The coordinates of  $A$  are  $\left(\frac{18}{7}, 0\right)$ .

For the coordinates of  $B$  substitute  $x = 0$ :

$$7(0) + 5y - 18 = 0$$

$$5y - 18 = 0$$

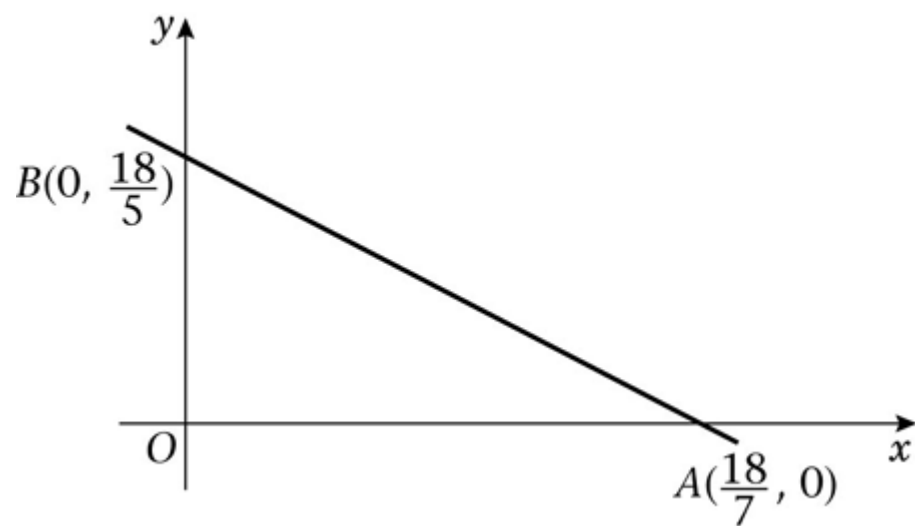
$$5y = 18$$

$$y = \frac{18}{5}$$

The coordinates of  $B$  are  $\left(0, \frac{18}{5}\right)$ .

The area of  $\triangle OAB$  is

$$\frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{162}{35}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 13

#### Question:

The points  $A$  and  $B$  have coordinates  $(k, 1)$  and  $(8, 2k - 1)$  respectively, where  $k$  is a constant. Given that the gradient of  $AB$  is  $\frac{1}{3}$ ,

(a) Show that  $k = 2$ .

(b) Find an equation for the line through  $A$  and  $B$ . **[E]**

#### Solution:

(a) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

$$\frac{(2k - 1) - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 1 - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 2}{8 - k} = \frac{1}{3}$$

Multiply each side by  $(8 - k)$  :

$$2k - 2 = \frac{1}{3} (8 - k)$$

Multiply each term by 3:

$$6k - 6 = 8 - k$$

$$7k - 6 = 8$$

$$7k = 14$$

$$k = 2$$

(b)  $k = 2$

So  $A$  and  $B$  have coordinates  $(2, 1)$  and  $(8, 3)$ .

The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{3 - 1} = \frac{x - 2}{8 - 2}$$

$$\frac{y - 1}{2} = \frac{x - 2}{6}$$

Multiply each side by 2:

$$y - 1 = \frac{1}{3} (x - 2)$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 14

#### Question:

The straight line  $l_1$  has equation  $4y + x = 0$ .

The straight line  $l_2$  has equation  $y = 2x - 3$ .

(a) On the same axes, sketch the graphs of  $l_1$  and  $l_2$ . Show clearly the coordinates of all points at which the graphs meet the coordinate axes.

The lines  $l_1$  and  $l_2$  intersect at the point  $A$ .

(b) Calculate, as exact fractions, the coordinates of  $A$ .

(c) Find an equation of the line through  $A$  which is perpendicular to  $l_1$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. **[E]**

#### Solution:

(a) (1) Rearrange  $4y + x = 0$  into the form  $y = mx + c$ :

$$4y = -x$$

$$y = -\frac{1}{4}x$$

$l_1$  has gradient  $-\frac{1}{4}$  and it meets the coordinate axes at  $(0, 0)$ .

(2)  $l_2$  has gradient 2 and it meets the  $y$ -axis at  $(0, -3)$ .

$l_2$  meets the  $x$ -axis when  $y = 0$ .

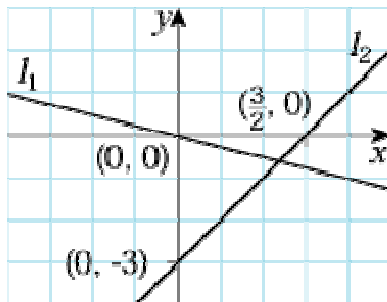
Substitute  $y = 0$ :

$$0 = 2x - 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$l_2$  meets the  $x$ -axis at  $\left(\frac{3}{2}, 0\right)$ .



(b) Solve  $4y + x = 0$  and  $y = 2x - 3$  simultaneously.

Substitute:

$$4(2x - 3) + x = 0$$

$$8x - 12 + x = 0$$

$$9x - 12 = 0$$

$$9x = 12$$

$$x = \frac{12}{9}$$

$$x = \frac{4}{3}$$

Substitute  $x = \frac{4}{3}$  into  $y = 2x - 3$ :

$$y = 2 \left( \frac{4}{3} \right) - 3 = \frac{8}{3} - 3 = -\frac{1}{3}$$

The coordinates of A are  $\left( \frac{4}{3}, -\frac{1}{3} \right)$ .

(c) The gradient of  $l_1$  is  $-\frac{1}{4}$ .

The gradient of a line perpendicular to  $l_1$  is  $-\frac{1}{-\frac{1}{4}} = 4$ .

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - \left( -\frac{1}{3} \right) = 4 \left( x - \frac{4}{3} \right)$$

$$y + \frac{1}{3} = 4x - \frac{16}{3}$$

$$y = 4x - \frac{17}{3}$$

Multiply each term by 3:

$$3y = 12x - 17$$

$$0 = 12x - 3y - 17$$

The equation of the line is  $12x - 3y - 17 = 0$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x, y) plane

#### Exercise F, Question 15

#### Question:

The points  $A$  and  $B$  have coordinates  $(4, 6)$  and  $(12, 2)$  respectively.

The straight line  $l_1$  passes through  $A$  and  $B$ .

(a) Find an equation for  $l_1$  in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

The straight line  $l_2$  passes through the origin and has gradient  $-4$ .

(b) Write down an equation for  $l_2$ .

The lines  $l_1$  and  $l_2$  intersect at the point  $C$ .

(c) Find the coordinates of  $C$ . **[E]**

#### Solution:

(a) The equation of  $l_1$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 6}{2 - 6} = \frac{x - 4}{12 - 4}$$

$$\frac{y - 6}{-4} = \frac{x - 4}{8}$$

Multiply each side by 8:

$$-2(y - 6) = x - 4 \quad (\text{Note: } \frac{8}{-4} = -2)$$

$$-2y + 12 = x - 4$$

$$-2y + 16 = x$$

$$16 = x + 2y$$

$$0 = x + 2y - 16$$

The equation of the line is  $x + 2y - 16 = 0$

(b) The equation of  $l_2$  is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - 0)$$

$$y = -4x$$

(c) Solve  $y = -4x$  and  $x + 2y = 16$  simultaneously.

Substitute:

$$x + 2(-4x) = 16$$

$$x - 8x = 16$$

$$-7x = 16$$

$$x = \frac{16}{-7}$$

$$x = -\frac{16}{7}$$

Substitute  $x = -\frac{16}{7}$  in  $y = -4x$ :

$$y = -4 \left( -\frac{16}{7} \right) = \frac{64}{7}$$

The coordinates of  $C$  are  $\left( -\frac{16}{7}, \frac{64}{7} \right)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

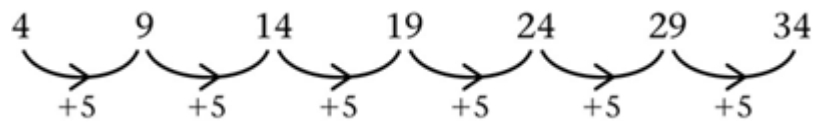
#### Exercise A, Question 1

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 9, 14, 19, ...

#### Solution:



“Add 5 to previous term”



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

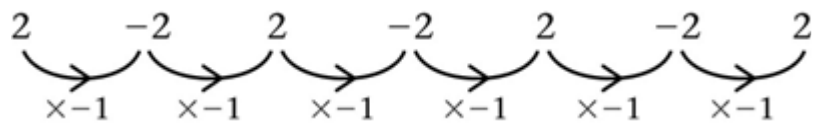
### Sequences and series

#### Exercise A, Question 2

**Question:**

Work out the next three terms of the following sequence. State the rule to find the next term:

2, -2, 2, -2, ...

**Solution:**

“Multiply previous term by  $-1$ ”

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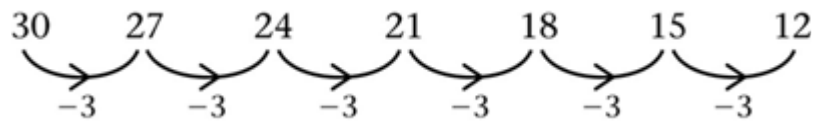
### Sequences and series

#### Exercise A, Question 3

**Question:**

Work out the next three terms of the following sequence. State the rule to find the next term:

30, 27, 24, 21, ...

**Solution:**

“Subtract 3 from previous term”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 4

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

2, 6, 18, 54, ...

#### Solution:



“Multiply previous term by 3”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 5

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$$4, -2, 1, -\frac{1}{2}, \dots$$

#### Solution:

$$\begin{array}{cccccccc} 4 & & -2 & & 1 & & -\frac{1}{2} & & +\frac{1}{4} & & -\frac{1}{8} & & +\frac{1}{16} \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright \\ \times -\frac{1}{2} & & \times -\frac{1}{2} & & \times -\frac{1}{2} & & \times -\frac{1}{2} & & \times -\frac{1}{2} & & \times -\frac{1}{2} & & \times -\frac{1}{2} \end{array}$$

“Multiply previous term by  $-\frac{1}{2}$ ” (or “divide by  $-2$ ”)

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 6

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 2, 5, 14, ...

#### Solution:

$$\begin{array}{cccccccc} 1 & & 2 & & 5 & & 14 & & 41 & & 122 & & 365 \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright \\ \times 3 - 1 & & \times 3 - 1 & & \times 3 - 1 & & \times 3 - 1 & & \times 3 - 1 & & \times 3 - 1 & & \times 3 - 1 \end{array}$$

“Multiply previous term by 3 then subtract 1”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 7

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 1, 2, 3, 5, ...

#### Solution:

$$\begin{array}{ccccccc} 1 & & 1 & & 2 & & 3 & & 5 & & 8 & & 13 & & 21 \\ & & \nearrow & & & & \nearrow & & \nearrow & & & & \nearrow & & \\ 1 + 1 = & & & & 3 + 5 = & & 8 + 13 = & & & & & & & & \end{array}$$

“Add together the two previous terms”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 8

**Question:**

Work out the next three terms of the following sequence. State the rule to find the next term:

$$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \dots$$

**Solution:**

$$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}$$

“Add 1 to previous numerator, 2 to previous denominator”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise A, Question 9

#### Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 3, 2.5, 2.25, 2.125, ...

#### Solution:

$$\begin{array}{cccccccc} 4 & & 3 & & 2.5 & & 2.25 & & 2.125 & & 2.0625 & & 2.03125 & & 2.015625 \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright \\ \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 & & \div 2 + 1 \end{array}$$

“Divide previous term by 2 then add 1”



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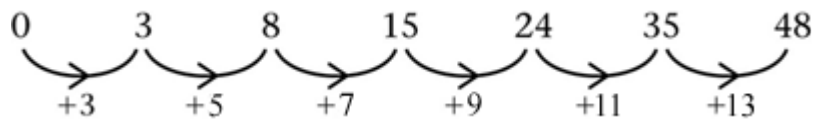
### Sequences and series

#### Exercise A, Question 10

**Question:**

Work out the next three terms of the following sequence. State the rule to find the next term:

0, 3, 8, 15, ...

**Solution:**

“Add consecutive odd numbers to previous term”

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 1

#### Question:

Find the  $U_1$ ,  $U_2$ ,  $U_3$  and  $U_{10}$  of the following sequences, where:

(a)  $U_n = 3n + 2$

(b)  $U_n = 10 - 3n$

(c)  $U_n = n^2 + 5$

(d)  $U_n = (n - 3)^2$

(e)  $U_n = (-2)^n$

(f)  $U_n = \frac{n}{n+2}$

(g)  $U_n = (-1)^n \frac{n}{n+2}$

(h)  $U_n = (n - 2)^3$

#### Solution:

(a)  $U_1 = 3 \times 1 + 2 = 5$ ,  $U_2 = 3 \times 2 + 2 = 8$ ,  $U_3 = 3 \times 3 + 2 = 11$ ,  $U_{10} = 3 \times 10 + 2 = 32$

(b)  $U_1 = 10 - 3 \times 1 = 7$ ,  $U_2 = 10 - 3 \times 2 = 4$ ,  $U_3 = 10 - 3 \times 3 = 1$ ,  $U_{10} = 10 - 3 \times 10 = -20$

(c)  $U_1 = 1^2 + 5 = 6$ ,  $U_2 = 2^2 + 5 = 9$ ,  $U_3 = 3^2 + 5 = 14$ ,  $U_{10} = 10^2 + 5 = 105$

(d)  $U_1 = (1 - 3)^2 = 4$ ,  $U_2 = (2 - 3)^2 = 1$ ,  $U_3 = (3 - 3)^2 = 0$ ,  $U_{10} = (10 - 3)^2 = 49$

(e)  $U_1 = (-2)^1 = -2$ ,  $U_2 = (-2)^2 = 4$ ,  $U_3 = (-2)^3 = -8$ ,  $U_{10} = (-2)^{10} = 1024$

(f)  $U_1 = \frac{1}{1+2} = \frac{1}{3}$ ,  $U_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$ ,  $U_3 = \frac{3}{3+2} = \frac{3}{5}$ ,  $U_{10} = \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(g)  $U_1 = (-1)^1 \frac{1}{1+2} = -\frac{1}{3}$ ,  $U_2 = (-1)^2 \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$ ,  $U_3 = (-1)^3 \frac{3}{3+2} = -\frac{3}{5}$ ,  $U_{10} = (-1)^{10} \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(h)  $U_1 = (1 - 2)^3 = (-1)^3 = -1$ ,  $U_2 = (2 - 2)^3 = 0$ ,  $U_3 = (3 - 2)^3 = 1$ ,  $U_{10} = (10 - 2)^3 = 8^3 = 512$

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 2

#### Question:

Find the value of  $n$  for which  $U_n$  has the given value:

(a)  $U_n = 2n - 4$ ,  $U_n = 24$

(b)  $U_n = (n - 4)^2$ ,  $U_n = 25$

(c)  $U_n = n^2 - 9$ ,  $U_n = 112$

(d)  $U_n = \frac{2n+1}{n-3}$ ,  $U_n = \frac{19}{6}$

(e)  $U_n = n^2 + 5n - 6$ ,  $U_n = 60$

(f)  $U_n = n^2 - 4n + 11$ ,  $U_n = 56$

(g)  $U_n = n^2 + 4n - 5$ ,  $U_n = 91$

(h)  $U_n = (-1)^n \frac{n}{n+4}$ ,  $U_n = \frac{7}{9}$

(i)  $U_n = \frac{n^3+3}{5}$ ,  $U_n = 13.4$

(j)  $U_n = \frac{n^3}{5} + 3$ ,  $U_n = 28$

#### Solution:

(a)  $24 = 2n - 4$   
 $28 = 2n \quad ( + 4 )$   
 $14 = n \quad ( \div 2 )$   
 $n = 14$

(b)  $25 = (n - 4)^2$   
 $\pm 5 = (n - 4) \quad ( \sqrt{\quad} )$   
 $9, -1 = n \quad ( + 4 )$   
 $n = 9 \quad ( \text{it must be positive} )$

(c)  $112 = n^2 - 9$   
 $121 = n^2 \quad ( + 9 )$   
 $\pm 11 = n \quad ( \sqrt{\quad} )$   
 $n = 11$

(d)  $\frac{19}{6} = \frac{2n+1}{n-3} \quad ( \text{cross multiply} )$

$19(n - 3) = 6(2n + 1)$

$$19n - 57 = 12n + 6 \quad ( - 12n )$$

$$7n - 57 = 6 \quad ( + 57 )$$

$$7n = 63$$

$$n = 9$$

(e)  $60 = n^2 + 5n - 6 \quad ( - 60 )$

$$0 = n^2 + 5n - 66 \quad (\text{factorise})$$

$$0 = (n + 11) (n - 6)$$

$$n = -11, 6$$

$$n = 6$$

(f)  $56 = n^2 - 4n + 11 \quad ( - 56 )$

$$0 = n^2 - 4n - 45 \quad (\text{factorise})$$

$$0 = (n - 9) (n + 5)$$

$$n = 9, -5$$

$$n = 9$$

(g)  $91 = n^2 + 4n - 5 \quad ( - 91 )$

$$0 = n^2 + 4n - 96 \quad (\text{factorise})$$

$$0 = (n + 12) (n - 8)$$

$$n = -12, 8$$

$$n = 8$$

(h)  $\frac{7}{9} = (-1)^n \frac{n}{n+4}$

$n$  must be even

$$\frac{7}{9} = \frac{n}{n+4}$$

$$7(n+4) = 9n$$

$$7n + 28 = 9n$$

$$28 = 2n$$

$$n = 14$$

(i)  $13.4 = \frac{n^3 + 3}{5} \quad ( \times 5 )$

$$67 = n^3 + 3 \quad ( - 3 )$$

$$64 = n^3 \quad ( \sqrt[3]{ } )$$

$$n = 4$$

(j)  $28 = \frac{n^3}{5} + 3 \quad ( - 3 )$

$$25 = \frac{n^3}{5} \quad ( \times 5 )$$

$$125 = n^3 \quad ( \sqrt[3]{ } )$$

$$n = 5$$

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 3

#### Question:

Prove that the  $(2n + 1)$  th term of the sequence  $U_n = n^2 - 1$  is a multiple of 4.

#### Solution:

$$\begin{aligned} & (2n + 1) \text{ th term} \\ &= (2n + 1)^2 - 1 \\ &= (2n + 1)(2n + 1) - 1 \\ &= 4n^2 + 4n + 1 - 1 \\ &= 4n^2 + 4n \\ &= 4n(n + 1) \\ &= 4 \times n(n + 1) \\ &= \text{multiple of 4 because it is } 4 \times \text{ whole number.} \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series Exercise B, Question 4

#### Question:

Prove that the terms of the sequence  $U_n = n^2 - 10n + 27$  are all positive. For what value of  $n$  is  $U_n$  smallest?

#### Solution:

$$U_n = n^2 - 10n + 27 = (n - 5)^2 - 25 + 27 = (n - 5)^2 + 2$$

$(n - 5)^2$  is always positive (or zero) because it is a square.

$$\therefore U_n \geq 0 + 2$$

Smallest value of  $U_n$  is 2.

(It occurs when  $n = 5$ .)

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 5

#### Question:

A sequence is generated according to the formula  $U_n = an + b$ , where  $a$  and  $b$  are constants. Given that  $U_3 = 14$  and  $U_5 = 38$ , find the values of  $a$  and  $b$ .

#### Solution:

$$U_n = an + b$$

$$\text{when } n = 3, U_3 = 14 \Rightarrow 14 = 3a + b \text{ ①}$$

$$\text{when } n = 5, U_5 = 38 \Rightarrow 38 = 5a + b \text{ ②}$$

$$\text{②} - \text{①}: 24 = 2a \Rightarrow a = 12$$

$$\text{substitute } a = 12 \text{ in ①: } 14 = 3 \times 12 + b \Rightarrow 14 = 36 + b \Rightarrow b = -22$$

$$\therefore U_n = 12n - 22$$

$$(\text{check: when } n = 3, U_3 = 12 \times 3 - 22 = 36 - 22 = 14 \checkmark)$$

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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 6

#### Question:

A sequence is generated according to the formula  $U_n = an^2 + bn + c$ , where  $a$ ,  $b$  and  $c$  are constants. If  $U_1 = 4$ ,  $U_2 = 10$  and  $U_3 = 18$ , find the values of  $a$ ,  $b$  and  $c$ .

#### Solution:

$$U_n = an^2 + bn + c$$

$$\text{when } n = 1, U_1 = 4 \Rightarrow 4 = a \times 1^2 + b \times 1 + c \Rightarrow 4 = a + b + c$$

$$\text{when } n = 2, U_2 = 10 \Rightarrow 10 = a \times 2^2 + b \times 2 + c \Rightarrow 10 = 4a + 2b + c$$

$$\text{when } n = 3, U_3 = 18 \Rightarrow 18 = a \times 3^2 + b \times 3 + c \Rightarrow 18 = 9a + 3b + c$$

we need to solve simultaneously

$$a + b + c = 4 \text{ ①}$$

$$4a + 2b + c = 10 \text{ ②}$$

$$9a + 3b + c = 18 \text{ ③}$$

$$\text{②} - \text{①}: 3a + b = 6 \text{ ④}$$

$$\text{③} - \text{②}: 5a + b = 8 \text{ ⑤}$$

$$\text{⑤} - \text{④}: 2a = 2 \Rightarrow a = 1$$

$$\text{Substitute } a = 1 \text{ in ④: } 3 + b = 6 \Rightarrow b = 3$$

$$\text{Substitute } a = 1, b = 3 \text{ in ①: } 1 + 3 + c = 4 \Rightarrow c = 0$$

$$\therefore U_n = 1n^2 + 3n + 0 = n^2 + 3n$$



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## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise B, Question 7

#### Question:

A sequence is generated from the formula  $U_n = pn^3 + q$ , where  $p$  and  $q$  are constants. Given that  $U_1 = 6$  and  $U_3 = 19$ , find the values of the constants  $p$  and  $q$ .

#### Solution:

$$U_n = pn^3 + q$$

$$\text{when } n = 1, U_1 = 6 \Rightarrow 6 = p \times 1^3 + q \Rightarrow 6 = p + q$$

$$\text{when } n = 3, U_3 = 19 \Rightarrow 19 = p \times 3^3 + q \Rightarrow 19 = 27p + q$$

Solve simultaneously:

$$p + q = 6 \text{ ①}$$

$$27p + q = 19 \text{ ②}$$

$$\text{②} - \text{①}: 26p = 13 \Rightarrow p = \frac{1}{2}$$

$$\text{substitute } p = \frac{1}{2} \text{ in ①: } \frac{1}{2} + q = 6 \Rightarrow q = 5 \frac{1}{2}$$

$$\therefore U_n = \frac{1}{2}n^3 + 5 \frac{1}{2} \text{ or } \frac{1}{2}n^3 + \frac{11}{2} \text{ or } \frac{n^3 + 11}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise C, Question 1

#### Question:

Find the first four terms of the following recurrence relationships:

(a)  $U_{n+1} = U_n + 3, U_1 = 1$

(b)  $U_{n+1} = U_n - 5, U_1 = 9$

(c)  $U_{n+1} = 2U_n, U_1 = 3$

(d)  $U_{n+1} = 2U_n + 1, U_1 = 2$

(e)  $U_{n+1} = \frac{U_n}{2}, U_1 = 10$

(f)  $U_{n+1} = (U_n)^2 - 1, U_1 = 2$

(g)  $U_{n+2} = 2U_{n+1} + U_n, U_1 = 3, U_2 = 5$

#### Solution:

(a)  $U_{n+1} = U_n + 3, U_1 = 1$

$n = 1 \Rightarrow U_2 = U_1 + 3 = 1 + 3 = 4$

$n = 2 \Rightarrow U_3 = U_2 + 3 = 4 + 3 = 7$

$n = 3 \Rightarrow U_4 = U_3 + 3 = 7 + 3 = 10$

Terms are 1, 4, 7, 10, ...

(b)  $U_{n+1} = U_n - 5, U_1 = 9$

$n = 1 \Rightarrow U_2 = U_1 - 5 = 9 - 5 = 4$

$n = 2 \Rightarrow U_3 = U_2 - 5 = 4 - 5 = -1$

$n = 3 \Rightarrow U_4 = U_3 - 5 = -1 - 5 = -6$

Terms are 9, 4, -1, -6, ...

(c)  $U_{n+1} = 2U_n, U_1 = 3$

$n = 1 \Rightarrow U_2 = 2U_1 = 2 \times 3 = 6$

$n = 2 \Rightarrow U_3 = 2U_2 = 2 \times 6 = 12$

$n = 3 \Rightarrow U_4 = 2U_3 = 2 \times 12 = 24$

Terms are 3, 6, 12, 24, ...

(d)  $U_{n+1} = 2U_n + 1, U_1 = 2$

$n = 1 \Rightarrow U_2 = 2U_1 + 1 = 2 \times 2 + 1 = 5$

$n = 2 \Rightarrow U_3 = 2U_2 + 1 = 2 \times 5 + 1 = 11$

$n = 3 \Rightarrow U_4 = 2U_3 + 1 = 2 \times 11 + 1 = 23$

Terms are 2, 5, 11, 23, ...

$$(e) U_{n+1} = \frac{U_n}{2}, U_1 = 10$$

$$n = 1 \Rightarrow U_2 = \frac{U_1}{2} = \frac{10}{2} = 5$$

$$n = 2 \Rightarrow U_3 = \frac{U_2}{2} = \frac{5}{2} = 2.5$$

$$n = 3 \Rightarrow U_4 = \frac{U_3}{2} = \frac{2.5}{2} = 1.25$$

Terms are 10, 5, 2.5, 1.25, ...

$$(f) U_{n+1} = (U_n)^2 - 1, U_1 = 2$$

$$n = 1 \Rightarrow U_2 = (U_1)^2 - 1 = 2^2 - 1 = 4 - 1 = 3$$

$$n = 2 \Rightarrow U_3 = (U_2)^2 - 1 = 3^2 - 1 = 9 - 1 = 8$$

$$n = 3 \Rightarrow U_4 = (U_3)^2 - 1 = 8^2 - 1 = 64 - 1 = 63$$

Terms are 2, 3, 8, 63, ...

$$(g) U_{n+2} = 2U_{n+1} + U_n, U_1 = 3, U_2 = 5$$

$$n = 1 \Rightarrow U_3 = 2U_2 + U_1 = 2 \times 5 + 3 = 13$$

$$n = 2 \Rightarrow U_4 = 2U_3 + U_2 = 2 \times 13 + 5 = 31$$

Terms are 3, 5, 13, 31, ...

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise C, Question 2

#### Question:

Suggest possible recurrence relationships for the following sequences (remember to state the first term):

(a) 3, 5, 7, 9, ...

(b) 20, 17, 14, 11, ...

(c) 1, 2, 4, 8, ...

(d) 100, 25, 6.25, 1.5625, ...

(e) 1, -1, 1, -1, 1, ...

(f) 3, 7, 15, 31, ...

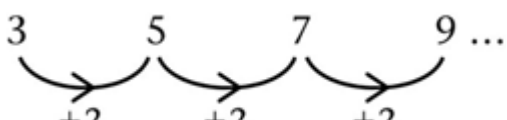
(g) 0, 1, 2, 5, 26, ...

(h) 26, 14, 8, 5, 3.5, ...

(i) 1, 1, 2, 3, 5, 8, 13, ...

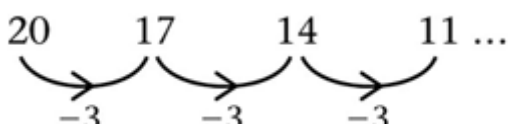
(j) 4, 10, 18, 38, 74, ...

#### Solution:

(a) 
  

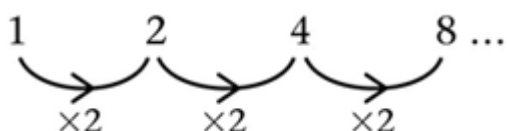
$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \dots$$

$$U_{n+1} = U_n + 2, U_1 = 3$$

(b) 
  

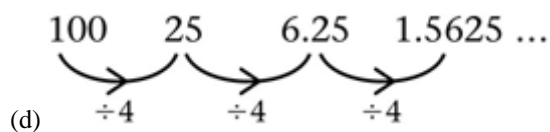
$$20 \xrightarrow{-3} 17 \xrightarrow{-3} 14 \xrightarrow{-3} 11 \dots$$

$$U_{n+1} = U_n - 3, U_1 = 20$$

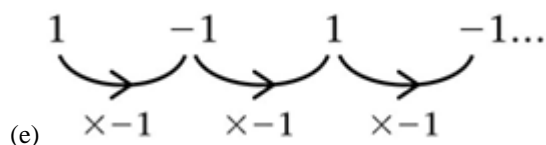
(c) 
  

$$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \dots$$

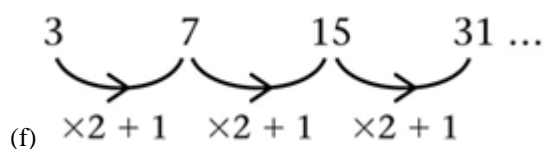
$$U_{n+1} = 2 \times U_n, U_1 = 1$$



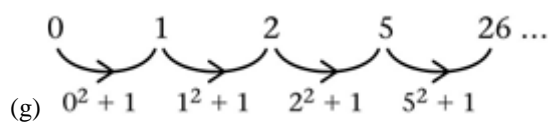
$$U_{n+1} = \frac{U_n}{4}, U_1 = 100$$



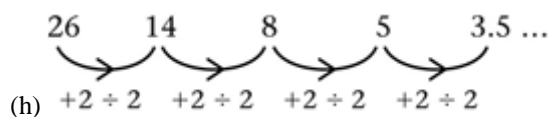
$$U_{n+1} = (-1) \times U_n, U_1 = 1$$



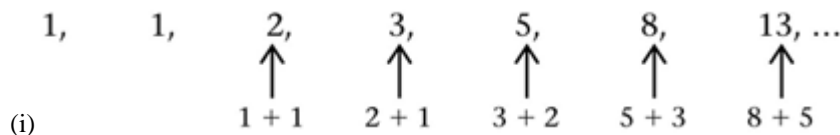
$$U_{n+1} = 2U_n + 1, U_1 = 3$$



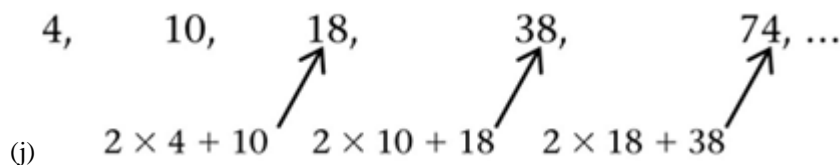
$$U_{n+1} = (U_n)^2 + 1, U_1 = 0$$



$$U_{n+1} = \frac{U_n + 2}{2}, U_1 = 26$$



$$U_{n+2} = U_{n+1} + U_n, U_1 = 1, U_2 = 1$$



$$U_{n+2} = U_{n+1} + 2U_n, U_1 = 4, U_2 = 10$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise C, Question 3

#### Question:

By writing down the first four terms or otherwise, find the recurrence formula that defines the following sequences:

(a)  $U_n = 2n - 1$

(b)  $U_n = 3n + 2$

(c)  $U_n = n + 2$

(d)  $U_n = \frac{n+1}{2}$

(e)  $U_n = n^2$

(f)  $U_n = (-1)^n n$

#### Solution:

(a)  $U_n = 2n - 1$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$U_1 = 1 \quad U_2 = 3 \quad U_3 = 5 \quad U_4 = 7$$

Recurrence formula is  $U_{n+1} = U_n + 2, U_1 = 1$ .

(b)  $U_n = 3n + 2$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$U_1 = 5 \quad U_2 = 8 \quad U_3 = 11 \quad U_4 = 14$$

Recurrence formula is  $U_{n+1} = U_n + 3, U_1 = 5$ .

(c)  $U_n = n + 2$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$U_1 = 3 \quad U_2 = 4 \quad U_3 = 5 \quad U_4 = 6$$

Recurrence formula is  $U_{n+1} = U_n + 1, U_1 = 3$ .

(d)  $U_n = \frac{n+1}{2}$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$\begin{array}{cccc}
 U_1 = 1 & U_2 = 1\frac{1}{2} & U_3 = 2 & U_4 = 2\frac{1}{2} \\
 \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \\
 +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} &
 \end{array}$$

Recurrence formula is  $U_{n+1} = U_n + \frac{1}{2}$ ,  $U_1 = 1$ .

(e)  $U_n = n^2$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$\begin{array}{cccc}
 U_1 = 1 & U_2 = 4 & U_3 = 9 & U_4 = 16 \\
 \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \\
 +3 & +5 & +7 & \\
 = 2 \times 1 + 1 & = 2 \times 2 + 1 & = 2 \times 3 + 1 &
 \end{array}$$

$U_{n+1} = U_n + 2n + 1$ ,  $U_1 = 1$ .

(f)  $U_n = (-1)^n n$ . Substituting  $n = 1, 2, 3$  and  $4$  gives

$$\begin{array}{cccc}
 U_1 = -1 & U_2 = 2 & U_3 = -3 & U_4 = 4 \\
 \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \underbrace{\quad \rightarrow \quad} & \\
 +3 & -5 & +7 & \\
 = 2 \times 1 + 1 & = -(2 \times 2 + 1) & = 2 \times 3 + 1 &
 \end{array}$$

$U_{n+1} = U_n - (-1)^n (2n + 1)$ ,  $U_1 = 1$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise C, Question 4

#### Question:

A sequence of terms  $\{ U_n \}$  is defined  $n \geq 1$  by the recurrence relation  $U_{n+1} = kU_n + 2$ , where  $k$  is a constant. Given that  $U_1 = 3$ :

(a) Find an expression in terms of  $k$  for  $U_2$ .

(b) Hence find an expression for  $U_3$ .

Given that  $U_3 = 42$ :

(c) Find possible values of  $k$ .

#### Solution:

$$U_{n+1} = kU_n + 2$$

(a) Substitute  $n = 1 \Rightarrow U_2 = kU_1 + 2$

As  $U_1 = 3 \Rightarrow U_2 = 3k + 2$

(b) Substitute  $n = 2 \Rightarrow U_3 = kU_2 + 2$

As  $U_2 = 3k + 2 \Rightarrow U_3 = k(3k + 2) + 2$

$$\Rightarrow U_3 = 3k^2 + 2k + 2$$

(c) We are given  $U_3 = 42$

$$\Rightarrow 3k^2 + 2k + 2 = 42 \quad ( - 42 )$$

$$\Rightarrow 3k^2 + 2k - 40 = 0$$

$$\Rightarrow (3k - 10)(k + 4) = 0$$

$$\Rightarrow k = \frac{10}{3}, -4$$

Possible values of  $k$  are  $\frac{10}{3}, -4$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise C, Question 5

#### Question:

A sequence of terms  $\{U_k\}$  is defined  $k \geq 1$  by the recurrence relation  $U_{k+2} = U_{k+1} - pU_k$ , where  $p$  is a constant. Given that  $U_1 = 2$  and  $U_2 = 4$ :

(a) Find an expression in terms of  $p$  for  $U_3$ .

(b) Hence find an expression in terms of  $p$  for  $U_4$ .

Given also that  $U_4$  is twice the value of  $U_3$ :

(c) Find the value of  $p$ .

#### Solution:

$$(a) U_{k+2} = U_{k+1} - pU_k$$

$$\text{Let } k = 1, \text{ then } U_3 = U_2 - pU_1$$

$$\text{Substitute } U_1 = 2, U_2 = 4: U_3 = 4 - p \times 2 \Rightarrow U_3 = 4 - 2p$$

$$(b) U_{k+2} = U_{k+1} - pU_k$$

$$\text{Let } k = 2, \text{ then } U_4 = U_3 - pU_2$$

$$\text{Substitute } U_2 = 4, U_3 = 4 - 2p: U_4 = (4 - 2p) - p \times 4 = 4 - 2p - 4p = 4 - 6p$$

(c) We are told  $U_4$  is twice  $U_3$ , so

$$U_4 = 2 \times U_3$$

$$4 - 6p = 2(4 - 2p)$$

$$4 - 6p = 8 - 4p$$

$$-4 = 2p$$

$$-2 = p$$

$$\text{Hence } p = -2.$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise D, Question 1

#### Question:

Which of the following sequences are arithmetic?

(a) 3, 5, 7, 9, 11, ...

(b) 10, 7, 4, 1, ...

(c)  $y, 2y, 3y, 4y, \dots$

(d) 1, 4, 9, 16, 25, ...

(e) 16, 8, 4, 2, 1, ...

(f) 1, -1, 1, -1, 1, ...

(g)  $y, y^2, y^3, y^4, \dots$

(h)  $U_{n+1} = U_n + 2, U_1 = 3$

(i)  $U_{n+1} = 3U_n - 2, U_1 = 4$

(j)  $U_{n+1} = (U_n)^2, U_1 = 2$

(k)  $U_n = n(n+1)$

(l)  $U_n = 2n + 3$

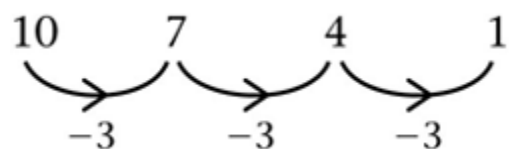
#### Solution:

(a)



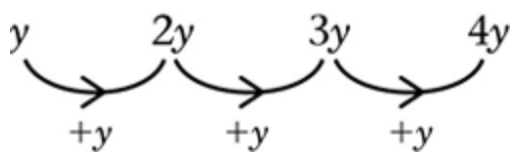
Arithmetic ( + 2 )

(b)



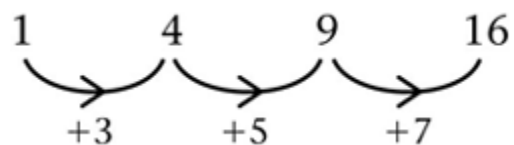
Arithmetic ( - 3 )

(c)



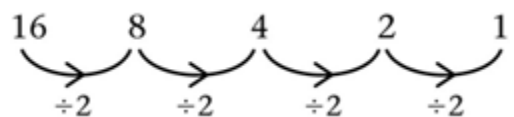
Arithmetic (  $+y$  )

(d)



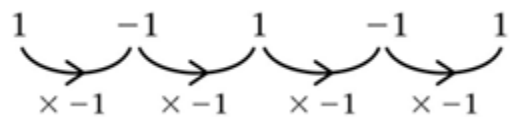
Not arithmetic

(e)



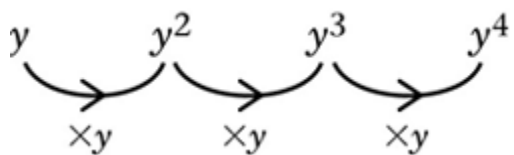
Not arithmetic

(f)



Not arithmetic

(g)



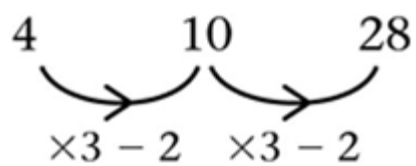
Not arithmetic

(h)  $U_{n+1} = U_n + 2$



Arithmetic (  $+2$  )

(i)  $U_{n+1} = 3U_n - 2$



Not arithmetic

(j)  $U_{n+1} = (U_n)^2, U_1 = 2$

2, 4, 16, 256

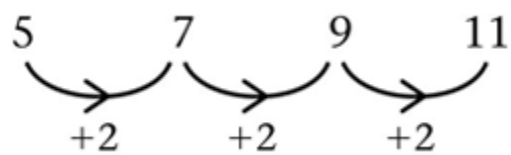
Not arithmetic

(k)  $U_n = n(n + 1)$

2, 6, 12, 20

Not arithmetic

(l)  $U_n = 2n + 3$



Arithmetic ( + 2 )

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise D, Question 2

#### Question:

Find the 10th and  $n$ th terms in the following arithmetic progressions:

(a) 5, 7, 9, 11, ...

(b) 5, 8, 11, 14, ...

(c) 24, 21, 18, 15, ...

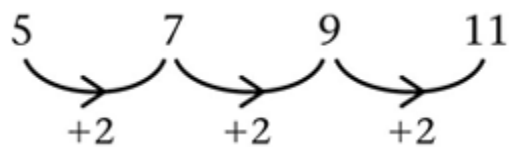
(d)  $-1, 3, 7, 11, \dots$

(e)  $x, 2x, 3x, 4x, \dots$

(f)  $a, a + d, a + 2d, a + 3d, \dots$

#### Solution:

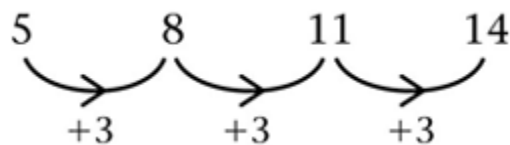
(a)



$$10\text{th term} = 5 + 9 \times 2 = 5 + 18 = 23$$

$$n\text{th term} = 5 + (n - 1) \times 2 = 5 + 2n - 2 = 2n + 3$$

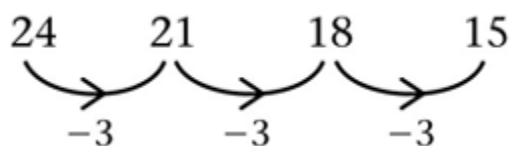
(b)



$$10\text{th term} = 5 + 9 \times 3 = 5 + 27 = 32$$

$$n\text{th term} = 5 + (n - 1) \times 3 = 5 + 3n - 3 = 3n + 2$$

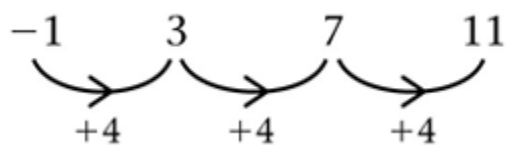
(c)



$$10\text{th term} = 24 + 9 \times -3 = 24 - 27 = -3$$

$$n\text{th term} = 24 + (n - 1) \times -3 = 24 - 3n + 3 = 27 - 3n$$

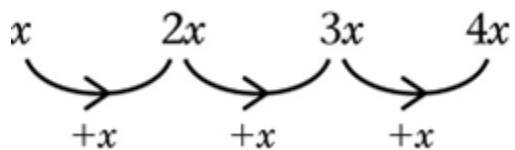
(d)



$$10\text{th term} = -1 + 9 \times 4 = -1 + 36 = 35$$

$$n\text{th term} = -1 + (n - 1) \times 4 = -1 + 4n - 4 = 4n - 5$$

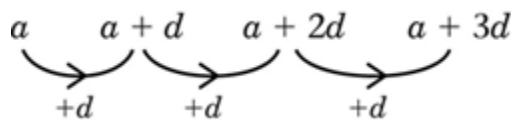
(e)



$$10\text{th term} = x + 9 \times x = 10x$$

$$n\text{th term} = x + (n - 1)x = nx$$

(f)



$$10\text{th term} = a + 9d$$

$$n\text{th term} = a + (n - 1)d$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise D, Question 3

#### Question:

An investor puts £4000 in an account. Every month thereafter she deposits another £200. How much money in total will she have invested at the start of **a** the 10th month and **b** the  $m$ th month? (Note that at the start of the 6th month she will have made only 5 deposits of £200.)

#### Solution:

(a) Initial amount = £ 4000 (start of month 1)

Start of month 2 = £ ( 4000 + 200 )

Start of month 3 = £ ( 4000 + 200 + 200 ) = £ ( 4000 + 2 × 200 )

⋮

Start of month 10 = £ ( 4000 + 9 × 200 ) = £ ( 4000 + 1800 ) = £ 5800

(b) Start of  $m$ th month

= £ [ 4000 + (  $m - 1$  ) × 200 ]

= £ ( 4000 + 200 $m$  - 200 )

= £ ( 3800 + 200 $m$  )

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise D, Question 4

#### Question:

Calculate the number of terms in the following arithmetic sequences:

(a) 3, 7, 11, ... , 83, 87

(b) 5, 8, 11, ... , 119, 122

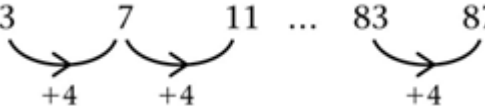
(c) 90, 88, 86, ... , 16, 14

(d) 4, 9, 14, ... , 224, 229

(e)  $x, 3x, 5x, \dots, 35x$

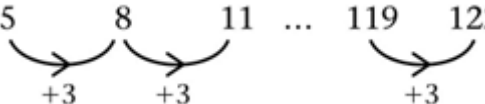
(f)  $a, a + d, a + 2d, \dots, a + (n - 1)d$

#### Solution:

(a)  $3 \quad 7 \quad 11 \quad \dots \quad 83 \quad 87$   


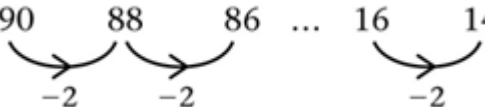
$$\text{number of jumps} = \frac{87 - 3}{4} = 21$$

$$\text{therefore number of terms} = 21 + 1 = 22.$$

(b)  $5 \quad 8 \quad 11 \quad \dots \quad 119 \quad 122$   


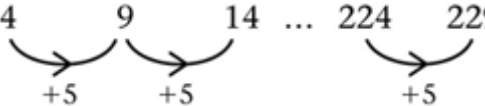
$$\text{number of jumps} = \frac{122 - 5}{3} = 39$$

$$\text{therefore number of terms} = 40$$

(c)  $90 \quad 88 \quad 86 \quad \dots \quad 16 \quad 14$   


$$\text{number of jumps} = \frac{90 - 14}{2} = 38$$

$$\text{therefore number of terms} = 39$$

(d)  $4 \quad 9 \quad 14 \quad \dots \quad 224 \quad 229$   


$$\text{number of jumps} = \frac{229 - 4}{5} = 45$$

$$\text{therefore number of terms} = 46$$



(e)  $x \quad 3x \quad 5x \quad \dots \quad 35x$

$+2x \quad +2x$

$$\text{number of jumps} = \frac{35x - x}{2x} = 17$$

$$\text{number of terms} = 18$$

(f)  $a \quad a + d \quad a + 2d \quad \dots \quad a + (n - 1)d$

$+d \quad +d$

$$\text{number of jumps} = \frac{a + (n - 1)d - a}{d} = \frac{(n - 1)d}{d} = n - 1$$

$$\text{number of terms} = n$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise E, Question 1

#### Question:

Find **i** the 20th and **ii** the  $n$ th terms of the following arithmetic series:

(a)  $2 + 6 + 10 + 14 + 18 \dots$

(b)  $4 + 6 + 8 + 10 + 12 + \dots$

(c)  $80 + 77 + 74 + 71 + \dots$

(d)  $1 + 3 + 5 + 7 + 9 + \dots$

(e)  $30 + 27 + 24 + 21 + \dots$

(f)  $2 + 5 + 8 + 11 + \dots$

(g)  $p + 3p + 5p + 7p + \dots$

(h)  $5x + x + (-3x) + (-7x) + \dots$

#### Solution:

(a)  $2 + 6 + 10 + 14 + 18$

$a = 2, d = 4$

(i) 20th term  $= a + 19d = 2 + 19 \times 4 = 78$

(ii)  $n$ th term  $= a + (n - 1)d = 2 + (n - 1) \times 4 = 4n - 2$

(b)  $4 + 6 + 8 + 10 + 12$

$a = 4, d = 2$

(i) 20th term  $= a + 19d = 4 + 19 \times 2 = 42$

(ii)  $n$ th term  $= a + (n - 1)d = 4 + (n - 1) \times 2 = 2n + 2$

(c)  $80 + 77 + 74 + 71 +$

$a = 80, d = -3$

(i) 20th term  $= a + 19d = 80 + 19 \times -3 = 23$

(ii)  $n$ th term  $= a + (n - 1)d = 80 + (n - 1) \times -3 = 83 - 3n$

(d)  $1 + 3 + 5 + 7 + 9$

$a = 1, d = 2$

(i) 20th term  $= a + 19d = 1 + 19 \times 2 = 39$

(ii)  $n$ th term  $= a + (n - 1)d = 1 + (n - 1) \times 2 = 2n - 1$

(e)  $30 + 27 + 24 + 21$

$a = 30, d = -3$

(i) 20th term  $= a + 19d = 30 + 19 \times -3 = -27$

(ii)  $n$ th term  $= a + (n - 1)d = 30 + (n - 1) \times -3 = 33 - 3n$

(f)  $2 + 5 + 8 + 11$

$a = 2, d = 3$

(i) 20th term  $= a + 19d = 2 + 19 \times 3 = 59$

(ii)  $n$ th term  $= a + (n - 1)d = 2 + (n - 1) \times 3 = 3n - 1$

(g)  $p + 3p + 5p + 7p$

$a = p, d = 2p$

$$(i) 20\text{th term} = a + 19d = p + 19 \times 2p = 39p$$

$$(ii) n\text{th term} = a + (n - 1)d = p + (n - 1) \times 2p = 2pn - p = (2n - 1)p$$

$$(h) 5x + x + (-3x) + (-7x)$$

$$a = 5x, d = -4x$$

$$(i) 20\text{th term} = a + 19d = 5x + 19 \times -4x = -71x$$

$$(ii) n\text{th term} = a + (n - 1)d = 5x + (n - 1) \times -4x = 9x - 4nx = (9 - 4n)x$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise E, Question 2

#### Question:

Find the number of terms in the following arithmetic series:

(a)  $5 + 9 + 13 + 17 + \dots + 121$

(b)  $1 + 1.25 + 1.5 + 1.75 + \dots + 8$

(c)  $-4 + -1 + 2 + 5 + \dots + 89$

(d)  $70 + 61 + 52 + 43 + \dots + (-200)$

(e)  $100 + 95 + 90 + \dots + (-1000)$

(f)  $x + 3x + 5x + \dots + 153x$

#### Solution:

(a)  $5 + 9 + 13 + 17 + \dots + 121$

$$nth \text{ term} = a + (n - 1)d$$

$$121 = 5 + (n - 1) \times 4$$

$$116 = (n - 1) \times 4$$

$$29 = (n - 1)$$

$$30 = n$$

$$n = 30 \text{ (30 terms)}$$

(b)  $1 + 1.25 + 1.5 + 1.75 + \dots + 8$

$$nth \text{ term} = a + (n - 1)d$$

$$8 = 1 + (n - 1) \times 0.25$$

$$7 = (n - 1) \times 0.25$$

$$28 = (n - 1)$$

$$29 = n$$

$$n = 29 \text{ (29 terms)}$$

(c)  $-4 + -1 + 2 + 5 + \dots + 89$

$$nth \text{ term} = a + (n - 1)d$$

$$89 = -4 + (n - 1) \times 3$$

$$93 = (n - 1) \times 3$$

$$31 = (n - 1)$$

$$32 = n$$

$$n = 32 \text{ (32 terms)}$$

(d)  $70 + 61 + 52 + 43 + \dots + (-200)$

$$nth \text{ term} = a + (n - 1)d$$

$$-200 = 70 + (n - 1) \times -9$$

$$-270 = (n - 1) \times -9$$

$$+30 = (n - 1)$$

$$31 = n$$

$$n = 31 \text{ (31 terms)}$$

(e)  $100 + 95 + 90 + \dots + (-1000)$

$$nth \text{ term} = a + (n - 1)d$$

$$-1000 = 100 + (n - 1) \times -5$$

$$-1100 = (n - 1) \times -5$$

$$+220 = (n - 1)$$

$$221 = n$$

$$n = 221 \text{ (221 terms)}$$

$$(f) x + 3x + 5x + \dots + 153x$$

$$nth \text{ term} = a + (n - 1) d$$

$$153x = x + (n - 1) \times 2x$$

$$152x = (n - 1) \times 2x$$

$$76 = (n - 1)$$

$$77 = n$$

$$n = 77 \text{ (77 terms)}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series Exercise E, Question 3

**Question:**

The first term of an arithmetic series is 14. If the fourth term is 32, find the common difference.

**Solution:**

Let the common difference be  $d$ .

4th term =  $a + 3d = 14 + 3d$  (first term = 14)

we are told the 4th term is 32

$$\Rightarrow 14 + 3d = 32$$

$$\Rightarrow 3d = 18$$

$$\Rightarrow d = 6$$

Common difference is 6.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise E, Question 4

#### Question:

Given that the 3rd term of an arithmetic series is 30 and the 10th term is 9 find  $a$  and  $d$ .  
Hence find which term is the first one to become negative.

#### Solution:

Let  $a$  = first term and  $d$  = common difference in the arithmetic series.

$$\text{If 3rd term} = 30 \Rightarrow a + 2d = 30 \text{ ①}$$

$$\text{If 10th term} = 9 \Rightarrow a + 9d = 9 \text{ ②}$$

$$\text{②} - \text{①}: 7d = -21 \Rightarrow d = -3$$

Substitute  $d = -3$  into equation ①:

$$a + 2 \times -3 = 30 \Rightarrow a = 36$$

$$n\text{th term in series} = 36 + (n - 1) \times -3 = 36 - 3n + 3 = 39 - 3n$$

$$\text{when } n = 13, n\text{th term} = 39 - 39 = 0$$

$$\text{when } n = 14, n\text{th term} = 39 - 42 = -3$$

The 14th term is the first to be negative.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise E, Question 5

#### Question:

In an arithmetic series the 20th term is 14 and the 40th term is  $-6$ . Find the 10th term.

#### Solution:

Let  $a$  = first term in the series and  $d$  = common difference in the series.

$$\text{20th term in series is } 14 \Rightarrow a + 19d = 14 \text{ ①}$$

$$\text{40th term in series is } -6 \Rightarrow a + 39d = -6 \text{ ②}$$

$$\text{Equation ②} - \text{①}: 20d = -20 \Rightarrow d = -1$$

Substitute  $d = -1$  into equation ①:

$$a + 19 \times -1 = 14 \Rightarrow a = 33$$

$$\text{10th term} = a + 9d = 33 + 9 \times -1 = 33 - 9 = 24$$

The 10th term in the series is 24.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series Exercise E, Question 6

#### Question:

The first three terms of an arithmetic series are  $5x$ ,  $20$  and  $3x$ . Find the value of  $x$  and hence the values of the three terms.

#### Solution:

$$5x, 20, 3x, \dots$$

$$\text{Term}_2 - \text{Term}_1 = \text{Term}_3 - \text{Term}_2$$

$$20 - 5x = 3x - 20$$

$$40 = 8x$$

$$5 = x$$

Substituting  $x = 5$  into the expressions gives

$$5 \times 5, 20, 3 \times 5$$

$$25, 20, 15$$

1st, 2nd, 3rd term

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise E, Question 7

#### Question:

For which values of  $x$  would the expression  $-8$ ,  $x^2$  and  $17x$  form the first three terms of an arithmetic series?

#### Solution:

$$\begin{aligned}
 & -8, x^2, 17x \\
 \text{Term2} - \text{Term1} &= \text{Term3} - \text{Term2} \\
 x^2 - (-8) &= 17x - x^2 \\
 x^2 + 8 &= 17x - x^2 \\
 2x^2 - 17x + 8 &= 0 \\
 (2x - 1)(x - 8) &= 0 \\
 x &= +\frac{1}{2}, +8
 \end{aligned}$$

Values of  $x$  are  $+\frac{1}{2}$  or  $+8$

Check:

$x = \frac{1}{2}$  gives terms

$$\begin{array}{ccc}
 -8 & & \frac{1}{4} & & 8\frac{1}{2} \\
 \curvearrowright & & \curvearrowright & & \\
 +8\frac{1}{4} & & +8\frac{1}{4} & & 
 \end{array}$$

$x = 8$  gives terms

$$\begin{array}{ccc}
 -8 & & 64 & & 136 \\
 \curvearrowright & & \curvearrowright & & \\
 +72 & & +72 & & 
 \end{array}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 1

#### Question:

Find the sums of the following series:

(a)  $3 + 7 + 11 + 14 + \dots$  (20 terms)

(b)  $2 + 6 + 10 + 14 + \dots$  (15 terms)

(c)  $30 + 27 + 24 + 21 + \dots$  (40 terms)

(d)  $5 + 1 + -3 + -7 + \dots$  (14 terms)

(e)  $5 + 7 + 9 + \dots + 75$

(f)  $4 + 7 + 10 + \dots + 91$

(g)  $34 + 29 + 24 + 19 + \dots + -111$

(h)  $(x + 1) + (2x + 1) + (3x + 1) + \dots + (21x + 1)$

#### Solution:

(a)  $3 + 7 + 11 + 14 + \dots$  (for 20 terms)

Substitute  $a = 3$ ,  $d = 4$  and  $n = 20$  into

$$S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right] = \frac{20}{2} \left( 6 + 19 \times 4 \right) = 10 \times 82 = 820$$

(b)  $2 + 6 + 10 + 14 + \dots$  (for 15 terms)

Substitute  $a = 2$ ,  $d = 4$  and  $n = 15$  into

$$S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right] = \frac{15}{2} \left( 4 + 14 \times 4 \right) = \frac{15}{2} \times 60 = 450$$

(c)  $30 + 27 + 24 + 21 + \dots$  (for 40 terms)

Substitute  $a = 30$ ,  $d = -3$  and  $n = 40$  into

$$S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right] = \frac{40}{2} \left( 60 + 39 \times -3 \right) = 20 \times -57 = -1140$$

(d)  $5 + 1 + -3 + -7 + \dots$  (for 14 terms)

Substitute  $a = 5$ ,  $d = -4$  and  $n = 14$  into

$$S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right] = \frac{14}{2} \left( 10 + 13 \times -4 \right) = 7 \times -42 = -294$$

(e)  $5 + 7 + 9 + \dots + 75$

Here  $a = 5$ ,  $d = 2$  and  $L = 75$ .

Use  $L = a + (n - 1)d$  to find the number of terms  $n$ .

$$75 = 5 + (n - 1) \times 2$$

$$70 = (n - 1) \times 2$$

$$35 = n - 1$$

$$n = 36 \text{ (36 terms)}$$

Substitute  $a = 5$ ,  $d = 2$ ,  $n = 36$  and  $L = 75$  into

$$S_n = \frac{n}{2} \left( a + L \right) = \frac{36}{2} \left( 5 + 75 \right) = 18 \times 80 = 1440$$

(f)  $4 + 7 + 10 + \dots + 91$

Here  $a = 4$ ,  $d = 3$  and  $L = 91$ .

Use  $L = a + (n - 1)d$  to find the number of terms  $n$ .

$$91 = 4 + (n - 1) \times 3$$

$$87 = (n - 1) \times 3$$

$$29 = (n - 1)$$

$$n = 30 \text{ (30 terms)}$$

Substitute  $a = 4$ ,  $d = 3$ ,  $L = 91$  and  $n = 30$  into

$$S_n = \frac{n}{2} \left( a + L \right) = \frac{30}{2} \left( 4 + 91 \right) = 15 \times 95 = 1425$$

(g)  $34 + 29 + 24 + 19 + \dots + -111$

Here  $a = 34$ ,  $d = -5$  and  $L = -111$ .

Use  $L = a + (n - 1)d$  to find the number of terms  $n$ .

$$-111 = 34 + (n - 1) \times -5$$

$$-145 = (n - 1) \times -5$$

$$29 = (n - 1)$$

$$30 = n \text{ (30 terms)}$$

Substitute  $a = 34$ ,  $d = -5$ ,  $L = -111$  and  $n = 30$  into

$$S_n = \frac{n}{2} \left( a + L \right) = \frac{30}{2} \left( 34 + -111 \right) = 15 \times -77 = -1155$$

(h)  $(x + 1) + (2x + 1) + (3x + 1) + \dots + (21x + 1)$

Here  $a = x + 1$ ,  $d = x$  and  $L = 21x + 1$ .

Use  $L = a + (n - 1)d$  to find the number of terms  $n$ .

$$21x + 1 = x + 1 + (n - 1) \times x$$

$$20x = (n - 1) \times x$$

$$20 = (n - 1)$$

$$21 = n \text{ (21 terms)}$$

Substitute  $a = x + 1$ ,  $d = x$ ,  $L = 21x + 1$  and  $n = 21$  into

$$S_n = \frac{n}{2} \left( a + L \right) = \frac{21}{2} \left( x + 1 + 21x + 1 \right) = \frac{21}{2} \times \left( 22x + 2 \right) = 21 \left( 11x + 1 \right)$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 2

#### Question:

Find how many terms of the following series are needed to make the given sum:

(a)  $5 + 8 + 11 + 14 + \dots = 670$

(b)  $3 + 8 + 13 + 18 + \dots = 1575$

(c)  $64 + 62 + 60 + \dots = 0$

(d)  $34 + 30 + 26 + 22 + \dots = 112$

#### Solution:

(a)  $5 + 8 + 11 + 14 + \dots = 670$

Substitute  $a = 5$ ,  $d = 3$ ,  $S_n = 670$  into

$$S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$670 = \frac{n}{2} \left[ 10 + (n-1) \times 3 \right]$$

$$670 = \frac{n}{2} (3n + 7)$$

$$1340 = n(3n + 7)$$

$$0 = 3n^2 + 7n - 1340$$

$$0 = (n - 20)(3n + 67)$$

$$n = 20 \text{ or } -\frac{67}{3}$$

Number of terms is 20

(b)  $3 + 8 + 13 + 18 + \dots = 1575$

Substitute  $a = 3$ ,  $d = 5$ ,  $S_n = 1575$  into

$$S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$1575 = \frac{n}{2} \left[ 6 + (n-1) \times 5 \right]$$

$$1575 = \frac{n}{2} (5n + 1)$$

$$3150 = n(5n + 1)$$

$$0 = 5n^2 + n - 3150$$

$$0 = (5n + 126)(n - 25)$$

$$n = -\frac{126}{5}, 25$$

Number of terms is 25

(c)  $64 + 62 + 60 + \dots = 0$

Substitute  $a = 64$ ,  $d = -2$  and  $S_n = 0$  into

$$S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$0 = \frac{n}{2} \left[ 128 + (n-1) \times -2 \right]$$

$$0 = \frac{n}{2} (130 - 2n)$$

$$0 = n(65 - n)$$

$$n = 0 \text{ or } 65$$

Number of terms is 65

(d)  $34 + 30 + 26 + 22 + \dots = 112$

Substitute  $a = 34$ ,  $d = -4$  and  $S_n = 112$  into

$$S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$112 = \frac{n}{2} \left[ 68 + (n-1) \times -4 \right]$$

$$112 = \frac{n}{2} (72 - 4n)$$

$$112 = n(36 - 2n)$$

$$2n^2 - 36n + 112 = 0$$

$$n^2 - 18n + 56 = 0$$

$$(n-4)(n-14) = 0$$

$$n = 4 \text{ or } 14$$

Number of terms is 4 or 14

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series Exercise F, Question 3

**Question:**

Find the sum of the first 50 even numbers.

**Solution:**

$$S = \underbrace{2 + 4 + 6 + 8 + \dots}_{50 \text{ terms}}$$

This is an arithmetic series with  $a = 2$ ,  $d = 2$  and  $n = 50$ .

$$\text{Use } S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$\text{So } S = \frac{50}{2}(4 + 49 \times 2) = 25 \times 102 = 2550$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 4

#### Question:

Carol starts a new job on a salary of £20000. She is given an annual wage rise of £500 at the end of every year until she reaches her maximum salary of £25000. Find the total amount she earns (assuming no other rises),

(a) in the first 10 years and

(b) over 15 years.

#### Solution:

$$\begin{array}{ccccccc}
 \text{Total salary} & & & & & & \\
 20\,000 & + & 20\,500 & + & 21\,000 & + & 21\,500 + \dots \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 = & & & & & & \\
 \text{Year 1} & & \text{Year 2} & & \text{Year 3} & & \text{Year 4} \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & & & & \\
 \text{1st increment} & \text{2nd increment} & \text{3rd increment} & & & & 
 \end{array}$$

Carol will reach her maximum salary after

$$\frac{25000 - 20000}{500} = 10 \text{ increments}$$

This will be after 11 years.

(a) Total amount after 10 years

$$= \underbrace{20\,000 + 20\,500 + 21\,000 + \dots}$$

This is an arithmetic series with  $a = 20000$ ,  $d = 500$  and  $n = 10$ . Use  $S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$ .

$$= \frac{10}{2} \left( 40000 + 9 \times 500 \right)$$

$$= 5 \times 44500$$

$$= \text{£ } 222\,500$$

(b) From year 11 to year 15 she will continue to earn £ 25 000.

Total in this time =  $5 \times 25000 = \text{£ } 125000$ .

Total amount in the first 15 years is

$$\text{£ } 222\,500 + \text{£ } 125000 = \text{£ } 347\,500$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 5

#### Question:

Find the sum of the multiples of 3 less than 100. Hence or otherwise find the sum of the numbers less than 100 which are not multiples of 3.

#### Solution:

Sum of multiples of 3 less than 100

$$= \underbrace{3 + 6 + 9 + 12 \dots + 96 + 99}_{\text{Arithmetic series}}$$

This is an arithmetic series with  $a = 3$ ,  $d = 3$  and  $n = \frac{99-3}{3} + 1 = 33$  terms.

$$\text{Use } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{33}{2} \left[ 2 \times 3 + \left( 33 - 1 \right) \times 3 \right]$$

$$= \frac{33}{2} (6 + 96)$$

$$= 33 \times 51$$

$$= 1683$$

Sum of numbers less than 100 that are not multiples of 3

$$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + \dots + 97 + 98$$

$$= (1 + 2 + 3 + \dots + 97 + 98 + 99) - (3 + 6 + \dots + 96 + 99)$$

$$= \frac{99}{2} \left[ 2 + \left( 99 - 1 \right) \times 1 \right] - 1683$$

$$= \frac{99}{2} \times 100 - 1683$$

$$= 4950 - 1683$$

$$= 3267$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 6

#### Question:

James decides to save some money during the six-week holiday. He saves 1p on the first day, 2p on the second, 3p on the third and so on. How much will he have at the end of the holiday (42 days)? If he carried on, how long would it be before he has saved £100?

#### Solution:

Amount saved by James

$$= \underbrace{1 + 2 + 3 + \dots + 42}$$

This is an arithmetic series with  $a = 1$ ,  $d = 1$ ,  $n = 42$  and  $L = 42$ .

$$\text{Use } S_n = \frac{n}{2} (a + L)$$

$$= \frac{42}{2} (1 + 42)$$

$$= 21 \times 43$$

$$= 903\text{p}$$

$$= \text{£ } 9.03$$

To save £100 we need

$$\underbrace{1 + 2 + 3 + \dots}_{\text{Sum to } n \text{ terms}} = 10000$$

$$\frac{n}{2} \left[ 2 \times 1 + (n - 1) \times 1 \right] = 10000$$

$$\frac{n}{2} (n + 1) = 10000$$

$$n(n + 1) = 20000$$

$$n^2 + n - 20000 = 0$$

$$n = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times (-20000)}}{2}$$

$$n = 140.9 \text{ or } -141.9$$

It takes James 141 days to save £100.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 7

#### Question:

The first term of an arithmetic series is 4. The sum to 20 terms is  $-15$ . Find, in any order, the common difference and the 20th term.

#### Solution:

Let common difference =  $d$ .

Substitute  $a = 4$ ,  $n = 20$ , and  $S_{20} = -15$  into

$$S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$-15 = \frac{20}{2} \left[ 8 + (20-1)d \right]$$

$$-15 = 10(8 + 19d)$$

$$-1.5 = 8 + 19d$$

$$19d = -9.5$$

$$d = -0.5$$

The common difference is  $-0.5$ .

Use  $n$ th term =  $a + (n-1)d$  to find

$$20\text{th term} = a + 19d = 4 + 19 \times -0.5 = 4 - 9.5 = -5.5$$

20th term is  $-5.5$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 8

#### Question:

The sum of the first three numbers of an arithmetic series is 12. If the 20th term is  $-32$ , find the first term and the common difference.

#### Solution:

Let the first term be  $a$  and the common difference  $d$ .

Sum of first three terms is 12, so

$$a + (a + d) + (a + 2d) = 12$$

$$3a + 3d = 12$$

$$a + d = 4 \text{ ①}$$

20th term is  $-32$ , so

$$a + 19d = -32 \text{ ②}$$

Equation ②  $-$  equation ①:

$$18d = -36$$

$$d = -2$$

Substitute  $d = -2$  into equation ①:

$$a + -2 = 4$$

$$a = 6$$

Therefore, first term is 6 and common difference is  $-2$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 9

#### Question:

Show that the sum of the first  $2n$  natural numbers is  $n(2n + 1)$ .

#### Solution:

Sum required

$$= \underbrace{1 + 2 + 3 + \dots + 2n}$$

Arithmetic series with  $a = 1$ ,  $d = 1$  and  $n = 2n$ .

$$\text{Use } S_n = \frac{n}{2} \left[ 2a + \binom{n-1}{1} d \right]$$

$$= \frac{2n}{2} \left[ 2 \times 1 + \binom{2n-1}{1} \times 1 \right]$$

$$= \frac{2n}{2} (2n + 1)$$

$$= n(2n + 1)$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise F, Question 10

#### Question:

Prove that the sum of the first  $n$  odd numbers is  $n^2$ .

#### Solution:

Required sum

$$= \underbrace{1 + 3 + 5 + 7 + \dots}_{n \text{ terms}}$$

This is an arithmetic series with  $a = 1$ ,  $d = 2$  and  $n = n$ .

$$\text{Use } S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$= \frac{n}{2} \left[ 2 \times 1 + (n-1) \times 2 \right]$$

$$= \frac{n}{2} (2 + 2n - 2)$$

$$= \frac{n \times 2n}{2}$$

$$= n \times n$$

$$= n^2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise G, Question 1

#### Question:

Rewrite the following sums using  $\Sigma$  notation:

(a)  $4 + 7 + 10 + \dots + 31$

(b)  $2 + 5 + 8 + 11 + \dots + 89$

(c)  $40 + 36 + 32 + \dots + 0$

(d) The multiples of 6 less than 100

#### Solution:

(a)  $4 + 7 + 10 + \dots + 31$

Here  $a = 4$  and  $d = 3$ ,

$$n\text{th term} = 4 + (n - 1) \times 3 = 3n + 1$$

4 is the 1st term ( $3 \times 1 + 1$ )

31 is the 10th term ( $3 \times 10 + 1$ )

$$10$$

Hence series is  $\sum_{r=1} (3r + 1)$ .

$$r = 1$$

(b)  $2 + 5 + 8 + 11 + \dots + 89$

Here  $a = 2$  and  $d = 3$ ,

$$n\text{th term} = 2 + (n - 1) \times 3 = 3n - 1$$

2 is the 1st term ( $3 \times 1 - 1$ )

89 is the 30th term ( $3 \times 30 - 1$ )

$$30$$

Hence series is  $\sum_{r=1} (3r - 1)$ .

$$r = 1$$

(c)  $40 + 36 + 32 + \dots + 0$

Here  $a = 40$  and  $d = -4$ ,

$$n\text{th term} = 40 + (n - 1) \times -4 = 44 - 4n$$

40 is the 1st term ( $44 - 4 \times 1$ )

0 is the 11th term ( $44 - 4 \times 11$ )

$$11$$

Hence series is  $\sum_{r=1} (44 - 4r)$ .

$$r = 1$$

(d) Multiples of 6 less than 100 =  $6 + 12 + 18 + \dots + 96$

6 is the 1st multiple

96 is the 16th multiple

$$16$$

Hence series is  $\sum_{r=1} 6r$ .

$$r = 1$$





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise G, Question 2

#### Question:

Calculate the following:

$$(a) \sum_{r=1}^5 3r$$

$$(b) \sum_{r=1}^{10} (4r - 1)$$

$$(c) \sum_{r=1}^{20} (5r - 2)$$

$$(d) \sum_{r=0}^5 r(r + 1)$$

#### Solution:

$$(a) \sum_{r=1}^5 3r = 3 + 6 + \dots + 15$$

Arithmetic series with  $a = 3$ ,  $d = 3$ ,  $n = 5$ ,  $L = 15$

$$\text{Use } S_n = \frac{n}{2} (a + L)$$

$$= \frac{5}{2} (3 + 15)$$

$$= 45$$

$$(b) \sum_{r=1}^{10} (4r - 1) = 3 + 7 + 11 + \dots + 39$$

Arithmetic series with  $a = 3$ ,  $d = 4$ ,  $n = 10$ ,  $L = 39$

$$\text{Use } S_n = \frac{n}{2} (a + L)$$

$$= \frac{10}{2} (3 + 39)$$

$$= 5 \times 42$$

$$= 210$$

$$(c) \sum_{r=1}^{20} (5r-2) = (5 \times 1 - 2) + (5 \times 2 - 2) + (5 \times 3 - 2) + \dots + (5 \times 20 - 2)$$

$\langle \text{semantics} \rangle$

$$= 3 + 8 + 13 + \dots + 98$$

Arithmetic series with  $a = 3, d = 5, n = 20, L = 98$

$$\text{Use } S_n = \frac{n}{2} (a + L)$$

$$= \frac{20}{2} (3 + 98)$$

$$= 10 \times 101$$

$$= 1010$$

$$(d) \sum_{r=0}^5 r(r+1) \langle \text{semantics} \rangle \text{ is not an arithmetic series, so simply add the terms}$$

$$\langle \text{semantics} \rangle \sum_{r=0}^5 r(r+1) = 0 + 2 + 6 + 12 + 20 + 30 \langle \text{semantics} \rangle$$

$$= 70$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise G, Question 3

#### Question:

For what value of  $n$  does  $\sum_{r=1}^n (5r + 3)$  first exceed 1000?

#### Solution:

$$\begin{aligned} & \sum_{r=1}^n (5r + 3) \\ &= (5 \times 1 + 3) + (5 \times 2 + 3) + (5 \times 3 + 3) + \dots + (5 \times n + 3) \\ &= \underbrace{8 + 13 + 18 + \dots + 5n + 3} \end{aligned}$$

Arithmetic series with  $a = 8$ ,  $d = 5$  and  $n = n$ .

$$\begin{aligned} \text{Use } S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{n}{2} [16 + (n-1) \times 5] \\ &= \frac{n}{2} (5n + 11) \end{aligned}$$

If sum exceeds 1000 then

$$\frac{n}{2} (5n + 11) > 1000$$

$$n(5n + 11) > 2000$$

$$5n^2 + 11n - 2000 > 0$$

Solve equality  $5n^2 + 11n - 2000 = 0$

$$n = \frac{-11 \pm \sqrt{(11)^2 - 4 \times 5 \times -2000}}{2 \times 5} = \frac{-11 \pm 200.30 \dots}{10} = 18.93 \text{ or } -21.13$$

The sum has to be bigger than 1000

$$\Rightarrow n = 19$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise G, Question 4

#### Question:

For what value of  $n$  would  $\sum_{r=1}^n (100 - 4r) = 0$ ?

#### Solution:

$$\begin{aligned} & \sum_{r=1}^n (100 - 4r) \\ &= (100 - 4 \times 1) + (100 - 4 \times 2) + (100 - 4 \times 3) + \dots + (100 - 4n) \\ &= \underbrace{96 + 92 + 88 + \dots + (100 - 4n)} \end{aligned}$$

Arithmetic series with  $a = 96$ ,  $d = -4$  and  $n = n$ .

$$\text{Use the sum formula } S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]$$

$$= \frac{n}{2} \left[ 192 + (n-1) \times -4 \right]$$

$$= \frac{n}{2} (196 - 4n)$$

$$= n(98 - 2n)$$

we require the sum to be zero, so

$$n(98 - 2n) = 0 \Rightarrow n = 0 \text{ or } \frac{98}{2}$$

Hence the value of  $n$  is 49.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 1

**Question:**

The  $r$ th term in a sequence is  $2 + 3r$ . Find the first three terms of the sequence.

**Solution:**

Substitute  $r = 1$  in  $2 + 3r = 2 + 3 \times 1 = 5$

1st term = 5

Substitute  $r = 2$  in  $2 + 3r = 2 + 3 \times 2 = 2 + 6 = 8$

2nd term = 8

Substitute  $r = 3$  in  $2 + 3r = 2 + 3 \times 3 = 2 + 9 = 11$

3rd term = 11

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 2

#### Question:

The  $r$ th term in a sequence is  $(r + 3)(r - 4)$ . Find the value of  $r$  for the term that has the value 78.

#### Solution:

$$r\text{th term} = (r + 3)(r - 4)$$

$$\text{when } r\text{th term} = 78$$

$$78 = (r + 3)(r - 4)$$

$$78 = r^2 - 1r - 12$$

$$0 = r^2 - 1r - 90$$

$$0 = (r - 10)(r + 9)$$

$$r = 10, -9$$

$r$  must be 10.

[Check: Substitute  $r = 10$  in  $(r + 3)(r - 4)$

$$\Rightarrow (10 + 3)(10 - 4) = 13 \times 6 = 78 \checkmark]$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series Exercise H, Question 3

**Question:**

A sequence is formed from an inductive relationship:

$$U_{n+1} = 2U_n + 5$$

Given that  $U_1 = 2$ , find the first four terms of the sequence.

**Solution:**

$$U_{n+1} = 2U_n + 5$$

$$\text{Substitute } n = 1 \Rightarrow U_2 = 2U_1 + 5$$

$$U_1 = 2 \Rightarrow U_2 = 2 \times 2 + 5 = 9$$

$$\text{Substitute } n = 2 \Rightarrow U_3 = 2U_2 + 5$$

$$U_2 = 9 \Rightarrow U_3 = 2 \times 9 + 5 = 23$$

$$\text{Substitute } n = 3 \Rightarrow U_4 = 2U_3 + 5$$

$$U_3 = 23 \Rightarrow U_4 = 2 \times 23 + 5 = 51$$

The first four terms of the sequence are 2, 9, 23 and 51.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 4

#### Question:

Find a rule that describes the following sequences:

(a) 5, 11, 17, 23, ...

(b) 3, 6, 9, 12, ...

(c) 1, 3, 9, 27, ...

(d) 10, 5, 0, -5, ...

(e) 1, 4, 9, 16, ...

(f) 1, 1.2, 1.44, 1.728

Which of the above are arithmetic sequences?

For the ones that are, state the values of  $a$  and  $d$ .

#### Solution:

(a) 
$$\begin{array}{ccccccc} 5 & & 11 & & 17 & & 23 \dots \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ +6 & & +6 & & +6 & & \end{array}$$

“Add 6 to the previous term.”

(b) 
$$\begin{array}{ccccccc} 3 & & 6 & & 9 & & 12 \dots \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ +3 & & +3 & & +3 & & \end{array}$$

“Add 3 to the previous term.”

(c) 
$$\begin{array}{ccccccc} 1 & & 3 & & 9 & & 27 \dots \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ \times 3 & & \times 3 & & \times 3 & & \end{array}$$

“Multiply the previous term by 3.”

(d) 
$$\begin{array}{ccccccc} 10 & & 5 & & 0 & & -5 \dots \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ -5 & & -5 & & -5 & & \end{array}$$

“Subtract 5 from the previous term.”

(e) 
$$\begin{array}{ccccccc} 1 & & 4 & & 9 & & 16 \dots \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ +3 & & +5 & & +7 & & \end{array}$$



“Add consecutive odd numbers to each term.” or “They are the square numbers.”

(f)

$$\begin{array}{ccccccc} 1 & & 1.2 & & 1.44 & & 1.728 \dots \\ \downarrow & \searrow & \downarrow & \searrow & \downarrow & \searrow & \\ \times 1.2 & & \times 1.2 & & \times 1.2 & & \end{array}$$

“Multiply the previous term by 1.2.”

The arithmetic sequences are (a) where  $a = 5$ ,  $d = 6$ , (b) where  $a = 3$ ,  $d = 3$ ,

(d) where  $a = 10$ ,  $d = -5$ .

Alternatively you could give the  $n$ th terms of the series as (a)  $6n - 1$  (b)  $3n$  (c)  $3^{n-1}$  (d)  $15 - 5n$  (e)  $n^2$  (f)  $1.2^{n-1}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

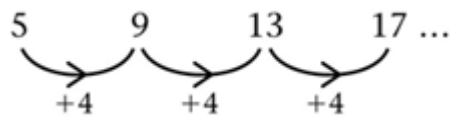
### Sequences and series

#### Exercise H, Question 5

#### Question:

For the arithmetic series  $5 + 9 + 13 + 17 + \dots$   
Find **a** the 20th term, and **b** the sum of the first 20 terms.

#### Solution:



The above sequence is arithmetic with  $a = 5$  and  $d = 4$ .

(a) As  $n$ th term  $= a + (n - 1)d$

20th term  $= a + (20 - 1)d = a + 19d$

Substitute  $a = 5, d = 4 \Rightarrow$  20th term  $= 5 + 19 \times 4 = 5 + 76 = 81$

(b) As sum to  $n$  terms  $S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$

$$S_{20} = \frac{20}{2} \left[ 2a + \left( 20 - 1 \right) d \right] = 10 \left( 2a + 19d \right)$$

Substitute  $a = 5, d = 4 \Rightarrow S_{20} = 10 ( 2 \times 5 + 19 \times 4 ) = 10 \times ( 10 + 76 ) = 10 \times 86 = 860$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 6

#### Question:

(a) Prove that the sum of the first  $n$  terms in an arithmetic series is

$$S = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$$

where  $a$  = first term and  $d$  = common difference.

(b) Use this to find the sum of the first 100 natural numbers.

#### Solution:

$$(a) S = a + (a + d) + (a + 2d) + \dots + [a + (n - 2)d] + [a + (n - 1)d]$$

Turning series around:

$$S = [a + (n - 1)d] + [a + (n - 2)d] + \dots + (a + d) + a$$

Adding the two sums:

$$2S = [2a + (n - 1)d] + [2a + (n - 1)d] + \dots + [2a + (n - 1)d] + [2a + (n - 1)d]$$

There are  $n$  lots of  $[2a + (n - 1)d]$  :

$$2S = n \times [2a + (n - 1)d]$$

$$(\div 2) S = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$$

(b) The first 100 natural numbers are 1,2,3, ... 100.

We need to find  $S = 1 + 2 + 3 + \dots + 99 + 100$ .

This series is arithmetic with  $a = 1$ ,  $d = 1$ ,  $n = 100$ .

Using  $S = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$  with  $a = 1$ ,  $d = 1$  and  $n = 100$  gives

$$S = \frac{100}{2} \left[ 2 \times 1 + \left( 100 - 1 \right) \times 1 \right] = \frac{100}{2} \left( 2 + 99 \times 1 \right) = 50 \times 101 = 5050$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 7

#### Question:

Find the least value of  $n$  for which  $\sum_{r=1}^n (4r - 3) > 2000$ .

#### Solution:

$$\begin{aligned} \sum_{r=1}^n (4r - 3) &= (4 \times 1 - 3) + (4 \times 2 - 3) + (4 \times 3 - 3) \dots (4 \times n - 3) \\ &= \underbrace{1 + 5 + 9 + \dots + 4n - 3} \end{aligned}$$

Arithmetic series with  $a = 1, d = 4$ .

Using  $S_n = \frac{n}{2} [2a + (n - 1)d]$  with  $a = 1, d = 4$  gives

$$S_n = \frac{n}{2} [2 \times 1 + (n - 1) \times 4] = \frac{n}{2} (2 + 4n - 4) = \frac{n}{2} (4n - 2) = n(2n - 1)$$

Solve  $S_n = 2000$ :

$$n(2n - 1) = 2000$$

$$2n^2 - n = 2000$$

$$2n^2 - n - 2000 = 0$$

$$n = \frac{1 \pm \sqrt{1 - 4 \times 2 \times -2000}}{2 \times 2} = 31.87 \text{ or } -31.37$$

$n$  must be positive, so  $n = 31.87$ .

If the sum has to be greater than 2000 then  $n = 32$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 8

#### Question:

A salesman is paid commission of £10 per week for each life insurance policy that he has sold. Each week he sells one new policy so that he is paid £10 commission in the first week, £20 commission in the second week, £30 commission in the third week and so on.

(a) Find his total commission in the first year of 52 weeks.

(b) In the second year the commission increases to £11 per week on new policies sold, although it remains at £10 per week for policies sold in the first year. He continues to sell one policy per week. Show that he is paid £542 in the second week of his second year.

(c) Find the total commission paid to him in the second year. **[E]**

#### Solution:

(a) Total commission

$$= \underbrace{10 + 20 + 30 + \dots + 520}$$

Arithmetic series with  $a = 10$ ,  $d = 10$ ,  $n = 52$ .

$$\begin{aligned} &= \frac{52}{2} \left[ 2 \times 10 + (52 - 1) \times 10 \right] \text{ using } S_n = \frac{n}{2} \left[ 2a + (n - 1)d \right] \\ &= 26 ( 20 + 51 \times 10 ) \\ &= 26 ( 20 + 510 ) \\ &= 26 \times 530 \\ &= \text{£ } 13780 \end{aligned}$$

(b) Commission = policies for year 1 + policies for 2nd week of year 2 =  $520 + 22 = \text{£ } 542$

(c) Total commission for year 2

$$\begin{aligned} &= \text{Commission for year 1 policies} + \text{Commission for year 2 policies} \\ &= 520 \times 52 + ( 11 + 22 + 33 + \dots + 52 \times 11 ) \end{aligned}$$

$$\text{Use } S_n = \frac{n}{2} \left[ 2a + (n - 1)d \right] \text{ with } n = 52, a = 11, d = 11$$

$$\begin{aligned} &= 27040 + \frac{52}{2} \left[ 2 \times 11 + (52 - 1) \times 11 \right] \\ &= \text{£ } 27040 + 26 \times ( 22 + 51 \times 11 ) \\ &= \text{£ } 27\,040 + \text{£ } 15\,158 \\ &= \text{£ } 42\,198 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 9

#### Question:

The sum of the first two terms of an arithmetic series is 47.  
The thirtieth term of this series is  $-62$ . Find:

- (a) The first term of the series and the common difference.  
(b) The sum of the first 60 terms of the series. **[E]**

#### Solution:

Let  $a =$  first term and  $d =$  common difference.

Sum of the first two terms  $= 47$

$$\Rightarrow a + a + d = 47$$

$$\Rightarrow 2a + d = 47$$

30th term  $= -62$

Using  $n$ th term  $= a + (n - 1)d$

$$\Rightarrow a + 29d = -62 \text{ (Note: } a + 12d \text{ is a common error here)}$$

Our two simultaneous equations are

$$2a + d = 47 \text{ ①}$$

$$a + 29d = -62 \text{ ②}$$

$$2a + 58d = -124 \text{ ③ ( ② } \times 2 \text{ )}$$

$$57d = -171 \text{ ( ③ } - \text{ ① )}$$

$$d = -3 \text{ ( } \div 57 \text{ )}$$

$$\text{Substitute } d = -3 \text{ into ①: } 2a - 3 = 47 \Rightarrow 2a = 50 \Rightarrow a = 25$$

Therefore, (a) first term  $= 25$  and common difference  $= -3$

$$\text{(b) using } S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$$

$$S_{60} = \frac{60}{2} \left[ 2a + \left( 60 - 1 \right) d \right] = 30 \left( 2a + 59d \right)$$

Substituting  $a = 25$ ,  $d = -3$  gives

$$S_{60} = 30 ( 2 \times 25 + 59 \times -3 ) = 30 ( 50 - 177 ) = 30 \times -127 = -3810$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 10

#### Question:

- (a) Find the sum of the integers which are divisible by 3 and lie between 1 and 400.
- (b) Hence, or otherwise, find the sum of the integers, from 1 to 400 inclusive, which are **not** divisible by 3. **[E]**

#### Solution:

- (a) Sum of integers divisible by 3 which lie between 1 and 400

$$= 3 + 6 + 9 + 12 + \dots + 399$$

This is an arithmetic series with  $a = 3$ ,  $d = 3$  and  $L = 399$ .

$$\text{Using } L = a + (n - 1)d$$

$$399 = 3 + (n - 1) \times 3$$

$$399 = 3 + 3n - 3$$

$$399 = 3n$$

$$n = 133$$

Therefore, there are 133 of these integers up to 400.

$$S_n = \frac{n}{2} (a + L) = \frac{133}{2} (3 + 399) = \frac{133}{2} \times 402 = 26\,733$$

- (b) Sum of integers not divisible by 3

$$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 \dots 400$$

$$= \underbrace{(1 + 2 + 3 + 4 \dots + 399 + 400)}_{\text{Arithmetic series with } a = 1, d = 1, L = 400, n = 400} - \underbrace{(3 + 6 + 9 + \dots + 399)}_{\text{From part (a). This equals 26733}}$$

$$\begin{aligned} S_n &= \frac{400}{2} (1 + 400) \\ &= 200 \times 401 \\ &= 80200 \end{aligned}$$

$$\begin{aligned} &= 80200 - 26733 \\ &= 53467 \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 11

#### Question:

A polygon has 10 sides. The lengths of the sides, starting with the smallest, form an arithmetic series. The perimeter of the polygon is 675 cm and the length of the longest side is twice that of the shortest side. Find, for this series:

(a) The common difference.

(b) The first term. **[E]**

#### Solution:

If we let the smallest side be  $a$ , the other sides would be  $a + d$ ,  $a + 2d$ , ... . The longest side would be  $a + 9d$ .

If perimeter = 675, then

$$a + (a + d) + (a + 2d) + \dots + (a + 9d) = 675$$

$$\frac{10}{2} \left[ 2a + \left( 10 - 1 \right) d \right] = 675 \text{ (Sum to 10 terms of an arithmetic series)}$$

$$5(2a + 9d) = 675 \quad (\div 5)$$

$$2a + 9d = 135$$

The longest side is double the shortest side

$$\Rightarrow a + 9d = 2 \times a \quad (-a)$$

$$\Rightarrow 9d = a$$

The simultaneous equations we need to solve are

$$2a + 9d = 135 \text{ ①}$$

$$9d = a \text{ ②}$$

Substitute  $9d = a$  into ①:

$$2a + a = 135$$

$$3a = 135$$

$$a = 45$$

Substitute back into ②:

$$9d = 45$$

$$d = 5$$

Therefore (a) the common difference = 5 and (b) the first term = 45.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 12

#### Question:

A sequence of terms  $\{ U_n \}$  is defined for  $n \geq 1$ , by the recurrence relation  $U_{n+2} = 2kU_{n+1} + 15U_n$ , where  $k$  is a constant. Given that  $U_1 = 1$  and  $U_2 = -2$ :

- (a) Find an expression, in terms of  $k$ , for  $U_3$ .  
 (b) Hence find an expression, in terms of  $k$ , for  $U_4$ .  
 (c) Given also that  $U_4 = -38$ , find the possible values of  $k$ . **[E]**

#### Solution:

$$U_{n+2} = 2kU_{n+1} + 15U_n$$

(a) Replacing  $n$  by 1 gives

$$U_3 = 2kU_2 + 15U_1$$

We know  $U_1 = 1$  and  $U_2 = -2$ , therefore

$$U_3 = 2k \times -2 + 15 \times 1$$

$$U_3 = -4k + 15$$

(b) Replacing  $n$  by 2 gives

$$U_4 = 2kU_3 + 15U_2$$

We know  $U_2 = -2$  and  $U_3 = -4k + 15$ , therefore

$$U_4 = 2k(-4k + 15) + 15 \times -2$$

$$U_4 = -8k^2 + 30k - 30$$

(c) We are told that  $U_4 = -38$ , therefore

$$-8k^2 + 30k - 30 = -38 \quad ( + 38 )$$

$$-8k^2 + 30k + 8 = 0 \quad ( \div -2 )$$

$$4k^2 - 15k - 4 = 0 \quad (\text{factorise})$$

$$(4k + 1)(k - 4) = 0$$

$$k = -\frac{1}{4}, 4$$

Possible values of  $k$  are  $-\frac{1}{4}, 4$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 13

#### Question:

Prospectors are drilling for oil. The cost of drilling to a depth of 50 m is £500. To drill a further 50 m costs £640 and, hence, the total cost of drilling to a depth of 100 m is £1140. Each subsequent extra depth of 50 m costs £140 more to drill than the previous 50 m.

(a) Show that the cost of drilling to a depth of 500 m is £11300.

(b) The total sum of money available for drilling is £76000. Find, to the nearest 50 m, the greatest depth that can be drilled. **[E]**

#### Solution:

(a) Cost of drilling to 500 m

$$= \underset{\substack{\uparrow \\ \text{1st} \\ \text{50 m}}}{500} + \underset{\substack{\uparrow \\ \text{2nd} \\ \text{50 m}}}{640} + \underset{\substack{\uparrow \\ \text{3rd} \\ \text{50 m}}}{780} + \dots$$

There would be 10 terms because there are 10 lots of 50 m in 500 m.

Arithmetic series with  $a = 500$ ,  $d = 140$  and  $n = 10$ .

$$\begin{aligned} \text{Using } S_n &= \frac{n}{2} \left[ 2a + (n-1)d \right] \\ &= \frac{10}{2} \left[ 2 \times 500 + (10-1) \times 140 \right] \\ &= 5 (1000 + 9 \times 140) \\ &= 5 \times 2260 \\ &= \text{£ } 11300 \end{aligned}$$

(b) This time we are given  $S = 76\,000$ . The first term will still be 500 and  $d$  remains 140.

$$\text{Use } S = \frac{n}{2} \left[ 2a + (n-1)d \right] \text{ with } S = 76000, a = 500, d = 140 \text{ and solve for } n.$$

$$76000 = \frac{n}{2} \left[ 2 \times 500 + (n-1) \times 140 \right]$$

$$76000 = \frac{n}{2} \left[ 1000 + 140(n-1) \right]$$

$$76000 = n [ 500 + 70(n-1) ]$$

$$76000 = n ( 500 + 70n - 70 )$$

$$76000 = n ( 70n + 430 ) \text{ (multiply out)}$$

$$76000 = 70n^2 + 430n \text{ (} \div 10 \text{)}$$

$$7600 = 7n^2 + 43n$$

$$0 = 7n^2 + 43n - 7600$$

$$n = \frac{-43 \pm \sqrt{(43)^2 - 4 \times 7 \times (-7600)}}{2 \times 7} \text{ (using } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{)}$$

$$n = 30.02, ( -36.16 )$$

only accept the positive answer.

There are 30 terms (to the nearest term).

So the greatest depth that can be drilled is  $30 \times 50 = 1500$  m (to the nearest 50 m)

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 14

#### Question:

Prove that the sum of the first  $2n$  multiples of 4 is  $4n(2n + 1)$ . [E]

#### Solution:

$$\text{Sum} = \begin{array}{ccccccc} 4 & + & 8 & + & 12 & + & \dots & + & 8n \\ \uparrow & & \uparrow & & \uparrow & & & & \uparrow \\ \text{1st} & & \text{2nd} & & \text{3rd} & & & & \text{2nth} \end{array}$$

This is an arithmetic series with  $a = 4$ ,  $d = 4$  and  $n = 2n$ .

$$\text{Using } S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$$

$$\begin{aligned} S_{2n} &= \frac{\cancel{2n}}{\cancel{2}} [2 \times 4 + (2n - 1) \times 4] \\ &= n(8 + 8n - 4) \\ &= n(8n + 4) \\ &= n \times 4(2n + 1) \\ &= 4n(2n + 1) \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 15

#### Question:

A sequence of numbers  $\{ U_n \}$  is defined, for  $n \geq 1$ , by the recurrence relation  $U_{n+1} = kU_n - 4$ , where  $k$  is a constant. Given that  $U_1 = 2$ :

- (a) Find expressions, in terms of  $k$ , for  $U_2$  and  $U_3$ .
- (b) Given also that  $U_3 = 26$ , use algebra to find the possible values of  $k$ . **[E]**

#### Solution:

(a) Replacing  $n$  with 1  $\Rightarrow U_2 = kU_1 - 4$

$$U_1 = 2 \Rightarrow U_2 = 2k - 4$$

Replacing  $n$  with 2  $\Rightarrow U_3 = kU_2 - 4$

$$U_2 = 2k - 4 \Rightarrow U_3 = k(2k - 4) - 4 \Rightarrow U_3 = 2k^2 - 4k - 4$$

(b) Substitute  $U_3 = 26$

$$\Rightarrow 2k^2 - 4k - 4 = 26$$

$$\Rightarrow 2k^2 - 4k - 30 = 0 \quad ( \div 2 )$$

$$\Rightarrow k^2 - 2k - 15 = 0 \quad (\text{factorise})$$

$$\Rightarrow (k - 5)(k + 3) = 0$$

$$\Rightarrow k = 5, -3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 16

#### Question:

Each year, for 40 years, Anne will pay money into a savings scheme. In the first year she pays in £500. Her payments then increase by £50 each year, so that she pays in £550 in the second year, £600 in the third year, and so on.

(a) Find the amount that Anne will pay in the 40th year.

(b) Find the total amount that Anne will pay in over the 40 years.

(c) Over the same 40 years, Brian will also pay money into the savings scheme. In the first year he pays in £890 and his payments then increase by £ $d$  each year. Given that Brian and Anne will pay in exactly the same amount over the 40 years, find the value of  $d$ .  
[E]

#### Solution:

(a) 1<sup>st</sup> year = £ 500

2<sup>nd</sup> year = £ 550 = £ ( 500 + 1 × 50 )

3<sup>rd</sup> year = £ 600 = £ ( 500 + 2 × 50 )

⋮

40<sup>th</sup> year = £ 500 + 39 × 50 = £ 2450

(b) Total amount paid in

$$= \underbrace{\pounds 500 + \pounds 550 + \pounds 600 + \dots + \pounds 2450}$$

This is an arithmetic series with  $a = 500$ ,  $d = 50$ ,  $L = 2450$  and  $n = 40$ .

$$= \frac{n}{2} \left( a + L \right)$$

$$= \frac{40}{2} \left( 500 + 2450 \right)$$

$$= 20 \times 2950$$

$$= \pounds 59000$$

(c) Brian's amount

$$= \underbrace{890 + (890 + d) + (890 + 2d) + \dots}_{40 \text{ years}}$$

$$\text{Use } S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right] \text{ with } n = 40, a = 890 \text{ and } d.$$

$$= \frac{40}{2} \left[ 2 \times 890 + \left( 40 - 1 \right) d \right]$$

$$= 20 ( 1780 + 39d )$$

Use the fact that

Brian's savings = Anne's savings

$$20 ( 1780 + 39d ) = 59000 \quad ( \div 20 )$$

$$1780 + 39d = 2950 \quad ( - 1780 )$$

$$39d = 1170 \quad ( \div 39 )$$

$$d = 30$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 17

#### Question:

The fifth term of an arithmetic series is 14 and the sum of the first three terms of the series is  $-3$ .

(a) Use algebra to show that the first term of the series is  $-6$  and calculate the common difference of the series.

(b) Given that the  $n$ th term of the series is greater than 282, find the least possible value of  $n$ . **[E]**

#### Solution:

(a) Use  $n$ th term  $= a + (n - 1)d$ :

$$5\text{th term is } 14 \Rightarrow a + 4d = 14$$

Use 1st term  $= a$ , 2nd term  $= a + d$ , 3rd term  $= a + 2d$ :

$$\text{sum of 1st three terms} = -3$$

$$\Rightarrow a + a + d + a + 2d = -3$$

$$\Rightarrow 3a + 3d = -3 \quad (\div 3)$$

$$\Rightarrow a + d = -1$$

Our simultaneous equations are

$$a + 4d = 14 \quad \textcircled{1}$$

$$a + d = -1 \quad \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}: 3d = 15 \quad (\div 3)$$

$$d = 5$$

Common difference  $= 5$

Substitute  $d = 5$  back in  $\textcircled{2}$ :

$$a + 5 = -1$$

$$a = -6$$

$$\text{First term} = -6$$

(b)  $n$ th term must be greater than 282

$$\Rightarrow a + (n - 1)d > 282$$

$$\Rightarrow -6 + 5(n - 1) > 282 \quad (+6)$$

$$\Rightarrow 5(n - 1) > 288 \quad (\div 5)$$

$$\Rightarrow (n - 1) > 57.6 \quad (+1)$$

$$n > 58.6$$

$\therefore$  least value of  $n = 59$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Sequences and series

#### Exercise H, Question 18

#### Question:

The fourth term of an arithmetic series is  $3k$ , where  $k$  is a constant, and the sum of the first six terms of the series is  $7k + 9$ .

(a) Show that the first term of the series is  $9 - 8k$ .

(b) Find an expression for the common difference of the series in terms of  $k$ .  
Given that the seventh term of the series is 12, calculate:

(c) The value of  $k$ .

(d) The sum of the first 20 terms of the series. **[E]**

#### Solution:

(a) We know  $n$ th term  $= a + (n - 1)d$

$$4\text{th term is } 3k \Rightarrow a + (4 - 1)d = 3k \Rightarrow a + 3d = 3k$$

$$\text{We know } S_n = \frac{n}{2} \left[ 2a + \left( n - 1 \right) d \right]$$

Sum to 6 terms is  $7k + 9$ , therefore

$$\frac{6}{2} \left[ 2a + \left( 6 - 1 \right) d \right] = 7k + 9$$

$$3(2a + 5d) = 7k + 9$$

$$6a + 15d = 7k + 9$$

The simultaneous equations are

$$a + 3d = 3k \text{ ①}$$

$$6a + 15d = 7k + 9 \text{ ②}$$

$$\text{①} \times 5: 5a + 15d = 15k \text{ ③}$$

$$\text{②} - \text{③}: 1a = -8k + 9 \Rightarrow a = 9 - 8k$$

First term is  $9 - 8k$

(b) Substituting this is ① gives

$$9 - 8k + 3d = 3k$$

$$3d = 11k - 9$$

$$d = \frac{11k - 9}{3}$$

Common difference is  $\frac{11k - 9}{3}$ .

(c) If the 7th term is 12, then

$$a + 6d = 12$$

Substitute values of  $a$  and  $d$ :

$$-8k + 9 + 6 \times \left( \frac{11k - 9}{3} \right) = 12$$

$$-8k + 9 + 2(11k - 9) = 12$$

$$-8k + 9 + 22k - 18 = 12$$

$$14k - 9 = 12$$

$$14k = 21$$

$$k = \frac{21}{14} = 1.5$$

(d) Calculate values of  $a$  and  $d$  first:

$$a = 9 - 8k = 9 - 8 \times 1.5 = 9 - 12 = -3$$

$$d = \frac{11k - 9}{3} = \frac{11 \times 1.5 - 9}{3} = \frac{16.5 - 9}{3} = \frac{7.5}{3} = 2.5$$

$$S_{20} = \frac{20}{2} \left[ 2a + \left( 20 - 1 \right) d \right]$$

$$= 10 ( 2a + 19d )$$

$$= 10 ( 2 \times -3 + 19 \times 2.5 )$$

$$= 10 ( -6 + 47.5 )$$

$$= 10 \times 41.5$$

$$= 415$$

Sum to 20 terms is 415.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise A, Question 1

#### Question:

$F$  is the point with co-ordinates  $(3, 9)$  on the curve with equation  $y = x^2$ .

(a) Find the gradients of the chords joining the point  $F$  to the points with coordinates:

(i)  $(4, 16)$

(ii)  $(3.5, 12.25)$

(iii)  $(3.1, 9.61)$

(iv)  $(3.01, 9.0601)$

(v)  $(3 + h, (3 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point  $(3, 9)$ ?

#### Solution:

$$\text{a (i) Gradient} = \frac{16 - 9}{4 - 3} = \frac{7}{1} = 7$$

$$\text{(ii) Gradient} = \frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$$

$$\text{(iii) Gradient} = \frac{9.61 - 9}{3.1 - 3} = \frac{0.61}{0.1} = 6.1$$

$$\text{(iv) Gradient} = \frac{9.0601 - 9}{3.01 - 3} = \frac{0.0601}{0.01} = 6.01$$

$$\text{(v) Gradient} = \frac{(3 + h)^2 - 9}{(3 + h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = \frac{6h + h^2}{h} = \frac{h(6 + h)}{h} = 6 + h$$

(b) The gradient at the point  $(3, 9)$  is the value of  $6 + h$  as  $h$  becomes very small, i.e. the gradient is 6.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise A, Question 2

#### Question:

$G$  is the point with coordinates  $(4, 16)$  on the curve with equation  $y = x^2$ .

(a) Find the gradients of the chords joining the point  $G$  to the points with coordinates:

(i)  $(5, 25)$

(ii)  $(4.5, 20.25)$

(iii)  $(4.1, 16.81)$

(iv)  $(4.01, 16.0801)$

(v)  $(4 + h, (4 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point  $(4, 16)$ ?

#### Solution:

(a) (i) Gradient =  $\frac{25 - 16}{5 - 4} = \frac{9}{1} = 9$

(ii) Gradient =  $\frac{20.25 - 16}{4.5 - 4} = \frac{4.25}{0.5} = 8.5$

(iii) Gradient =  $\frac{16.81 - 16}{4.1 - 4} = \frac{0.81}{0.1} = 8.1$

(iv) Gradient =  $\frac{16.0801 - 16}{4.01 - 4} = \frac{0.0801}{0.01} = 8.01$

(v) Gradient =  $\frac{(4 + h)^2 - 16}{4 + h - 4} = \frac{16 + 8h + h^2 - 16}{h} = \frac{8h + h^2}{h} = \frac{h(8 + h)}{h} = 8 + h$

(b) When  $h$  is small the gradient of the chord is close to the gradient of the tangent, and  $8 + h$  is close to the value 8. So the gradient of the tangent at  $(4, 16)$  is 8.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 1

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^7$$

#### Solution:

$$f(x) = x^7$$

$$f'(x) = 7x^6$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 2

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^8$$

#### Solution:

$$f(x) = x^8$$

$$f'(x) = 8x^7$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 3

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^4$$

#### Solution:

$$f(x) = x^4$$

$$f'(x) = 4x^3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 4

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^{\frac{1}{3}}$$

#### Solution:

$$f(x) = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 5

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^{\frac{1}{4}}$$

#### Solution:

$$f(x) = x^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}x^{\frac{1}{4} - 1} = \frac{1}{4}x^{-\frac{3}{4}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 6

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$\sqrt[3]{x}$$

#### Solution:

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 7

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^{-3}$$

#### Solution:

$$f(x) = x^{-3}$$

$$f'(x) = -3x^{-3-1} = -3x^{-4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 8

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^{-4}$$

#### Solution:

$$f(x) = x^{-4}$$

$$f'(x) = -4x^{-4-1} = -4x^{-5}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 9

**Question:**

Find the derived function, given that  $f(x)$  equals:

$$\frac{1}{x^2}$$

**Solution:**

$$f(x) = \frac{1}{x^2} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 10

**Question:**

Find the derived function, given that  $f(x)$  equals:

$$\frac{1}{x^5}$$

**Solution:**

$$f(x) = \frac{1}{x^5} = x^{-5}$$

$$f'(x) = -5x^{-5-1} = -5x^{-6}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 11

**Question:**

Find the derived function, given that  $f(x)$  equals:

$$\frac{1}{\sqrt[3]{x}}$$

**Solution:**

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$$

$$f'(x) = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 12

**Question:**

Find the derived function, given that  $f(x)$  equals:

$$\frac{1}{\sqrt{x}}$$

**Solution:**

$$f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 13

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$\frac{x^2}{x^4}$$

#### Solution:

$$f(x) = \frac{x^2}{x^4} = x^{2-4} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 14

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$\frac{x^3}{x^2}$$

#### Solution:

$$f(x) = \frac{x^3}{x^2} = x^{3-2} = x^1$$

$$f'(x) = 1x^{1-1} = 1x^0 = 1$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 15

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$\frac{x^6}{x^3}$$

#### Solution:

$$f(x) = \frac{x^6}{x^3} = x^{6-3} = x^3$$

$$f'(x) = 3x^2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 16

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^3 \times x^6$$

#### Solution:

$$f(x) = x^3 \times x^6 = x^{3+6} = x^9$$

$$f'(x) = 9x^8$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 17

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x^2 \times x^3$$

#### Solution:

$$f(x) = x^2 \times x^3 = x^{2+3} = x^5$$

$$f'(x) = 5x^4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise B, Question 18

#### Question:

Find the derived function, given that  $f(x)$  equals:

$$x \times x^2$$

#### Solution:

$$f(x) = x \times x^2 = x^{1+2} = x^3$$

$$f'(x) = 3x^2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 1

#### Question:

Find  $\frac{dy}{dx}$  when y equals:

(a)  $2x^2 - 6x + 3$

(b)  $\frac{1}{2}x^2 + 12x$

(c)  $4x^2 - 6$

(d)  $8x^2 + 7x + 12$

(e)  $5 + 4x - 5x^2$

#### Solution:

(a)  $y = 2x^2 - 6x + 3$

$$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$$

(b)  $y = \frac{1}{2}x^2 + 12x$

$$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$$

(c)  $y = 4x^2 - 6$

$$\frac{dy}{dx} = 4(2x) - 0 = 8x$$

(d)  $y = 8x^2 + 7x + 12$

$$\frac{dy}{dx} = 8(2x) + 7 + 0 = 16x + 7$$

(e)  $y = 5 + 4x - 5x^2$

$$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 2

#### Question:

Find the gradient of the curve whose equation is

(a)  $y = 3x^2$  at the point  $(2, 12)$

(b)  $y = x^2 + 4x$  at the point  $(1, 5)$

(c)  $y = 2x^2 - x - 1$  at the point  $(2, 5)$

(d)  $y = \frac{1}{2}x^2 + \frac{3}{2}x$  at the point  $(1, 2)$

(e)  $y = 3 - x^2$  at the point  $(1, 2)$

(f)  $y = 4 - 2x^2$  at the point  $(-1, 2)$

#### Solution:

(a)  $y = 3x^2$

$$\frac{dy}{dx} = 6x$$

At the point  $(2, 12)$ ,  $x = 2$ .

Substitute  $x = 2$  into the gradient expression  $\frac{dy}{dx} = 6x$  to give

$$\text{gradient} = 6 \times 2 = 12.$$

(b)  $y = x^2 + 4x$

$$\frac{dy}{dx} = 2x + 4$$

At the point  $(1, 5)$ ,  $x = 1$ .

Substitute  $x = 1$  into  $\frac{dy}{dx} = 2x + 4$  to give

$$\text{gradient} = 2 \times 1 + 4 = 6$$

(c)  $y = 2x^2 - x - 1$

$$\frac{dy}{dx} = 4x - 1$$

At the point  $(2, 5)$ ,  $x = 2$ .

Substitute  $x = 2$  into  $\frac{dy}{dx} = 4x - 1$  to give

$$\text{gradient} = 4 \times 2 - 1 = 7$$

(d)  $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$$\frac{dy}{dx} = x + \frac{3}{2}$$

At the point  $(1, 2)$ ,  $x = 1$ .

Substitute  $x = 1$  into  $\frac{dy}{dx} = x + \frac{3}{2}$  to give

$$\text{gradient} = 1 + \frac{3}{2} = 2\frac{1}{2}$$

(e)  $y = 3 - x^2$

$$\frac{dy}{dx} = -2x$$

At  $(1, 2)$ ,  $x = 1$ .

Substitute  $x = 1$  into  $\frac{dy}{dx} = -2x$  to give

$$\text{gradient} = -2 \times 1 = -2$$

(f)  $y = 4 - 2x^2$

$$\frac{dy}{dx} = -4x$$

At  $(-1, 2)$ ,  $x = -1$ .

Substitute  $x = -1$  into  $\frac{dy}{dx} = -4x$  to give

$$\text{gradient} = -4 \times -1 = +4$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 3

#### Question:

Find the  $y$ -coordinate and the value of the gradient at the point P with  $x$ -coordinate 1 on the curve with equation  $y = 3 + 2x - x^2$ .

#### Solution:

$$y = 3 + 2x - x^2$$

When  $x = 1$ ,  $y = 3 + 2 - 1$   
 $\Rightarrow y = 4$  when  $x = 1$

Differentiate to give

$$\frac{dy}{dx} = 0 + 2 - 2x$$

When  $x = 1$ ,  $\frac{dy}{dx} = 2 - 2$

$$\Rightarrow \frac{dy}{dx} = 0 \text{ when } x = 1$$

Therefore, the  $y$ -coordinate is 4 and the gradient is 0 when the  $x$ -coordinate is 1 on the given curve.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 4

#### Question:

Find the coordinates of the point on the curve with equation  $y = x^2 + 5x - 4$  where the gradient is 3.

#### Solution:

$$y = x^2 + 5x - 4$$

$$\frac{dy}{dx} = 2x + 5$$

$$\text{Put } \frac{dy}{dx} = 3$$

$$\text{Then } 2x + 5 = 3$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -1$$

Substitute  $x = -1$  into  $y = x^2 + 5x - 4$ :

$$y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$$

Therefore,  $(-1, -8)$  is the point where the gradient is 3.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 5

#### Question:

Find the gradients of the curve  $y = x^2 - 5x + 10$  at the points  $A$  and  $B$  where the curve meets the line  $y = 4$ .

#### Solution:

The curve  $y = x^2 - 5x + 10$  meets the line  $y = 4$  when

$$x^2 - 5x + 10 = 4$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x = 3 \text{ or } x = 2$$

The gradient function for the curve is given by

$$\frac{dy}{dx} = 2x - 5$$

$$\text{when } x = 3, \frac{dy}{dx} = 2 \times 3 - 5 = 1$$

$$\text{when } x = 2, \frac{dy}{dx} = 2 \times 2 - 5 = -1$$

So the gradients are  $-1$  and  $1$  at  $(2, 4)$  and  $(3, 4)$  respectively.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise C, Question 6

#### Question:

Find the gradients of the curve  $y = 2x^2$  at the points  $C$  and  $D$  where the curve meets the line  $y = x + 3$ .

#### Solution:

The curve  $y = 2x^2$  meets the line  $y = x + 3$  when

$$2x^2 = x + 3$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = 1.5 \text{ or } -1$$

The gradient of the curve is given by the equation  $\frac{dy}{dx} = 4x$ .

The gradient at the point where  $x = -1$  is  $4 \times -1 = -4$ .

The gradient at the point where  $x = 1.5$  is  $4 \times 1.5 = 6$ .

So the gradient is  $-4$  at  $(-1, 2)$  and is  $6$  at  $(1.5, 4.5)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise D, Question 1

**Question:**

Use standard results to differentiate:

(a)  $x^4 + x^{-1}$

(b)  $\frac{1}{2}x^{-2}$

(c)  $2x^{-\frac{1}{2}}$

**Solution:**

(a)  $f(x) = x^4 + x^{-1}$   
 $f'(x) = 4x^3 + (-1)x^{-2}$

(b)  $f(x) = \frac{1}{2}x^{-2}$   
 $f'(x) = \frac{1}{2}(-2)x^{-3} = -x^{-3}$

(c)  $f(x) = 2x^{-\frac{1}{2}}$   
 $f'(x) = 2 \left( -\frac{1}{2} \right) x^{-1\frac{1}{2}} = -x^{-\frac{3}{2}}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise D, Question 2

#### Question:

Find the gradient of the curve with equation  $y = f(x)$  at the point  $A$  where:

(a)  $f(x) = x^3 - 3x + 2$  and  $A$  is at  $(-1, 4)$

(b)  $f(x) = 3x^2 + 2x^{-1}$  and  $A$  is at  $(2, 13)$

#### Solution:

(a)  $f(x) = x^3 - 3x + 2$

$$f'(x) = 3x^2 - 3$$

At  $(-1, 4)$ ,  $x = -1$ .

Substitute  $x = -1$  to find  $f'(-1) = 3(-1)^2 - 3 = 0$

Therefore, gradient = 0.

(b)  $f(x) = 3x^2 + 2x^{-1}$

$$f'(x) = 6x + 2(-1)x^{-2} = 6x - 2x^{-2}$$

At  $(2, 13)$ ,  $x = 2$ .

$$f'(2) = 6(2) - 2(2)^{-2} = 12 - \frac{2}{4} = 11 \frac{1}{2}$$

Therefore, gradient =  $11 \frac{1}{2}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise D, Question 3

#### Question:

Find the point or points on the curve with equation  $y = f(x)$ , where the gradient is zero:

(a)  $f(x) = x^2 - 5x$

(b)  $f(x) = x^3 - 9x^2 + 24x - 20$

(c)  $f(x) = x^{\frac{3}{2}} - 6x + 1$

(d)  $f(x) = x^{-1} + 4x$

#### Solution:

(a)  $f(x) = x^2 - 5x$

$f'(x) = 2x - 5$

When gradient is zero,  $f'(x) = 0$ .

$$\Rightarrow 2x - 5 = 0$$

$$\Rightarrow x = 2.5$$

As  $y = f(x)$ ,  $y = f(2.5)$  when  $x = 2.5$ .

$$\Rightarrow y = (2.5)^2 - 5(2.5) = -6.25$$

Therefore,  $(2.5, -6.25)$  is the point on the curve where the gradient is zero.

(b)  $f(x) = x^3 - 9x^2 + 24x - 20$

$f'(x) = 3x^2 - 18x + 24$

When gradient is zero,  $f'(x) = 0$ .

$$\Rightarrow 3x^2 - 18x + 24 = 0$$

$$\Rightarrow 3(x^2 - 6x + 8) = 0$$

$$\Rightarrow 3(x - 4)(x - 2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = 2$$

As  $y = f(x)$ ,  $y = f(4)$  when  $x = 4$ .

$$\Rightarrow y = 4^3 - 9 \times 4^2 + 24 \times 4 - 20 = -4$$

Also  $y = f(2)$  when  $x = 2$ .

$$\Rightarrow y = 2^3 - 9 \times 2^2 + 24 \times 2 - 20 = 0.$$

Therefore, at  $(4, -4)$  and at  $(2, 0)$  the gradient is zero.

(c)  $f(x) = x^{\frac{3}{2}} - 6x + 1$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 6$

When gradient is zero,  $f'(x) = 0$ .

$$\Rightarrow \frac{3}{2}x^{\frac{1}{2}} - 6 = 0$$

$$\Rightarrow x^{\frac{1}{2}} = 4$$

$$\Rightarrow x = 16$$

As  $y = f(x)$ ,  $y = f(16)$  when  $x = 16$ .

$$\Rightarrow y = 16^{\frac{3}{2}} - 6 \times 16 + 1 = -31$$

Therefore, at  $(16, -31)$  the gradient is zero.

$$(d) f(x) = x^{-1} + 4x$$

$$f'(x) = -1x^{-2} + 4$$

For zero gradient,  $f'(x) = 0$ .

$$\Rightarrow -x^{-2} + 4 = 0$$

$$\Rightarrow \frac{1}{x^2} = 4$$

$$\Rightarrow x = \pm \frac{1}{2}$$

$$\text{When } x = \frac{1}{2}, y = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{-1} + 4\left(\frac{1}{2}\right) = 2 + 2 = 4$$

$$\text{When } x = -\frac{1}{2}, y = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{-1} + 4\left(-\frac{1}{2}\right) = -2 - 2 = -4$$

Therefore,  $\left(\frac{1}{2}, 4\right)$  and  $\left(-\frac{1}{2}, -4\right)$  are points on the curve where the gradient is zero.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise E, Question 1

**Question:**

Use standard results to differentiate:

(a)  $2\sqrt{x}$

(b)  $\frac{3}{x^2}$

(c)  $\frac{1}{3x^3}$

(d)  $\frac{1}{3}x^3(x-2)$

(e)  $\frac{2}{x^3} + \sqrt{x}$

(f)  $3\sqrt{x} + \frac{1}{2x}$

(g)  $\frac{2x+3}{x}$

(h)  $\frac{3x^2-6}{x}$

(i)  $\frac{2x^3+3x}{\sqrt{x}}$

(j)  $x(x^2-x+2)$

(k)  $3x^2(x^2+2x)$

(l)  $(3x-2)\left(4x + \frac{1}{x}\right)$

**Solution:**

(a)  $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 2 \left(\frac{1}{2}\right) x^{-\frac{1}{2}} = x^{-\frac{1}{2}}$$



$$(b) y = \frac{3}{x^2} = 3x^{-2}$$

$$\frac{dy}{dx} = 3(-2)x^{-3} = -6x^{-3}$$

$$(c) y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$$

$$\frac{dy}{dx} = \frac{1}{3}(-3)x^{-4} = -x^{-4}$$

$$(d) y = \frac{1}{3}x^3(x-2) = \frac{1}{3}x^4 - \frac{2}{3}x^3$$

$$\frac{dy}{dx} = \frac{4}{3}x^3 - \frac{2}{3} \times 3x^2 = \frac{4}{3}x^3 - 2x^2$$

$$(e) y = \frac{2}{x^3} + \sqrt{x} = 2x^{-3} + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$(f) y = \sqrt[3]{x} + \frac{1}{2x} = x^{\frac{1}{3}} + \frac{1}{2}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$$

$$(g) y = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1}$$

$$\frac{dy}{dx} = 0 - 3x^{-2}$$

$$(h) y = \frac{3x^2-6}{x} = \frac{3x^2}{x} - \frac{6}{x} = 3x - 6x^{-1}$$

$$\frac{dy}{dx} = 3 + 6x^{-2}$$

$$(i) y = \frac{2x^3+3x}{\sqrt{x}} = \frac{2x^3}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} = 2x^2 \frac{1}{2} + 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 5x^{\frac{1}{2}} + 1.5x^{-\frac{1}{2}}$$

$$(j) y = x(x^2 - x + 2) = x^3 - x^2 + 2x$$

$$\frac{dy}{dx} = 3x^2 - 2x + 2$$

$$(k) y = 3x^2(x^2 + 2x) = 3x^4 + 6x^3$$

$$\frac{dy}{dx} = 12x^3 + 18x^2$$

$$(1) y = (3x - 2)\left(4x + \frac{1}{x}\right) = 12x^2 - 8x + 3 - \frac{2}{x} = 12x^2 - 8x + 3 - 2x^{-1}$$

$$\frac{dy}{dx} = 24x - 8 + 2x^{-2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise E, Question 2

#### Question:

Find the gradient of the curve with equation  $y = f(x)$  at the point  $A$  where:

(a)  $f(x) = x(x + 1)$  and  $A$  is at  $(0, 0)$

(b)  $f(x) = \frac{2x-6}{x^2}$  and  $A$  is at  $(3, 0)$

(c)  $f(x) = \frac{1}{\sqrt{x}}$  and  $A$  is at  $\left(\frac{1}{4}, 2\right)$

(d)  $f(x) = 3x - \frac{4}{x^2}$  and  $A$  is at  $(2, 5)$

#### Solution:

(a)  $f(x) = x(x + 1) = x^2 + x$

$f'(x) = 2x + 1$

At  $(0, 0)$ ,  $x = 0$ .

Therefore, gradient  $= f'(0) = 1$

(b)  $f(x) = \frac{2x-6}{x^2} = \frac{2x}{x^2} - \frac{6}{x^2} = \frac{2}{x} - 6x^{-2} = 2x^{-1} - 6x^{-2}$

$f'(x) = -2x^{-2} + 12x^{-3}$

At  $(3, 0)$ ,  $x = 3$ .

Therefore, gradient  $= f'(3) = -\frac{2}{3^2} + \frac{12}{3^3} = -\frac{2}{9} + \frac{12}{27} = \frac{2}{9}$

(c)  $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

At  $\left(\frac{1}{4}, 2\right)$ ,  $x = \frac{1}{4}$ .

Therefore, gradient  $= f'\left(\frac{1}{4}\right) = -\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}} = -\frac{1}{2} \times 2^3 = -4$

(d)  $f(x) = 3x - \frac{4}{x^2} = 3x - 4x^{-2}$

$f'(x) = 3 + 8x^{-3}$

At  $(2, 5)$ ,  $x = 2$ .

Therefore, gradient  $= f'(2) = 3 + 8(2)^{-3} = 3 + \frac{8}{8} = 4$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise F, Question 1

#### Question:

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $y$  equals:

$$12x^2 + 3x + 8$$

#### Solution:

$$y = 12x^2 + 3x + 8$$

$$\frac{dy}{dx} = 24x + 3$$

$$\frac{d^2y}{dx^2} = 24$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise F, Question 2

#### Question:

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $y$  equals:

$$15x + 6 + \frac{3}{x}$$

#### Solution:

$$y = 15x + 6 + \frac{3}{x} = 15x + 6 + 3x^{-1}$$

$$\frac{dy}{dx} = 15 - 3x^{-2}$$

$$\frac{d^2y}{dx^2} = 0 + 6x^{-3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise F, Question 3

#### Question:

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $y$  equals:

$$9\sqrt{x} - \frac{3}{x^2}$$

#### Solution:

$$y = 9\sqrt{x} - \frac{3}{x^2} = 9x^{\frac{1}{2}} - 3x^{-2}$$

$$\frac{dy}{dx} = 4\frac{1}{2}x^{-\frac{1}{2}} + 6x^{-3}$$

$$\frac{d^2y}{dx^2} = -2\frac{1}{4}x^{-\frac{3}{2}} - 18x^{-4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise F, Question 4

#### Question:

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $y$  equals:

$$(5x + 4)(3x - 2)$$

#### Solution:

$$y = (5x + 4)(3x - 2) = 15x^2 + 2x - 8$$

$$\frac{dy}{dx} = 30x + 2$$

$$\frac{d^2y}{dx^2} = 30$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise F, Question 5

#### Question:

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $y$  equals:

$$\frac{3x+8}{x^2}$$

#### Solution:

$$y = \frac{3x+8}{x^2} = \frac{3x}{x^2} + \frac{8}{x^2} = \frac{3}{x} + 8x^{-2} = 3x^{-1} + 8x^{-2}$$

$$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$$

$$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 1

#### Question:

Find  $\frac{d\theta}{dt}$  where  $\theta = t^2 - 3t$

#### Solution:

$$\theta = t^2 - 3t$$

$$\frac{d\theta}{dt} = 2t - 3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 2

#### Question:

Find  $\frac{dA}{dr}$  where  $A = 2 \pi r$

#### Solution:

$$A = 2 \pi r$$

$$\frac{dA}{dr} = 2 \pi$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 3

#### Question:

Find  $\frac{dr}{dt}$  where  $r = \frac{12}{t}$

#### Solution:

$$r = \frac{12}{t} = 12t^{-1}$$

$$\frac{dr}{dt} = -12t^{-2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 4

#### Question:

Find  $\frac{dv}{dt}$  where  $v = 9.8t + 6$

#### Solution:

$$v = 9.8t + 6$$

$$\frac{dv}{dt} = 9.8$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 5

#### Question:

Find  $\frac{dR}{dr}$  where  $R = r + \frac{5}{r}$

#### Solution:

$$R = r + \frac{5}{r} = r + 5r^{-1}$$

$$\frac{dR}{dr} = 1 - 5r^{-2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 6

#### Question:

Find  $\frac{dx}{dt}$  where  $x = 3 - 12t + 4t^2$

#### Solution:

$$x = 3 - 12t + 4t^2$$
$$\frac{dx}{dt} = 0 - 12 + 8t$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise G, Question 7

#### Question:

Find  $\frac{dA}{dx}$  where  $A = x(10 - x)$

#### Solution:

$$A = x(10 - x) = 10x - x^2$$

$$\frac{dA}{dx} = 10 - 2x$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise H, Question 1

#### Question:

Find the equation of the tangent to the curve:

(a)  $y = x^2 - 7x + 10$  at the point  $(2, 0)$

(b)  $y = x + \frac{1}{x}$  at the point  $\left(2, 2\frac{1}{2}\right)$

(c)  $y = 4\sqrt{x}$  at the point  $(9, 12)$

(d)  $y = \frac{2x-1}{x}$  at the point  $(1, 1)$

(e)  $y = 2x^3 + 6x + 10$  at the point  $(-1, 2)$

(f)  $y = x^2 + \frac{-7}{x^2}$  at the point  $(1, -6)$

#### Solution:

(a)  $y = x^2 - 7x + 10$

$$\frac{dy}{dx} = 2x - 7$$

At  $(2, 0)$ ,  $x = 2$ , gradient  $= 2 \times 2 - 7 = -3$ .

Therefore, equation of tangent is

$$y - 0 = -3(x - 2)$$

$$y = -3x + 6$$

$$y + 3x - 6 = 0$$

(b)  $y = x + \frac{1}{x} = x + x^{-1}$

$$\frac{dy}{dx} = 1 - x^{-2}$$

At  $\left(2, 2\frac{1}{2}\right)$ ,  $x = 2$ , gradient  $= 1 - 2^{-2} = \frac{3}{4}$ .

Therefore, equation of tangent is

$$y - 2\frac{1}{2} = \frac{3}{4}(x - 2)$$

$$y = \frac{3}{4}x - 1\frac{1}{2} + 2\frac{1}{2}$$

$$y = \frac{3}{4}x + 1$$

$$4y - 3x - 4 = 0$$

(c)  $y = 4\sqrt{x} = 4x^{\frac{1}{2}}$



$$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$$

At ( 9 , 12 ) ,  $x = 9$ , gradient  $= 2 \times 9^{-\frac{1}{2}} = \frac{2}{3}$ .

Therefore, equation of tangent is

$$y - 12 = \frac{2}{3}(x - 9)$$

$$y = \frac{2}{3}x - 6 + 12$$

$$y = \frac{2}{3}x + 6$$

$$3y - 2x - 18 = 0$$

(d)  $y = \frac{2x-1}{x} = \frac{2x}{x} - \frac{1}{x} = 2 - x^{-1}$

$$\frac{dy}{dx} = 0 + x^{-2}$$

At ( 1 , 1 ) ,  $x = 1$ , gradient  $= 1^{-2} = 1$ .

Therefore, equation of tangent is

$$y - 1 = 1 \times (x - 1)$$

$$y = x$$

(e)  $y = 2x^3 + 6x + 10$

$$\frac{dy}{dx} = 6x^2 + 6$$

At ( - 1 , 2 ) ,  $x = -1$ , gradient  $= 6(-1)^2 + 6 = 12$ .

Therefore, equation of tangent is

$$y - 2 = 12 [ x - (-1) ]$$

$$y - 2 = 12x + 12$$

$$y = 12x + 14$$

(f)  $y = x^2 - \frac{7}{x^2} = x^2 - 7x^{-2}$

$$\frac{dy}{dx} = 2x + 14x^{-3}$$

At ( 1 , - 6 ) ,  $x = 1$ , gradient  $= 2 + 14 = 16$ .

Therefore, equation of tangent is

$$y - (-6) = 16(x - 1)$$

$$y + 6 = 16x - 16$$

$$y = 16x - 22$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise H, Question 2

#### Question:

Find the equation of the normal to the curves:

(a)  $y = x^2 - 5x$  at the point  $(6, 6)$

(b)  $y = x^2 - \frac{8}{\sqrt{x}}$  at the point  $(4, 12)$

#### Solution:

(a)  $y = x^2 - 5x$

$$\frac{dy}{dx} = 2x - 5$$

At  $(6, 6)$ ,  $x = 6$ , gradient of curve is  $2 \times 6 - 5 = 7$ .

Therefore, gradient of normal is  $-\frac{1}{7}$ .

The equation of the normal is

$$y - 6 = -\frac{1}{7}(x - 6)$$

$$7y - 42 = -x + 6$$

$$7y + x - 48 = 0$$

(b)  $y = x^2 - \frac{8}{\sqrt{x}} = x^2 - 8x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 2x + 4x^{-\frac{3}{2}}$$

At  $(4, 12)$ ,  $x = 4$ , gradient of curve is  $2 \times 4 + 4(4)^{-\frac{3}{2}} = 8 + \frac{4}{8} = \frac{17}{2}$

Therefore, gradient of normal is  $-\frac{2}{17}$ .

The equation of the normal is

$$y - 12 = -\frac{2}{17}(x - 4)$$

$$17y - 204 = -2x + 8$$

$$17y + 2x - 212 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise H, Question 3

#### Question:

Find the coordinates of the point where the tangent to the curve  $y = x^2 + 1$  at the point  $(2, 5)$  meets the normal to the same curve at the point  $(1, 2)$ .

#### Solution:

$$y = x^2 + 1$$

$$\frac{dy}{dx} = 2x$$

At  $(2, 5)$ ,  $x = 2$ ,  $\frac{dy}{dx} = 4$ .

The tangent at  $(2, 5)$  has gradient 4.

Its equation is

$$y - 5 = 4(x - 2)$$

$$y = 4x - 3 \text{ ①}$$

The curve has gradient 2 at the point  $(1, 2)$ .

The normal is perpendicular to the curve. Its gradient is  $-\frac{1}{2}$ .

The equation of the normal is

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + 2\frac{1}{2} \text{ ②}$$

Solve Equations ① and ② to find where the tangent and the normal meet.

Equation ① – Equation ②:

$$0 = 4\frac{1}{2}x - 5\frac{1}{2}$$

$$x = \frac{11}{9}$$

Substitute into Equation ① to give  $y = \frac{44}{9} - 3 = \frac{17}{9}$ .

Therefore, the tangent at  $(2, 5)$  meets the normal at  $(1, 2)$  at  $\left(\frac{11}{9}, \frac{17}{9}\right)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise H, Question 4

#### Question:

Find the equations of the normals to the curve  $y = x + x^3$  at the points  $(0, 0)$  and  $(1, 2)$ , and find the coordinates of the point where these normals meet.

#### Solution:

$$y = x + x^3$$

$$\frac{dy}{dx} = 1 + 3x^2$$

At  $(0, 0)$  the curve has gradient  $1 + 3 \times 0^2 = 1$ .

The gradient of the normal at  $(0, 0)$  is  $-\frac{1}{1} = -1$ .

The equation of the normal at  $(0, 0)$  is

$$y - 0 = -1(x - 0)$$

$$y = -x \text{ ①}$$

At  $(1, 2)$  the curve has gradient  $1 + 3 \times 1^2 = 4$ .

The gradient of the normal at  $(1, 2)$  is  $-\frac{1}{4}$ .

The equation of the normal at  $(1, 2)$  is

$$y - 2 = -\frac{1}{4}(x - 1)$$

$$4y - 8 = -x + 1$$

$$4y + x - 9 = 0 \text{ ②}$$

Solve Equations ① and ② to find where the normals meet.

Substitute  $y = -x$  into Equation ②:

$$-4x + x = 9 \Rightarrow x = -3 \text{ and } y = +3.$$

Therefore, the normals meet at  $(-3, 3)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise H, Question 5

#### Question:

For  $f(x) = 12 - 4x + 2x^2$ , find an equation of the tangent and normal at the point where  $x = -1$  on the curve with equation  $y = f(x)$ . [E]

#### Solution:

$$y = 12 - 4x + 2x^2$$

$$\frac{dy}{dx} = 0 - 4 + 4x$$

$$\text{when } x = -1, \frac{dy}{dx} = -4 - 4 = -8.$$

The gradient of the curve is  $-8$  when  $x = -1$ .

As  $y = f(x)$ , when  $x = -1$

$$y = f(-1) = 12 + 4 + 2 = 18$$

The tangent at  $(-1, 18)$  has gradient  $-8$ . So its equation is

$$y - 18 = -8(x + 1)$$

$$y - 18 = -8x - 8$$

$$y = 10 - 8x$$

The normal at  $(-1, 18)$  has gradient  $\frac{-1}{-8} = \frac{1}{8}$ . So its equation is

$$y - 18 = \frac{1}{8} (x + 1)$$

$$8y - 144 = x + 1$$

$$8y - x - 145 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 1

#### Question:

A curve is given by the equation  $y = 3x^2 + 3 + \frac{1}{x^2}$ , where  $x > 0$ .

At the points  $A$ ,  $B$  and  $C$  on the curve,  $x = 1$ ,  $2$  and  $3$  respectively.  
Find the gradients at  $A$ ,  $B$  and  $C$ . **[E]**

#### Solution:

$$y = 3x^2 + 3 + \frac{1}{x^2} = 3x^2 + 3 + x^{-2}$$

$$\frac{dy}{dx} = 6x - 2x^{-3} = 6x - \frac{2}{x^3}$$

$$\text{When } x = 1, \frac{dy}{dx} = 6 \times 1 - \frac{2}{1^3} = 4$$

$$\text{When } x = 2, \frac{dy}{dx} = 6 \times 2 - \frac{2}{2^3} = 12 - \frac{2}{8} = 11 \frac{3}{4}$$

$$\text{When } x = 3, \frac{dy}{dx} = 6 \times 3 - \frac{2}{3^3} = 18 - \frac{2}{27} = 17 \frac{25}{27}$$

The gradients at points  $A$ ,  $B$  and  $C$  are  $4$ ,  $11 \frac{3}{4}$  and  $17 \frac{25}{27}$ , respectively.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 2

#### Question:

Taking  $f(x) = \frac{1}{4}x^4 - 4x^2 + 25$ , find the values of  $x$  for which  $f'(x) = 0$ . **[E]**

#### Solution:

$$f(x) = \frac{1}{4}x^4 - 4x^2 + 25$$

$$f'(x) = x^3 - 8x$$

$$\text{When } f'(x) = 0,$$

$$x^3 - 8x = 0$$

$$x(x^2 - 8) = 0$$

$$x = 0 \text{ or } x^2 = 8$$

$$x = 0 \text{ or } \pm \sqrt{8}$$

$$x = 0 \text{ or } \pm 2\sqrt{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 3

#### Question:

A curve is drawn with equation  $y = 3 + 5x + x^2 - x^3$ . Find the coordinates of the two points on the curve where the gradient of the curve is zero. **[E]**

#### Solution:

$$y = 3 + 5x + x^2 - x^3$$

$$\frac{dy}{dx} = 5 + 2x - 3x^2$$

Put  $\frac{dy}{dx} = 0$ . Then

$$5 + 2x - 3x^2 = 0$$

$$(5 - 3x)(1 + x) = 0$$

$$x = -1 \text{ or } x = \frac{5}{3}$$

Substitute to obtain

$$y = 3 - 5 + 1 - (-1)^3 \text{ when } x = -1, \text{ i.e.}$$

$$y = 0 \text{ when } x = -1$$

and

$$y = 3 + 5 \left( \frac{5}{3} \right) + \left( \frac{5}{3} \right)^2 - \left( \frac{5}{3} \right)^3 \text{ when } x = \frac{5}{3}, \text{ i.e.}$$

$$y = 3 + \frac{25}{3} + \frac{25}{9} - \frac{125}{27} = 9 \frac{13}{27} \text{ when } x = \frac{5}{3}$$

So the points have coordinates  $(-1, 0)$  and  $\left(1 \frac{2}{3}, 9 \frac{13}{27}\right)$ .



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 4

#### Question:

Calculate the  $x$ -coordinates of the points on the curve with equation  $y = 7x^2 - x^3$  at which the gradient is equal to 16. **[E]**

#### Solution:

$$y = 7x^2 - x^3$$

$$\frac{dy}{dx} = 14x - 3x^2$$

$$\text{Put } \frac{dy}{dx} = 16, \text{ i.e.}$$

$$14x - 3x^2 = 16$$

$$3x^2 - 14x + 16 = 0$$

$$(3x - 8)(x - 2) = 0$$

$$x = \frac{8}{3} \text{ or } x = 2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 5

#### Question:

Find the  $x$ -coordinates of the two points on the curve with equation  $y = x^3 - 11x + 1$  where the gradient is 1. Find the corresponding  $y$ -coordinates. **[E]**

#### Solution:

$$y = x^3 - 11x + 1$$

$$\frac{dy}{dx} = 3x^2 - 11$$

As gradient is 1, put  $\frac{dy}{dx} = 1$ , then

$$3x^2 - 11 = 1$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

Substitute these values into  $y = x^3 - 11x + 1$ :

$$y = 2^3 - 11 \times 2 + 1 = -13 \text{ when } x = 2 \text{ and}$$

$$y = (-2)^3 - 11(-2) + 1 = 15 \text{ when } x = -2$$

The gradient is 1 at the points  $(2, -13)$  and  $(-2, 15)$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 6

#### Question:

The function  $f$  is defined by  $f(x) = x + \frac{9}{x}$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$ .

(a) Find  $f'(x)$ .

(b) Solve  $f'(x) = 0$ . **[E]**

#### Solution:

(a)  $f(x) = x + \frac{9}{x} = x + 9x^{-1}$

$$f'(x) = 1 - 9x^{-2} = 1 - \frac{9}{x^2}$$

(b) When  $f'(x) = 0$ ,

$$1 - \frac{9}{x^2} = 0$$

$$\frac{9}{x^2} = 1$$

$$x^2 = 9$$

$$x = \pm 3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 7

#### Question:

Given that

$$y = x^{\frac{3}{2}} + \frac{48}{x}, x > 0,$$

find the value of  $x$  and the value of  $y$  when  $\frac{dy}{dx} = 0$ . [E]

#### Solution:

$$y = x^{\frac{3}{2}} + \frac{48}{x} = x^{\frac{3}{2}} + 48x^{-1}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 48x^{-2}$$

Put  $\frac{dy}{dx} = 0$ , then

$$\frac{3}{2}x^{\frac{1}{2}} - \frac{48}{x^2} = 0$$

$$\frac{3}{2}x^{\frac{1}{2}} = \frac{48}{x^2}$$

Multiply both sides by  $x^2$ :

$$\frac{3}{2}x^2 \cdot \frac{1}{2} = 48$$

$$x^2 \cdot \frac{1}{2} = 32$$

$$x = (32)^{\frac{2}{5}}$$

$$x = 4$$

Substitute to give  $y = 4^{\frac{3}{2}} + \frac{48}{4} = 8 + 12 = 20$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 8

#### Question:

Given that

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}, x > 0,$$

find  $\frac{dy}{dx}$ . [E]

#### Solution:

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{4}{2}x^{-\frac{3}{2}} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 9

#### Question:

A curve has equation  $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$ .

(a) Show that  $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$

(b) Find the coordinates of the point on the curve where the gradient is zero. **[E]**

#### Solution:

(a)  $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = 12 \left( \frac{1}{2} \right) x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$$

(b) The gradient is zero when  $\frac{dy}{dx} = 0$ :

$$\frac{3}{2}x^{-\frac{1}{2}}(4 - x) = 0$$

$$x = 4$$

Substitute into  $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$  to obtain

$$y = 12 \times 2 - 2^3 = 16$$

The gradient is zero at the point with coordinates ( 4 , 16 ) .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 10

#### Question:

(a) Expand  $\left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right)$ .

(b) A curve has equation  $y = \left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right)$ ,  $x > 0$ . Find  $\frac{dy}{dx}$ .

(c) Use your answer to **b** to calculate the gradient of the curve at the point where  $x = 4$ . **[E]**

#### Solution:

(a)  $\left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right) = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

(b)  $y = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

$$\frac{dy}{dx} = 1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

(c) When  $x = 4$ ,  $\frac{dy}{dx} = 1 + \frac{3}{2} \times 2 + \frac{1}{2} \times \frac{1}{4^{\frac{3}{2}}} = 1 + 3 + \frac{1}{16} = 4 \frac{1}{16}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 11

#### Question:

Differentiate with respect to  $x$ :

$$2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2} \text{ [E]}$$

#### Solution:

$$\text{Let } y = 2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$$

$$\Rightarrow y = 2x^3 + x^{\frac{1}{2}} + \frac{x^2}{x^2} + \frac{2x}{x^2}$$

$$\Rightarrow y = 2x^3 + x^{\frac{1}{2}} + 1 + 2x^{-1}$$

$$\frac{dy}{dx} = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2} = 6x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 12

#### Question:

The volume,  $V \text{ cm}^3$ , of a tin of radius  $r \text{ cm}$  is given by the formula  $V = \pi (40r - r^2 - r^3)$ . Find the positive value of  $r$  for which  $\frac{dV}{dr} = 0$ , and find the value of  $V$  which corresponds to this value of  $r$ . **[E]**

#### Solution:

$$V = \pi (40r - r^2 - r^3)$$

$$\frac{dV}{dr} = 40\pi - 2\pi r - 3\pi r^2$$

Put  $\frac{dV}{dr} = 0$ , then

$$\begin{aligned}\pi (40 - 2r - 3r^2) &= 0 \\ (4 + r)(10 - 3r) &= 0\end{aligned}$$

$$r = \frac{10}{3} \text{ or } -4$$

As  $r$  is positive,  $r = \frac{10}{3}$ .

Substitute into the given expression for  $V$ :

$$V = \pi \left( 40 \times \frac{10}{3} - \frac{100}{9} - \frac{1000}{27} \right) = \frac{2300}{27} \pi$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 13

#### Question:

The total surface area of a cylinder  $A \text{ cm}^2$  with a fixed volume of 1000 cubic cm is given by the formula  $A = 2\pi x^2 + \frac{2000}{x}$ , where  $x$  cm is the radius. Show that when the rate of change of the area with respect to the radius is zero,  $x^3 =$

$$\frac{500}{\pi}. \text{ [E]}$$

#### Solution:

$$A = 2\pi x^2 + \frac{2000}{x} = 2\pi x^2 + 2000x^{-1}$$

$$\frac{dA}{dx} = 4\pi x - 2000x^{-2} = 4\pi x - \frac{2000}{x^2}$$

$$\text{When } \frac{dA}{dx} = 0,$$

$$4\pi x = \frac{2000}{x^2}$$

$$x^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 14

#### Question:

The curve with equation  $y = ax^2 + bx + c$  passes through the point  $(1, 2)$ . The gradient of the curve is zero at the point  $(2, 1)$ . Find the values of  $a$ ,  $b$  and  $c$ . **[E]**

#### Solution:

The point  $(1, 2)$  lies on the curve with equation  $y = ax^2 + bx + c$ .  
Therefore, substitute  $x = 1$ ,  $y = 2$  into the equation to give

$$2 = a + b + c \text{ ①}$$

The point  $(2, 1)$  also lies on the curve.  
Therefore, substitute  $x = 2$ ,  $y = 1$  to give

$$1 = 4a + 2b + c \text{ ②}$$

Eliminate  $c$  by subtracting Equation ② – Equation ①:

$$-1 = 3a + b \text{ ③}$$

The gradient of the curve is zero at  $(2, 1)$  so substitute  $x = 2$  into the expression for  $\frac{dy}{dx} = 0$ .

$$\text{As } y = ax^2 + bx + c$$

$$\frac{dy}{dx} = 2ax + b$$

At  $(2, 1)$

$$0 = 4a + b \text{ ④}$$

Solve Equations ③ and ④ by subtracting ④ – ③:

$$1 = a$$

Substitute  $a = 1$  into Equation ③ to give  $b = -4$ .

Then substitute  $a$  and  $b$  into Equation ① to give  $c = 5$ .

Therefore,  $a = 1$ ,  $b = -4$ ,  $c = 5$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 15

#### Question:

A curve  $C$  has equation  $y = x^3 - 5x^2 + 5x + 2$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $x$ .

(b) The points  $P$  and  $Q$  lie on  $C$ . The gradient of  $C$  at both  $P$  and  $Q$  is 2. The  $x$ -coordinate of  $P$  is 3.

(i) Find the  $x$ -coordinate of  $Q$ .

(ii) Find an equation for the tangent to  $C$  at  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants.

(iii) If this tangent intersects the coordinate axes at the points  $R$  and  $S$ , find the length of  $RS$ , giving your answer as a surd. **[E]**

#### Solution:

$$y = x^3 - 5x^2 + 5x + 2$$

(a)  $\frac{dy}{dx} = 3x^2 - 10x + 5$

(b) Given that the gradient is 2,  $\frac{dy}{dx} = 2$

$$3x^2 - 10x + 5 = 2$$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3} \text{ or } 3$$

(i) At  $P$ ,  $x = 3$ . Therefore, at  $Q$ ,  $x = \frac{1}{3}$ .

(ii) At the point  $P$ ,  $x = 3$ ,  $y = 3^3 - 5 \times 3^2 + 5 \times 3 + 2 = 27 - 45 + 15 + 2 = -1$

The gradient of the curve is 2.

The equation of the tangent at  $P$  is

$$y - (-1) = 2(x - 3)$$

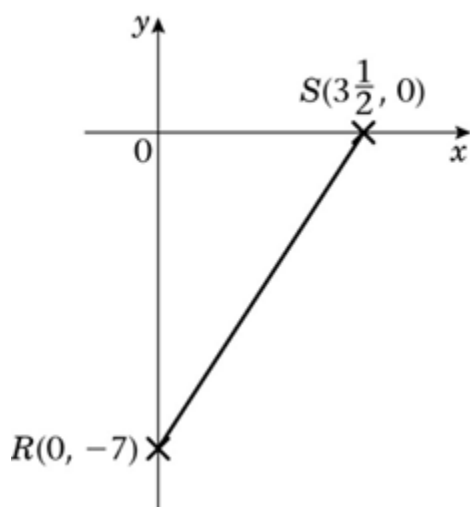
$$y + 1 = 2x - 6$$

$$y = 2x - 7$$

(iii) This tangent meets the axes when  $x = 0$  and when  $y = 0$ .

When  $x = 0$ ,  $y = -7$ . When  $y = 0$ ,  $x = 3\frac{1}{2}$ .

The tangent meets the axes at  $(0, -7)$  and  $(3\frac{1}{2}, 0)$ .



The distance  $RS = \sqrt{\left(3\frac{1}{2} - 0\right)^2 + [0 - (-7)]^2} = \sqrt{\frac{49}{4} + 49} = \frac{7}{2}\sqrt{1+4} = \frac{7}{2}\sqrt{5}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 16

#### Question:

Find an equation of the tangent and the normal at the point where  $x = 2$  on the curve with equation  $y = \frac{8}{x} - x + 3x^2$ ,  $x > 0$ . [E]

#### Solution:

$$y = \frac{8}{x} - x + 3x^2 = 8x^{-1} - x + 3x^2$$

$$\frac{dy}{dx} = -8x^{-2} - 1 + 6x = -\frac{8}{x^2} - 1 + 6x$$

$$\text{when } x = 2, \frac{dy}{dx} = -\frac{8}{4} - 1 + 12 = 9$$

$$\text{At } x = 2, y = \frac{8}{2} - 2 + 3 \times 2^2 = 14$$

So the equation of the tangent through the point  $(2, 14)$  with gradient 9 is

$$y - 14 = 9(x - 2)$$

$$y = 9x - 18 + 14$$

$$y = 9x - 4$$

The gradient of the normal is  $-\frac{1}{9}$ , as the normal is at right angles to the tangent.

So the equation of the normal is

$$y - 14 = -\frac{1}{9}(x - 2)$$

$$9y + x = 128$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Differentiation

#### Exercise I, Question 17

#### Question:

The normals to the curve  $2y = 3x^3 - 7x^2 + 4x$ , at the points  $O(0, 0)$  and  $A(1, 0)$ , meet at the point  $N$ .

- (a) Find the coordinates of  $N$ .
- (b) Calculate the area of triangle  $OAN$ . **[E]**

#### Solution:

$$(a) 2y = 3x^3 - 7x^2 + 4x$$

$$y = \frac{3}{2}x^3 - \frac{7}{2}x^2 + 2x$$

$$\frac{dy}{dx} = \frac{9}{2}x^2 - 7x + 2$$

At  $(0, 0)$ ,  $x = 0$ , gradient of curve is  $0 - 0 + 2 = 2$ .

The gradient of the normal at  $(0, 0)$  is  $-\frac{1}{2}$ .

The equation of the normal at  $(0, 0)$  is  $y = -\frac{1}{2}x$ .

At  $(1, 0)$ ,  $x = 1$ , gradient of curve is  $\frac{9}{2} - 7 + 2 = -\frac{1}{2}$ .

The gradient of the normal at  $(1, 0)$  is 2.

The equation of the normal at  $(1, 0)$  is  $y = 2(x - 1)$ .

The normals meet when  $y = 2x - 2$  and  $y = -\frac{1}{2}x$ :

$$2x - 2 = -\frac{1}{2}x$$

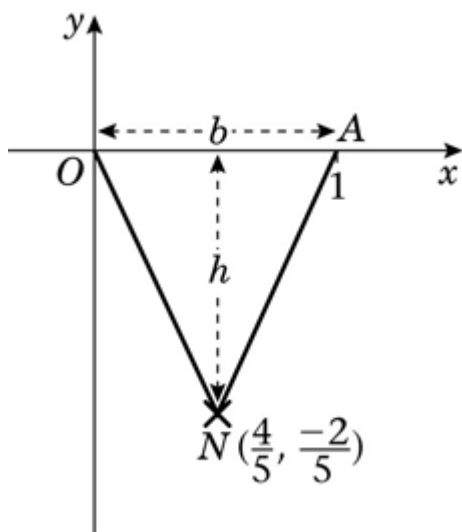
$$2\frac{1}{2}x = 2$$

$$x = 2 \div 2\frac{1}{2} = \frac{4}{5}$$

Substitute into  $y = 2x - 2$  to obtain  $y = -\frac{2}{5}$  and check in  $y = -\frac{1}{2}x$ .

$N$  has coordinates  $\left(\frac{4}{5}, -\frac{2}{5}\right)$ .

(b)



The area of  $\triangle OAN = \frac{1}{2}$  base  $\times$  height

$$\text{base } (b) = 1$$

$$\text{height}(h) = \frac{2}{5}$$

$$\text{Area} = \frac{1}{2} \times 1 \times \frac{2}{5} = \frac{1}{5}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 1

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$x^5$$

#### Solution:

$$\frac{dy}{dx} = x^5$$

$$y = \frac{x^6}{6} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 2

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$10x^4$$

#### Solution:

$$\frac{dy}{dx} = 10x^4$$

$$y = 10 \frac{x^5}{5} + c$$

$$y = 2x^5 + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 3

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$3x^2$$

#### Solution:

$$\frac{dy}{dx} = 3x^2$$

$$y = 3 \frac{x^3}{3} + c$$

$$y = x^3 + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 4

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-x^{-2}$$

#### Solution:

$$\frac{dy}{dx} = -x^{-2}$$

$$y = -\frac{x^{-1}}{-1} + c$$

$$y = x^{-1} + c \text{ or}$$

$$y = \frac{1}{x} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 5

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-4x^{-3}$$

#### Solution:

$$\frac{dy}{dx} = -4x^{-3}$$

$$y = -4 \frac{x^{-2}}{-2} + c$$

$$y = 2x^{-2} + c \text{ or}$$

$$y = \frac{2}{x^2} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 6

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$x^{-\frac{2}{3}}$$

#### Solution:

$$\frac{dy}{dx} = x^{-\frac{2}{3}}$$

$$y = \frac{x^{-\frac{2}{3} + 1}}{-\frac{2}{3} + 1} + c$$

$$y = \frac{3}{5}x^{\frac{1}{3}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 7

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$4x^{\frac{1}{2}}$$

#### Solution:

$$\frac{dy}{dx} = 4x^{\frac{1}{2}}$$

$$y = 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = \frac{8}{3}x^{\frac{3}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 8

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-2x^6$$

#### Solution:

$$\frac{dy}{dx} = -2x^6$$

$$y = -2 \frac{x^7}{7} + c$$

$$y = -\frac{2}{7}x^7 + c$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 9

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$3x^5$$

#### Solution:

$$\frac{dy}{dx} = 3x^5$$

$$y = 3 \frac{x^6}{6} + c$$

$$y = \frac{1}{2}x^6 + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 10

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$3x^{-4}$$

#### Solution:

$$\frac{dy}{dx} = 3x^{-4}$$

$$y = 3 \frac{x^{-3}}{-3} + c$$

$$y = -x^{-3} + c \text{ or}$$

$$y = -\frac{1}{x^3} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 11

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$x^{-\frac{1}{2}}$$

#### Solution:

$$\frac{dy}{dx} = x^{-\frac{1}{2}}$$

$$y = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = 2x^{\frac{1}{2}} + c \text{ or}$$

$$y = 2\sqrt{x} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 12

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$5x^{-\frac{3}{2}}$$

#### Solution:

$$\frac{dy}{dx} = 5x^{-\frac{3}{2}}$$

$$y = 5 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$y = -10x^{-\frac{1}{2}} + c \text{ or}$$

$$y = \frac{-10}{\sqrt{x}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 13

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-2x^{-\frac{3}{2}}$$

#### Solution:

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}}$$

$$y = -2 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$y = 4x^{-\frac{1}{2}} + c \text{ or}$$

$$y = \frac{4}{\sqrt{x}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 14

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$6x^{\frac{1}{3}}$$

#### Solution:

$$\frac{dy}{dx} = 6x^{\frac{1}{3}}$$

$$y = 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$y = \frac{18}{4} x^{\frac{4}{3}} + c$$

$$y = \frac{9}{2} x^{\frac{4}{3}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 15

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$36x^{11}$$

#### Solution:

$$\frac{dy}{dx} = 36x^{11}$$

$$y = 36 \frac{x^{12}}{12} + c$$

$$y = 3x^{12} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 16

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-14x^{-8}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 17

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$-3x^{-\frac{2}{3}}$$

#### Solution:

$$\frac{dy}{dx} = -3x^{-\frac{2}{3}}$$

$$y = -3 \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$y = -9x^{\frac{1}{3}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 18

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$- 5$$

#### Solution:

$$\frac{dy}{dx} = - 5 = - 5x^0$$

$$y = - 5 \frac{x^1}{1} + c$$

$$y = - 5x + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 19

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$6x$$

#### Solution:

$$\frac{dy}{dx} = 6x$$

$$y = 6 \frac{x^2}{2} + c$$

$$y = 3x^2 + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise A, Question 20

#### Question:

Find an expression for  $y$  when  $\frac{dy}{dx}$  is:

$$2x^{-0.4}$$

#### Solution:

$$\frac{dy}{dx} = 2x^{-0.4}$$

$$y = 2 \frac{x^{0.6}}{0.6} + c$$

$$y = \frac{20}{6}x^{0.6} + c$$

$$y = \frac{10}{3}x^{0.6} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise B, Question 1

#### Question:

Find  $y$  when  $\frac{dy}{dx}$  is given by the following expressions. In each case simplify your answer:

(a)  $4x - x^{-2} + 6x^{\frac{1}{2}}$

(b)  $15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

(c)  $x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

(d)  $4x^3 + x^{-\frac{2}{3}} - x^{-2}$

(e)  $4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

(f)  $5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

(g)  $-\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

(h)  $5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

#### Solution:

(a)  $\frac{dy}{dx} = 4x - x^{-2} + 6x^{\frac{1}{2}}$

$$y = 4 \frac{x^2}{2} - \frac{x^{-1}}{-1} + 6 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = 2x^2 + x^{-1} + 4x^{\frac{3}{2}} + c$$

(b)  $\frac{dy}{dx} = 15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

$$y = 15 \frac{x^3}{3} + 6 \frac{x^{-2}}{-2} - 3 \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + c$$

$$y = 5x^3 - 3x^{-2} + 2x^{-\frac{3}{2}} + c$$

$$(c) \frac{dy}{dx} = x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$$

$$y = \frac{x^4}{4} - \frac{3}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6 \frac{x^{-1}}{-1} + c$$

$$y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$$

$$(d) \frac{dy}{dx} = 4x^3 + x^{-\frac{2}{3}} - x^{-2}$$

$$y = 4 \frac{x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c$$

$$y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$$

$$(e) \frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$$

$$y = 4x - 12 \frac{x^{-3}}{-3} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$$

$$(f) \frac{dy}{dx} = 5x^{\frac{2}{3}} - 10x^4 + x^{-3}$$

$$y = 5 \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - 10 \frac{x^5}{5} + \frac{x^{-2}}{-2} + c$$

$$y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$$

$$(g) \frac{dy}{dx} = -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$$

$$y = -\frac{4}{3} \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + 8 \frac{x^2}{2} + c$$

$$y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$$

$$(h) \frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$$

$$y = 5 \frac{x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 12 \frac{x^{-4}}{-4} + c$$

$$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise B, Question 2

#### Question:

Find  $f(x)$  when  $f'(x)$  is given by the following expressions. In each case simplify your answer:

(a)  $12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

(b)  $6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

(c)  $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

(d)  $10x + 8x^{-3}$

(e)  $2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

(f)  $9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

(g)  $x^2 + x^{-2} + x^{\frac{1}{2}}$

(h)  $-2x^{-3} - 2x + 2x^{\frac{1}{2}}$

#### Solution:

(a)  $f'(x) = 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

$$f(x) = 12 \frac{x^2}{2} + \frac{3}{2} \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c$$

$$f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$$

(b)  $f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

$$f(x) = 6 \frac{x^6}{6} + 6 \frac{x^{-6}}{-6} - \frac{1}{6} \frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$$

$$f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$$



$$(c) f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$f(x) = \frac{1}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2} \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$$

$$(d) f'(x) = 10x + 8x^{-3}$$

$$f(x) = 10 \frac{x^2}{2} + 8 \frac{x^{-2}}{-2} + c$$

$$f(x) = 5x^2 - 4x^{-2} + c$$

$$(e) f'(x) = 2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$$

$$f(x) = 2 \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 4 \frac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + c$$

$$f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$$

$$(f) f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$$

$$f(x) = 9 \frac{x^3}{3} + 4 \frac{x^{-2}}{-2} + \frac{1}{4} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$$

$$(g) f'(x) = x^2 + x^{-2} + x^{\frac{1}{2}}$$

$$f(x) = \frac{x^3}{3} + \frac{x^{-1}}{-1} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$f(x) = \frac{1}{3}x^3 - x^{-1} + \frac{2}{3}x^{\frac{3}{2}} + c$$

$$(h) f'(x) = -2x^{-3} - 2x + 2x^{\frac{1}{2}}$$

$$f(x) = -2 \frac{x^{-2}}{-2} - 2 \frac{x^2}{2} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$f(x) = x^{-2} - x^2 + \frac{4}{3}x^{\frac{3}{2}} + c$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 1

#### Question:

Find the following integral:

$$\int (x^3 + 2x) \, dx$$

#### Solution:

$$\begin{aligned}\int (x^3 + 2x) \, dx \\ &= \frac{x^4}{4} + 2 \frac{x^2}{2} + c \\ &= \frac{1}{4}x^4 + x^2 + c\end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 2

#### Question:

Find the following integral:

$$\int (2x^{-2} + 3) dx$$

#### Solution:

$$\int (2x^{-2} + 3) dx$$

$$= 2 \frac{x^{-1}}{-1} + 3x + c$$

$$= -2x^{-1} + 3x + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 3

#### Question:

Find the following integral:

$$\int \left( 5x^{\frac{3}{2}} - 3x^2 \right) dx$$

#### Solution:

$$\int \left( 5x^{\frac{3}{2}} - 3x^2 \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 3 \frac{x^3}{3} + c$$

$$= 2x^{\frac{5}{2}} - x^3 + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 4

#### Question:

Find the following integral:

$$\int \left( 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

#### Solution:

$$\int \left( 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

$$= 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 4x + c$$

$$= \frac{4}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 5

#### Question:

Find the following integral:

$$\int (4x^3 - 3x^{-4} + r) dx$$

#### Solution:

$$\begin{aligned} & \int (4x^3 - 3x^{-4} + r) dx \\ &= 4 \frac{x^4}{4} - 3 \frac{x^{-3}}{-3} + rx + c \\ &= x^4 + x^{-3} + rx + c \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 6

#### Question:

Find the following integral:

$$\int (3t^2 - t^{-2}) dt$$

#### Solution:

$$\begin{aligned} \int (3t^2 - t^{-2}) dt \\ &= 3 \frac{t^3}{3} - \frac{t^{-1}}{-1} + c \\ &= t^3 + t^{-1} + c \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 7

#### Question:

Find the following integral:

$$\int \left( 2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

#### Solution:

$$\int \left( 2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

$$= 2 \frac{t^3}{3} - 3 \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$$

$$= \frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 8

#### Question:

Find the following integral:

$$\int \left( x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

#### Solution:

$$\int \left( x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

$$= \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$= \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 9

#### Question:

Find the following integral:

$$\int (px^4 + 2t + 3x^{-2}) dx$$

#### Solution:

$$\begin{aligned} \int (px^4 + 2t + 3x^{-2}) dx \\ &= p \frac{x^5}{5} + 2tx + 3 \frac{x^{-1}}{-1} + c \\ &= \frac{p}{5}x^5 + 2tx - 3x^{-1} + c \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise C, Question 10

#### Question:

Find the following integral:

$$\int (pt^3 + q^2 + px^3) dt$$

#### Solution:

$$\begin{aligned} \int (pt^3 + q^2 + px^3) dt \\ = p \frac{t^4}{4} + q^2t + px^3t + c \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise D, Question 1

#### Question:

Find the following integrals:

$$(a) \int (2x + 3) x^2 dx$$

$$(b) \int \frac{(2x^2 + 3)}{x^2} dx$$

$$(c) \int (2x + 3)^2 dx$$

$$(d) \int (2x + 3)(x - 1) dx$$

$$(e) \int (2x + 3) \sqrt{x} dx$$

#### Solution:

$$\begin{aligned} (a) \int (2x + 3) x^2 dx &= \int (2x^3 + 3x^2) dx \\ &= 2 \frac{x^4}{4} + 3 \frac{x^3}{3} + c \\ &= \frac{1}{2} x^4 + x^3 + c \end{aligned}$$

$$\begin{aligned} (b) \int \frac{(2x^2 + 3)}{x^2} dx &= \int \left( \frac{2x^2}{x^2} + \frac{3}{x^2} \right) dx \\ &= \int (2 + 3x^{-2}) dx \\ &= 2x + 3 \frac{x^{-1}}{-1} + c \\ &= 2x - 3x^{-1} + c \\ \text{or } &= 2x - \frac{3}{x} + c \end{aligned}$$

$$\begin{aligned} (c) \int (2x + 3)^2 dx &= \int (4x^2 + 12x + 9) dx \\ &= 4 \frac{x^3}{3} + 12 \frac{x^2}{2} + 9x + c \\ &= \frac{4}{3} x^3 + 6x^2 + 9x + c \end{aligned}$$

$$\begin{aligned} (d) \int (2x + 3)(x - 1) dx &= \int (2x^2 + x - 3) dx \\ &= 2 \frac{x^3}{3} + \frac{x^2}{2} - 3x + c \end{aligned}$$

$$= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$$

$$(e) \int (2x + 3) \sqrt{x} \, dx$$

$$= \int (2x + 3) x^{\frac{1}{2}} \, dx$$

$$= \int \left( 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) \, dx$$

$$= 2 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{4}{5}\sqrt{x^5} + 2\sqrt{x^3} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise D, Question 2

#### Question:

Find  $\int f(x)dx$  when  $f(x)$  is given by the following:

(a)  $(x + 2)^2$

(b)  $\left(x + \frac{1}{x}\right)^2$

(c)  $(\sqrt{x+2})^2$

(d)  $\sqrt{x(x+2)}$

(e)  $\left(\frac{x+2}{\sqrt{x}}\right)$

(f)  $\left(\frac{1}{\sqrt{x}} + 2\sqrt{x}\right)$

#### Solution:

$$\begin{aligned} \text{(a)} \quad & \int (x + 2)^2 dx \\ &= \int (x^2 + 4x + 4) dx \\ &= \frac{1}{3}x^3 + \frac{4}{2}x^2 + 4x + c \\ &= \frac{1}{3}x^3 + 2x^2 + 4x + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int \left(x + \frac{1}{x}\right)^2 dx \\ &= \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx \\ &= \int (x^2 + 2 + x^{-2}) dx \\ &= \frac{1}{3}x^3 + 2x + \frac{x^{-1}}{-1} + c \\ &= \frac{1}{3}x^3 + 2x - x^{-1} + c \\ \text{or} \quad &= \frac{1}{3}x^3 + 2x - \frac{1}{x} + c \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \int (\sqrt{x+2})^2 dx \\ &= \int (x + 4\sqrt{x+2}) dx \end{aligned}$$

$$= \int \left( x + 4x^{\frac{1}{2}} + 4 \right) dx$$

$$= \frac{1}{2}x^2 + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$$

$$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$$

$$(d) \int \sqrt{x(x+2)} dx$$

$$= \int \left( x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + c$$

$$(e) \int \left( \frac{x+2}{\sqrt{x}} \right) dx$$

$$= \int \left( \frac{x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} \right) dx$$

$$= \int \left( x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

$$\text{or} = \frac{2}{3}\sqrt{x^3} + 4\sqrt{x} + c$$

$$(f) \int \left( \frac{1}{\sqrt{x}} + 2\sqrt{x} \right) dx$$

$$= \int \left( x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$



$$= 2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

$$\text{or } = 2\sqrt{x} + \frac{4}{3}\sqrt{x^3} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise D, Question 3

#### Question:

Find the following integrals:

$$(a) \int \left( 3\sqrt{x} + \frac{1}{x^2} \right) dx$$

$$(b) \int \left( \frac{2}{\sqrt{x}} + 3x^2 \right) dx$$

$$(c) \int \left( x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$$

$$(d) \int \left( \frac{2+x}{x^3} + 3 \right) dx$$

$$(e) \int (x^2 + 3)(x - 1) dx$$

$$(f) \int \left( \frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$$

$$(g) \int (x - 3)^2 dx$$

$$(h) \int \frac{(2x+1)^2}{\sqrt{x}} dx$$

$$(i) \int \left( 3 + \frac{\sqrt{x+6x^3}}{x} \right) dx$$

$$(j) \int \sqrt{x}(\sqrt{x+3})^2 dx$$

#### Solution:

$$(a) \int \left( 3\sqrt{x} + \frac{1}{x^2} \right) dx$$

$$= \int \left( 3x^{\frac{1}{2}} + x^{-2} \right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + c$$

$$= 2x^{\frac{3}{2}} - x^{-1} + c$$

$$\text{or} = 2\sqrt{x^3} - \frac{1}{x} + c$$

$$\text{(b)} \int \left( \frac{2}{\sqrt{x}} + 3x^2 \right) dx$$

$$= \int \left( 2x^{-\frac{1}{2}} + 3x^2 \right) dx$$

$$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3}{3}x^3 + c$$

$$= 4x^{\frac{1}{2}} + x^3 + c$$

$$\text{or} = 4\sqrt{x} + x^3 + c$$

$$\text{(c)} \int \left( x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$$

$$= \int \left( x^{\frac{2}{3}} + 4x^{-3} \right) dx$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 4 \frac{x^{-2}}{-2} + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} - 2x^{-2} + c$$

$$\text{or} = \frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$$

$$\text{(d)} \int \left( \frac{2+x}{x^3} + 3 \right) dx$$

$$= \int (2x^{-3} + x^{-2} + 3) dx$$

$$= 2 \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$$

$$= -x^{-2} - x^{-1} + 3x + c$$

$$\text{or} = -\frac{1}{x^2} - \frac{1}{x} + 3x + c$$

$$\text{(e)} \int (x^2 + 3)(x - 1) dx$$

$$= \int (x^3 - x^2 + 3x - 3) dx$$

$$= \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$$

$$\text{(f)} \int \left( \frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$$

$$= \int \left( 2x^{-\frac{1}{2}} + 3x^{\frac{3}{2}} \right) dx$$

$$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 3 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 4x^{\frac{1}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$$

$$\text{or } = 4\sqrt{x} + \frac{6}{5}x^2\sqrt{x} + c$$

$$\begin{aligned} \text{(g)} \int (x-3)^2 dx &= \int (x^2 - 6x + 9) dx \\ &= \frac{1}{3}x^3 - \frac{6}{2}x^2 + 9x + c \\ &= \frac{1}{3}x^3 - 3x^2 + 9x + c \end{aligned}$$

$$\begin{aligned} \text{(h)} \int \frac{(2x+1)^2}{\sqrt{x}} dx &= \int x^{-\frac{1}{2}} \left( 4x^2 + 4x + 1 \right) dx \\ &= \int \left( 4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx \\ &= 4 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c \\ \text{or } &= \frac{8}{5}\sqrt{x^5} + \frac{8}{3}\sqrt{x^3} + 2\sqrt{x} + c \end{aligned}$$

$$\begin{aligned} \text{(i)} \int \left( 3 + \frac{\sqrt{x+6x^3}}{x} \right) dx &= \int \left( 3 + x^{-\frac{1}{2}} + 6x^2 \right) dx \\ &= 3x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6}{3}x^3 + c \\ &= 3x + 2x^{\frac{1}{2}} + 2x^3 + c \\ \text{or } &= 3x + 2\sqrt{x} + 2x^3 + c \end{aligned}$$

$$\text{(j)} \int \sqrt{x} (\sqrt{x+3})^2 dx$$

$$= \int x^{\frac{1}{2}} \left( x + 6x^{\frac{1}{2}} + 9 \right) dx$$

$$= \int \left( x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{6}{2}x^2 + 9 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{2}{5}\sqrt{x^5} + 3x^2 + 6\sqrt{x^3} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise E, Question 1

#### Question:

Find the equation of the curve with the given  $\frac{dy}{dx}$  that passes through the given point:

(a)  $\frac{dy}{dx} = 3x^2 + 2x$ ; point ( 2 , 10 )

(b)  $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$ ; point ( 1 , 4 )

(c)  $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$ ; point ( 4 , 11 )

(d)  $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$ ; point ( 4 , 0 )

(e)  $\frac{dy}{dx} = (x + 2)^2$ ; point ( 1 , 7 )

(f)  $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$ ; point ( 0 , 1 )

#### Solution:

(a)  $\frac{dy}{dx} = 3x^2 + 2x$

$$\Rightarrow y = \frac{3}{3}x^3 + \frac{2}{2}x^2 + c$$

So  $y = x^3 + x^2 + c$

$x = 2, y = 10 \Rightarrow 10 = 8 + 4 + c$

So  $c = -2$

So equation is  $y = x^3 + x^2 - 2$

(b)  $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$

$$\Rightarrow y = \frac{4}{4}x^4 - \frac{2}{2}x^{-2} + 3x + c$$

So  $y = x^4 - x^{-2} + 3x + c$

$x = 1, y = 4 \Rightarrow 4 = 1 - 1 + 3 + c$

So  $c = 1$

So equation is  $y = x^4 - x^{-2} + 3x + 1$

(c)  $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$

$$\Rightarrow y = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{4} \frac{x^3}{3} + c$$

$$\text{So } y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + c$$

$$x = 4, y = 11 \Rightarrow 11 = \frac{2}{3} \times 2^3 + \frac{1}{12} \times 4^3 + c$$

$$\text{So } c = \frac{33}{3} - \frac{32}{3} = \frac{1}{3}$$

$$\text{So equation is } y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$$

$$(d) \frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$$

$$\Rightarrow y = 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2 + c$$

$$\text{So } y = 6\sqrt{x} - \frac{1}{2}x^2 + c$$

$$x = 4, y = 0 \Rightarrow 0 = 6 \times 2 - \frac{1}{2} \times 16 + c$$

$$\text{So } c = -4$$

$$\text{So equation is } y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$$

$$(e) \frac{dy}{dx} = (x + 2)^2 = x^2 + 4x + 4$$

$$\Rightarrow y = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$$x = 1, y = 7 \Rightarrow 7 = \frac{1}{3} + 2 + 4 + c$$

$$\text{So } c = \frac{2}{3}$$

$$\text{So equation is } y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$$

$$(f) \frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$$

$$\Rightarrow y = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\text{So } y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + c$$

$$x = 0, y = 1 \Rightarrow 1 = \frac{2}{5} \times 0 + 6 \times 0 + c$$

$$\text{So } c = 1$$

$$\text{So equation of curve is } y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$$





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise E, Question 2

#### Question:

The curve  $C$ , with equation  $y = f(x)$ , passes through the point  $(1, 2)$  and  $f'(x) = 2x^3 - \frac{1}{x^2}$ . Find the equation of  $C$  in the form  $y = f(x)$ .

#### Solution:

$$f'(x) = 2x^3 - \frac{1}{x^2} = 2x^3 - x^{-2}$$

$$\text{So } f(x) = \frac{2}{4}x^4 - \frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 + \frac{1}{x} + c$$

$$\text{But } f(1) = 2$$

$$\text{So } 2 = \frac{1}{2} + 1 + c$$

$$\Rightarrow c = \frac{1}{2}$$

$$\text{So } f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise E, Question 3

#### Question:

The gradient of a particular curve is given by  $\frac{dy}{dx} = \frac{\sqrt{x+3}}{x^2}$ . Given that the curve passes through the point  $(9, 0)$ , find an equation of the curve.

#### Solution:

$$\frac{dy}{dx} = \frac{\sqrt{x+3}}{x^2} = x^{-\frac{3}{2}} + 3x^{-2}$$

$$\Rightarrow y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 3 \frac{x^{-1}}{-1} + c$$

$$\text{So } y = -2x^{-\frac{1}{2}} - 3x^{-1} + c = -\frac{2}{\sqrt{x}} - \frac{3}{x} + c$$

$$x = 9, y = 0 \Rightarrow 0 = -\frac{2}{3} - \frac{3}{9} + c$$

$$\text{So } c = \frac{2}{3} + \frac{1}{3} = 1$$

$$\text{So equation is } y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise E, Question 4

#### Question:

A set of curves, that each pass through the origin, have equations  $y = f_1(x)$ ,  $y = f_2(x)$ ,  $y = f_3(x)$ ... where  $f_n'(x) = f_{n-1}(x)$  and  $f_1(x) = x^2$ .

(a) Find  $f_2(x)$ ,  $f_3(x)$ .

(b) Suggest an expression for  $f_n(x)$ .

#### Solution:

$$(a) f_2'(x) = f_1(x) = x^2$$

$$\text{So } f_2(x) = \frac{1}{3}x^3 + c$$

The curve passes through  $(0, 0)$  so  $f_2(0) = 0 \Rightarrow c = 0$ .

$$\text{So } f_2(x) = \frac{1}{3}x^3$$

$$f_3'(x) = \frac{1}{3}x^3$$

$$f_3(x) = \frac{1}{12}x^4 + c, \text{ but } c = 0 \text{ since } f_3(0) = 0.$$

$$\text{So } f_3(x) = \frac{1}{12}x^4$$

$$(b) f_2(x) = \frac{x^3}{3}, f_3(x) = \frac{x^4}{3 \times 4}$$

So power of  $x$  is  $n + 1$  for  $f_n(x)$ , denominator is  $3 \times 4 \times \dots$  up to  $n + 1$ :

$$f_n(x) = \frac{x^{n+1}}{3 \times 4 \times 5 \times \dots \times (n+1)}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise E, Question 5

#### Question:

A set of curves, with equations  $y = f_1(x)$ ,  $y = f_2(x)$ ,  $y = f_3(x)$  ... all pass through the point  $(0, 1)$  and they are related by the property  $f_n'(x) = f_{n-1}(x)$  and  $f_1(x) = 1$ .

Find  $f_2(x)$ ,  $f_3(x)$ ,  $f_4(x)$ .

#### Solution:

$$f_2'(x) = f_1(x) = 1$$

$$\Rightarrow f_2(x) = x + c$$

$$\text{But } f_2(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_2(x) = x + 1$$

$$f_3'(x) = f_2(x) = x + 1$$

$$\Rightarrow f_3(x) = \frac{1}{2}x^2 + x + c$$

$$\text{But } f_3(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_3(x) = \frac{1}{2}x^2 + x + 1$$

$$f_4'(x) = f_3(x) = \frac{1}{2}x^2 + x + 1$$

$$\Rightarrow f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + c$$

$$\text{But } f_4(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 1

#### Question:

Find:

$$(a) \int (x + 1)(2x - 5) dx$$

$$(b) \int \left( x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx.$$

#### Solution:

$$\begin{aligned} (a) \int (x + 1)(2x - 5) dx &= \int (2x^2 - 3x - 5) dx \\ &= 2 \frac{x^3}{3} - 3 \frac{x^2}{2} - 5x + c \\ &= \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x + c \end{aligned}$$

$$\begin{aligned} (b) \int \left( x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx &= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c \\ &= \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 2

#### Question:

The gradient of a curve is given by  $f'(x) = x^2 - 3x - \frac{2}{x^2}$ . Given that the curve passes through the point  $(1, 1)$ , find the equation of the curve in the form  $y = f(x)$ .

#### Solution:

$$f'(x) = x^2 - 3x - \frac{2}{x^2} = x^2 - 3x - 2x^{-2}$$

$$\text{So } f(x) = \frac{x^3}{3} - 3 \frac{x^2}{2} - 2 \frac{x^{-1}}{-1} + c$$

$$\text{So } f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + c$$

$$\text{But } f\left(\begin{matrix} 1 \\ 1 \end{matrix}\right) = 1 \Rightarrow \frac{1}{3} - \frac{3}{2} + 2 + c = 1$$

$$\text{So } c = \frac{1}{6}$$

$$\text{So the equation is } y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 3

#### Question:

Find

$$(a) \int (8x^3 - 6x^2 + 5) dx$$

$$(b) \int (5x + 2) x^{\frac{1}{2}} dx.$$

#### Solution:

$$(a) \int (8x^3 - 6x^2 + 5) dx$$

$$= 8 \frac{x^4}{4} - 6 \frac{x^3}{3} + 5x + c$$

$$= 2x^4 - 2x^3 + 5x + c$$

$$(b) \int (5x + 2) x^{\frac{1}{2}} dx$$

$$= \int \left( 5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 4

#### Question:

Given  $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$ , find  $\int y dx$ .

#### Solution:

$$y = \frac{(x+1)(2x-3)}{\sqrt{x}}$$

$$y = \left( 2x^2 - x - 3 \right) x^{-\frac{1}{2}}$$

$$y = 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$\int y dx = \int \left( 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \right) dx$$

$$= 2 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{4}{5} x^{\frac{5}{2}} - \frac{2}{3} x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 5

#### Question:

Given that  $\frac{dx}{dt} = 3t^2 - 2t + 1$  and that  $x = 2$  when  $t = 1$ , find the value of  $x$  when  $t = 2$ .

#### Solution:

$$\frac{dx}{dt} = 3t^2 - 2t + 1$$

$$\Rightarrow x = 3 \frac{t^3}{3} - 2 \frac{t^2}{2} + t + c$$

$$\text{So } x = t^3 - t^2 + t + c$$

But when  $t = 1$ ,  $x = 2$ .

$$\text{So } 2 = 1 - 1 + 1 + c$$

$$\Rightarrow c = 1$$

$$\text{So } x = t^3 - t^2 + t + 1$$

$$\text{When } t = 2, x = 8 - 4 + 2 + 1$$

$$\text{So } x = 7$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 6

#### Question:

Given  $y = 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$ ,  $x > 0$ , find  $\int y dx$ .

#### Solution:

$$\int y dx = \int \left( 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 7

#### Question:

Given that  $\frac{dx}{dt} = (t + 1)^2$  and that  $x = 0$  when  $t = 2$ , find the value of  $x$  when  $t = 3$ .

#### Solution:

$$\frac{dx}{dt} = (t + 1)^2 = t^2 + 2t + 1$$

$$\Rightarrow x = \frac{t^3}{3} + 2 \frac{t^2}{2} + t + c$$

But  $x = 0$  when  $t = 2$ .

$$\text{So } 0 = \frac{8}{3} + 4 + 2 + c$$

$$\Rightarrow c = -\frac{26}{3}$$

$$\text{So } x = \frac{1}{3}t^3 + t^2 + t - \frac{26}{3}$$

$$\text{When } t = 3, x = \frac{27}{3} + 9 + 3 - \frac{26}{3}$$

$$\text{So } x = 12 \frac{1}{3} \text{ or } \frac{37}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 8

#### Question:

Given that  $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$ :

(a) Show that  $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$ , where  $A$  and  $B$  are constants to be found.

(b) Hence find  $\int y dx$ . **[E]**

#### Solution:

$$(a) y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$$

$$\text{So } y = \left( x^{\frac{1}{3}} + 3 \right)^2$$

$$\text{So } y = \left( x^{\frac{1}{3}} \right)^2 + 6x^{\frac{1}{3}} + 9$$

$$\text{So } y = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9$$

$$(A = 6, B = 9)$$

$$(b) \int y dx = \int \left( x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \right) dx$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 9

#### Question:

Given that  $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$  ( $x > 0$ ):

(a) Find  $\frac{dy}{dx}$ .

(b) Find  $\int y dx$ . [E]

#### Solution:

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$$

$$(a) \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 4 \times \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$\text{So } \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

$$(b) \int y dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Integration

#### Exercise F, Question 10

#### Question:

$$\text{Find } \int \left( x^{\frac{1}{2}} - 4 \right) \left( x^{-\frac{1}{2}} - 1 \right) dx. \text{[E]}$$

#### Solution:

$$\int \left( x^{\frac{1}{2}} - 4 \right) \left( x^{-\frac{1}{2}} - 1 \right) dx$$

$$= \int \left( 1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 \right) dx$$

$$= \int \left( 5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) dx$$

$$= 5x - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 1

**Question:**

(a) Write down the value of  $16^{\frac{1}{2}}$ . (1)

(b) Hence find the value of  $16^{\frac{3}{2}}$ . (2)

**Solution:**

(a)  $16^{\frac{1}{2}} = \sqrt{16} = 4$

(b)  $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 2

**Question:**

Find  $\int (6x^2 + \sqrt{x}) dx$ . (4)

**Solution:**

$$\int \left( 6x^2 + x^{\frac{1}{2}} \right) dx$$

$$= 6 \frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^3 + \frac{2}{3}x^{\frac{3}{2}} + c$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 3

**Question:**

A sequence  $a_1, a_2, a_3, \dots, a_n$  is defined by

$$a_1 = 2, a_{n+1} = 2a_n - 1.$$

(a) Write down the value of  $a_2$  and the value of  $a_3$ . (2)

5

(b) Calculate  $\sum_{r=1}^5 a_r$ . (2)

**Solution:**

$$(a) a_2 = 2a_1 - 1 = 4 - 1 = 3$$

$$a_3 = 2a_2 - 1 = 6 - 1 = 5$$

$$(b) a_4 = 2a_3 - 1 = 10 - 1 = 9$$

$$a_5 = 2a_4 - 1 = 18 - 1 = 17$$

5

$$\sum_{r=1}^5 a_r = a_1 + a_2 + a_3 + a_4 + a_5 = 2 + 3 + 5 + 9 + 17 = 36$$
$$r = 1$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 4

**Question:**

(a) Express  $(5 + \sqrt{2})^2$  in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers. (3)

(b) Hence, or otherwise, simplify  $(5 + \sqrt{2})^2 - (5 - \sqrt{2})^2$ . (2)

**Solution:**

$$(a) (5 + \sqrt{2})^2 = (5 + \sqrt{2})(5 + \sqrt{2}) = 25 + 10\sqrt{2} + 2 = 27 + 10\sqrt{2}$$

$$(b) (5 - \sqrt{2})^2 = (5 - \sqrt{2})(5 - \sqrt{2}) = 25 - 10\sqrt{2} + 2 = 27 - 10\sqrt{2}$$

$$\begin{aligned} & (5 + \sqrt{2})^2 - (5 - \sqrt{2})^2 \\ &= (27 + 10\sqrt{2}) - (27 - 10\sqrt{2}) \\ &= 27 + 10\sqrt{2} - 27 + 10\sqrt{2} \\ &= 20\sqrt{2} \end{aligned}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1 Exercise 1, Question 5

#### Question:

Solve the simultaneous equations:

$$x - 3y = 6$$

$$3xy + x = 24 \quad (7)$$

#### Solution:

$$x - 3y = 6$$

$$x = 6 + 3y$$

Substitute into  $3xy + x = 24$ :

$$3y(6 + 3y) + (6 + 3y) = 24$$

$$18y + 9y^2 + 6 + 3y = 24$$

$$9y^2 + 21y - 18 = 0$$

Divide by 3:

$$3y^2 + 7y - 6 = 0$$

$$(3y - 2)(y + 3) = 0$$

$$y = \frac{2}{3}, y = -3$$

Substitute into  $x = 6 + 3y$ :

$$y = \frac{2}{3} \Rightarrow x = 6 + 2 = 8$$

$$y = -3 \Rightarrow x = 6 - 9 = -3$$

$$x = -3, y = -3 \text{ or } x = 8, y = \frac{2}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 6

#### Question:

The points  $A$  and  $B$  have coordinates  $(-3, 8)$  and  $(5, 4)$  respectively.  
The straight line  $l_1$  passes through  $A$  and  $B$ .

- (a) Find an equation for  $l_1$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (4)
- (b) Another straight line  $l_2$  is perpendicular to  $l_1$  and passes through the origin. Find an equation for  $l_2$ . (2)
- (c) The lines  $l_1$  and  $l_2$  intersect at the point  $P$ . Use algebra to find the coordinates of  $P$ . (3)

#### Solution:

$$(a) \text{ Gradient of } l_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{5 - (-3)} = -\frac{4}{8} = -\frac{1}{2}$$

Equation for  $l_1$ :

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2} \left( x - 5 \right)$$

$$y - 4 = -\frac{1}{2}x + \frac{5}{2}$$

$$\frac{1}{2}x + y - \frac{13}{2} = 0$$

$$x + 2y - 13 = 0$$

(b) For perpendicular lines,  $m_1 m_2 = -1$

$$m_1 = -\frac{1}{2}, \text{ so } m_2 = 2$$

Equation for  $l_2$  is  $y = 2x$

(c) Substitute  $y = 2x$  into  $x + 2y - 13 = 0$ :

$$x + 4x - 13 = 0$$

$$5x = 13$$

$$x = 2\frac{3}{5}$$

$$y = 2x = 5\frac{1}{5}$$

Coordinates of  $P$  are  $\left( 2\frac{3}{5}, 5\frac{1}{5} \right)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 7

#### Question:

On separate diagrams, sketch the curves with equations:

(a)  $y = \frac{2}{x}$ ,  $-2 \leq x \leq 2, x \neq 0$  (2)

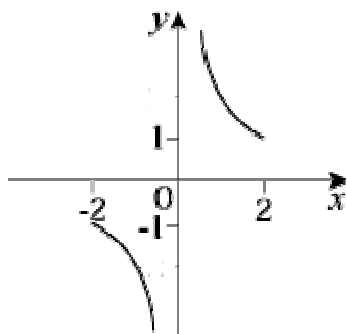
(b)  $y = \frac{2}{x} - 4$ ,  $-2 \leq x \leq 2, x \neq 0$  (3)

(c)  $y = \frac{2}{x+1}$ ,  $-2 \leq x \leq 2, x \neq -1$  (3)

In each part, show clearly the coordinates of any point at which the curve meets the  $x$ -axis or the  $y$ -axis.

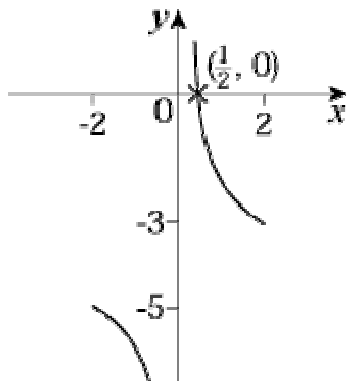
#### Solution:

(a)



$$y = \frac{2}{x}$$

(b) Translation of  $-4$  units parallel to the  $y$ -axis.



$$y = \frac{2}{x} - 4$$

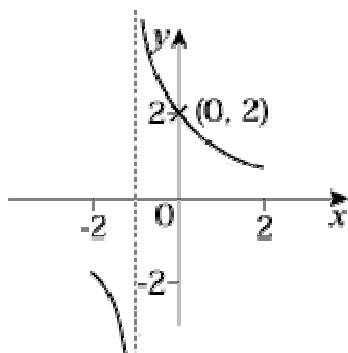
Curve crosses the  $x$ -axis where  $y = 0$ :

$$\frac{2}{x} - 4 = 0$$

$$\frac{2}{x} = 4$$

$$x = \frac{1}{2}$$

(c) Translation of  $-1$  unit parallel to the  $x$ -axis.



$$y = \frac{2}{x+1}$$

The line  $x = -1$  is an asymptote.

Curve crosses the  $y$ -axis where  $x = 0$ :

$$y = \frac{2}{0+1} = 2$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 8

#### Question:

In the year 2007, a car dealer sold 400 new cars. A model for future sales assumes that sales will increase by  $x$  cars per year for the next 10 years, so that  $(400 + x)$  cars are sold in 2008,  $(400 + 2x)$  cars are sold in 2009, and so on. Using this model with  $x = 30$ , calculate:

- (a) The number of cars sold in the year 2016. (2)
- (b) The total number of cars sold over the 10 years from 2007 to 2016. (3)  
The dealer wants to sell at least 6000 cars over the 10-year period.  
Using the same model:
- (c) Find the least value of  $x$  required to achieve this target. (4)

#### Solution:

(a)  $a = 400, d = x = 30$   
 $T_{10} = a + 9d = 400 + 270 = 670$   
 670 cars sold in 2016

$$(b) S_n = \frac{1}{2}n \left[ 2a + \left( n - 1 \right) d \right]$$

So  $S_{10} = 5 [ (2 \times 400) + (9 \times 30) ] = 5 \times 1070 = 5350$   
 5350 cars sold from 2007 to 2016

(c)  $S_{10}$  required to be at least 6000:

$$\frac{1}{2}n \left[ 2a + \left( n - 1 \right) d \right] \geq 6000$$

$$5(800 + 9x) \geq 6000$$

$$4000 + 45x \geq 6000$$

$$45x \geq 2000$$

$$x \geq 44 \frac{4}{9}$$

To achieve the target,  $x = 45$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 9

#### Question:

(a) Given that

$$x^2 + 4x + c = (x + a)^2 + b$$

where  $a$ ,  $b$  and  $c$  are constants:

(i) Find the value of  $a$ . (1)

(ii) Find  $b$  in terms of  $c$ . (2)

Given also that the equation  $x^2 + 4x + c = 0$  has unequal real roots:

(iii) Find the range of possible values of  $c$ . (2)

(b) Find the set of values of  $x$  for which:

(i)  $3x < 20 - x$ , (2)

(ii)  $x^2 + 4x - 21 > 0$ , (4)

(iii) both  $3x < 20 - x$  and  $x^2 + 4x - 21 > 0$ . (2)

#### Solution:

(a) (i)  $x^2 + 4x + c = (x + 2)^2 - 4 + c = (x + 2)^2 + (c - 4)$   
So  $a = 2$

(ii)  $b = c - 4$

(iii) For unequal real roots:

$$(x + 2)^2 - 4 + c = 0$$

$$(x + 2)^2 = 4 - c$$

$$4 - c > 0$$

$$c < 4$$

(b) (i)  $3x < 20 - x$

$$3x + x < 20$$

$$4x < 20$$

$$x < 5$$

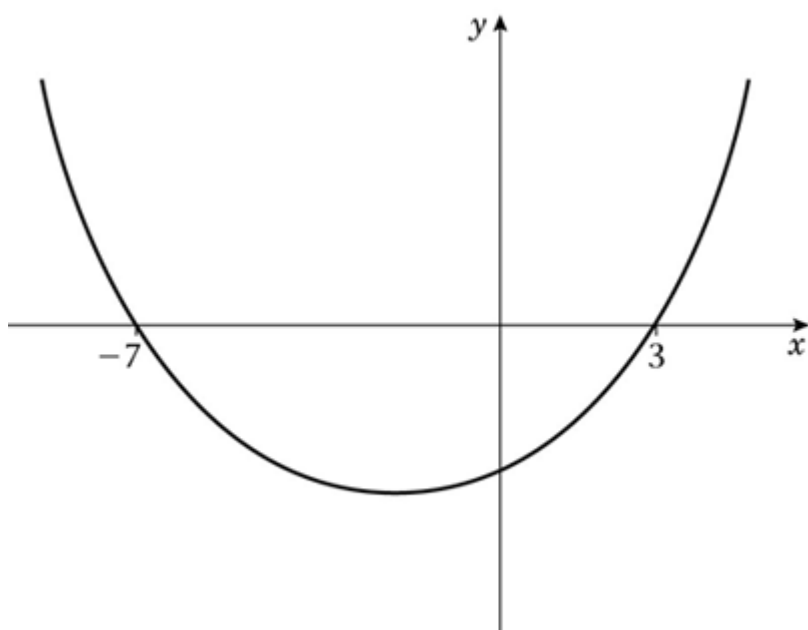
(ii) Solve  $x^2 + 4x - 21 = 0$ :

$$(x + 7)(x - 3) = 0$$

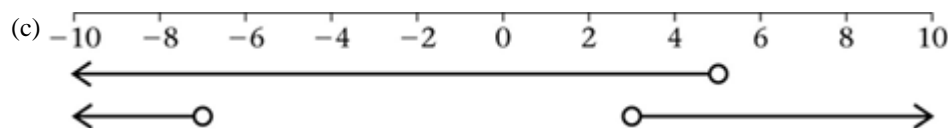
$$x = -7, x = 3$$

Sketch of  $y = x^2 + 4x - 21$ :





$$x^2 + 4x - 21 > 0 \text{ when } x < -7 \text{ or } x > 3$$



Both inequalities are true when  
 $x < -7$  or  $3 < x < 5$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Practice paper C1

#### Exercise 1, Question 10

#### Question:

(a) Show that  $\frac{(3x-4)^2}{x^2}$  may be written as  $P + \frac{Q}{x} + \frac{R}{x^2}$  where  $P$ ,  $Q$  and  $R$  are constants to be found. (3)

(b) The curve  $C$  has equation  $y = \frac{(3x-4)^2}{x^2}$ ,  $x \neq 0$ . Find the gradient of the tangent to  $C$  at the point on  $C$  where  $x = -2$ . (5)

(c) Find the equation of the normal to  $C$  at the point on  $C$  where  $x = -2$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (5)

#### Solution:

$$(a) (3x-4)^2 = (3x-4)(3x-4) = 9x^2 - 24x + 16$$

$$\frac{(3x-4)^2}{x^2} = \frac{9x^2 - 24x + 16}{x^2} = 9 - \frac{24}{x} + \frac{16}{x^2}$$

$$P = 9, Q = -24, R = 16$$

$$(b) y = 9 - 24x^{-1} + 16x^{-2}$$

$$\frac{dy}{dx} = 24x^{-2} - 32x^{-3}$$

$$\text{Where } x = -2, \frac{dy}{dx} = \frac{24}{(-2)^2} - \frac{32}{(-2)^3} = \frac{24}{4} + \frac{32}{8} = 10$$

Gradient of the tangent is 10.

$$(c) \text{ Where } x = -2, y = 9 - \frac{24}{(-2)} + \frac{16}{(-2)^2} = 9 + 12 + 4 = 25$$

$$\text{Gradient of the normal} = \frac{-1}{\text{Gradient of tangent}} = -\frac{1}{10}$$

The equation of the normal at  $(-2, 25)$  is

$$y - 25 = -\frac{1}{10} \left[ x - \begin{pmatrix} -2 \\ \end{pmatrix} \right]$$

Multiply by 10:

$$10y - 250 = -x - 2$$

$$x + 10y - 248 = 0$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 1

#### Question:

Factorise completely

(a)  $2x^3 - 13x^2 - 7x$

(b)  $9x^2 - 16$

(c)  $x^4 + 7x^2 - 8$

#### Solution:

(a)

$$\begin{aligned} 2x^3 - 13x^2 - 7x & \\ &= x(2x^2 - 13x - 7) \\ &= x(2x^2 + x - 14x - 7) \\ &= x[x(2x + 1) - 7(2x + 1)] \\ &= x(2x + 1)(x - 7) \end{aligned}$$

$x$  is a common factor

So take  $x$  outside the bracket.

For the quadratic,  $ac = -14$  and

$$1 - 14 = -13 = b$$

Factorise

(b)

$$\begin{aligned} 9x^2 - 16 & \\ &= (3x)^2 - 4^2 \\ &= (3x + 4)(3x - 4) \end{aligned}$$

This is a difference of two

squares,  $(3x)^2$  and  $4^2$

$$\text{Use } x^2 - y^2 = (x + y)(x - y)$$

(c)

$$\begin{aligned} x^4 + 7x^2 - 8 & \\ &= y^2 + 7y - 8 \\ &= y^2 - y + 8y - 8 \\ &= y(y - 1) + 8(y - 1) \\ &= (y - 1)(y + 8) \\ &= (x^2 - 1)(x^2 + 8) \\ &= (x + 1)(x - 1) \\ & \quad (x^2 + 8) \end{aligned}$$

squares,

$$\text{Let } y = x^2$$

$$ac = -8 \text{ and } -1 + 8 = +7 = b$$

Factorise

Replace  $y$  by  $x^2$

$x^2 - 1$  is a difference of two

$$\text{so use } x^2 - y^2 = (x + y)(x - y)$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 2

#### Question:

Find the value of

(a)  $81^{\frac{1}{2}}$

(b)  $81^{\frac{3}{4}}$

(c)  $81^{\frac{3}{4}}$ .

#### Solution:

(a)

$$\begin{aligned} 81^{1/2} \\ &= \sqrt{81} \\ &= 9 \end{aligned}$$

Use  $a^{\frac{1}{m}} = m\sqrt[m]{a}$ , so  $a^{\frac{1}{2}} = \sqrt{a}$

(b)

$$\begin{aligned} 81^{\frac{3}{4}} \\ &= (\sqrt[4]{81})^3 \quad \text{then cube this} \\ &= 3^3 \quad \sqrt{81} = 3 \text{ because } 3 \times 3 \times 3 \times 3 = 81 \\ &= 27 \end{aligned}$$

$$a^{\frac{n}{m}} = m\sqrt[m]{(a^n)} \quad \text{or} \quad (m\sqrt[m]{a})^n$$

It is easier to find the fourth root,

4

(c)

$$\begin{aligned} 81^{-\frac{3}{4}} &= \frac{1}{81^{3/4}} \\ &= \frac{1}{27} \end{aligned}$$

Use  $a^{-m} = \frac{1}{a^m}$

Use the answer from part (b)

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 3

#### Question:

(a) Write down the value of  $8^{\frac{1}{3}}$ .

(b) Find the value of  $8^{-\frac{2}{3}}$ .

#### Solution:

(a)

$$\begin{aligned} 8^{\frac{1}{3}} &= \sqrt[3]{8} \\ &= 2 \end{aligned}$$

$$\text{Use } a^{\frac{1}{m}} = \sqrt[m]{a}, \text{ so } 8^{\frac{1}{3}} = \sqrt[3]{8}$$

$$\sqrt[3]{8} = 2 \text{ because } 2 \times 2 \times 2 = 8$$

(b)

$$\begin{aligned} 8^{-\frac{2}{3}} &= \left( \sqrt[3]{8} \right)^{-2} && \left( \sqrt[m]{a} \right)^n \\ &= 2^{-2} = \frac{1}{2^2} \\ &= \frac{1}{4} \end{aligned}$$

$$\text{First find } 8^{\frac{2}{3}} \quad a^{\frac{n}{m}} = \sqrt[m]{(a^n)} \text{ or}$$

$$\text{Use } a^{-m} = \frac{1}{a^m}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 4

#### Question:

(a) Find the value of  $125^{\frac{4}{3}}$ .

(b) Simplify  $24x^2 \div 18x^{\frac{4}{3}}$ .

#### Solution:

(a)

$$\begin{aligned} 125^{\frac{4}{3}} &= ({}^3\sqrt{125})^4 \\ &= 5^4 \\ &= 625 \end{aligned}$$

$$a^{\frac{n}{m}} = {}^m\sqrt{(a^n)} \quad \text{or} \quad ({}^m\sqrt{a})^n$$

It is easier to find the cube root,  
then the fourth power

$${}^3\sqrt{125} = 5 \quad \text{because} \quad 5 \times 5 \times 5 = 125$$

(b)

$$24x^2 \div 18x^{\frac{4}{3}}$$

$$= \frac{24x^2}{18x^{\frac{4}{3}}} = \frac{4x^2}{3x^{\frac{4}{3}}}$$

by 6

Divide

$$= \frac{4x^{\frac{2}{3}}}{3}$$

$$\text{Use } a^m \div a^n = a^{m-n}$$

$$\left( \text{or } \frac{4}{3}x^{\frac{2}{3}} \right)$$

$$\frac{2}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 5

#### Question:

- (a) Express  $\sqrt{80}$  in the form  $a\sqrt{5}$ , where  $a$  is an integer.
- (b) Express  $(4 - \sqrt{5})^2$  in the form  $b + c\sqrt{5}$ , where  $b$  and  $c$  are integers.

#### Solution:

$$\begin{aligned} \text{(a)} \quad \sqrt{80} &= \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \quad (a = 4) \end{aligned}$$

$$\text{Use } \sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

$$\begin{aligned} \text{(b)} \quad (4 - \sqrt{5})^2 &= (4 - \sqrt{5})(4 - \sqrt{5}) \\ &= 4(4 - \sqrt{5}) - \sqrt{5}(4 - \sqrt{5}) \\ &= 16 - 4\sqrt{5} - 4\sqrt{5} + 5 \\ &= 21 - 8\sqrt{5} \end{aligned}$$

$$\begin{aligned} &\text{Multiply the brackets.} \\ &\sqrt{5} \times \sqrt{5} = 5 \end{aligned}$$

$$(b = 21 \text{ and } c = -8)$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 6

#### Question:

- (a) Expand and simplify  $(4 + \sqrt{3})(4 - \sqrt{3})$ .
- (b) Express  $\frac{26}{4 + \sqrt{3}}$  in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are integers.

#### Solution:

(a)

$$\begin{aligned} & (4 + \sqrt{3})(4 - \sqrt{3}) \\ &= 4(4 - \sqrt{3}) + \sqrt{3}(4 - \sqrt{3}) \\ &= 16 - 4\sqrt{3} + 4\sqrt{3} - 3 \\ &= 13 \end{aligned}$$

Multiply the brackets.  
 $\sqrt{3} \times \sqrt{3} = 3$

(b)

$$\frac{26}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}}$$

To rationalise the denominator, multiply top and

$$\begin{aligned} & \text{bottom by } 4 - \sqrt{3} \\ &= \frac{26(4 - \sqrt{3})}{(4 + \sqrt{3})(4 - \sqrt{3})} \end{aligned}$$

$$= \frac{26(4 - \sqrt{3})}{13}$$

Use the answer from part (a)

$$\begin{aligned} &= 2(4 - \sqrt{3}) \\ &= 8 - 2\sqrt{3} \end{aligned}$$

Use the

Divide by 13

$$(a = 8 \text{ and } b = -2)$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 7

#### Question:

- (a) Express  $\sqrt{108}$  in the form  $a\sqrt{3}$ , where  $a$  is an integer.
- (b) Express  $(2 - \sqrt{3})^2$  in the form  $b + c\sqrt{3}$ , where  $b$  and  $c$  are integers to be found.

#### Solution:

(a)

$$\begin{aligned}\sqrt{108} &= \sqrt{36} \times \sqrt{3} \\ &= 6\sqrt{3} \quad (a = 6)\end{aligned}$$

Use  $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

(b)

$$\begin{aligned}(2 - \sqrt{3})^2 &= (2 - \sqrt{3})(2 - \sqrt{3}) \\ &= 2(2 - \sqrt{3}) - \sqrt{3}(2 - \sqrt{3}) \\ &= 4 - 2\sqrt{3} - 2\sqrt{3} + 3 \\ &= 7 - 4\sqrt{3}\end{aligned}$$

Multiply the brackets  
 $\sqrt{3} \times \sqrt{3} = 3$

$(b = 7 \text{ and } c = -4)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 8

#### Question:

- (a) Express  $(2\sqrt{7})^3$  in the form  $a\sqrt{7}$ , where  $a$  is an integer.
- (b) Express  $(8 + \sqrt{7})(3 - 2\sqrt{7})$  in the form  $b + c\sqrt{7}$ , where  $b$  and  $c$  are integers.
- (c) Express  $\frac{6 + 2\sqrt{7}}{3 - \sqrt{7}}$  in the form  $d + e\sqrt{7}$ , where  $d$  and  $e$  are integers.

#### Solution:

(a)

$$\begin{aligned} (2\sqrt{7})^3 &= 2\sqrt{7} \times 2\sqrt{7} \times 2\sqrt{7} && \text{Multiply the 2s.} \\ &= 8(\sqrt{7} \times \sqrt{7} \times \sqrt{7}) \\ &= 8(7\sqrt{7}) \\ &= 56\sqrt{7} \quad (a = 56) \end{aligned}$$

$$\sqrt{7} \times \sqrt{7} = 7$$

(b)

$$\begin{aligned} (8 + \sqrt{7})(3 - 2\sqrt{7}) &= 8(3 - 2\sqrt{7}) + \sqrt{7}(3 - 2\sqrt{7}) \\ &= 24 - 16\sqrt{7} + 3\sqrt{7} - 14 \\ &= 10 - 13\sqrt{7} \\ (b = 10 \text{ and } c = -13) \end{aligned}$$

$$\sqrt{7} \times 2\sqrt{7} = 2 \times 7$$

(c)

$$\frac{6 + 2\sqrt{7}}{3 - \sqrt{7}} \times \frac{3 + \sqrt{7}}{3 + \sqrt{7}}$$

To rationalise the denominator, multiply top and bottom

$$\begin{aligned} &\text{by } 3 + \sqrt{7} \\ &= \frac{(6 + 2\sqrt{7})(3 + \sqrt{7})}{(3 - \sqrt{7})(3 + \sqrt{7})} \\ &= \frac{6(3 + \sqrt{7}) + 2\sqrt{7}(3 + \sqrt{7})}{3(3 + \sqrt{7}) - \sqrt{7}(3 + \sqrt{7})} \\ &= \frac{18 + 6\sqrt{7} + 6\sqrt{7} + 14}{9 + 3\sqrt{7} - 3\sqrt{7} - 7} \\ &= \frac{32 + 12\sqrt{7}}{2} = 16 + 6\sqrt{7} \end{aligned}$$

Divide by 2

$(d = 16 \text{ and } e = 6)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 9

#### Question:

Solve the equations

(a)  $x^2 - x - 72 = 0$

(b)  $2x^2 + 7x = 0$

(c)  $10x^2 + 9x - 9 = 0$

#### Solution:

(a)

$$x^2 - x - 72 = 0$$

$$(x + 8)(x - 9) = 0$$

$x + 8 = 0, x - 9 = 0$  the equation could be solved using the

$x = -8, x = 9$  formula or 'completing

Although  
quadratic

Factorise

the  
square', factorisation is quicker.

(b)

$$2x^2 + 7x = 0$$

$$x(2x + 7) = 0$$

$x = 0, 2x + 7 = 0$  forget the  $x = 0$  solution.

$$x = 0, x = -\frac{7}{2}$$

Use  
the factor x.

Don't

(b)

$$2x^2 + 7x = 0$$

$$x(2x + 7) = 0$$

$x = 0, 2x + 7 = 0$  forget the  $x = 0$  solution.

$$x = 0, x = -\frac{7}{2}$$

Use  
the factor x.

Don't

(c)

$$10x^2 + 9x - 9 = 0$$

$$(2x + 3)(5x - 3) = 0$$

$$2x + 3 = 0, 5x - 3 = 0$$

$$x = -\frac{3}{2}, x = \frac{3}{5}$$

Factorise

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 10

**Question:**

Solve the equations, giving your answers to 3 significant figures

(a)  $x^2 + 10x + 17 = 0$

(b)  $2x^2 - 5x - 1 = 0$

(c)  $(2x - 3)^2 = 7$

**Solution:**

(a)

$$x^2 + 10x + 17 = 0$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

quadratic will not factorise.

$$a = 1, b = 10, c = 17$$

$$x = \frac{-10 \pm \sqrt{(100 - 68)}}{2}$$

the formula first.

Since the question requires answers to

3 significant figures, you know that the

$$= \frac{-10 \pm \sqrt{32}}{2}$$

$$= \frac{-10 \pm 5.656 \dots}{2}$$

at least 4 sig. figs.

$$= \frac{-10 + 5.656 \dots}{2}, \frac{-10 - 5.656 \dots}{2}$$

$$x = -2.17, x = -7.83$$

Alternative method:

$$x^2 + 10x + 17 = 0$$

$$x^2 + 10x = -17$$

$$(x + 5)^2 - 25 = -17$$

$$(x + 5)^2 = -17 + 25$$

$$(x + 5)^2 = 8$$

$$x + 5 = \pm \sqrt{8}$$

$$x = -5 \pm \sqrt{8}$$

$$x = -5 + \sqrt{8}, x = -5 - \sqrt{8}$$

$$x = -2.17, x = -7.83$$

Use the quadratic formula, quoting

Intermediate working should be to

Divide by 2, and round to 3 sig. figs.

Subtract 17 to get LHS in the required form.

Complete the square for  $x^2 + 10x$

Add 25 to both sides

Square root both sides.

Subtract 5 from both sides.

(b)

$$2x^2 - 5x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2, b = -5, c = -1$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 2 \times -1)}}{4}$$

Use the quadratic formula, quoting

the formula first.

$$= \frac{5 \pm \sqrt{(25 + 8)}}{4} = \frac{5 \pm \sqrt{33}}{4}$$

$$= \frac{5 + 5.744 \dots}{4}, \frac{5 - 5.744 \dots}{4}$$

$$x = 2.69, x = -0.186$$

Divide by 4, and round to 3 sig. figs.

(c)

$$(2x - 3)^2 = 7$$

$$2x - 3 = \pm \sqrt{7}$$

The quickest method is to take the square root

of both sides.

$$2x = 3 \pm \sqrt{7}$$

sides.

Add 3 to both

$$x = \frac{3 + \sqrt{7}}{2}, x =$$

Divide both

$$\frac{3 - \sqrt{7}}{2}$$

sides by 2

$$x = 2.82, x = 0.177$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 11

#### Question:

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where  $a$  and  $b$  are constants.

(a) Find the value of  $a$  and the value of  $b$ .

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are  $c \pm d\sqrt{5}$ , where  $c$  and  $d$  are integers to be found.

#### Solution:

(a)

$$x^2 - 8x = (x - 4)^2 - 16 \quad \text{for } x^2 - 8x \quad \text{Complete the square}$$

$$\begin{aligned} x^2 - 8x - 29 &= (x - 4)^2 - 16 - 29 \\ &= (x - 4)^2 - 45 \end{aligned}$$

$$(a = -4 \text{ and } b = -45)$$

(b)

$$x^2 - 8x - 29 = 0$$

$$(x - 4)^2 - 45 = 0$$

$$(x - 4)^2 = 45$$

$$x - 4 = \pm \sqrt{45}$$

$$x = 4 \pm \sqrt{45}$$

$$\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{9} \times \sqrt{5} = 3$$

$$= \sqrt{a}\sqrt{b}$$

$$\text{Use } \sqrt{(ab)}$$

$$\text{Roots are } 4 \pm 3\sqrt{5}$$

$$(c = 4 \text{ and } d = 3)$$

Use  
the result from part (a)

Take  
the square root of both sides.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 12

#### Question:

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express  $f(x)$  in the form  $(x - a)^2 + b$ , where  $a$  and  $b$  are integers.

The curve  $C$  with equation  $y = f(x)$ ,  $x \geq 0$ , meets the  $y$ -axis at  $P$  and has a minimum point at  $Q$ .

(b) Sketch the graph of  $C$ , showing the coordinates of  $P$  and  $Q$ .

The line  $y = 41$  meets  $C$  at the point  $R$ .

(c) Find the  $x$ -coordinate of  $R$ , giving your answer in the form  $p + q\sqrt{2}$ , where  $p$  and  $q$  are integers.

#### Solution:

(a)

$$f(x) = x^2 - 6x + 18$$

$$x^2 - 6x = (x - 3)^2 - 9 \quad \text{for } x^2 - 6x \quad \text{Complete the square}$$

$$\begin{aligned} x^2 - 6x + 18 &= (x - 3)^2 - 9 + 18 \\ &= (x - 3)^2 + 9 \end{aligned}$$

$$(a = 3 \text{ and } b = 9)$$

(b)

$$y = x^2 - 6x + 18$$

$$y = (x - 3)^2 + 9$$

$$(x - 3)^2 \geq 0$$

Squaring a number cannot give a negative result

The minimum value of  $(x - 3)^2$  is zero, when  $x = 3$ .


So the minimum value of  $y$  is  $0 + 9 = 9$ , when  $x = 3$ .

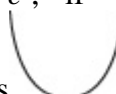
$Q$  is the point  $(3, 9)$

The curve crosses the  $y$ -axis where  $x = 0$ .

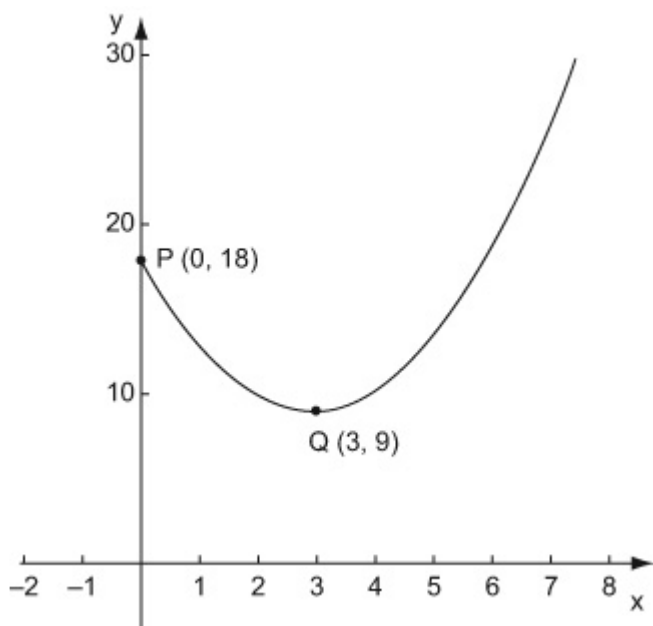
For  $x = 0$ ,  $y = 18$

$P$  is the point  $(0, 18)$

The graph of  $y = x^2 - 6x + 18$  is a  shape.

For  $y = ax^2 + bx + c$ , if  $a > 0$ , the shape is 





Use the information about P and Q to sketch the curve  $x \geq 0$ , so the part where  $x < 0$  is not needed.

(c)

$$y = (x - 3)^2 + 9$$

$$41 = (x - 3)^2 + 9$$

$$32 = (x - 3)^2$$

$$(x - 3)^2 = 32$$

$$x - 3 = \pm \sqrt{32}$$

$$x = 3 \pm \sqrt{32}$$

$$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$$

$$x = 3 \pm 4\sqrt{2}$$

x-coordinate of R is  $3 + 4\sqrt{2}$

The other value  $3 - 4\sqrt{2}$  is less than 0,

needed

so not

the equation of C.

both sides.

root of both sides.

$$= \sqrt{a}\sqrt{b}$$

Put  $y = 41$  into

Subtract 9 from

Take the square

Use  $\sqrt{(ab)}$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 13

#### Question:

Given that the equation  $kx^2 + 12x + k = 0$ , where  $k$  is a positive constant, has equal roots, find the value of  $k$ .

#### Solution:

$$Kx^2 + 12x + K = 0$$

$$a = K, b = 12, c = K$$

For equal roots,  $b^2 = 4ac$   
( or  $b^2 - 4ac = 0$  )

$$12^2$$

$$4K^2$$

$$K^2$$

$$K$$

$$\text{So } K$$

Write down the  
values of  $a$ ,  $b$  and  $c$

for the quadratic  
equation.

$$= 4 \times K \times K$$

$$= 144$$

$$= 36$$

$$= \pm 6$$

$$= 6$$

The question says  
that  $K$  is a positive constant.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 14

#### Question:

Given that

$$x^2 + 10x + 36 \equiv (x + a)^2 + b,$$

where  $a$  and  $b$  are constants,

- (a) find the value of  $a$  and the value of  $b$ .
- (b) Hence show that the equation  $x^2 + 10x + 36 = 0$  has no real roots.

The equation  $x^2 + 10x + k = 0$  has equal roots.

- (c) Find the value of  $k$ .
- (d) For this value of  $k$ , sketch the graph of  $y = x^2 + 10x + k$ , showing the coordinates of any points at which the graph meets the coordinate axes.

#### Solution:

(a)

$$x^2 + 10x + 36$$

$$x^2 + 10x = (x + 5)^2 - 25$$

Complete the square for  $x^2 + 10x$

$$\begin{aligned} x^2 + 10x + 36 &= (x + 5)^2 - 25 + 36 \\ &= (x + 5)^2 + 11 \end{aligned}$$

$$a = 5 \text{ and } b = 11$$

(b)

$$x^2 + 10x + 36 = 0$$

$$(x + 5)^2 + 11 = 0 \quad \text{used}$$

'Hence' implies that part (a) must be

$$(x + 5)^2 = -11$$

A real number squared cannot

be negative,  $\therefore$  no real roots

(c)

$$x^2 + 10x + K = 0$$

$$a = 1, b = 10, c = K$$


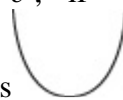
For equal roots,  $b^2 = 4ac$

$$10^2 = 4 \times 1 \times K$$

$$4K = 100$$

$$K = 25$$

(d)

The graph of  $y = x^2 + 10x + 25$  is a  shape. For  $y = ax^2 + bx + c$ , if  $a > 0$ , the shape is .

$x = 0 : y = 0 + 0 + 25 = 25$

Meets  $y$ -axis at  $(0, 25)$

Put  $x = 0$  to find intersections with the

$y = 0 : x^2 + 10x + 25 = 0$

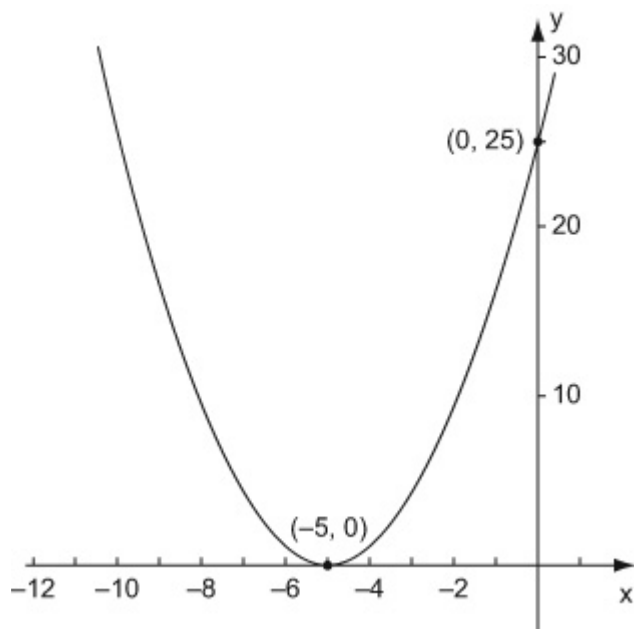
$y$ -axis, and  $y = 0$  to find intersections

$(x + 5)(x + 5) = 0$

with the  $x$ -axis.

$x = -5$

Meets  $x$ -axis at  $(-5, 0)$



The graph meets the  $x$ -axis at just one point, so it 'touches' the  $x$ -axis

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 15

#### Question:

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

- (a) Find the values of the constants  $a$  and  $b$ .
- (b) Sketch the graph of  $y = x^2 + 2x + 3$ , indicating clearly the coordinates of any intersections with the coordinate axes.
- (c) Find the value of the discriminant of  $x^2 + 2x + 3$ .  
Explain how the sign of the discriminant relates to your sketch in part (b).

The equation  $x^2 + kx + 3 = 0$ , where  $k$  is a constant, has no real roots.

- (d) Find the set of possible values of  $k$ , giving your answer in surd form.

#### Solution:

(a)

$$x^2 + 2x + 3$$

$$x^2 + 2x = (x + 1)^2 - 1 \quad \text{for } x^2 + 2x$$


$$x^2 + 2x + 3 = (x + 1)^2 - 1 + 3$$

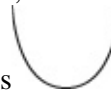
$$= (x + 1)^2 + 2$$

$$a = 1 \text{ and } b = 2$$

Complete the square

(b)

The graph of  $y = x^2 + 2x + 3$  is a  shape

For  $y = ax^2 + bx + c$ ,  
if  $a > 0$ , the shape is 

$$x = 0 : y = 0 + 0 + 3$$

Meets  $y$ -axis at  $(0, 3)$

Put  $x = 0$  to find  
intersections with the  $y$ -axis,

$$y = 0 : x^2 + 2x + 3 = 0$$

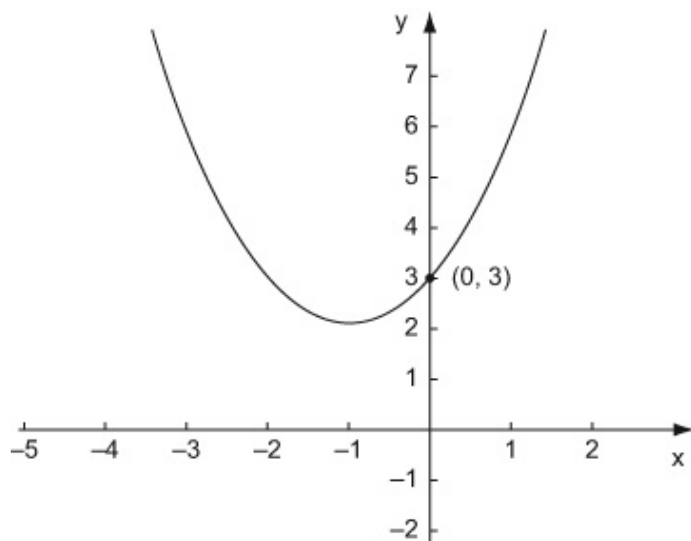
$$(x + 1)^2 + 2 = 0$$

$$(x + 1)^2 = -2$$

A real number squared cannot  
be negative,  $\therefore$

no real roots, so no intersection  
with  $x$ -axis.

and  $y = 0$  to find  
intersections with the  $x$ -axis.



The minimum value of  $(x + 1)^2$  is zero, when  $x = -1$ , so the minimum point on the graph is at  $x = -1$

(c)

$$x^2 + 2x + 3$$

$$a = 1, b = 2, c = 3$$

$$b^2 - 4ac = 2^2 - 4 \times 1 \times 3 = -8$$

Since the discriminant is negative

$$(b^2 - 4ac < 0), x^2 + 2x + 3 = 0$$

has no real roots, so the graph

does not cross the  $x$ -axis.

$$\text{real roots: } b^2 < 4ac$$

The discriminant is  $b^2 - 4ac$

No

(d)

$$x^2 + kx + 3 = 0$$

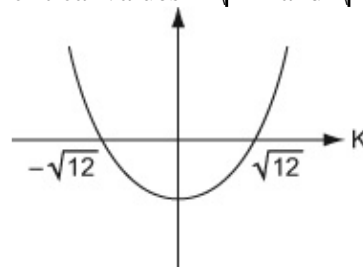
$$a = 1, b = k, c = 3$$

For no real roots,  $b^2 < 4ac$

$$k^2 < 12$$

$$k^2 - 12 < 0$$

This is a quadratic inequality with critical values  $-\sqrt{12}$  and  $\sqrt{12}$



$$(k + \sqrt{12})(k - \sqrt{12}) < 0$$

Critical values:

$$K = -\sqrt{12}, K = \sqrt{12}$$

$$-\sqrt{12} < K < \sqrt{12}$$

$$\left( \begin{array}{l} \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \\ -2\sqrt{3} < K < 2\sqrt{3} \end{array} \right)$$

$$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

The surds can be simplified using

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 16

#### Question:

Solve the simultaneous equations

$$\begin{aligned}x + y &= 2 \\x^2 + 2y &= 12\end{aligned}$$

#### Solution:

$$y = 2 - x$$

Rearrange  
the linear equation to get  $y = \dots$

$$x^2 + 2(2 - x) = 12$$

Substitute  
into the quadratic equation

$$x^2 + 4 - 2x = 12$$

$$x^2 - 2x + 4 - 12 = 0$$

$$x^2 - 2x - 8 = 0$$

Solve  
for  $x$  using factorisation

$$(x + 2)(x - 4) = 0$$

$$x = -2 \text{ or } x = 4$$

Substitute  
the  $x$  values back into  $y = 2 - x$

$$x = -2 : y = 2 - (-2) = 4$$

$$x = 4 : y = 2 - 4 = -2$$

**Solution:**  $x = -2, y = 4$   
and  $x = 4, y = -2$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 17

#### Question:

(a) By eliminating  $y$  from the equations

$$\begin{aligned} y &= x - 4, \\ 2x^2 - xy &= 8, \end{aligned}$$

show that

$$x^2 + 4x - 8 = 0.$$

(b) Hence, or otherwise, solve the simultaneous equations

$$\begin{aligned} y &= x - 4, \\ 2x^2 - xy &= 8, \end{aligned}$$

giving your answers in the form  $a \pm b\sqrt{3}$ , where  $a$  and  $b$  are integers.

#### Solution:

(a)

$$\frac{2x^2 - x}{(x - 4)} = 8 \text{ equation.}$$

Substitute  $y = x - 4$  into the quadratic

$$2x^2 - x^2 + 4x = 8$$

$$x^2 + 4x - 8 = 0$$

(b)

(a). The  $\sqrt{3}$   
factorisation  
quadratic

Solve the equation found in part  
in the given answer suggests that  
will not be possible, so use the  
formula, or complete the square.

$$\begin{aligned}
 x^2 + 4x - 8 &= 0 \\
 x^2 + 4x &= (x + 2)^2 - 4 \\
 (x + 2)^2 - 4 - 8 &= 0 \\
 (x + 2)^2 &= 12 \\
 x + 2 &= \pm \sqrt{12} \\
 x &= -2 \pm \sqrt{12} \\
 \sqrt{12} &= \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \\
 x &= -2 \pm 2\sqrt{3} \\
 (a = -2 \text{ and } b = 2) \\
 \text{Using } y &= x - 4, \\
 y &= (-2 \pm 2\sqrt{3}) \\
 &\quad - 4 \\
 &= -6 \pm 2\sqrt{3} \\
 \text{Solution: } x &= -2 \pm 2\sqrt{3} \\
 y &= -6 \pm 2\sqrt{3}
 \end{aligned}$$

Complete the square for  $x^2 + 4x$

$$\text{Use } \sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 18

#### Question:

Solve the simultaneous equations

$$\begin{aligned}2x - y - 5 &= 0 \\x^2 + xy - 2 &= 0\end{aligned}$$

#### Solution:

$y = 2x - 5$		Rearrange
	the linear	
	get $y = \dots$	equation to
$x^2 + x(2x - 5) - 2 = 0$		Substitute
$x^2 + 2x^2 - 5x - 2 = 0$		into the quadratic equation.
$3x^2 - 5x - 2 = 0$		
$(3x + 1)(x - 2) = 0$		Solve
$x = -\frac{1}{3}$ or $x = 2$		for $x$ using factorisation
$x = -\frac{1}{3} : y = -\frac{2}{3} - 5 = -\frac{17}{3}$	the $x$ values	Substitute
$x = 2 : y = 4 - 5 = -1$	into $y = 2x - 5$	back
Solution $x = -\frac{1}{3}, y = -\frac{17}{3}$		
and $x = 2, y = -1$		

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 19

#### Question:

Find the set of values of  $x$  for which

(a)  $3(2x + 1) > 5 - 2x$ ,

(b)  $2x^2 - 7x + 3 > 0$ ,

(c) both  $3(2x + 1) > 5 - 2x$  and  $2x^2 - 7x + 3 > 0$ .

#### Solution:

(a)

$$3(2x + 1) > 5 - 2x$$

$$6x + 3 > 5 - 2x$$

$$6x + 2x + 3 > 5$$

$$8x > 2$$

$$x > \frac{1}{4}$$

Multiply out

Add  $2x$  to both sides.

Subtract 3 from both sides

Divide both sides by 8

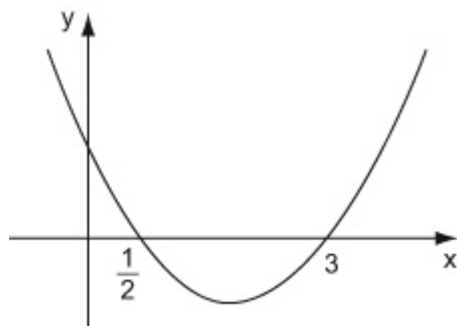
(b)

$$2x^2 - 7x + 3 = 0$$

$$(2x - 1)(x - 3) = 0 \text{ quadratic equation.}$$

$$x = \frac{1}{2}, x = 3$$

Factorise to solve the



Sketch the graph of  
 $y = 2x^2 - 7x + 3$ . The

shape is  The sketch  
does not need to be accurate.

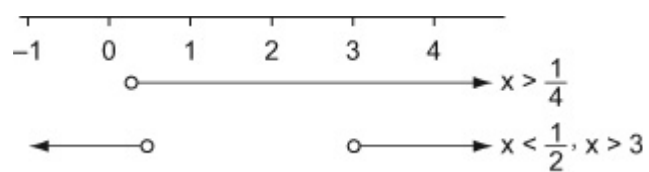
$2x^2 - 7x + 3 > 0$  where the part

$$x < \frac{1}{2} \text{ or } x > 3$$

$2x^2 - 7x + 3 > 0$  ( $y > 0$ ) for

of the graph above the  $x$ -axis

(c)



$$\frac{1}{4} < x <$$

$$\frac{1}{2}, x > 3$$

(a)

Use a number line. The

two sets of values (from part

and part (b)) overlap for

$$\frac{1}{4} < x < \frac{1}{2} \text{ and } x > 3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 20

#### Question:

Find the set of values of  $x$  for which

(a)  $x(x - 5) < 7x - x^2$

(b)  $x(3x + 7) > 20$

#### Solution:

(a)

$$x(x - 5) < 7x - x^2$$

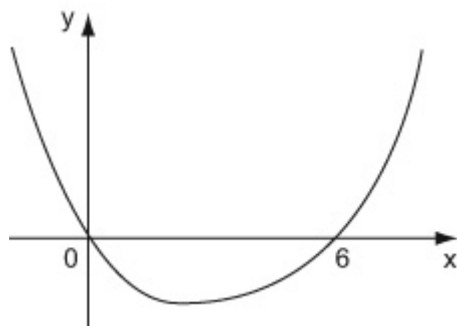
$$x^2 - 5x < 7x - x^2$$

$$2x^2 - 12x < 0$$

$$2x(x - 6) < 0$$

$$2x(x - 6) = 0$$

$$x = 0, x = 6$$



$$2x^2 - 12x < 0 \text{ where}$$

$$0 < x < 6$$

(b)

$$x(3x + 7) > 20$$

$$3x^2 + 7x > 20$$

$$3x^2 + 7x - 20 > 0$$

$$(3x - 5)(x + 4) > 0$$

$$(3x - 5)(x + 4) = 0$$

$$x = \frac{5}{3}, x = -4$$

Multiply out

Factorise using the common factor  $2x$

Solve the quadratic equation to find the critical values

Sketch the graph of  $y = 2x^2 - 12x$

$$2x^2 - 12x < 0 \quad (y < 0)$$

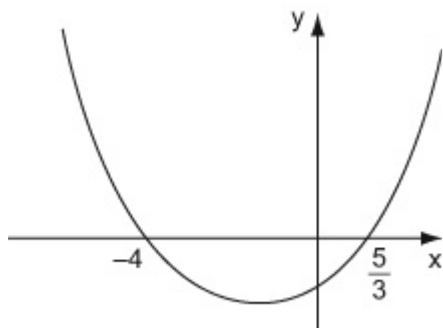
for the part of the graph below the  $x$ -axis

Multiply out

Factorise

Solve the quadratic equation to

find the critical values



$$3x^2 + 7x - 20 > 0 \text{ where}$$
$$x < -4 \text{ or } x > \frac{5}{3}$$

Sketch the graph of  
 $y = 3x^2 + 7x - 20$

$$3x^2 + 7x - 20 > 0 \quad (y > 0)$$

for the part of the graph

above the  $x$ -axis.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 21

#### Question:

(a) Solve the simultaneous equations

$$\begin{aligned}y + 2x &= 5, \\2x^2 - 3x - y &= 16.\end{aligned}$$

(b) Hence, or otherwise, find the set of values of  $x$  for which

$$2x^2 - 3x - 16 > 5 - 2x.$$

#### Solution:

(a)

$$y = 5 - 2x$$

the linear equation

Rearrange

to

$$\text{get } y = \dots$$

$$2x^2 - 3x - (5 - 2x) = 16$$

Substitute  
into the quadratic equation.

$$2x^2 - 3x - 5 + 2x = 16$$

$$2x^2 - x - 21 = 0$$

$$(2x - 7)(x + 3) = 0$$

Solve  
for  $x$  using factorisation.

$$x = 3\frac{1}{2} \text{ or } x = -3$$

$$x = 3$$

$$\frac{1}{2} : y = 5 - 7 = -2 \quad \text{the } x\text{-values back into}$$

Substitute

$$x = -3 : y = 5 + 6 = 11$$

$$y = 5 - 2x$$

Solution  $x = 3$

$$\frac{1}{2}, y = -2$$

$$\text{and } x = -3, y = 11$$

(b)

The equations in (a) could be written as

$$y = 5 - 2x \text{ and } y = 2x^2 - 3x - 16.$$

The solutions to  $2x^2 - 3x - 16 = 5 - 2x$  are the  $x$  solutions from (a). These are the critical values for  $2x^2 - 3x - 16 > 5 - 2x$ .

Critical values

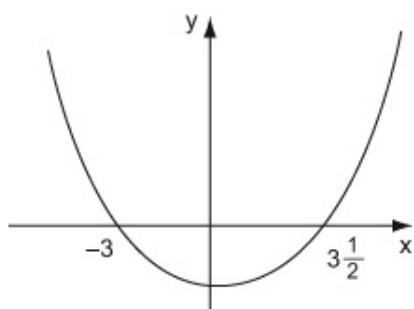
$$x = 3\frac{1}{2} \text{ and } x = -3.$$

$$2x^2 - 3x - 16 > 5 - 2x$$

$$(2x^2 - 3x - 16 - 5 + 2x > 0)$$

$$2x^2 - x - 21 > 0$$





$$x < -3 \text{ or } x > 3\frac{1}{2}$$

Sketch the graph of  
 $y = 2x^2 - x - 21$

$2x^2 - x - 21 > 0$  ( $y > 0$ ) for the  
part of the graph above the  $x$ -axis.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 22

#### Question:

The equation  $x^2 + kx + (k + 3) = 0$ , where  $k$  is a constant, has different real roots.

- (a) Show that  $k^2 - 4k - 12 > 0$ .
- (b) Find the set of possible values of  $k$ .

#### Solution:

(a)

$$x^2 + kx + (k + 3) = 0$$

$$a = 1, b = k, c = k + 3$$

$$b^2 > 4ac$$

$$k^2 > 4(k + 3)$$

$$k^2 > 4k + 12$$

$$k^2 - 4k - 12 > 0$$

Write down  $a$ ,  $b$  and  $c$  for the equation

For different real roots,  $b^2 > 4ac$

(b)

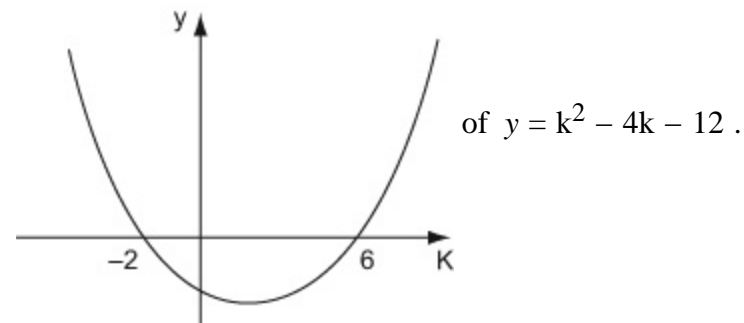
$$k^2 - 4k - 12 = 0 \text{ equation.}$$

$$(k + 2) = 0$$


$$(k - 6) = 0$$

$$k = -2, k = 6$$

Factorise to solve the quadratic



Sketch the graph

The shape is  The sketch does not need to be accurate

$$k^2 - 4k - 12 > 0 \text{ where}$$

$$k < -2 \text{ or } k > 6$$

$k^2 - 4k - 12 > 0$  ( $y > 0$ ) for the part of the graph above the  $k$ -axis.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 23

#### Question:

Given that the equation  $kx^2 + 3kx + 2 = 0$ , where  $k$  is a constant, has no real roots, find the set of possible values of  $k$ .

#### Solution:

$$kx^2 + 3kx + 2 = 0$$

$a = k$ ,  $b = 3k$ ,  $c = 2$  Write down  $a$ ,  $b$  and  $c$  for the equation.

$$b^2 < 4ac$$

$$(3k)^2 < 4 \times k \times 2 \quad \text{no real roots, } b^2 < 4ac.$$

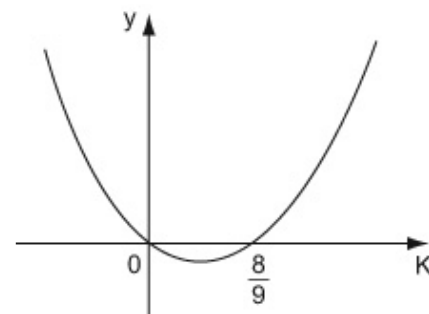
$$9k^2 < 8k$$

$$9k^2 - 8k < 0$$

$$9k^2 - 8k = 0$$

$$k(9k - 8) = 0$$

$$k = 0, k = \frac{8}{9}$$



$$9k^2 - 8k < 0 \quad \text{where}$$

$$0 < k < \frac{8}{9}$$

Write

For

Factorise to solve the quadratic equation

Sketch the graph of  $y = 9k^2 - 8k$ . The shape is



. The sketch does not need to be accurate.

$9k^2 - 8k < 0$  ( $y < 0$ ) for the part of the graph below the  $k$ -axis.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 24

#### Question:

The equation  $(2p + 5)x^2 + px + 1 = 0$ , where  $p$  is a constant, has different real roots.

- (a) Show that  $p^2 - 8p - 20 > 0$
- (b) Find the set of possible values of  $p$ .

Given that  $p = -3$ ,

- (c) find the exact roots of  $(2p + 5)x^2 + px + 1 = 0$ .

#### Solution:

(a)

$$(2p + 5)x^2 + px + 1 = 0$$

$$a = 2p + 5, b = p, c = 1$$

$$b^2 > 4ac$$

$$p^2 > 4(2p + 5)$$

$$p^2 > 8p + 20$$

$$p^2 - 8p - 20 > 0$$

Write down  $a$ ,  $b$  and  $c$  for the equation.

For different real roots,  $b^2 > 4ac$

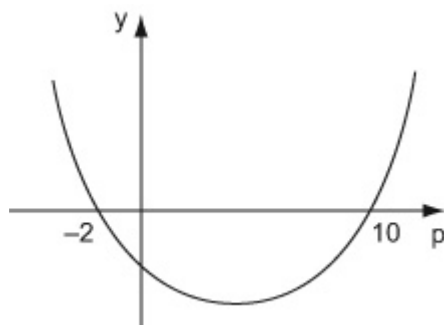
(b)

$$p^2 - 8p - 20 = 0$$

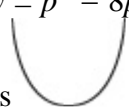
$$(p + 2)(p - 10) = 0 \text{ equation.}$$

$$p = -2, p = 10$$

Factorise to solve the quadratic



Sketch the graph of  
 $y = p^2 - 8p - 20$  The shape

is . The sketch does not need to be accurate

$$p^2 - 8p - 20 > 0 \text{ where}$$

$$p < -2 \text{ or } p > 10$$

$$p^2 - 8p - 20 > 0 \text{ (} y > 0 \text{) for the part}$$

$$\text{of the graph above the } p\text{-axis}$$

(c)

For  $p = -3$

$$(-6 + 5)x^2 - 3x + 1 = 0$$

$$-x^2 - 3x + 1 = 0$$

$$x^2 + 3x - 1 = 0$$

$$a = 1, b = 3, c = -1$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-3 \pm \sqrt{9 + 4}}{2}$$

$$x = \frac{1}{2} (-3 \pm \sqrt{13})$$

$\sqrt{13}$  cannot be simplified.

$$x = \frac{1}{2} (-3 + \sqrt{13}) \quad \text{or} \quad x =$$

$$\frac{1}{2} (-3 - \sqrt{13})$$

Substitute  $p = -3$  into the equation.

Multiply by  $-1$

The equation does not factorise,

so use the quadratics formula.

Quote the formula.

Exact roots are required.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 25

#### Question:

- (a) Factorise completely  $x^3 - 4x$
- (b) Sketch the curve with equation  $y = x^3 - 4x$ , showing the coordinates of the points where the curve crosses the  $x$ -axis.
- (c) On a separate diagram, sketch the curve with equation  

$$y = (x - 1)^3 - 4(x - 1)$$
 showing the coordinates of the points where the curve crosses the  $x$ -axis.

#### Solution:

(a)

$$x^3 - 4x$$

$$= x(x^2 - 4)$$

squares

$$= x(x + 2)$$

$$(x - 2)$$

$x$  is a common factor

$(x^2 - 4)$  is a difference of

(b)

Curve crosses  $x$ -axis where  $y = 0$

$$x(x + 2)(x - 2) = 0$$

$$x = 0, x = -2, x = 2$$

When  $x = 0, y = 0$

curve crosses

the  $y$ -axis.

When  $x \rightarrow \infty, y \rightarrow \infty$

large

When  $x \rightarrow -\infty, y \rightarrow -\infty$

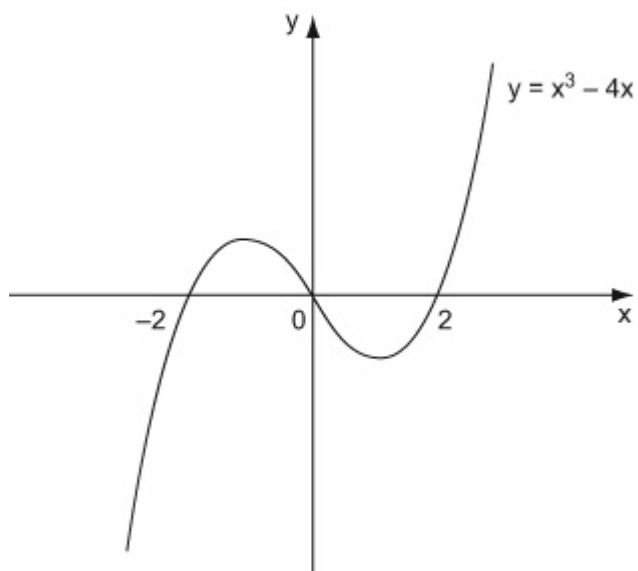
of  $x$

Put  $y = 0$  and solve for  $x$

Put  $x = 0$  to find where the

Check what happens to  $y$  for

positive and negative values



Crosses at  $(0, 0)$

Crosses  $x$ -axis at  $(-2, 0), (2, 0)$ .

(c)

$$y = x^3 - 4x$$

$$y = (x - 1)^3 - 4(x - 1)$$

This is a translation of  $+1$  in the  $x$ -direction.

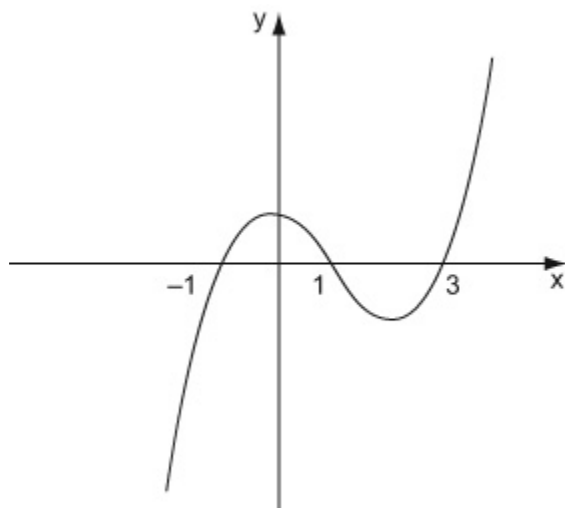
(b).

Compare with the equation from part

$x$  has been replaced by  $x - 1$ .

$f(x + a)$  is a translation of

$-a$  in the  $x$ -direction.



Crosses  $x$ -axis at  $(-1, 0), (1, 0), (3, 0)$

The shape is the same as in part (b).

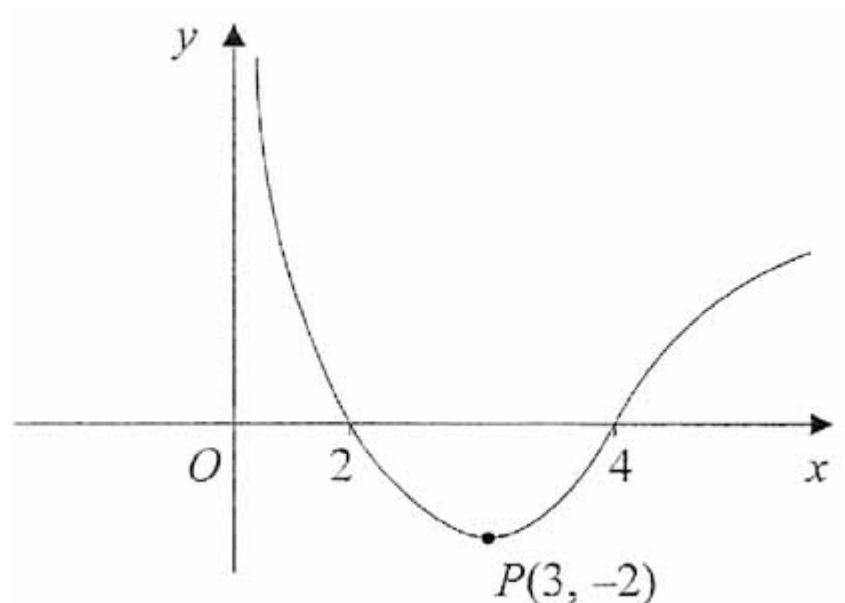
# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 26

#### Question:



The figure shows a sketch of the curve with equation  $y = f(x)$ . The curve crosses the  $x$ -axis at the points  $(2, 0)$  and  $(4, 0)$ . The minimum point on the curve is  $P(3, -2)$ .

In separate diagrams, sketch the curve with equation

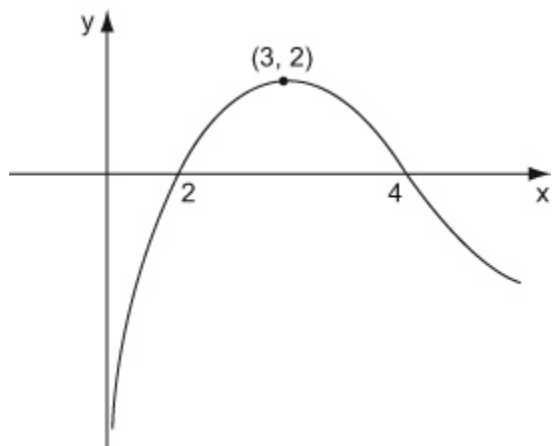
(a)  $y = -f(x)$

(b)  $y = f(2x)$

On each diagram, give the coordinates of the points at which the curve crosses the  $x$ -axis, and the coordinates of the image of  $P$  under the given transformation.

#### Solution:

(a)



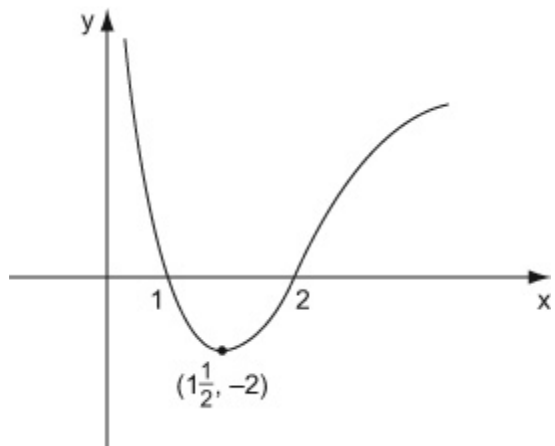
The transformation  $-f(x)$  multiplies the  $y$ -coordinates by  $-1$ . This turns the graph upside-down.



Crosses the  $x$ -axis at  $(2, 0)$ ,  $(4, 0)$

Image of  $P$  is  $(3, 2)$

(b)



Crosses the  $x$ -axis at  $(1, 0)$ ,  
 $(2, 0)$

Image of  $P$  is  $(1\frac{1}{2}, -2)$

unchanged.

$f(2x)$  is a stretch of  $\frac{1}{2}$   
in the  $x$ -direction. (Multiply

$x$ -coordinates by  $\frac{1}{2}$ .)

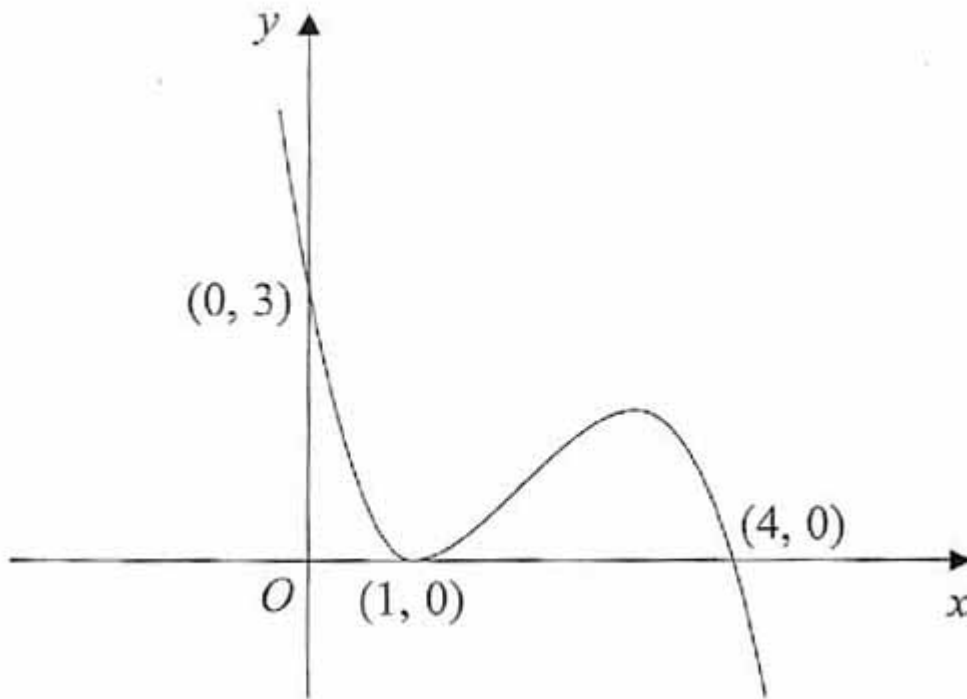
$y$ -coordinates are

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions  
Exercise A, Question 27

Question:



The figure shows a sketch of the curve with equation  $y = f(x)$ . The curve passes through the points  $(0, 3)$  and  $(4, 0)$  and touches the  $x$ -axis at the point  $(1, 0)$ .

On separate diagrams, sketch the curve with equation

(a)  $y = f(x + 1)$

(b)  $y = 2f(x)$

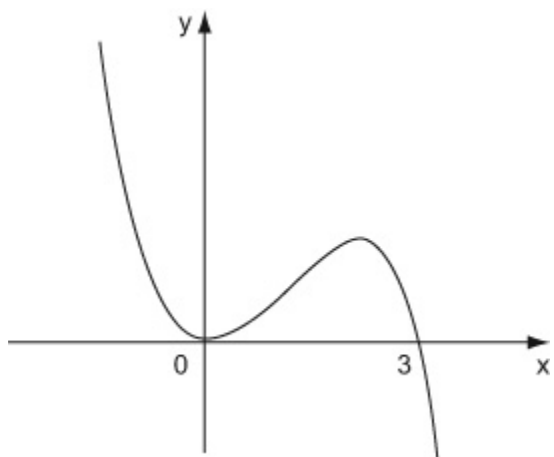
(c)  $y = f\left(\frac{1}{2}x\right)$

On each diagram, show clearly the coordinates of all the points where the curve meets the axes.

**Solution:**

(a)

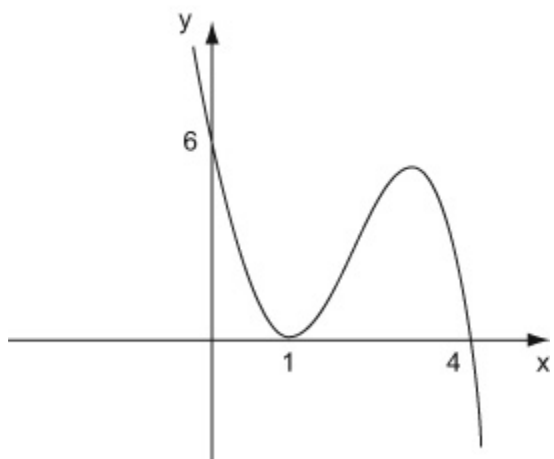
$f(x + 1)$  is a translation of



Meets the  $x$ -axis at  $(0, 0)$ ,  $(3, 0)$

Meets the  $y$ -axis at  $(0, 0)$

(b)



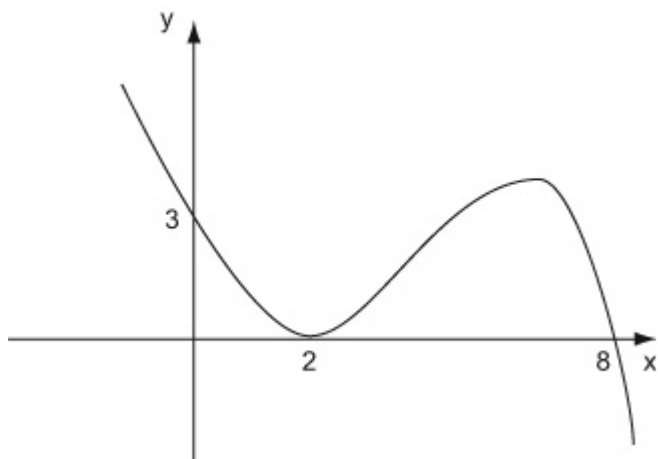
Meets the  $x$ -axis at  $(1, 0)$ ,  
 $(4, 0)$

Meets the  $y$ -axis at  $(0, 6)$

unchanged.

$x$ -coordinates are

(c)



$f\left(\frac{1}{2}x\right)$  is a stretch of scale

factor  $\frac{1}{\left(\frac{1}{2}\right)} = 2$  in the

$x$ -direction. (Multiply  
 $x$ -coordinates by 2)

Meets the  $x$ -axis at  $(2, 0)$ ,  
 $(8, 0)$

Meets the  $y$ -axis at  $(0, 3)$  unchanged.

$y$ -coordinates are

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 28

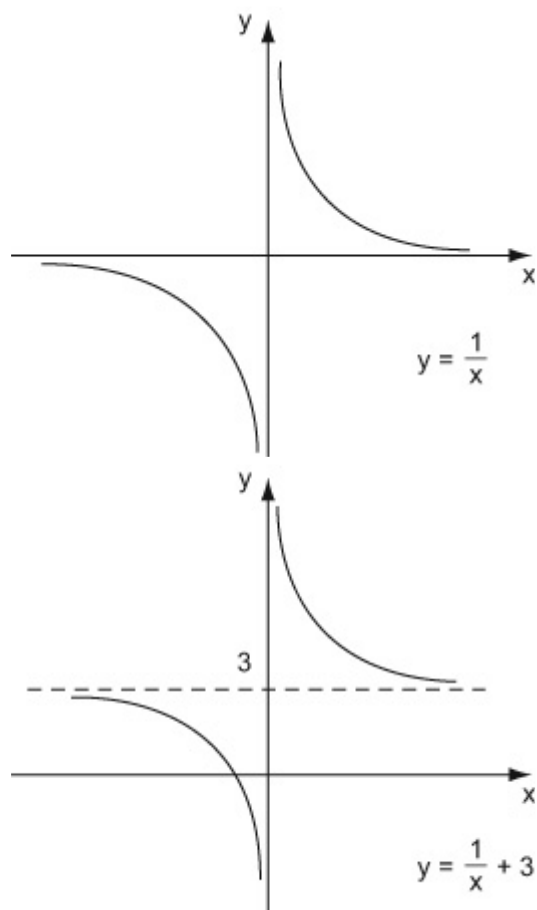
#### Question:

Given that  $f(x) = \frac{1}{x}$ ,  $x \neq 0$ ,

- (a) sketch the graph of  $y = f(x) + 3$  and state the equations of the asymptotes.
- (b) Find the coordinates of the point where  $y = f(x) + 3$  crosses a coordinate axis.

#### Solution:

(a)



You should know the shape of this curve.

$f(x) + 3$  is a translation of  $+3$  in the  $y$ -direction.

$y = 3$  is an asymptote  
 $x = 0$  is an asymptote

is  $x = 0$

The equation of the  $y$ -axis

(b)

The graph does not cross

get

the  $y$ -axis ( see sketch in  
( a ) ) .

undefined ,

Crosses the  $x$ -axis where  $y = 0$  :

$$\frac{1}{x} + 3$$

$$= 0$$

$$\frac{1}{x}$$

$$= -3$$

$$x$$

$$= -\frac{1}{3} \quad \left( -\frac{1}{3}, 0 \right)$$

If you used  $x = 0$  you would

$$y = \frac{1}{0} + 3 \text{ but } \frac{1}{0} \text{ is}$$

or infinite.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 29

#### Question:

Given that  $f(x) = (x^2 - 6x)(x - 2) + 3x$ ,

- (a) express  $f(x)$  in the form  $x(ax^2 + bx + c)$ , where  $a$ ,  $b$  and  $c$  are constants
- (b) hence factorise  $f(x)$  completely
- (c) sketch the graph of  $y = f(x)$ , showing the coordinates of each point at which the graph meets the axes

#### Solution:

(a)

$$\begin{aligned}
 f(x) &= (x^2 - 6x)(x - 2) + 3x && \text{Multiply} \\
 &= x^2(x - 2) - 6x(x - 2) && \text{out the bracket} \\
 &\quad + 3x \\
 &= x^3 - 2x^2 - 6x^2 + 12x + 3x \\
 &= x^3 - 8x^2 + 15x && \text{common factor} \\
 &= x(x^2 - 8x + 15) && x \text{ is a}
 \end{aligned}$$

$$(a = 1, b = -8, c = 15)$$

(b)

$$\begin{aligned}
 &x(x^2 - 8x + 15) && \text{Factorise the quadratic} \\
 f(x) &= x(x - 3)(x - 5)
 \end{aligned}$$

(c)

Curve meets  $x$ -axis  
where  $y = 0$ .

$$x(x - 3)(x - 5) = 0$$

$$x = 0, x = 3, x = 5$$

$$\text{When } x = 0, y = 0$$

$$\text{When } x \rightarrow \infty, y \rightarrow \infty$$

large

$$\text{When } x \rightarrow -\infty, y \rightarrow -\infty$$

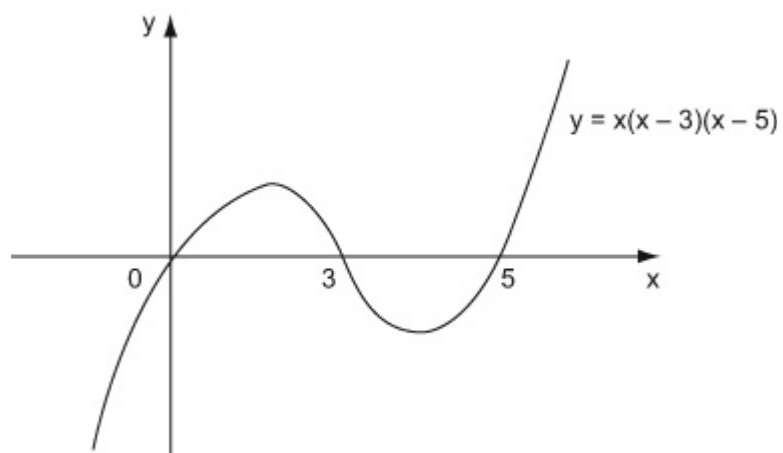
of  $x$ .

Put  $y = 0$  and solve for  $x$

Put  $x = 0$  to find where the curve crosses the  $y$ -axis

Check what happens to  $y$  for

positive and negative values



Meets  $x$ -axis at  $(0, 0)$ ,  $(3, 0)$ ,  $(5, 0)$

Meets  $y$ -axis at  $(0, 0)$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

Exercise A, Question 30

**Question:**

- (a) Sketch on the same diagram the graph of  $y = x(x + 2)(x - 4)$  and the graph of  $y = 3x - x^2$ , showing the coordinates of the points at which each graph meets the  $x$ -axis.
- (b) Find the exact coordinates of each of the intersection points of  $y = x(x + 2)(x - 4)$  and  $y = 3x - x^2$ .

**Solution:**

(a)

$$y = x(x + 2)(x - 4)$$

Curve meets  $x$ -axis where  $y = 0$ .

$$x(x + 2)(x - 4) = 0$$


$$x = 0, x = -2, x = 4$$

$$\text{When } x = 0, y = 0$$

$$\text{When } x \rightarrow \infty, y \rightarrow \infty$$

$$\text{When } x \rightarrow -\infty, y \rightarrow -\infty$$


$$y = 3x - x^2$$

The graph of  $y = 3x - x^2$  is a  shape

Put  $y = 0$  and solve for  $x$ .

Put  $x = 0$  to find where the curve crosses the  $y$ -axis

Check what happens to  $y$  for large positive and negative values of  $x$ .

For  $y = ax^2 + bx + c$ ,  
if  $a < 0$ , the shape is 

$$3x - x^2$$

$$= 0$$

$$x(3 - x)$$

$$= 0$$

$$x = 0, x = 3$$

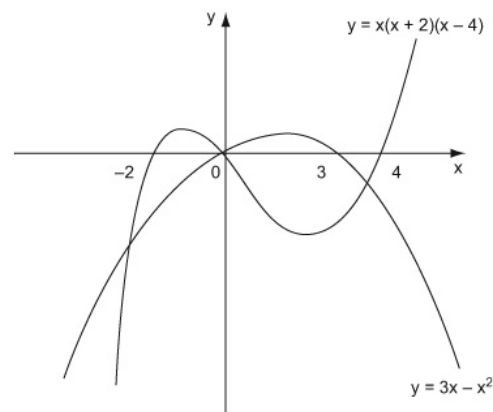
Put  $y = 0$  and solve for  $x$ .

$$\text{When } x = 0, y = 0$$

find where the curve crosses the  $y$ -axis.

Put  $x = 0$  to

the  $y$ -axis.



$y = x(x + 2)(x - 4)$  meets the  $x$ -axis at  $(-2, 0)$ ,  $(0, 0)$ ,  $(4, 0)$

$y = 3x - x^2$  meets the  $x$ -axis at  $(0, 0)$ ,  $(3, 0)$

(b)

$$x(x + 2)(x - 4) = 3x - x^2$$

$$x(x + 2)(x - 4) = x(3 - x)$$

to give an equation in  $x$ .

$$(x + 2)(x - 4) = 3 - x$$

One solution is  $x = 0$

$x = 0$  is a solution.

$$x^2 - 2x - 8 = 3 - x$$

$$x^2 - 2x + x - 8 - 3 = 0$$

$$x^2 - x - 11 = 0$$

To find where the graphs intersect, equate the two expressions for  $y$

If you divide by  $x$ , remember that

The equation does not factorise, so

use the quadratic formula.

$$a = 1, b = -1, c = -11$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quote the formula

$$x = \frac{1 \pm \sqrt{(-1)^2 - (4 \times 1 \times -11)}}{2}$$

$$= \frac{1 \pm \sqrt{45}}{2}$$

Exact values are required, not rounded

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

decimals, so leave the answers in surd form.

$$x = \frac{1}{2} (1 \pm 3\sqrt{5})$$

$$x = \frac{1}{2} (1 + 3\sqrt{5}) \text{ or } x = \frac{1}{2} (1 - 3\sqrt{5})$$

$x = 0 : y = 0$  The y-coordinates for the intersection

$$x = \frac{1}{2} (1 + 3\sqrt{5})$$

points are also needed.

$$y = \frac{3(1 + 3\sqrt{5})}{2} - \frac{(1 + 3\sqrt{5})^2}{4}$$

Use  $y = 3x - x^2$ , the simpler equation

$$\begin{aligned} (1 + 3\sqrt{5})^2 &= (1 + 3\sqrt{5})(1 + 3\sqrt{5}) \\ &= 1(1 + 3\sqrt{5}) + 3\sqrt{5}(1 + 3\sqrt{5}) \\ &= 1 + 3\sqrt{5} + 3\sqrt{5} + 45 \\ &= 46 + 6\sqrt{5} \end{aligned}$$

$$\sqrt{5} \times \sqrt{5} = 5$$

$$y = \frac{6(1 + 3\sqrt{5})}{4} - \frac{46 + 6\sqrt{5}}{4}$$

Use a common denominator 4.

$$= \frac{6 + 18\sqrt{5} - 46 - 6\sqrt{5}}{4}$$

$$= \frac{-40 + 12\sqrt{5}}{4} = -10 + 3\sqrt{5}$$

$$x = \frac{1}{2} (1 - 3\sqrt{5})$$

$$= \frac{3(1 - 3\sqrt{5})}{2} -$$

$$y = \frac{(1 - 3\sqrt{5})^2}{4}$$

$$y = \frac{6(1 - 3\sqrt{5})}{4} - \frac{46 - 6\sqrt{5}}{4}$$

that for

The working will be similar to

repeated.  $1 + 3\sqrt{5}$ , so need not be fully

$$= \frac{6 - 18\sqrt{5} - 46 + 6\sqrt{5}}{4}$$

$$= \frac{-40 - 12\sqrt{5}}{4} = -10 - 3\sqrt{5}$$

$\sqrt{5}$

Intersection points are :

Finally, write down the coordinates of all the

$$(0, 0), \left(\frac{1}{2}(1 + 3\sqrt{5}), -10 + 3\sqrt{5}\right)$$

points you have found. You can compare

these with your sketch, as a rough check.

$$\text{and } \left(\frac{1}{2}(1 - 3\sqrt{5}), -10 - 3\sqrt{5}\right)$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 1

#### Question:

The line  $L$  has equation  $y = 5 - 2x$ .

(a) Show that the point  $P(3, -1)$  lies on  $L$ .

(b) Find an equation of the line, perpendicular to  $L$ , which passes through  $P$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

(a)

For  $x = 3$ ,

$$y = 5 - (2 \times 3) = 5 - 6 = -1$$

So  $(3, -1)$  lies on  $L$ .

Substitute  $x = 3$   
into the equation of  $L$ .  
Give a conclusion.

(b)

$$y = -2x + 5$$

Gradient of  $L$  is  $-2$ .

Perpendicular to  $L$ ,

gradient is  $\frac{1}{2}$  (

$$\frac{1}{2} \times -2 = -1)$$

Compare with  
 $y = mx + c$  to find  
the gradient  $m$   
For a perpendicular

line, the gradient

$$\text{is } -\frac{1}{m}$$

Use  $y - y_1 = m$

$$y - (-1) = \frac{1}{2}(x - 3)$$

$$(x - x_1)$$

$$y + 1 = \frac{1}{2}x - \frac{3}{2}$$

Multiply by 2

$$2y + 2 = x - 3$$

$$0 = x - 2y - 5$$

$$x - 2y - 5 = 0$$

This is the required  
form  $ax + by + c = 0$ ,

$$(a = 1, b = -2, c = -5)$$

where  $a$ ,  $b$  and  $c$   
are integers.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 2

**Question:**

The points  $A$  and  $B$  have coordinates  $(-2, 1)$  and  $(5, 2)$  respectively.

(a) Find, in its simplest surd form, the length  $AB$ .

(b) Find an equation of the line through  $A$  and  $B$ , giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers.

The line through  $A$  and  $B$  meets the  $y$ -axis at the point  $C$ .

(c) Find the coordinates of  $C$ .

**Solution:**

(a)

$A : (-2, 1)$ ,  $B$   
 $(5, 2)$

$AB$

The distance between

$$= \sqrt{(5 - (-2))^2 + (2 - 1)^2}$$

$$= \sqrt{(7^2 + 1^2)} = \sqrt{50}$$

two points is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\sqrt{50}$   
 $AB$

Theorem )  
 $= \sqrt{(25 \times 2)} = 5\sqrt{2}$   
 $= 5\sqrt{2}$

(Pythagoras's

Use  $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

(b)

$$m = \frac{2 - 1}{5 - (-2)} = \frac{1}{7}$$

Find the gradient  
of the line, using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - 1 = \frac{1}{7}(x - (-2))$$

$$(x - x_1)$$

Use  $y - y_1 = m$

$$y - 1 = \frac{1}{7}x + \frac{2}{7}$$

Multiply by 7

$$7y - 7 = x + 2$$

$$0 = x - 7y + 9$$

$$x - 7y + 9 = 0$$

This is the required  
form  $ax + by + c = 0$ ,

$(a = 1, b = -7, c = 9)$

where  $a, b$  and  $c$   
are integers.

(c)

$x = 0$  :

Use  $x = 0$  to find

$$0 - 7y + 9 = 0$$

$$9 = 7y$$

where the line meets  
the  $y$ -axis.

$$y = \frac{9}{7} \text{ or } y = 1 \frac{2}{7}$$

$C$  is the point  $(0, 1 \frac{2}{7})$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 3

**Question:**

The line  $l_1$  passes through the point  $(9, -4)$  and has gradient  $\frac{1}{3}$ .

(a) Find an equation for  $l_1$  in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

The line  $l_2$  passes through the origin  $O$  and has gradient  $-2$ . The lines  $l_1$  and  $l_2$  intersect at the point  $P$ .

(b) Calculate the coordinates of  $P$ .

Given that  $l_1$  crosses the  $y$ -axis at the point  $C$ ,

(c) calculate the exact area of  $\triangle OCP$ .

**Solution:**

(a)

$$y - (-4) = \frac{1}{3}(x - 9) \quad (x - x_1) \quad \text{Use } y - y_1 = m$$

$$y + 4 = \frac{1}{3}(x - 9)$$

$$y + 4 = \frac{1}{3}x - 3 \quad \text{Multiply by 3}$$

$$3y + 12 = x - 9$$

$$0 = x - 3y - 21$$

$$x - 3y - 21 = 0$$

$$(a = 1, b = -3, c = -21)$$

required

form  $ax + by + c = 0$ ,  
where  $a$ ,  $b$  and  $c$   
are integers.

This is the

(b)

Equation of  $l_2 : y = -2x$

The equation of a  
straight line through  
the origin

is  $y = mx$ .

$$l_1 : x - 3y - 21 = 0$$

$$x - 3(-2x) - 21 = 0$$

$$x + 6x - 21 = 0$$

$$7x = 21$$

$$x = 3$$

$$y = -2 \times 3 = -6$$

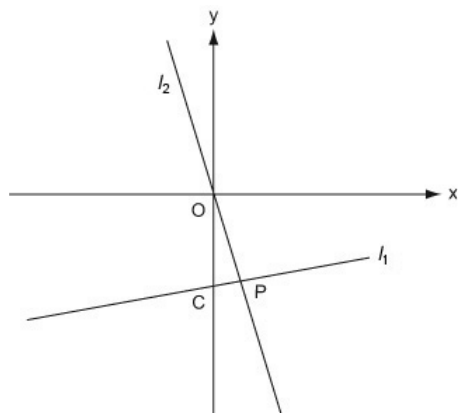
Substitute  $y = -2x$   
into the equation  
of  $l_1$

Substitute back  
into  $y = -2x$

Coordinates of  $P$  :

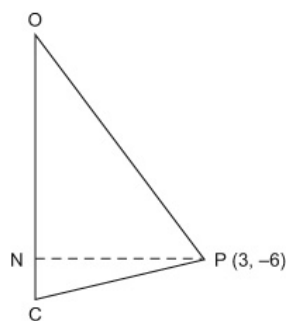
$$(3, -6)$$

(c)



Use a rough sketch to show  
the given information

Be careful not to make any  
wrong assumptions. Here, for  
example,  $\angle OPC$  is *not*  $90^\circ$



Use OC as the base and PN  
as the perpendicular height

Where  $l_1$  meets the  $y$ -axis,  $x = 0$ .

$$0 - 3y - 21 = 0$$

$$3y = -21$$

$$y = -7$$

So OC = 7 and PN = 3

Put  $x = 0$  in the  
equation of  $l_1$

The distance of  $P$   
from the  $y$ -axis is  
the same as its  
 $x$ -coordinate

$$\text{Area of } \triangle OCP = \frac{1}{2} (\text{base} \times \text{height})$$

$$= \frac{1}{2} (7 \times 3)$$

$$= 10 \frac{1}{2}$$

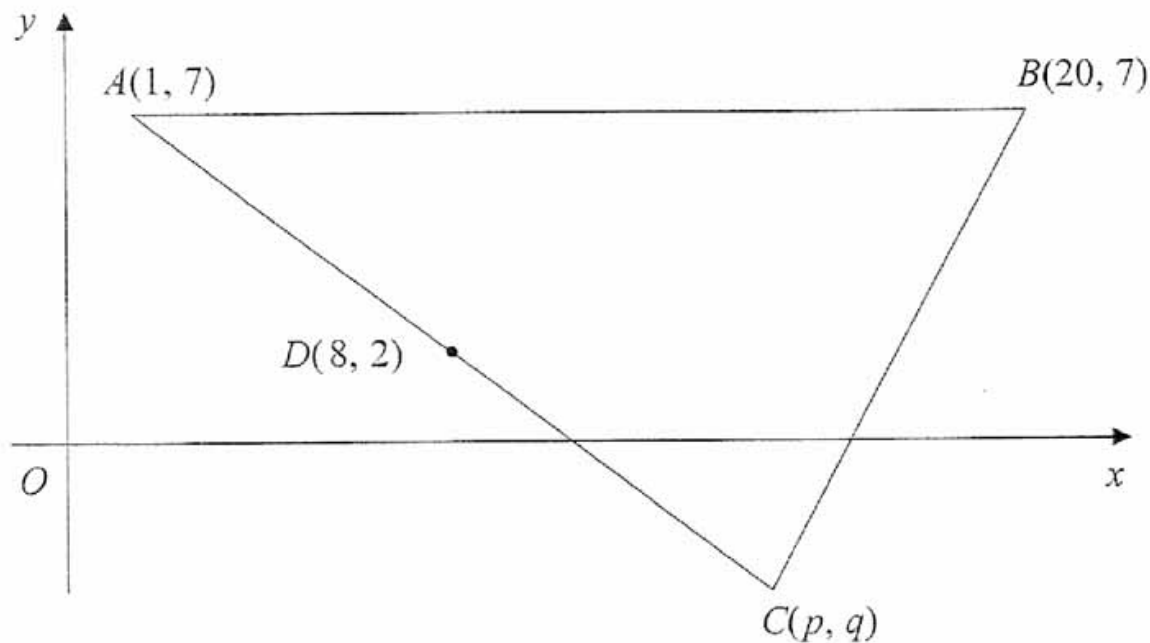
# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 4

Question:



The points  $A(1, 7)$ ,  $B(20, 7)$  and  $C(p, q)$  form the vertices of a triangle  $ABC$ , as shown in the figure. The point  $D(8, 2)$  is the mid-point of  $AC$ .

(a) Find the value of  $p$  and the value of  $q$ .

The line  $l$ , which passes through  $D$  and is perpendicular to  $AC$ , intersects  $AB$  at  $E$ .

(b) Find an equation for  $l$ , in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(c) Find the exact  $x$ -coordinate of  $E$ .

**Solution:**

(a)

$$\left( \frac{1+p}{2}, \frac{7+q}{2} \right) = (8, 2) \qquad \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

is the mid-point  
of the line from  
 $(x_1, y_1)$  to

$$(x_2, y_2)$$

$$\frac{1+p}{2} = 8 \qquad \text{Equate the } x\text{-coordinates}$$

$$1+p = 16$$

$$p = 15$$

$$\frac{7+q}{2} = 2 \qquad \text{Equate the } y\text{-coordinates}$$

$$7+q = 4$$

$$q = -3$$

(b)

Gradient of AC :

$$m = \frac{2-7}{8-1} = \frac{-5}{7}$$

Use the points A

and D, with

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

to find the gradient of AC ( or

AD ) .

For a perpendicular

Gradient of  $l$  is

$$-\frac{1}{\left(-\frac{5}{7}\right)} = \frac{7}{5}$$

gradient

line, the

$$\text{is } -\frac{1}{m}$$

The

$$y - 2 = \frac{7}{5}(x - 8)$$

line  $l$  passes

through  $D(8, 2)$

. So

use this point in

$$y - y_1 = m$$

$$y - 2 = \frac{7x}{5} - \frac{56}{5}$$

by 5

Multiply

$$5y - 10 = 7x - 56$$

$$0 = 7x - 5y - 46$$

$$7x - 5y - 46 = 0$$

in the

This is

$$(a = 7, b = -5, c = -46)$$

required form

$$ax + by + c = 0,$$

where  $a, b$  and  $c$  are integers.

(c)

The equation of AB

is  $y = 7$

At E :

Substitute  $y = 7$  into

$$7x - (5 \times 7) - 46 = 0$$

of  $l$  to

the equation

$$7x - 35 - 46 = 0$$

E.

find the point

$$7x = 81$$

$$x = 11 \frac{4}{7}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 5

#### Question:

The straight line  $l_1$  has equation  $y = 3x - 6$ .

The straight line  $l_2$  is perpendicular to  $l_1$  and passes through the point  $(6, 2)$ .

(a) Find an equation for  $l_2$  in the form  $y = mx + c$ , where  $m$  and  $c$  are constants.

The lines  $l_1$  and  $l_2$  intersect at the point  $C$ .

(b) Use algebra to find the coordinates of  $C$ .

The lines  $l_1$  and  $l_2$  cross the  $x$ -axis at the point  $A$  and  $B$  respectively.

(c) Calculate the exact area of triangle  $ABC$ .

#### Solution:

(a)

The gradient  
of  $l_1$  is 3.

with  $y = mx + c$ .

Compare

So the gradient

of  $l_2$  is  $-\frac{1}{3}$

For a perpendicular

line, the gradient

is  $-\frac{1}{m}$

Eqn. of  $l_2$  :

$$y - 2 = -\frac{1}{3}(x - 6) \quad (x - x_1)$$

Use  $y - y_1 = m$

$$y - 2 = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x + 4$$

This is the required

form  $y = mx + c$ .

(b)

$$y = 3x - 6$$

equations

Solve these

$$y = -\frac{1}{3}x + 4$$

simultaneously

$$3x - 6 = -\frac{1}{3}x + 4$$

$$3x + \frac{1}{3}x = 4 + 6$$

$$\frac{10}{3}x = 10$$

by 3 and

Multiply

$$x = 3$$

divide by 10

$y =$

$$(3 \times 3) - 6 = 3$$

Substitute back

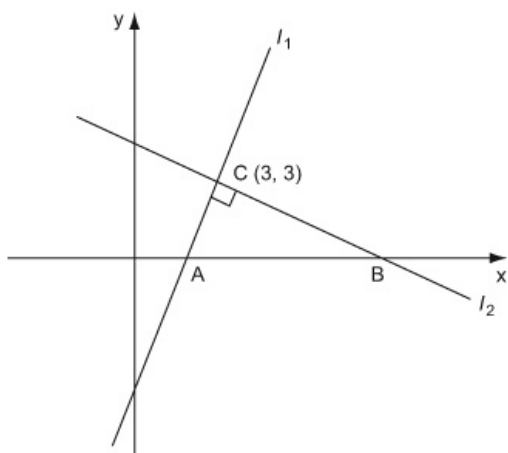
The point

$C$  is

$$(3, 3)$$

into  $y = 3x - 6$

(c)



Use a rough sketch to show the given information.

Where  $l_1$  meets the  $x$ -axis,  $y = 0$  :

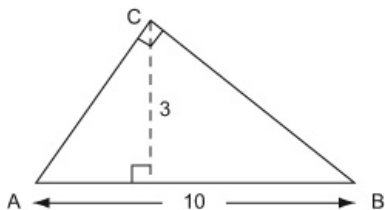
$$\begin{aligned} 0 &= 3x - 6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

A is the point  $(2, 0)$

Where  $l_2$  meets the  $x$ -axis,  $y = 0$  :

$$\begin{aligned} 0 &= -\frac{1}{3}x + 4 \\ \frac{1}{3}x &= 4 \\ x &= 12 \end{aligned}$$

B is the point  $(12, 0)$



$$AB = 10 (12 - 2)$$

The perpendicular height, using AB as the base, is 3

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} (10 \times 3) \\ &= 15 \end{aligned}$$

Put  $y = 0$  to find

where the lines meet the  $x$ -axis

Although  $\angle C$  is a right-angle, it is easier to use AB as the base.

The distance of C from the  $x$ -axis is the same as its  $y$ -coordinate.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 6

#### Question:

The line  $l_1$  has equation  $6x - 4y - 5 = 0$ .

The line  $l_2$  has equation  $x + 2y - 3 = 0$ .

(a) Find the coordinates of  $P$ , the point of intersection of  $l_1$  and  $l_2$ .

The line  $l_1$  crosses the  $y$ -axis at the point  $M$  and the line  $l_2$  crosses the  $y$ -axis at the point  $N$ .

(b) Find the area of  $\triangle MNP$ .

#### Solution:

(a)

$$6x - 4y - 5 = 0 \quad (\text{i})$$

$$x + 2y - 3 = 0 \quad (\text{ii})$$

$$x = 3 - 2y \quad \text{equation (ii)}$$

$$6(3 - 2y) - 4y - 5 = 0$$

$$18 - 12y - 4y - 5 = 0$$

$$18 - 5 = 12y + 4y$$

$$16y = 13$$

$$y = \frac{13}{16}$$

$$x = 3 - 2\left(\frac{13}{16}\right) = 3 - \frac{26}{16}$$

$$x = 1\frac{3}{8}$$

$P$  is the point  $\left(1\frac{3}{8}, \frac{13}{16}\right)$

$$\frac{13}{16}$$

(b)

Solve the equations

simultaneously

Find  $x$  in terms of  $y$  from

Substitute into equation (i)

Substitute back into  $x = 3 - 2y$

Where  $l_1$  meets the  $y$ -axis,  $x = 0$

$$0 - 4y - 5$$

$$4y$$

$$y$$

Put  $x = 0$  to find where the

$$= 0$$

$$= -5$$

$$= -\frac{5}{4}$$

lines meet the  $y$ -axis.

$M$  is the point  $(0, -\frac{5}{4})$

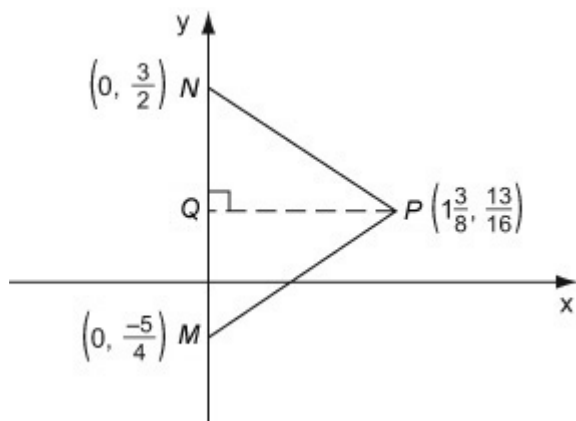
Where  $l_2$  meets the  $y$ -axis,  $x = 0$ :

$$0 + 2y - 3 = 0$$

$$2y = 3$$

$$y = \frac{3}{2}$$

$N$  is the point  $(0, \frac{3}{2})$



Use a rough sketch to show the information

Use  $MN$  as the base and  $PQ$  as the perpendicular height.

$$MN = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$$

the same as its

The distance of  $P$  from the  $y$ -axis is

$x$ -coordinate

$$PQ = 1 \frac{3}{8} = \frac{11}{8}$$

$$\begin{aligned} \text{Area of } \triangle MNP &= \frac{1}{2} \\ &= \frac{1}{2} \left( \frac{11}{4} \times \frac{11}{8} \right) \end{aligned}$$

$$= \frac{121}{64}$$

$$= 1 \frac{57}{64}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 7

#### Question:

The 5th term of an arithmetic series is 4 and the 15th term of the series is 39.

- (a) Find the common difference of the series.
- (b) Find the first term of the series.
- (c) Find the sum of the first 15 terms of the series.

#### Solution:

(a)

$$n^{\text{th}} \text{ term} = a + (n - 1)d$$

$$n = 5 : \quad a + 4d = 4 \quad (\text{i})$$

$$n = 15 : \quad a + 14d = 39 \quad (\text{ii}) \quad \text{formula.}$$

Substitute the given values into the  $n^{\text{th}}$  term

Subtract (ii)-(i)

$$10d = 35$$

Solve simultaneously.

$$d = 3 \frac{1}{2}$$

Common difference is  $3 \frac{1}{2}$

$$\frac{1}{2}$$

(b)

$$a + (4 \times 3 \frac{1}{2}) = 4$$

Substitute back into equation (i).

$$a + 14 = 4$$

$$a = -10$$

First term is  $-10$

(c)

$$S_n = \frac{1}{2}n(2a + (n-1)d)$$

$$n = 15, a$$

$$= -10, d = 3\frac{1}{2}$$

values

Substitute the

into the

sum formula.

$$S_{15}$$

$$= \frac{1}{2} \times 15 ( -20 +$$

$$( 14 \times 3\frac{1}{2} ) )$$

$$= \frac{15}{2} ( -20 + 49 )$$

$$= \frac{15}{2} \times 29$$

$$= 217\frac{1}{2}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 8

#### Question:

An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs farther than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term  $a$  km and common difference  $d$  km.

He runs 9 km on the 11<sup>th</sup> day, and he runs a total of 77 km over the 11 day period.

Find the value of  $a$  and the value of  $d$ .

#### Solution:

$n^{\text{th}}$  term =  $a + (n - 1)d$  distance run on the 11th day is the

$n = 11$  :  $a + 10d = 9$  term of the arithmetic sequence. 11th

$S_n = \frac{1}{2}n(2a + (n - 1)d)$  total distance run is the sum The

$S_n = 77$ ,  $n = 11$  : the arithmetic series. of

$$\frac{1}{2} \times 11(2a + 10d) = 77$$

$$\frac{1}{2}(2a + 10d) = 7$$

$$a + 5d = 7$$

$$a + 10d = 9 \quad (\text{i})$$

$$a + 5d = 7 \quad (\text{ii})$$

Subtract (i)-(ii):

$$5d = 2$$

$$d = \frac{2}{5}$$

$$a + (10 \times \frac{2}{5}) = 9$$

$$a + 4 = 9$$

$$a = 5$$

It is simpler to divide each side of the equation by 11. Solve simultaneously

Substitute back into equation (i).

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 9

#### Question:

The  $r$ th term of an arithmetic series is  $(2r - 5)$ .

- (a) Write down the first three terms of this series.
- (b) State the value of the common difference.

(c) Show that 
$$\sum_{r=1}^n (2r - 5) = n(n - 4).$$

#### Solution:

(a)

$$\begin{aligned} r = 1 : 2r - 5 &= -3 \\ r = 2 : 2r - 5 &= -1 \\ r = 3 : 2r - 5 &= 1 \end{aligned}$$

First three terms are  $-3, -1, 1$

(b)

Common difference  $d = 2$

The terms increase  
by 2 each time  
(  $U_{k+1} = U_k + 2$  )

(c)

$$\sum_{r=1}^n (2r - 5) = S_n$$

$(2r - 5)$  is just

$$S_n = \frac{1}{2}n(2a + (n - 1)d)$$

the

$a = -3, d = 2$  to  $n$  terms

series

$$\begin{aligned} S_n &= \frac{1}{2}n(-6 + 2(n - 1)) \\ &= \frac{1}{2}n(-6 + 2n - 2) \\ &= \frac{1}{2}n(2n - 8) \\ &= \frac{1}{2}n2(n - 4) \\ &= n(n - 4) \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 10

#### Question:

Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

- (a) Find the amount he plans to save in the year 2011.
- (b) Calculate his total planned savings over the 20 year period from 2001 to 2020.

Ben also plans to save money over the same 20 year period. He saves £ $A$  in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben's total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

- (c) calculate the value of  $A$ .

#### Solution:

- (a)
- |     |                          |                   |
|-----|--------------------------|-------------------|
| $a$ | $= 250$<br>( Year 2001 ) |                   |
| $d$ | $= 50$                   | arithmetic series |
- Write down the values  
of  $a$  and  $d$  for the

Taking 2001 as Year 1  
(  $n = 1$  ) ,

2011 is Year 11  
(  $n = 11$  ).

Year 11 savings:

$$\begin{aligned}
 a + (n - 1) d &= 250 + (11 - 1) 50 && \text{Use the term} \\
 &= 250 + (10 \times 50) && \text{formula } a + (n - 1) d \\
 &= 750
 \end{aligned}$$

Year 11 savings : £ 750

- (b)

$$S_n = \frac{1}{2}n(2a + (n-1)d)$$

Using  $n = 20$ ,

$$\begin{aligned} S_{20} &= \frac{1}{2} \times 20 (500 + \\ & (19 \times 50) ) \\ &= 10 (500 + 950) \\ &= 10 \times 1450 \\ &= 14500 \end{aligned}$$

series.

The total savings  
will be the sum of  
the arithmetic

Total savings : £ 14  
500

(c)

$$a = A \quad (\text{Year 2001})$$

$$d = 60$$

$$S_{20} = \frac{1}{2} \times 20 (2A + (19 \times 60) )$$

$$\begin{aligned} S_{20} &= 10 (2A + 1140) \\ &= 20A + 11400 \end{aligned}$$

$$20A + 11400 = 14500$$

$$20A = 14500 - 11400$$

$$20A = 3100$$

$$A = 155$$

Write down the values  
of  $a$  and  $d$  for Ben's series.

Use the sum formula.

Equate Ahmed's  
and Ben's total savings.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 11

#### Question:

A sequence  $a_1, a_2, a_3, \dots$  is defined by

$$a_1 = 3,$$

$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(a) Find the value of  $a_2$  and the value of  $a_3$ .

(b) Calculate the value of  $\sum_{r=1}^5 a_r$ .

#### Solution:

(a)

$$a_{n+1} = 3a_n - 5$$

$$n = 1 : a_2 = 3a_1 - 5$$

$$a_1 = 3, \text{ so } a_2 = 9 - 5$$

$$a_2 = 4$$

$$n = 2 : a_3 = 3a_2 - 5$$

$$a_2 = 4, \text{ so } a_3 = 12 - 5$$

$$a_3 = 7$$

Use the given  
formula, with  
 $n = 1$  and  $n = 2$

(b)

$$\sum_{a=1}^5 a_r = a_1 + a_2 + a_3 + a_4 + a_5$$

$$n = 3 : a_4 = 3a_3 - 5$$

$$a_3 = 7, \text{ so } a_4 = 21 - 5$$

$$a_4 = 16$$

$$n = 4 : a_5 = 3a_4 - 5$$

$$a_4 = 16, \text{ so } a_5 = 48 - 5$$

$$a_5 = 43$$

$$\sum_{a=1}^5 a_r = 3 + 4 + 7 + 16 + 43$$

$$= 73$$

This is not an arithmetic series.

The first three terms are 3, 4, 7.

The differences between

the terms are not the same.

You cannot use a standard formula,  
so work out each separate term and

then add them together to find

the required sum.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 12

#### Question:

A sequence  $a_1, a_2, a_3, \dots$  is defined by

$$\begin{aligned} a_1 &= k, \\ a_{n+1} &= 3a_n + 5, \quad n \geq 1, \end{aligned}$$

where  $k$  is a positive integer.

(a) Write down an expression for  $a_2$  in terms of  $k$ .

(b) Show that  $a_3 = 9k + 20$ .

(c) (i) Find  $\sum_{r=1}^4 a_r$  in terms of  $k$ .

(ii) Show that  $\sum_{r=1}^4 a_r$  is divisible by 10.

#### Solution:

(a)

$$a_{n+1} = 3a_n + 5$$

$$n = 1 : a_2 = 3a_1 + 5$$

$$a_2 = 3k + 5$$

Use the given

formula with  $n = 1$

(b)

$$n = 2 : a_3 = 3a_2 + 5$$

$$= 3(3k + 5) + 5$$

$$= 9k + 15 + 5$$

$$a_3 = 9k + 20$$

(c)(i)

$$\sum_{r=1}^4 a_r = a_1 + a_2 + a_3 + a_4$$

$$n = 3 : a_4 = 3a_3 + 5$$

$$= 3(9k + 20) + 5$$

$$= 27k + 65$$

$$\sum_{r=1}^4 a_r = k + (3k + 5) + (9k + 20) + (27k + 65)$$

$$= 40k + 90$$

(ii)

$$\sum_{r=1}^4 a_r = 10(4k + 9)$$

There is a factor 10, so the sum is divisible by 10.

This is *not* an arithmetic series.

You cannot use a standard formula, so work out each separate term and then add them together

to find the required sum.

Give a conclusion.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 13

#### Question:

A sequence  $a_1, a_2, a_3, \dots$  is defined by

$$\begin{aligned} a_1 &= k \\ a_{n+1} &= 2a_n - 3, \quad n \geq 1 \end{aligned}$$

(a) Show that  $a_5 = 16k - 45$

Given that  $a_5 = 19$ , find the value of

(b)  $k$

(c)  $\sum_{r=1}^6 a_r$

#### Solution:

(a)

$$\begin{aligned} a_{n+1} &= 2a_n - 3 \\ n = 1 : a_2 &= 2a_1 - 3 \\ &= 2k - 3 \\ n = 2 : a_3 &= 2a_2 - 3 \\ &= 2(2k - 3) - 3 \\ &= 4k - 6 - 3 \\ &= 4k - 9 \\ n = 3 : a_4 &= 2a_3 - 3 \\ &= 2(4k - 9) - 3 \\ &= 8k - 18 - 3 \\ &= 8k - 21 \\ n = 4 : a_5 &= 2a_4 - 3 \\ &= 2(8k - 21) - 3 \\ &= 16k - 42 - 3 \\ a_5 &= 16k - 45 \end{aligned}$$

Use the given formula  
with  $n = 1, 2, 3$  and 4.

(b)

$$a_5 = 19,$$

$$\text{so } 16k - 45 = 19$$

$$16k = 19 + 45$$

$$16k = 64$$

$$k = 4$$

(c)

6

$$\sum_{r=1} a_r = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

This  
is *not* an arithmetic series.

$$a_1 = k = 4$$

$$a_2 = 2k - 3 = 5$$

$$a_3 = 4k - 9 = 7$$

$$a_4 = 8k - 21 = 11$$

$$a_5 = 16k - 45 = 19$$

From the original formula,

$$a_6 = 2a_5 - 3 = (2 \times 19) - 3 = 35$$

6

$$\sum_{r=1} a_r = 4 + 5 + 7 + 11 + 19 + 35$$

$$= 81$$

You  
cannot use a standard  
formula,  
so work  
out each separate term and  
then add  
them together  
to find  
the required sum.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 14

#### Question:

An arithmetic sequence has first term  $a$  and common difference  $d$ .

(a) Prove that the sum of the first  $n$  terms of the series is

$$\frac{1}{2}n \left[ 2a + (n-1)d \right]$$

Sean repays a loan over a period of  $n$  months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the  $n$ th month, where  $n > 21$ .

(b) Find the amount Sean repays in the 21st month.

Over the  $n$  months, he repays a total of £5000.

(c) Form an equation in  $n$ , and show that your equation may be written as

$$n^2 - 150n + 5000 = 0$$

(d) Solve the equation in part (c).

(e) State, with a reason, which of the solutions to the equation in part (c) is not a sensible solution to the repayment problem.

#### Solution:

(a)

$$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

You need to know this proof. Make

Reversing the sum :

sure that you understand it, and do

$$S_n = (a+(n-1)d) + \dots + (a+2d) + (a+d) + a$$

not miss out any of the steps.

Adding these two :

When you add, each pair of terms

$$2S_n = (2a+(n-1)d) + \dots + (2a+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

adds up to  $2a+(n-1)d$ ,  
and there are  $n$  pairs of terms.

$$S_n = \frac{1}{2}n(2a+(n-1)d)$$

(b)

$$a = 149 \quad (\text{First month})$$

$$d = -2$$

Write down the values of  $a$  and  $d$  for the arithmetic series.

21st month:

$$\begin{aligned} a + (n - 1)d &= 149 + (20 \times -2) \\ &= 149 - 40 \\ &= 109 \end{aligned}$$

Use the term formula  $a + (n - 1)d$

He repays £ 109 in the 21st month

(c)

$$S_n = \frac{1}{2}n(2a + (n - 1)d)$$

sum of

The total he repays will be the sum of the arithmetic series.

$$\begin{aligned} &= \frac{1}{2}n(298 - 2(n - 1)) \\ &= \frac{1}{2}n(298 - 2n + 2) \\ &= \frac{1}{2}n(300 - 2n) \\ &= \frac{1}{2}n2(150 - n) \\ &= n(150 - n) \end{aligned}$$

$$n(150 - n) = 5000$$

Equate  $S_n$  to 5000

$$150n - n^2 = 5000$$

$$n^2 - 150n + 5000 = 0$$

(d)

$$\frac{(n - 50)}{(n - 100)} = 0$$

Always try to factorise the quadratic.

$n = 50$  or  $n = 100$  quadratic formula would be

The

awkward here with such large numbers.

(e)

$n = 100$  is not sensible .

For example, his repayment  
in month 100 ( $n = 100$ )

would be  $a + (n - 1)d$

Check back in the  
context of

$$= 149 + (99 \times -2)$$

$$= 149 - 198$$

$$= -49$$

the problem to see if  
the  
solution is sensible.

A negative repayment is not  
sensible .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 15

#### Question:

A sequence is given by

$$a_1 = 2$$

$$a_{n+1} = a_n^2 - ka_n, \quad n \geq 1,$$

where  $k$  is a constant.

(a) Show that  $a_3 = 6k^2 - 20k + 16$

Given that  $a_3 = 2$ ,

(b) find the possible values of  $k$ .

For the larger of the possible values of  $k$ , find the value of

(c)  $a_2$

(d)  $a_5$

(e)  $a_{100}$

#### Solution:

(a)

$$a_{n+1} = a_n^2 - ka_n$$

$$n = 1 : a_2 = a_1^2 - ka_1$$

$$= 4 - 2k$$

$$n = 2 : a_3 = a_2^2 - ka_2$$

$$= (4 - 2k)^2 - k(4 - 2k)$$

$$= 16 - 16k + 4k^2 - 4k + 2k^2$$

$$a_3 = 6k^2 - 20k + 16$$

Use the given formula  
with  $n = 1$  and  $2$ .

(b)

$$a_3 = 2 :$$

$$6k^2 - 20k + 16 = 2$$

$$6k^2 - 20k + 14 = 0$$

$$3k^2 - 10k + 7 = 0$$

$$\frac{(3k - 7)}{(k - 1)} = 0$$

$$k =$$

$$\frac{7}{3} \text{ or } k = 1 \quad \text{using the quadratic formula.}$$

by 2 to make solution easier

Divide

factorise the quadratic rather

Try to

than

(c)

The larger  $k$  value is  $\frac{7}{3}$

$$a_2 = 4 - 2k = 4 - \left( 2 \times \frac{7}{3} \right)$$

$$= 4 - \frac{14}{3} = -\frac{2}{3}$$

(d)

$$a_{n+1} = a_n^2 - \frac{7}{3}a_n$$

$$n = 3 : a_4 = a_3^2 - \frac{7}{3}a_3$$

But  $a_3 = 2$  is given, so

$$a_4 = 2^2 - \left( \frac{7}{3} \times 2 \right)$$

$$= 4 - \frac{14}{3} = \frac{-2}{3}$$

$$n = 4 : a_5 = a_4^2 - \frac{7}{3}a_4$$

$$= \left( \frac{-2}{3} \right)^2 - \left( \frac{7}{3} \times \frac{-2}{3} \right)$$

$$= \frac{4}{9} + \frac{14}{9} = \frac{18}{9}$$

$$a_5 = 2$$

(e)

$$a_2 = \frac{-2}{3}, a_3 = 2$$

$$a_4 = \frac{-2}{3}, a_5 = 2$$

For even values

$$\text{of } n, a_n = \frac{-2}{3}.$$

$$\text{So } a_{100} = \frac{-2}{3}$$

sequence is

the values

$$\frac{-2}{3} \text{ and}$$

$$\frac{-2}{3}.$$

Notice that the

“oscillating” between

If  $n$  is even,  $a_n =$

If  $n$  is odd,  $a_n = 2.$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 16

**Question:**

Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find  $\frac{dy}{dx}$ .

**Solution:**

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (4 \times 3x^2) + (2 \times \frac{1}{2}x^{-\frac{1}{2}})$$

$$\frac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = nx^{n-1}$$

the constant

zero.

write down an

version of the answer first

make a mistake

simplifying).

For  $y = x^n$ ,

Differentiating

$- 1$  gives

It is better to

unsimplified

(in case you

when

(  
Or:

$$\frac{dy}{dx} = 12x^2 +$$

$$\frac{1}{x^{\frac{1}{2}}}$$

is not necessary to change your

It

Or:

$$\frac{dy}{dx} = 12x^2 +$$

$$\frac{1}{\sqrt{x}}$$

one of these forms.

answer into

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 17

#### Question:

Given that  $y = 2x^2 - \frac{6}{x^3}$ ,  $x \neq 0$ ,

(a) find  $\frac{dy}{dx}$ ,

(b) find  $\int y \, dx$ .

#### Solution:

(a)

$$y = 2x^2 - \frac{6}{x^3}$$

$$= 2x^2 - 6x^{-3}$$

$$\frac{dy}{dx} = (2 \times 2x^1) - (6 \times -3x^{-4})$$

$$\frac{dy}{dx} = 4x + 18x^{-4}$$

$$\frac{dy}{dx} = 4x + 18x^{-4}$$

( Or:

$$\frac{dy}{dx} = 4x +$$

$$\frac{18}{x^4} )$$

is not necessary to change

It

into this form.

Use

For  $y = x^n$ ,

Write down

of the answer

first.

your answer

(b)

$$\int (2x^2 - 6x^{-3}) dx$$
$$= \frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C \quad \text{constant}$$

$$= \frac{2x^3}{3} + 3x^{-2} + C \quad \text{version}$$

$$\left( \text{Or: } \frac{2x^3}{3} + \frac{3}{x^2} + C \right)$$

$$\text{Use } \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Do not forget to include the

of integration, C.

Write down an unsimplified

of the answer first

It is not necessary to change

your answer into this form.



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 18

**Question:**

Given that  $y = 3x^2 + 4\sqrt{x}$ ,  $x > 0$ , find

(a)  $\frac{dy}{dx}$ ,

(b)  $\frac{d^2y}{dx^2}$ ,

(c)  $\int y \, dx$ .

**Solution:**

(a)

$$y = 3x^2 + 4\sqrt{x}$$

Use  $\sqrt{x} = x^{\frac{1}{2}}$

$$= 3x^2 + 4x^{\frac{1}{2}}$$

$$= (3 \times 2x^1) + (4 \times \frac{1}{2}x^{-\frac{1}{2}})$$

For  $y = x^n$ ,

$$\frac{dy}{dx} = nx^{n-1}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

$$= 6x + 2x^{-\frac{1}{2}}$$

an

Write down

version

unsimplified

first.

of the answer

(

$$\frac{dy}{dx} = 6x +$$

Or:

$$\frac{2}{x^{\frac{1}{2}}}$$

It

is not necessary to change

Or:

$$\frac{dy}{dx} = 6x +$$

$$\frac{2}{\sqrt{x}}$$

your answer

into one of these forms

(b)

$$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$$

again Differentiate

$$\frac{d^2y}{dx^2} = 6 + \left( 2 \times \frac{-1}{2} x^{-\frac{3}{2}} \right)$$

$$= 6 - x^{-\frac{3}{2}}$$

(

Or:

$$\frac{d^2y}{dx^2} = 6 -$$

$$\frac{1}{x^{\frac{3}{2}}}$$

$$x^{\frac{3}{2}}$$

is not necessary to change your

It

Or:

$$\frac{d^2y}{dx^2} = 6 -$$

$$\frac{1}{x\sqrt{x}}$$

answer

into one of these forms.

x

$$\frac{3}{2} = x^1 \times x^{\frac{1}{2}} = x\sqrt{x}$$

(c)

$$\int \left( 3x^2 + 4x^{\frac{1}{2}} \right) dx$$

$$= \frac{3x^3}{3} + \frac{4x^{\frac{3}{2}}}{\left( \frac{3}{2} \right)} + C$$

$$= x^3 + 4 \left( \frac{2}{3} \right) x^{\frac{3}{2}} + C$$

$$= x^3 + \frac{8}{3} x^{\frac{3}{2}} + C$$

$$\left( \text{Or: } x^3 + \frac{8}{3} x\sqrt{x} + C \right)$$

Use  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$  Do

not forget to include the constant

of integration, C

Write down an unsimplified version

of the answer first.

It is not necessary to change your answer into this form.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 19

#### Question:

(i) Given that  $y = 5x^3 + 7x + 3$ , find

(a)  $\frac{dy}{dx}$ ,

(b)  $\frac{d^2y}{dx^2}$ .

(ii) Find  $\int \left( 1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$ .

#### Solution:

(i)

$$y = 5x^3 + 7x + 3$$

(a)

$$\frac{dy}{dx} = (5 \times 3x^2) + (7 \times 1x^0)$$

$$\frac{dy}{dx} = nx^{n-1}.$$

For  $y = x^n$ ,

Differentiating the constant  
3 gives zero.

$$\frac{dy}{dx} = 15x^2 + 7$$

Use  $x^0 = 1$

Differentiating  $Kx$  gives  $K$ .

(b)

$$\frac{dy}{dx} = 15x^2 + 7$$

Differentiate again

$$\begin{aligned} \frac{d^2y}{dx^2} &= (15 \times 2x^1) \\ &= 30x \end{aligned}$$

(ii)

$$\int \left( 1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$$

$$= \int \left( 1 + 3x^{\frac{1}{2}} - x^{-2} \right) dx$$

include the  
integration C.

$$= x + \frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{x^{-1}}{(-1)} + C$$

$$= x + \left( 3 \times \frac{2}{3} x^{\frac{3}{2}} \right) + x^{-1} + C$$

$$= x + 2x^{\frac{3}{2}} + x^{-1} + C$$

$$\text{( Or: } x + 2x\sqrt{x} + \frac{1}{x} + C \text{ )}$$

$$\frac{1}{x^n} = x^{-n}$$

$$\frac{x^{n+1}}{n+1} + C .$$

Do not forget to  
constant of

Use  $\sqrt{x} = x^{\frac{1}{2}}$  and

Use  $\int x^n dx =$

change

form.

It is not necessary to

your answer into this

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 20

#### Question:

The curve  $C$  has equation  $y = 4x + 3x^{\frac{3}{2}} - 2x^2$ ,  $x > 0$ .

(a) Find an expression for  $\frac{dy}{dx}$ .

(b) Show that the point  $P(4, 8)$  lies on  $C$ .

(c) Show that an equation of the normal to  $C$  at the point  $P$  is  
 $3y = x + 20$ .

The normal to  $C$  at  $P$  cuts the  $x$ -axis at the point  $Q$ .

(d) Find the length  $PQ$ , giving your answer in a simplified surd form.

#### Solution:

(a)

$$y = 4x + 3x^{\frac{3}{2}} - 2x^2$$

$$\frac{dy}{dx} = (4 \times 1x^0) + (3 \times \frac{3}{2}x^{\frac{1}{2}}) - (2 \times 2x^1)$$

For  $y = x^n$ ,

$$\frac{dy}{dx} = nx^{n-1}$$

$$\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$$

(b)

For  $x = 4$ ,

$$y = (4 \times 4) + (3 \times 4^{\frac{3}{2}}) - (2 \times 4^2)$$

$$= 16 + (3 \times 4 \times 2) - 32$$

$$= 16 + 24 - 32 = 8$$

$$x^{\frac{3}{2}} = x^1 \times x$$

$$\frac{1}{2} = x \sqrt{x}$$

So  $P(4, 8)$  lies on  $C$

(c)

The value

For  $x = 4$ , of  $\frac{dy}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= 4 + \left( \frac{9}{2} \times 4 \frac{1}{2} \right) - (4 \times 4) \\ &= 4 + \left( \frac{9}{2} \times 2 \right) - 16 \\ &= 4 + 9 - 16 = -3 \end{aligned}$$

is the gradient of the tangent.

The gradient of the normal is perpendicular to the

The normal tangent, so

at P is  $\frac{1}{3}$  the gradient is  $-\frac{1}{m}$

Equation of the normal :

$$y - 8 = \frac{1}{3} (x - 4) \quad (x - x_1)$$

Use  $y - y_1 = m$

$$y - 8 = \frac{x}{3} - \frac{4}{3}$$

Multiply by 3

$$3y - 24 = x - 4$$

$$3y = x + 20$$

(d)

$$\begin{aligned} y = 0 : \quad 0 &= x + 20 \\ x &= -20 \end{aligned}$$

Use  $y = 0$  to find where the normal cuts

the  $x$ -axis.

Q is the point  $(-20, 0)$

$$PQ = \frac{\sqrt{(4 - (-20))^2 + (8 - 0)^2}}$$

points is

The distance between two

$$= \sqrt{24^2 + 8^2}$$

$$\frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{576 + 64}$$

$$= \sqrt{640}$$

$$= \sqrt{64} \times \sqrt{10}$$

$$= 8\sqrt{10}$$

To simplify the surd, find a factor which is an exact square ( here  $64 = 8^2$  )

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 21

#### Question:

The curve  $C$  has equation  $y = 4x^2 + \frac{5-x}{x}$ ,  $x \neq 0$ . The point  $P$  on  $C$  has  $x$ -coordinate 1.

(a) Show that the value of  $\frac{dy}{dx}$  at  $P$  is 3.

(b) Find an equation of the tangent to  $C$  at  $P$ .

This tangent meets the  $x$ -axis at the point  $(k, 0)$ .

(c) Find the value of  $k$ .

#### Solution:

(a)

$$y = 4x^2 + \frac{5-x}{x}$$

$$= 4x^2 + 5x^{-1} - 1$$

$$\frac{dy}{dx} = (4 \times 2x^1) + (5x - 1x^{-2})$$

constant  $-1$  gives zero

$$\frac{dy}{dx} = 8x - 5x^{-2}$$

At  $P$ ,  $x = 1$ , so

$$\frac{dy}{dx} = (8 \times 1) - (5 \times 1^{-2})$$

$$= 8 - 5 = 3$$

Divide  $5 - x$  by  $x$

$$\text{For } y = x^n, \frac{dy}{dx} = nx^{n-1}$$

Differentiating the

$$1^{-2} = \frac{1}{1^2} = \frac{1}{1} = 1$$

(b)

$$\text{At } x = 1, \frac{dy}{dx} = 3$$

The value of  $\frac{dy}{dx}$

is the gradient of the

tangent

$$\text{At } x = 1, y = (4 \times 1^2) + \frac{5-1}{1}$$

$$y = 4 + 4 = 8$$

Equation of the tangent :

$$y - 8 = 3(x - 1) \quad (x - x_1)$$

Use  $y - y_1 = m$

$$y = 3x + 5$$

(c)

$$y = 0 : 0 = 3x + 5$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

Use  $y = 0$  to find where the tangent

meets the  $x$ -axis

$$\text{So } k = -\frac{5}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 22

**Question:**

The curve  $C$  has equation  $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$ .

The point  $P$  has coordinates  $(3, 0)$ .

(a) Show that  $P$  lies on  $C$ .

(b) Find the equation of the tangent to  $C$  at  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants.

Another point  $Q$  also lies on  $C$ . The tangent to  $C$  at  $Q$  is parallel to the tangent to  $C$  at  $P$ .

(c) Find the coordinates of  $Q$ .

**Solution:**

(a)

$$y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$$

At  $x = 3$ ,

$$\begin{aligned}y &= \left(\frac{1}{3} \times 3^3\right) - (4 \times 3^2) + (8 \times 3) + 3 \\&= 9 - 36 + 24 + 3 \\&= 0\end{aligned}$$

So  $P(3, 0)$  lies on  $C$

(b)



$$\frac{dy}{dx} = \left( \frac{1}{3} \times 3x^2 \right) - (4 \times 2x^1) + (8 \times 1x^0)$$

For  $y = x^n$ ,  
 $\frac{dy}{dx} = nx^{n-1}$

Differentiating the constant 3 gives zero.

$$= x^2 - 8x + 8$$

At  $x = 3$ ,

$$\frac{dy}{dx} = 3^2 - (8 \times 3) + 8$$

$$= 9 - 24 + 8 = -7$$

The value of  $\frac{dy}{dx}$  is the gradient of the

tangent.

Equation of the tangent :

$$y - 0 = -7(x - 3)$$

$$(x - x_1)$$

Use  $y - y_1 = m$

$$y = -7x + 21$$

This is in the

required form  $y = mx + c$

(c)

At  $Q$ ,  $\frac{dy}{dx} = -7$

If the tangents are

parallel, they have the same gradient.

$$x^2 - 8x + 8 = -7$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ or } x = 5$$

$x = 3$  at the point P

For  $Q$ ,  $x = 5$

$$y = \left( \frac{1}{3} \times 5^3 \right) - (4 \times 5^2) + (8 \times 5) + 3$$

Substitute  $x = 5$

$$= \frac{125}{3} - 100 + 40 + 3$$

back into the equation

of C

$$= -15 \frac{1}{3}$$

Q is the point  $(5, -15 \frac{1}{3})$

$$\frac{1}{3}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 23

#### Question:

$$f\left(\frac{1}{x}\right) = \frac{(2x+1)(x+4)}{\sqrt{x}}, \quad x > 0$$

(a) Show that  $f(x)$  can be written in the form  $Px^{\frac{3}{2}} + Qx^{\frac{1}{2}} + Rx^{-\frac{1}{2}}$ , stating the values of the constants  $P$ ,  $Q$  and  $R$ .

(b) Find  $f'(x)$ .

(c) Show that the tangent to the curve with equation  $y = f(x)$  at the point where  $x = 1$  is parallel to the line with equation  $2y = 11x + 3$ .

#### Solution:

(a)

$$f\left(\frac{1}{x}\right) = \frac{(2x+1)(x+4)}{\sqrt{x}}$$

$$= \frac{2x^2 + 9x + 4}{\sqrt{x}}$$

Divide each term by

$x$

$$\frac{1}{2}, \text{ remembering}$$

$$= 2x^{\frac{3}{2}} + 9x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}.$$

that  $x^m \div x^n = x^{m-n}$

$$P = 2, \quad Q = 9, \quad R = 4$$

(b)

$$f'(x) = \left(2 \times \frac{3}{2}x^{\frac{1}{2}}\right) + \left(9 \times \frac{1}{2}x^{-\frac{1}{2}}\right) + \left(4 \times \frac{-1}{2}x^{-\frac{3}{2}}\right)$$

$f'(x)$  is the derivative of  $f(x)$ ,

$$f'(x) = 3x^{\frac{1}{2}} + \frac{9}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$$

so differentiate

(c)

At  $x = 1$ ,

$$f'(1) = (3 \times 1^{\frac{1}{2}}) + (\frac{9}{2} \times 1^{\frac{-1}{2}}) - (2 \times 1^{\frac{-3}{2}})$$

of the tangent at  $x = 1$

$$= 3 + \frac{9}{2} - 2 = \frac{11}{2}$$

The line  $2y$

$$= 11x + 3 \text{ is}$$

$y$

$$= \frac{11}{2}x + \frac{3}{2}$$

The gradient is  $\frac{11}{2}$

So the tangent to the curve where

$x = 1$  is parallel to this line,

since the gradients are equal.

$f'(1)$  is the gradient

$$1^n = 1 \text{ for any } n.$$

Compare with  $y = mx + c$

Give a conclusion,

with a reason.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 24

#### Question:

The curve  $C$  with equation  $y = f(x)$  passes through the point  $(3, 5)$ .

Given that  $f'(x) = x^2 + 4x - 3$ , find  $f(x)$ .

#### Solution:

$$f'(x) = x^2 + 4x - 3$$

To find  $f(x)$  from  $f'(x)$ , integrate.

$$f(x) = \frac{x^3}{3} + \frac{4x^2}{2} - 3x + C$$

Use  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ .

$$= \frac{x^3}{3} + 2x^2 - 3x + C$$

Do not forget to include the constant of integration  $C$ .

The curve passes through  $(3, 5)$ , so

$$\frac{3^3}{3} + (2 \times 3^2) - (3 \times 3) + C = 5$$

$$9 + 18 - 9 + C = 5$$

$$C = -13$$

so  $f(3) = 5$ .

$$f(x) = \frac{x^3}{3} + 2x^2 - 3x - 13$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 25

#### Question:

The curve with equation  $y = f(x)$  passes through the point  $(1, 6)$ . Given that

$$f'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find  $f(x)$  and simplify your answer.

#### Solution:

$$f'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}$$

Divide  $5x^2 + 2$  by  $x^{\frac{1}{2}}$ ,

remembering that

$$x^m \div x^n = x^{m-n}$$

$$= 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$f'(x)$ , integrate.

To find  $f(x)$  from

$$f(x) = 3x + \frac{5x^{\frac{5}{2}}}{(\frac{5}{2})} + \frac{2x^{\frac{1}{2}}}{(\frac{1}{2})} + C$$

Use  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ .

$$= 3x + (5 \times \frac{2}{5} x^{\frac{5}{2}}) + (2 \times \frac{2}{1} x^{\frac{1}{2}}) + C$$

Do not forget to include

$$= 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$$

the constant of integration  $C$ .

When  $x = 1$ ,  $f(x) = 6$ , so

The curve passes

$$(3 \times 1) + (2 \times 1^{\frac{5}{2}}) +$$

through  $(1, 6)$ ,

$$(4 \times 1^{\frac{1}{2}}) + C = 6$$

$$\text{so } f(1) = 6$$

$$3 + 2 + 4 + C$$

$$= 6$$

$$1^n = 1 \text{ for any } n.$$

$$C$$

$$= -3$$

$$f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 26

#### Question:

For the curve  $C$  with equation  $y = f(x)$ ,

$$\frac{dy}{dx} = x^3 + 2x - 7$$

(a) Find  $\frac{d^2y}{dx^2}$

(b) Show that  $\frac{d^2y}{dx^2} \geq 2$  for all values of  $x$ .

Given that the point  $P(2, 4)$  lies on  $C$ ,

(c) find  $y$  in terms of  $x$ ,

(d) find an equation for the normal to  $C$  at  $P$  in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

(a)

$$\frac{dy}{dx} = x^3 + 2x - 7$$

Differentiate to find

$$\frac{d^2y}{dx^2} = 3x^2 + 2$$

the second derivative

(b)

$$x^2 \geq 0 \text{ for any (real) } x.$$

The square of a

$$\text{So } 3x^2 \geq 0$$

real number

$$\text{So } 3x^2 + 2 \geq 2$$

cannot be negative.

$$\text{So } \frac{d^2y}{dx^2} \geq 2 \text{ for all values of } x.$$

Give a conclusion.

(c)

$$\frac{dy}{dx} = x^3 + 2x - 7$$

Integrate  $\frac{dy}{dx}$  to

find  $y$  in terms

of  $x$ .

$$y = \frac{x^4}{4} + \frac{2x^2}{2} - 7x + C$$

include

Do not forget to

$$= \frac{x^4}{4} + x^2 - 7x + C$$

integration  $C$ .

the constant of

When  $x = 2$ ,  $y = 4$ , so

Use the fact that

$$4 = \frac{2^4}{4} + 2^2 - (7 \times 2) + C$$

the curve.

$P(2, 4)$  lies on

$$4 = 4 + 4 - 14 + C$$

$$C = +10$$

$$y = \frac{x^4}{4} + x^2 + 7x + 10$$

(d)

For  $x = 2$ ,

$$\frac{dy}{dx} = 2^3 + (2 \times 2) - 7$$

$$= 8 + 4 - 7 = 5$$

The gradient of the normal

at P is  $-\frac{1}{5}$

Equation of the normal :

$$y - 4 = \frac{-1}{5} (x - 2)$$

$$y - 4 = \frac{-x}{5} + \frac{2}{5}$$

$$5y - 20 = -x + 2$$

$$x + 5y - 22 = 0$$

The normal is

perpendicular to the tangent,

so the gradient is  $-\frac{1}{m}$

This is in the required form

$ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

The value of

$$\frac{dy}{dx}$$

is the gradient

of the tangent .

Use  $y - y_1 = m$

Multiply by 5

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 27

#### Question:

For the curve  $C$  with equation  $y = f(x)$ ,

$$\frac{dy}{dx} = \frac{1-x^2}{x^4}$$

Given that  $C$  passes through the point  $\left(\frac{1}{2}, \frac{2}{3}\right)$ ,

(a) find  $y$  in terms of  $x$ .

(b) find the coordinates of the point on  $C$  at which  $\frac{dy}{dx} = 0$ .

#### Solution:

(a)

$$\frac{dy}{dx} = \frac{1-x^2}{x^4}$$

$$= x^{-4} - x^{-2}$$

$$y = \frac{x^{-3}}{-3} - \frac{x^{-1}}{-1} + C$$

$$= \frac{-x^{-3}}{3} + x^{-1} + C$$

constant of integration  $C$ .

$$y = \frac{-1}{3x^3} + \frac{1}{x} + C$$

will make it easier

calculate values

the next stage.

When  $x =$

$$\frac{1}{2}, y =$$

$$\frac{2}{3}, \text{ so}$$

$$\frac{2}{3} = -\frac{8}{3} + 2 + C$$

$$C = \frac{2}{3} + \frac{8}{3} - 2 = \frac{4}{3}$$

$$y = \frac{-1}{3x^3} + \frac{1}{x} + \frac{4}{3}$$

(b)

Divide  $1 - x^2$  by  $x^4$

Integrate  $\frac{dy}{dx}$  to

of  $x$ . Do not forget

find  $y$  in terms

to include

the

Use  $x^{-n} = \frac{1}{x^n}$ .

This

to

at

Use the fact that

$\left(\frac{1}{2}, \frac{2}{3}\right)$  lies on

the curve.



$$\frac{1-x^2}{x^4} = 0$$

to zero, its

$$1 - x^2 = 0$$

$$x^2 = 1$$

$$x = 1 \text{ or } x = -1$$

$$x = 1 : y = \frac{-1}{3} + 1 + \frac{4}{3}$$

$$y = 2$$

$$x = -1 : y = \frac{1}{3} - 1 + \frac{4}{3}$$

$$y = \frac{2}{3}$$

The points are

$(1, 2)$

and  $(-1, \frac{2}{3})$

is equal to zero, its numerator must be zero.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 28

#### Question:

The curve  $C$  with equation  $y = f(x)$  passes through the point  $(5, 65)$ .

Given that  $f'(x) = 6x^2 - 10x - 12$ ,

(a) use integration to find  $f(x)$ .

(b) Hence show that  $f(x) = x(2x + 3)(x - 4)$ .

(c) Sketch  $C$ , showing the coordinates of the points where  $C$  crosses the  $x$ -axis.

#### Solution:

(a)

$$f'(x) = 6x^2 - 10x - 12 \quad \text{find } f(x) \text{ from } f'(x), \text{ integrate} \quad \text{To}$$

$$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C \quad \text{not forget to} \quad \text{Do}$$

When  $x = 5$ ,  $y = 65$ , so include the constant of integration  $C$ .

$$65 = \frac{6 \times 125}{3} - \frac{10 \times 25}{2} - 60 + C \quad \text{the fact that} \quad \text{Use}$$

$$65 = 250 - 125 - 60 + C \quad \text{the curve passes through } (5, 65)$$

$$C = 65 + 125 + 60 - 250$$

$$C = 0$$

$$f(x) = 2x^3 - 5x^2 - 12x$$

(b)

$$f(x) = x(2x^2 - 5x - 12)$$

$$f(x) = x(2x + 3)(x - 4)$$

(c)

Curve meets  $x$ -axis where  $y = 0$

$$x(2x + 3)(x - 4) = 0$$

$$x = 0, x = -\frac{3}{2}, x = 4$$

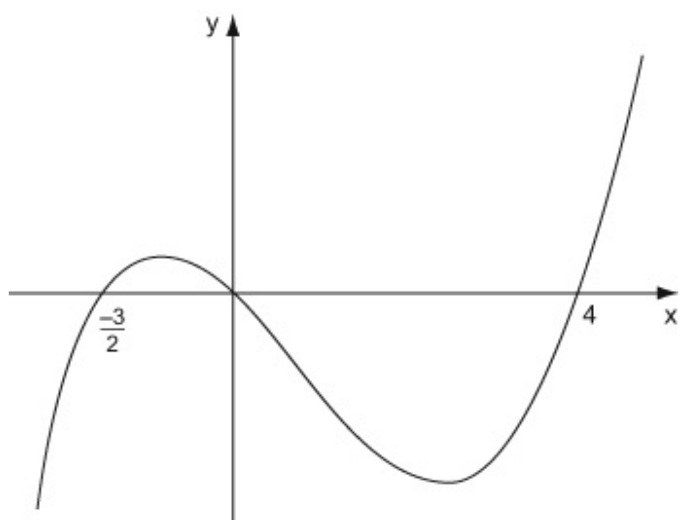
When  $x \rightarrow \infty$ ,  $y \rightarrow \infty$

When  $x \rightarrow -\infty$ ,  $y \rightarrow -\infty$

Put  $y = 0$  and

solve for  $x$

Check what happens to  $y$  for large positive and negative values of  $x$ .



Crosses  $x$ -axis at  $(-\frac{3}{2}, 0)$ ,  $(0, 0)$ ,  $(4, 0)$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 29

#### Question:

The curve  $C$  has equation  $y = x^2 \left( x - 6 \right) + \frac{4}{x}, x > 0$ .

The points  $P$  and  $Q$  lie on  $C$  and have  $x$ -coordinates 1 and 2 respectively.

(a) Show that the length of  $PQ$  is  $\sqrt{170}$ .

(b) Show that the tangents to  $C$  at  $P$  and  $Q$  are parallel.

(c) Find an equation for the normal to  $C$  at  $P$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

(a)

$$y = x^2 (x - 6) + \frac{4}{x}$$

At  $P$ ,  $x = 1$ ,

$$y = 1(1 - 6) + \frac{4}{1} = -1$$

$P$  is  $(1, -1)$

At  $Q$ ,  $x = 2$ ,

$$y = 4(2 - 6) + \frac{4}{2} = -14$$

$Q$  is  $(2, -14)$

$$\begin{aligned} PQ &= \sqrt{(2 - 1)^2 + (-14 - (-1))^2} \\ &= \sqrt{1^2 + (-13)^2} \\ &= \sqrt{1 + 169} = \sqrt{170} \end{aligned}$$

The distance between two points is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(b)

$$y = x^3 - 6x^2 + 4x^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 - (6 \times 2x^{-1}) + (4x - 1x^{-2}) \\ &= 3x^2 - 12x - 4x^{-2} \end{aligned}$$

At  $x = 1$ ,

The value of  $\frac{dy}{dx}$

$$\frac{dy}{dx} = 3 - 12 - 4 = -13$$

is the gradient of

the tangent.

At  $x = 2$ ,

$$\begin{aligned} \frac{dy}{dx} &= (3 \times 4) - (12 \times 2) - (4 \times 2^{-2}) \\ &= 12 - 24 - \frac{4}{4} = -13 \end{aligned}$$

At  $P$  and also at  $Q$  the gradient is  $-13$ , so the tangents are parallel (equal gradients).

Give a conclusion

(c)

The gradient of the normal is perpendicular to the tangent at P is –

$$\frac{1}{-13} = \frac{1}{13} \quad \text{the gradient is } -\frac{1}{m}$$

Equation of the normal:

$$y - (-1) = \frac{1}{13}(x - 1)$$

$$y + 1 = \frac{x}{13} - \frac{1}{13}$$

$$13y + 13 = x - 1$$

$$x - 13y - 14 = 0$$

integers.

The normal

tangent, so

$b$  and  $c$  are

$$\text{Use } y - y_1 = m(x - x_1)$$

Multiply by 13

This is in the required form  $ax + by + c = 0$ , where  $a$ ,

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebraic fractions

#### Exercise A, Question 30

#### Question:

- (a) Factorise completely  $x^3 - 7x^2 + 12x$ .
- (b) Sketch the graph of  $y = x^3 - 7x^2 + 12x$ , showing the coordinates of the points at which the graph crosses the  $x$ -axis.

The graph of  $y = x^3 - 7x^2 + 12x$  crosses the positive  $x$ -axis at the points  $A$  and  $B$ .

The tangents to the graph at  $A$  and  $B$  meet at the point  $P$ .

- (c) Find the coordinates of  $P$ .

#### Solution:

(a)

$$x^3 - 7x^2 + 12x$$

$$= x(x^2 - 7x + 12)$$

$$= x(x - 3)(x - 4)$$

$x$  is a common factor

(b)

Curve meets  $x$ -axis where  $y = 0$ .

$$x(x - 3)(x - 4) = 0$$

$$x = 0, x = 3, x = 4$$

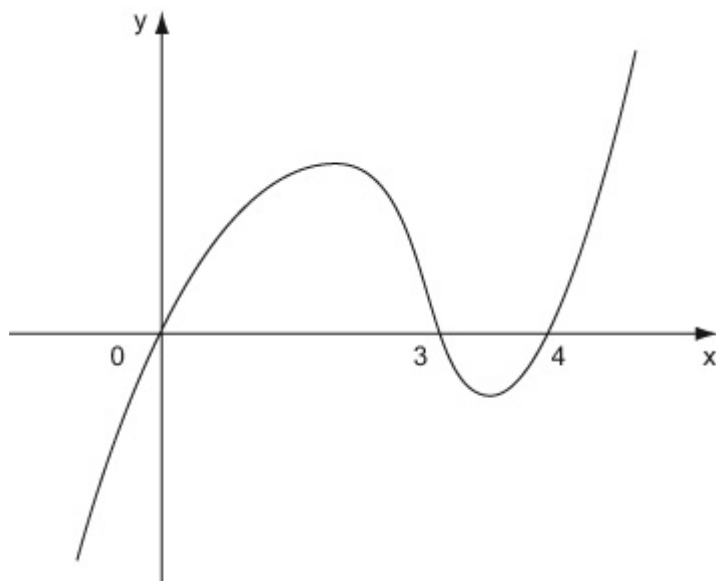
When  $x \rightarrow \infty, y \rightarrow \infty$

When  $x \rightarrow -\infty, y \rightarrow -\infty$

Put  $y = 0$  and  
solve for  $x$ .

Check what happens to  
 $y$  for large

positive and negative values of  $x$



Crosses  $x$ -axis at  $(0, 0)$ ,  $(3, 0)$ ,  $(4, 0)$

(c)

$A$  and  $B$  are

$$(3, 0)$$

and

$$(4, 0)$$

$$\frac{dy}{dx} = 3x^2 - 14x + 12$$

At  $x = 3$ ,

(A)

value of  $\frac{dy}{dx}$

The

$$\frac{dy}{dx} = 27 - 42 + 12 = -3$$

of the tangent.

is the gradient

At  $x = 4$

(B)

$$\frac{dy}{dx} = 48 - 56 + 12 = 4$$

Tangent at A:

$$y - 0 = -3(x - 3)$$

( $x - x_1$ )

Use  $y - y_1 = m$

$$y = -3x + 9 \quad (\text{i})$$

Tangent at B:

$$y - 0 = 4(x - 4)$$

$$y = 4x - 16 \quad (\text{ii})$$

Subtract

(ii) -

(i) :

$$0 = 7x - 25$$

simultaneously to

Solve (i) and (ii)

$$x = \frac{25}{7}$$

intersection

find the

point

of the tangents

Substituting

back into (i):

$$y = -\frac{75}{7} + 9 = -\frac{12}{7}$$

P is the

point  $(\frac{25}{7},$

$\frac{-12}{7})$