

1	a = 7 d = $\frac{2}{5}$ g = 2 j = $\sqrt{\frac{25}{16}} = \frac{5}{4}$ or $1\frac{1}{4}$	b = 11 e = 0.1 h = 10 k = $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ or 0.5	c = $\frac{1}{3}$ f = 0.3 i = 3 l = $\sqrt[3]{\frac{125}{8}} = \frac{5}{2}$ or $2\frac{1}{2}$
2	a = 7 e = $4\sqrt{2}$ i = $\sqrt{16} = 4$	b = 20 f = $24\sqrt{3}$ j = $\sqrt{\frac{1}{4}} = \frac{1}{2}$	c = 27 g = $\sqrt{16} = 4$ k = 6 d = 36 h = $2\sqrt{81} = 18$ l = 54
3	a = $\sqrt{9} \times \sqrt{2} = 3\sqrt{2}$ d = $\sqrt{49} \times \sqrt{2} = 7\sqrt{2}$	b = $\sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ e = $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$	c = $\sqrt{4} \times \sqrt{2} = 2\sqrt{2}$ f = $\sqrt{81} \times \sqrt{2} = 9\sqrt{2}$
4	a = $\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ d = $\sqrt{9} \times \sqrt{3} = 3\sqrt{3}$ g = $\sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ j = $\sqrt{16} \times \sqrt{7} = 4\sqrt{7}$ m = $\sqrt{36} \times \sqrt{6} = 6\sqrt{6}$ p = $\sqrt{4} \times \sqrt{15} = 2\sqrt{15}$	b = $\sqrt{4} \times \sqrt{7} = 2\sqrt{7}$ e = $\sqrt{4} \times \sqrt{6} = 2\sqrt{6}$ h = $\sqrt{4} \times \sqrt{10} = 2\sqrt{10}$ k = $\sqrt{9} \times \sqrt{11} = 3\sqrt{11}$ n = $\sqrt{400} \times \sqrt{2} = 20\sqrt{2}$ q = $\sqrt{121} \times \sqrt{3} = 11\sqrt{3}$	c = $\sqrt{16} \times \sqrt{5} = 4\sqrt{5}$ f = $\sqrt{64} \times \sqrt{2} = 8\sqrt{2}$ i = $\sqrt{25} \times \sqrt{3} = 5\sqrt{3}$ l = $\sqrt{49} \times \sqrt{3} = 7\sqrt{3}$ o = $\sqrt{36} \times \sqrt{5} = 6\sqrt{5}$ r = $\sqrt{16} \times \sqrt{13} = 4\sqrt{13}$
5	a = $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$ d = $6\sqrt{10} - 4\sqrt{10} = 2\sqrt{10}$	b = $4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$ e = $2\sqrt{5} - 3\sqrt{5} + 6\sqrt{5} = 5\sqrt{5}$	c = $4\sqrt{2} + 6\sqrt{2} = 10\sqrt{2}$ f = $2\sqrt{6} + 5\sqrt{6} - 8\sqrt{6} = -\sqrt{6}$
6	a = $3 + 2\sqrt{3}$ d = $4 + 8\sqrt{3} + \sqrt{3} + 6$ = $10 + 9\sqrt{3}$	b = $4 - \sqrt{3} - 2 + 2\sqrt{3}$ = $2 + \sqrt{3}$ e = $27 - 24\sqrt{3} + 16$ = $43 - 24\sqrt{3}$	c = $2 + \sqrt{3} + 2\sqrt{3} + 3$ = $5 + 3\sqrt{3}$ f = $6\sqrt{3} - 45 + 2 - 5\sqrt{3}$ = $-43 + \sqrt{3}$
7	a = $10 + 3\sqrt{5} + 2\sqrt{5} + 3$ = $13 + 5\sqrt{5}$ d = $12 + 15\sqrt{2} - 2\sqrt{2} - 5$ = $7 + 13\sqrt{2}$	b = $4\sqrt{2} - 3 - 8 + 3\sqrt{2}$ = $7\sqrt{2} - 11$ e = $5 + 2\sqrt{10} - \sqrt{10} - 4$ = $1 + \sqrt{10}$	c = $28 + 12\sqrt{7} + 9$ = $37 + 12\sqrt{7}$ f = $(3 - 2\sqrt{2})(4 + \sqrt{2})$ = $12 + 3\sqrt{2} - 8\sqrt{2} - 4$ = $8 - 5\sqrt{2}$

$$\begin{array}{lll}
 8 \quad \mathbf{a} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5} \sqrt{5} & \mathbf{b} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3} \sqrt{3} & \mathbf{c} = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4} \sqrt{2} \\
 \mathbf{d} = \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7} & \mathbf{e} = \frac{3\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{6} & \mathbf{f} = \frac{\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \sqrt{3} \\
 \mathbf{g} = \frac{1}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{1}{21} \sqrt{7} & \mathbf{h} = \frac{12}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} & \mathbf{i} = \frac{1}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{20} \sqrt{5} \\
 \mathbf{j} = \frac{3}{6\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{12} \sqrt{6} & \mathbf{k} = \frac{8\sqrt{5}}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4}{9} \sqrt{10} & \mathbf{l} = \frac{15\sqrt{7}}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{6} \sqrt{21}
 \end{array}$$

$$\begin{array}{lll}
 9 \quad \mathbf{a} = 2\sqrt{2} + \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} & \mathbf{b} = 4\sqrt{3} - \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} & \mathbf{c} = \frac{6-2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 = 2\sqrt{2} + 3\sqrt{2} & = 4\sqrt{3} - \frac{10}{3}\sqrt{3} & = \frac{6\sqrt{2}-4}{2} \\
 = 5\sqrt{2} & = \frac{2}{3}\sqrt{3} & = 3\sqrt{2} - 2 \\
 \mathbf{d} = \frac{3\sqrt{5}-5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} & \mathbf{e} = \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} & \mathbf{f} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}} \\
 = \frac{15-5\sqrt{5}}{10} & = \frac{1}{6}\sqrt{2} + \frac{1}{8}\sqrt{2} & = \frac{2}{3}\sqrt{3} - \frac{1}{6}\sqrt{3} \\
 = \frac{1}{2}(3 - \sqrt{5}) & = \frac{7}{24}\sqrt{2} & = \frac{1}{2}\sqrt{3}
 \end{array}$$

$$\begin{array}{ll}
 10 \quad \mathbf{a} \quad x^2 + 4x = 4x + 32 & \mathbf{b} \quad x - 4\sqrt{3} = 2\sqrt{3} - 2x \\
 x^2 = 32 & 3x = 6\sqrt{3} \\
 x = \pm\sqrt{32} & x = 2\sqrt{3} \\
 x = \pm 4\sqrt{2} & \\
 \mathbf{c} \quad 3\sqrt{2}x - 4 = 2\sqrt{2} & \mathbf{d} \quad \sqrt{5}x + 2 = 2\sqrt{5}(x-1) \\
 6x - 4\sqrt{2} = 4 & 5x + 2\sqrt{5} = 10(x-1) \\
 6x = 4 + 4\sqrt{2} & 5x = 10 + 2\sqrt{5} \\
 x = \frac{2}{3}(1 + \sqrt{2}) & x = 2 + \frac{2}{5}\sqrt{5}
 \end{array}$$

$$\begin{array}{ll}
 11 \quad \mathbf{a} = 4 - (\sqrt{3})^2 = 4 - 3 = 1 & \\
 \mathbf{b} = \frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{1} = 4 + 2\sqrt{3} &
 \end{array}$$

$$\begin{array}{ll}
 12 \quad \mathbf{a} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1 & \\
 \mathbf{b} = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1) & \\
 \mathbf{c} = \frac{1}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2) \text{ or } \frac{1}{2}\sqrt{6}+1 & \\
 \mathbf{d} = \frac{3}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3}) & \\
 \mathbf{e} = \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2 &
 \end{array}$$

$$f = \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2 + \sqrt{2}$$

$$g = \frac{6}{\sqrt{7}+3} \times \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$$

$$h = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$i = \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{16-12} = \frac{1}{2}(2+\sqrt{3}) \text{ or } 1+\frac{1}{2}\sqrt{3}$$

$$j = \frac{3}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4} = \frac{3(3\sqrt{2}-4)}{18-16} = \frac{3}{2}(3\sqrt{2}-4) \text{ or } \frac{9}{2}\sqrt{2}-6$$

$$k = \frac{2\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$$

$$l = \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$$

$$13 \quad 3x = \sqrt{5}x + 2\sqrt{5}$$

$$x(3 - \sqrt{5}) = 2\sqrt{5}$$

$$x = \frac{2\sqrt{5}}{3-\sqrt{5}} = \frac{2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$$

$$x = \frac{6\sqrt{5}+10}{4} = \frac{3}{2} + \frac{3}{2}\sqrt{5}$$

$$14 \quad l = \frac{6}{3\sqrt{2}-3} = \frac{6}{3\sqrt{2}-3} \times \frac{3\sqrt{2}+3}{3\sqrt{2}+3} = \frac{6(3\sqrt{2}+3)}{18-9}$$

$$l = \frac{18(\sqrt{2}+1)}{9} = 2\sqrt{2} + 2$$

$$15 \quad a = \frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}} \times \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{\sqrt{2}(\sqrt{2}-\sqrt{6})}{2-6} = -\frac{1}{4}(2-2\sqrt{3}) = \frac{1}{2}(\sqrt{3}-1)$$

$$b = \frac{1+\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{(1+\sqrt{3})(2-\sqrt{3})}{4-3} = 2-\sqrt{3}+2\sqrt{3}-3 = \sqrt{3}-1$$

$$c = \frac{1+\sqrt{10}}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \frac{(1+\sqrt{10})(\sqrt{10}+3)}{10-9} = \sqrt{10}+3+10+3\sqrt{10} = 13+4\sqrt{10}$$

$$d = \frac{3-\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(3-\sqrt{2})(4-3\sqrt{2})}{16-18} = \frac{12-9\sqrt{2}-4\sqrt{2}+6}{-2} = \frac{1}{2}(13\sqrt{2}-18) \text{ or } \frac{13}{2}\sqrt{2}-9$$

$$e = \frac{1-\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{(1-\sqrt{2})(3+2\sqrt{2})}{9-8} = 3+2\sqrt{2}-3\sqrt{2}-4 = -1-\sqrt{2}$$

$$f = \frac{\sqrt{3}-5}{2\sqrt{3}-4} \times \frac{2\sqrt{3}+4}{2\sqrt{3}+4} = \frac{(\sqrt{3}-5)(2\sqrt{3}+4)}{12-16} = \frac{6+4\sqrt{3}-10\sqrt{3}-20}{-4} = \frac{1}{2}(7+3\sqrt{3})$$

$$g = \frac{2\sqrt{3}+3}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(2\sqrt{3}+3)(3+\sqrt{3})}{9-3} = \frac{6\sqrt{3}+6+9+3\sqrt{3}}{6} = \frac{1}{2}(3\sqrt{3}+5)$$

$$h = \frac{3\sqrt{7}-2}{2\sqrt{7}-5} \times \frac{2\sqrt{7}+5}{2\sqrt{7}+5} = \frac{(3\sqrt{7}-2)(2\sqrt{7}+5)}{28-25} = \frac{42+15\sqrt{7}-4\sqrt{7}-10}{3} = \frac{1}{3}(32+11\sqrt{7})$$

- 6 **a** $= \sqrt{4} \times \sqrt[3]{27}$
 $= 2 \times 3 = 6$ **b** $= \sqrt[4]{16} + \sqrt{25}$
 $= 2 + 5 = 7$ **c** $= \frac{1}{\sqrt[3]{8}} \div \sqrt{36}$
 $= \frac{1}{2} \div 6 = \frac{1}{12}$ **d** $= \sqrt[3]{-64} \times (\sqrt{9})^3$
 $= -4 \times 27 = -108$
- e** $= 3^2 - \sqrt[3]{-8}$
 $= 9 - (-2) = 11$ **f** $= \sqrt{\frac{1}{25}} \times 4^2$
 $= \frac{1}{5} \times 16 = \frac{16}{5}$ or $3\frac{1}{5}$ **g** $= (\sqrt[4]{81})^3 - \sqrt{49}$
 $= 27 - 7 = 20$ **h** $= \sqrt[3]{27} \times (\sqrt{\frac{9}{4}})^3$
 $= 3 \times \frac{27}{8} = \frac{81}{8}$ or $10\frac{1}{8}$
- i** $= \sqrt{9} \times (\sqrt[5]{-32})^3$
 $= 3 \times (-8) = -24$ **j** $= \sqrt{121} + \sqrt[5]{32}$
 $= 11 + 2 = 13$ **k** $= \sqrt{100} \div (\sqrt{\frac{1}{4}})^3$
 $= 10 \div \frac{1}{8} = 80$ **l** $= \frac{1}{\sqrt[4]{16}} \times (\sqrt[5]{243})^2$
 $= \frac{1}{2} \times 9 = \frac{9}{2}$ or $4\frac{1}{2}$
- 7 **a** $= x^2$ **b** $= y^{-6}$ **c** $= 3p^{-4}$ **d** $= 8x^{-12}$
- e** $= y^{\frac{5}{2}}$ **f** $= 8b^{\frac{2}{3} + \frac{1}{4}} = 8b^{\frac{11}{12}}$ **g** $= x^{\frac{3}{5} - \frac{1}{3}} = x^{\frac{4}{15}}$ **h** $= a^{\frac{1}{2} - \frac{4}{3}} = a^{-\frac{5}{6}}$
- i** $= p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$ **j** $= 9x^{\frac{4}{5}}$ **k** $= y^{1 + \frac{5}{6} - \frac{3}{2}} = y^{\frac{1}{3}}$ **l** $= \frac{1}{3}t$
- m** $= b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$ **n** $= y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$ **o** $= 2x^{\frac{2}{3} + (-\frac{1}{6}) - \frac{3}{4}} = 2x^{-\frac{1}{4}}$ **p** $= \frac{1}{4}a^{1 + \frac{3}{4} - (-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$
- 8 **a** $x = 6^2 = 36$ **b** $x = 5^3 = 125$ **c** $x^{\frac{1}{2}} = \frac{1}{2}$
 $x = (\frac{1}{2})^2 = \frac{1}{4}$ **d** $x^{\frac{1}{4}} = 3$
 $x = 3^4 = 81$
- e** $x^{\frac{1}{2}} = \sqrt[3]{8} = 2$ **f** $x^{\frac{1}{3}} = \pm \sqrt{16} = \pm 4$
 $x = 2^2 = 4$ $x = (\pm 4)^3 = \pm 64$ **g** $x^{\frac{1}{3}} = \pm \sqrt[4]{81} = \pm 3$
 $x = (\pm 3)^3 = \pm 27$ **h** $x^{\frac{3}{2}} = \frac{1}{27}$
 $x^{\frac{1}{2}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$
 $x = (\frac{1}{3})^2 = \frac{1}{9}$
- 9 **a** $= x^{\frac{1}{2}}$ **b** $= x^{-\frac{1}{3}}$ **c** $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$ **d** $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$
- e** $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$ **f** $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$ **g** $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$ **h** $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$
- 10 **a** $4x^{-\frac{1}{2}}$ **b** $\frac{1}{2}x^{-1}$ **c** $\frac{3}{4}x^{-3}$ **d** $\frac{1}{9}x^{-2}$ **e** $\frac{2}{5}x^{-\frac{1}{3}}$ **f** $\frac{1}{3}x^{-\frac{2}{3}}$
- 11 **a** $= (2^3)^2 = 2^6$ **b** $= (2^{-2})^{-2} = 2^4$ **c** $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$
- d** $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$ **e** $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$ **f** $= (2^{-5})^{-3} = 2^{15}$
- 12 **a** $= (3^2)^x = 3^{2x}$ **b** $= (3^4)^{x+1} = 3^{4x+4}$ **c** $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$
- d** $= (3^{-1})^x = 3^{-x}$ **e** $= (3^2)^{2x-1} = 3^{4x-2}$ **f** $= (3^{-3})^{x+2} = 3^{-3x-6}$
- 13 **a** $= 2 \times 2^x = 2^y$ **b** $= 2^{-2} \times 2^x = \frac{1}{4}y$ **c** $= (2^x)^2 = y^2$
- d** $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$ **e** $= 2^3 \times 2^{4x} = 8y^4$ **f** $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \frac{8}{y}$

- 14**
- a** $2^x = 2^6$
 $x = 6$
- b** $5^{x-1} = 5^3$
 $x - 1 = 3$
 $x = 4$
- c** $3^{x+4} = 27 = 3^3$
 $x + 4 = 3$
 $x = -1$
- d** $(2^3)^x = 2^{3x} = 2$
 $3x = 1$
 $x = \frac{1}{3}$
- e** $3^{2x-1} = 3^2$
 $2x - 1 = 2$
 $x = \frac{3}{2}$
- f** $16 = 4^2 = 4^{3x-2}$
 $2 = 3x - 2$
 $x = \frac{4}{3}$
- g** $(3^2)^{x-2} = 3^{2x-4} = 3^3$
 $2x - 4 = 3$
 $x = \frac{7}{2}$
- h** $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
 $6x + 3 = 4$
 $x = \frac{1}{6}$
- i** $(7^2)^{x+1} = 7^{2x+2} = 7^{\frac{1}{2}}$
 $2x + 2 = \frac{1}{2}$
 $x = -\frac{3}{4}$
- j** $3^{3x-2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$
 $3x - 2 = \frac{2}{3}$
 $x = \frac{8}{9}$
- k** $(6^{-1})^{x+3} = 6^{-x-3} = 6^2$
 $-x - 3 = 2$
 $x = -5$
- l** $(2^{-1})^{3x-1} = 2^{1-3x} = 2^3$
 $1 - 3x = 3$
 $x = -\frac{2}{3}$
- 15**
- a** $2^{x+3} = (2^2)^x = 2^{2x}$
 $x + 3 = 2x$
 $x = 3$
- b** $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
 $3x = 2x + 2$
 $x = 2$
- c** $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
 $4x = x - 3$
 $x = -1$
- d** $(4^2)^x = 4^{2x} = 4^{1-x}$
 $2x = 1 - x$
 $x = \frac{1}{3}$
- e** $(2^2)^{x+2} = (2^3)^x$
 $2^{2x+4} = 2^{3x}$
 $2x + 4 = 3x$
 $x = 4$
- f** $(3^3)^{2x} = (3^2)^{3-x}$
 $3^{6x} = 3^{6-2x}$
 $6x = 6 - 2x$
 $x = \frac{3}{4}$
- g** $6^{3x-1} = (6^2)^{x+2}$
 $6^{3x-1} = 6^{2x+4}$
 $3x - 1 = 2x + 4$
 $x = 5$
- h** $(2^3)^x = (2^4)^{2x-1}$
 $2^{3x} = 2^{8x-4}$
 $3x = 8x - 4$
 $x = \frac{4}{5}$
- i** $(5^3)^x = 5^{x-3}$
 $5^{3x} = 5^{x-3}$
 $3x = x - 3$
 $x = -\frac{3}{2}$
- j** $(3^{-1})^x = 3^{x-4}$
 $3^{-x} = 3^{x-4}$
 $-x = x - 4$
 $x = 2$
- k** $(2^{-1})^{1-x} = (2^{-3})^{2x}$
 $2^{x-1} = 2^{-6x}$
 $x - 1 = -6x$
 $x = \frac{1}{7}$
- l** $(2^{-2})^{x+1} = (2^3)^x$
 $2^{-2x-2} = 2^{3x}$
 $-2x - 2 = 3x$
 $x = -\frac{2}{5}$
- 16**
- a** $= x^3 - 1$
- b** $= 2x^2 + 6x^3$
- c** $= 3 - x^2$
- d** $= 12x^3 + 8x$
- e** $= 3x^3 + 2x$
- f** $= 3 - 3x^2$
- g** $= 5x^{\frac{1}{2}} + x^2$
- h** $= 3x^2 - x^{-1}$
- i** $= x^6 + x^4 - 3x^2 - 3$
- j** $= 2x^9 + 6x^5 + x^5 + 3x$
 $= 2x^9 + 7x^5 + 3x$
- k** $= x^3 - 1 - 2 + 2x^{-3}$
 $= x^3 - 3 + 2x^{-3}$
- l** $= x^3 - x^{\frac{5}{2}} - x^{\frac{5}{2}} + x^2$
 $= x^3 - 2x^{\frac{5}{2}} + x^2$
- 17**
- a** $= x^2 + 2$
- b** $= 2t^3 - 3t$
- c** $= x - 3x^{\frac{1}{2}}$
- d** $= \frac{y^5 - 6y^2}{3y}$
 $= \frac{1}{3}y^4 - 2y$
- e** $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$
- f** $= 2w^{\frac{3}{2}} - \frac{1}{2}w$
- g** $= \frac{x^{\frac{1}{2}}(x+1)}{x+1}$
 $= x^{\frac{1}{2}}$
- h** $= \frac{t^{\frac{1}{2}} \times 2t(t^2 - 2)}{t^2 - 2}$
 $= 2t^{\frac{3}{2}}$

$$\begin{aligned}
 1 \quad \mathbf{a} &= \sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{2} \\
 &= 10\sqrt{2} + 3\sqrt{3} \\
 \mathbf{b} &= \sqrt{18} - \sqrt{48} \\
 &= \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3} \\
 &= 3\sqrt{2} - 4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad x^2 - 2x &= 12 - 2x \\
 x^2 &= 12 \\
 x &= \pm\sqrt{12} = \pm 2\sqrt{3} \\
 x > 0 \quad \therefore x &= 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad 25^x &= (5^2)^x = 5^{4x+1} \\
 5^{2x} &= 5^{4x+1} \\
 2x &= 4x+1 \\
 x &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \mathbf{a} &= \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3} \\
 \mathbf{b} \quad \sqrt[3]{81} &= \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3} \\
 \therefore \sqrt[3]{24} + \sqrt[3]{81} &= 2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3} \\
 &= \sqrt[3]{125 \times 3} = \sqrt[3]{375} \\
 \therefore n &= 375
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \frac{10\sqrt{3}}{\sqrt{15}} &= \frac{10\sqrt{3}}{\sqrt{5}\sqrt{3}} = \frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = 2\sqrt{5} \\
 \frac{4}{\sqrt{5}-\sqrt{7}} \times \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} &= \frac{4(\sqrt{5}+\sqrt{7})}{5-7} = -2\sqrt{5} - 2\sqrt{7} \\
 \therefore \frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}} &= 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{7} \\
 &= -2\sqrt{7} \quad [k = -2]
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \mathbf{a} &= \sqrt{\frac{75}{2}} = \frac{5\sqrt{3}}{\sqrt{2}} = \frac{5\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5}{2}\sqrt{6} \\
 \mathbf{b} &= \sqrt{\frac{48}{5}} - \sqrt{\frac{20}{3}} = \frac{4\sqrt{3}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}} \\
 &= \frac{4\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{4}{5}\sqrt{15} - \frac{2}{3}\sqrt{15} \\
 &= \frac{2}{15}\sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad \mathbf{i} \quad xy &= 2^{t-1} \times 2^{3t} = 2^{4t-1} \\
 \mathbf{ii} \quad 2y^2 &= 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1} \\
 \mathbf{b} \quad 2^{6t+1} - 2^{4t-1} &= 0 \\
 2^{6t+1} &= 2^{4t-1} \\
 6t+1 &= 4t-1 \\
 t &= -1
 \end{aligned}$$

$$\begin{aligned}
 8 \quad 3x\sqrt{2} - \sqrt{2} &= 4x+6 \\
 x(3\sqrt{2} - 4) &= 6 + \sqrt{2} \\
 x &= \frac{6+\sqrt{2}}{3\sqrt{2}-4} = \frac{6+\sqrt{2}}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{(6+\sqrt{2})(3\sqrt{2}+4)}{18-16} \\
 &= \frac{1}{2}(18\sqrt{2} + 24 + 6 + 4\sqrt{2}) \\
 &= \frac{1}{2}(30 + 22\sqrt{2}) \\
 &= 15 + 11\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \mathbf{a} \quad 6^{y+1} &= 36^{x-2} = (6^2)^{x-2} \\
 6^{y+1} &= 6^{2x-4} \\
 y+1 &= 2x-4 \\
 y &= 2x-5 \\
 \mathbf{b} \quad x - \frac{1}{2}y &= x - \frac{1}{2}(2x-5) = x - x + \frac{5}{2} = \frac{5}{2} \\
 \therefore 4^{x-\frac{1}{2}y} &= 4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = 32
 \end{aligned}$$

$$\begin{aligned}
 10 \quad \mathbf{a} &= 3 + 3\sqrt{2} - \sqrt{2} - 2 \\
 &= 1 + 2\sqrt{2} \\
 \mathbf{b} &= \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} \\
 &= \sqrt{2}(\sqrt{2}+1) \\
 &= 2 + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 11 \quad (2^4)^{x+1} &= (2^3)^{2x+1} \\
 2^{4x+4} &= 2^{6x+3} \\
 4x+4 &= 6x+3 \\
 x &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad a^2 - 4a\sqrt{3} + 12 &= b - 20\sqrt{3} \\
 a \text{ and } b \text{ integers } \therefore -4a &= -20 \\
 a &= 5 \\
 \text{also } a^2 + 12 &= b \\
 b &= 37
 \end{aligned}$$

$$\begin{aligned}
 13 \quad \mathbf{a} \quad (2^{-2})^{t-3} &= 2^3 \\
 2^{6-2t} &= 2^3 \\
 6-2t &= 3 \\
 t &= \frac{3}{2} \\
 \mathbf{b} \quad (3^{-1})^y &= (3^3)^{y+1} \\
 3^{-y} &= 3^{3y+3} \\
 -y &= 3y+3 \\
 y &= -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \mathbf{a} \quad &= 2\sqrt{5}(\sqrt{5}-3) \\
 &= 10 - 6\sqrt{5} \\
 \mathbf{b} \quad &= 3 + 2\sqrt{5} - 3\sqrt{5} - 10 \\
 &= -7 - \sqrt{5} \\
 \mathbf{c} \quad &= \frac{1+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(1+\sqrt{5})(\sqrt{5}+2)}{5-4} \\
 &= (1+\sqrt{5})(\sqrt{5}+2) \\
 &= \sqrt{5} + 2 + 5 + 2\sqrt{5} \\
 &= 7 + 3\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \mathbf{a} \quad a &= (b^{\frac{3}{4}})^3 = b^{\frac{9}{4}} \\
 a^{\frac{1}{2}} &= (b^{\frac{9}{4}})^{\frac{1}{2}} = b^{\frac{9}{8}} \\
 \mathbf{b} \quad b &= (a^{\frac{1}{3}})^{\frac{4}{3}} = a^{\frac{4}{9}} \\
 b^{\frac{1}{2}} &= (a^{\frac{4}{9}})^{\frac{1}{2}} = a^{\frac{2}{9}}
 \end{aligned}$$

$$\begin{aligned}
 16 \quad \mathbf{a} \quad \text{area} &= \frac{1}{2}(2\sqrt{3}-1)(\sqrt{3}+2) \\
 &= \frac{1}{2}(6+4\sqrt{3}-\sqrt{3}-2) \\
 &= \frac{1}{2}(4+3\sqrt{3}) \text{ or } 2 + \frac{3}{2}\sqrt{3} \\
 \mathbf{b} \quad AC^2 &= (2\sqrt{3}-1)^2 + (\sqrt{3}+2)^2 \\
 &= 12 - 4\sqrt{3} + 1 + 3 + 4\sqrt{3} + 4 = 20 \\
 \therefore AC &= \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5} \\
 \mathbf{c} \quad \tan(\angle ACB) &= \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4} \\
 &= -(2\sqrt{3}-1)(\sqrt{3}-2) \\
 &= -(6-4\sqrt{3}-\sqrt{3}+2) \\
 &= -(8-5\sqrt{3}) = 5\sqrt{3}-8
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \mathbf{a} \quad \mathbf{i} \quad 2^{x+2} &= 2^2 \times 2^x = 4y \\
 \mathbf{ii} \quad 4^x &= (2^2)^x = 2^{2x} = (2^x)^2 = y^2 \\
 \mathbf{b} \quad y^2 - 4y &= 0 \\
 y(y-4) &= 0 \\
 y &= 0 \text{ or } 4 \\
 2^x &= 0 \text{ (no solutions) or } 2^x = 4 \\
 x &= 2
 \end{aligned}$$

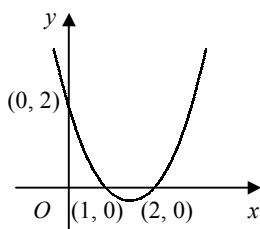
$$\begin{aligned}
 18 \quad 5\sqrt{3} &= 2(1+\sqrt{3})^2 + p(1+\sqrt{3}) + q \\
 5\sqrt{3} &= 2 + 4\sqrt{3} + 6 + p + p\sqrt{3} + q \\
 p, q \text{ rational } \therefore 5\sqrt{3} &= 4\sqrt{3} + p\sqrt{3} \\
 p &= 1 \\
 \text{and } 0 &= 2 + 6 + p + q \\
 q &= -9
 \end{aligned}$$

- 1
- a** $= 3x^2 + 8x + 3$
- b** $= x^3 + 5x^2 - 2x + 1$
- c** $= -3x^3 + 6x^2 - 2x + 7$
- d** $= x^5 - x^4 + 8x^3 - 5x^2 - 4x - 8$
- e** $= 3x^3 - 7x^2 + 2 - x^3 - 2x^2 - x + 6$
- f** $= x^5 + 3x^4 - x^2 - 3 - x^4 - 2x^3 + 3x - 2$
- $= 2x^3 - 9x^2 - x + 8$
- $= x^5 + 2x^4 - 2x^3 - x^2 + 3x - 5$
- g** $= 2x^7 - 9x^5 + x^3 + x - 3x^6 + 4x^3 - x - 5$
- h** $= 2x^4 + 8x^2 - 6 + x^4 + 3x^3 - 8$
- $= 2x^7 - 3x^6 - 9x^5 + 5x^3 - 5$
- $= 3x^4 + 3x^3 + 8x^2 - 14$
- i** $= 21 + 12x - 3x^2 - 6x^3 - 10 - 15x + 5x^3$
- j** $= 6x^3 + 30x^2 - 12 - 6x^3 + 3x^2 + 3x$
- $= -x^3 - 3x^2 - 3x + 11$
- $= 33x^2 + 3x - 12$
- k** $= 8x^4 + 16x^2 - 32x - 8 - 10 + 6x - 2x^3$
- l** $= 7x^6 + 21x^3 + 7x^2 - 28 - 8x^6 - 4x^5 + 12x + 28$
- $= 8x^4 - 2x^3 + 16x^2 - 26x - 18$
- $= -x^6 - 4x^5 + 21x^3 + 7x^2 + 12x$
- 2
- a** $= 3y^3 - 2y^2 + y + 6$
- b** $= 3t^4 - 3t^3 + 12t + 6 - t - 3t^3 + 2t^4 - 4t^2 + 8$
- $= 5t^4 - 6t^3 - 4t^2 + 11t + 14$
- c** $= x^3 - 6x^2 + 8 + 5x^2 - x + 1 - 2x^3 - 3x^2 - x + 7$
- $= -x^3 - 4x^2 - 2x + 16$
- d** $= 6 + 2m + 14m^2 - 6m^5 + 6 - 6m^2 + 12m^4 - 5m^5 - 15m^3 + 5m^2 - 10$
- $= 2 + 2m + 13m^2 - 15m^3 + 12m^4 - 11m^5$
- e** $= \frac{1}{3} - \frac{2}{3}u + \frac{1}{5}u^2 + u^4 - 1 + \frac{1}{2}u - \frac{1}{3}u^2 + \frac{1}{4}u^3$
- $= -\frac{2}{3} - \frac{1}{6}u - \frac{2}{15}u^2 + \frac{1}{4}u^3 + u^4$
- 3
- a** $= 2x - 3x^2 + x^3 + 4 + 8x^2 - 4x^3$
- b** $= x^5 + 7x^3 - 5x^2 + 9x - 2x^4 + 8x^3 + 6$
- $= 4 + 2x + 5x^2 - 3x^3$
- $= 6 + 9x - 5x^2 + 15x^3 - 2x^4 + x^5$
- c** $= -10x + 8x^2 - 2x^4 + 14 - 21x^2 + 7x^4$
- d** $= 8x^2 + 2x^3 + x^4 - 15 - 12x^2 - 3x^3$
- $= 14 - 10x - 13x^2 + 5x^4$
- $= -15 - 4x^2 - x^3 + x^4$
- e** $= 3x^3 + 9x^2 - x^4 - 4x^3 + 5x^3 - 10x$
- f** $= 6x^2 - x^3 + 5x^4 + 14x - 7x^4 + 4 - 12x - 4x^2$
- $= -10x + 9x^2 + 4x^3 - x^4$
- $= 4 + 2x + 2x^2 - x^3 - 2x^4$
- 4
- a** LHS $= (3x + 1)(x^2 - 2x + 4)$
- $= 3x(x^2 - 2x + 4) + (x^2 - 2x + 4)$
- $= 3x^3 - 6x^2 + 12x + x^2 - 2x + 4$
- $= 3x^3 - 5x^2 + 10x + 4 = \text{RHS}$
- b** LHS $= (1 + 2x - x^2)(1 - 2x + x^2)$
- $= (1 - 2x + x^2) + 2x(1 - 2x + x^2) - x^2(1 - 2x + x^2)$
- $= 1 - 2x + x^2 + 2x - 4x^2 + 2x^3 - x^2 + 2x^3 - x^4$
- $= 1 - 4x^2 + 4x^3 - x^4 = \text{RHS}$
- c** LHS $= (3 - x)^3$
- $= (3 - x)(9 - 6x + x^2)$
- $= 3(9 - 6x + x^2) - x(9 - 6x + x^2)$
- $= 27 - 18x + 3x^2 - 9x + 6x^2 - x^3$
- $= 27 - 27x + 9x^2 - x^3 = \text{RHS}$

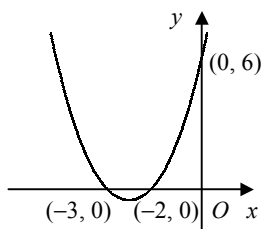
- 5
- a** $= x(x^2 + 5x - 6) + (x^2 + 5x - 6)$
 $= x^3 + 5x^2 - 6x + x^2 + 5x - 6$
 $= x^3 + 6x^2 - x - 6$
- b** $= 2x(x^2 - 3x + 7) - 5(x^2 - 3x + 7)$
 $= 2x^3 - 6x^2 + 14x - 5x^2 + 15x - 35$
 $= 2x^3 - 11x^2 + 29x - 35$
- c** $= 4(2 + 5x - x^2) - 7x(2 + 5x - x^2)$
 $= 8 + 20x - 4x^2 - 14x - 35x^2 + 7x^3$
 $= 7x^3 - 39x^2 + 6x + 8$
- d** $= (3x - 2)(3x - 2)^2 = (3x - 2)(9x^2 - 12x + 4)$
 $= 3x(9x^2 - 12x + 4) - 2(9x^2 - 12x + 4)$
 $= 27x^3 - 36x^2 + 12x - 18x^2 + 24x - 8$
 $= 27x^3 - 54x^2 + 36x - 8$
- e** $= x^2(2x^2 - x + 9) + 3(2x^2 - x + 9)$
 $= 2x^4 - x^3 + 9x^2 + 6x^2 - 3x + 27$
 $= 2x^4 - x^3 + 15x^2 - 3x + 27$
- f** $= 4x(x^4 - 3x^2 + 5x + 2) - (x^4 - 3x^2 + 5x + 2)$
 $= 4x^5 - 12x^3 + 20x^2 + 8x - x^4 + 3x^2 - 5x - 2$
 $= 4x^5 - x^4 - 12x^3 + 23x^2 + 3x - 2$
- g** $= x^2(x^2 + 3x + 1) + 2x(x^2 + 3x + 1) + 5(x^2 + 3x + 1)$
 $= x^4 + 3x^3 + x^2 + 2x^3 + 6x^2 + 2x + 5x^2 + 15x + 5$
 $= x^4 + 5x^3 + 12x^2 + 17x + 5$
- h** $= x^2(2x^2 - x + 4) + x(2x^2 - x + 4) - 3(2x^2 - x + 4)$
 $= 2x^4 - x^3 + 4x^2 + 2x^3 - x^2 + 4x - 6x^2 + 3x - 12$
 $= 2x^4 + x^3 - 3x^2 + 7x - 12$
- i** $= 3x^2(2x^2 - 4x - 8) - 5x(2x^2 - 4x - 8) + 2(2x^2 - 4x - 8)$
 $= 6x^4 - 12x^3 - 24x^2 - 10x^3 + 20x^2 + 40x + 4x^2 - 8x - 16$
 $= 6x^4 - 22x^3 + 32x^2 - 8x - 16$
- j** $= x^2(x^2 + 2x - 6) + 2x(x^2 + 2x - 6) - 6(x^2 + 2x - 6)$
 $= x^4 + 2x^3 - 6x^2 + 2x^3 + 4x^2 - 12x - 6x^2 - 12x + 36$
 $= x^4 + 4x^3 - 8x^2 - 24x + 36$
- k** $= x^3(2x^4 + x^2 + 3) + 4x^2(2x^4 + x^2 + 3) + (2x^4 + x^2 + 3)$
 $= 2x^7 + x^5 + 3x^3 + 8x^6 + 4x^4 + 12x^2 + 2x^4 + x^2 + 3$
 $= 2x^7 + 8x^6 + x^5 + 6x^4 + 3x^3 + 13x^2 + 3$
- l** $= 6(3 + x^2 - x^3 + 2x^4) - 2x(3 + x^2 - x^3 + 2x^4) + x^3(3 + x^2 - x^3 + 2x^4)$
 $= 18 + 6x^2 - 6x^3 + 12x^4 - 6x - 2x^3 + 2x^4 - 4x^5 + 3x^3 + x^5 - x^6 + 2x^7$
 $= 2x^7 - x^6 - 3x^5 + 14x^4 - 5x^3 + 6x^2 - 6x + 18$
- 6
- a** $= (p^2 - 1)(2p^2 + 11p + 12)$
 $= p^2(2p^2 + 11p + 12) - (2p^2 + 11p + 12)$
 $= 2p^4 + 11p^3 + 12p^2 - 2p^2 - 11p - 12$
 $= 2p^4 + 11p^3 + 10p^2 - 11p - 12$
- b** $= t(t^2 + 3t + 5) + 2(t^2 + 3t + 5) + t(t^2 + t + 7) + 4(t^2 + t + 7)$
 $= t^3 + 3t^2 + 5t + 2t^2 + 6t + 10 + t^3 + t^2 + 7t + 4t^2 + 4t + 28$
 $= 2t^3 + 10t^2 + 22t + 38$
- c** $= 2x^2(x^2 + x - 4) - 6(x^2 + x - 4) + 3x(4x^3 + 2x^2 - x + 6) - (4x^3 + 2x^2 - x + 6)$
 $= 2x^4 + 2x^3 - 8x^2 - 6x^2 - 6x + 24 + 12x^4 + 6x^3 - 3x^2 + 18x - 4x^3 - 2x^2 + x - 6$
 $= 14x^4 + 4x^3 - 19x^2 + 13x + 18$
- d** $= u(u^3 - 4u^2 - 3) + 2(u^3 - 4u^2 - 3) - 2u^3(u^2 + 5u - 3) - u(u^2 + 5u - 3) + (u^2 + 5u - 3)$
 $= u^4 - 4u^3 - 3u + 2u^3 - 8u^2 - 6 - 2u^5 - 10u^4 + 6u^3 - u^3 - 5u^2 + 3u + u^2 + 5u - 3$
 $= -2u^5 - 9u^4 + 3u^3 - 12u^2 + 5u - 9$

- 1** **a** $(x+1)(x+3)$ **b** $(x+2)(x+5)$ **c** $(y-1)(y-2)$ **d** $(x-3)^2$
e $(y+1)(y-2)$ **f** $(a+4)(a-2)$ **g** $(x+1)(x-1)$ **h** $(p+2)(p+7)$
i $(x+3)(x-5)$ **j** $(m-2)(m-8)$ **k** $(t+6)(t-3)$ **l** $(y-5)(y-8)$
m $(r+4)(r-4)$ **n** $(y+7)(y-9)$ **o** $(a+11)^2$ **p** $(x+12)(x-6)$
q $(x-2)(x-13)$ **r** $(s+8)(s+15)$ **s** $(p+17)(p-3)$ **t** $(m-10)(m+9)$
- 2** **a** $(2x+1)(x+1)$ **b** $(3p+1)(p+2)$ **c** $(2y-3)(y-1)$ **d** $(2+m)(1-m)$
e $(3r+1)(r-1)$ **f** $(5+y)(1-4y)$ **g** $(3a-1)(a-4)$ **h** $(5x+2)(x-2)$
i $(2x+1)(2x+3)$ **j** $(3s-1)^2$ **k** $(2m+5)(2m-5)$ **l** $(2+3y)(1-2y)$
m $(4u+1)(u+4)$ **n** $(3p+4)(2p-1)$ **o** $(8x+3)(x+2)$ **p** $(6r-5)(2r+3)$
- 3** **a** $(x-1)(x-3)=0$
 $x=1$ or 3 **b** $(x+4)(x+2)=0$
 $x=-4$ or -2 **c** $(x+5)(x-1)=0$
 $x=-5$ or 1 **d** $x^2-7x-8=0$
 $(x+1)(x-8)=0$
 $x=-1$ or 8
e $(x+5)(x-5)=0$ **f** $x^2-x-42=0$
 $x=-5$ or 5 $(x+6)(x-7)=0$
 $x=-6$ or 7 **g** $x^2-3x=0$
 $x(x-3)=0$
 $x=0$ or 3 **h** $(x+9)(x+3)=0$
 $x=-9$ or -3
i $x^2+4x-60=0$ **j** $x^2-5x-14=0$
 $(x+10)(x-6)=0$ $(x+2)(x-7)=0$
 $x=-10$ or 6 $x=-2$ or 7 **k** $(2x-1)(x-1)=0$
 $x=\frac{1}{2}$ or 1 **l** $x^2-x=6x-12$
 $x^2-7x+12=0$
 $(x-3)(x-4)=0$
 $x=3$ or 4
m $3x^2+11x-4=0$ **n** $2x^2-3x-5=0$ **o** $4x^2-23x-6=0$ **p** $6x^2-19x+10=0$
 $(3x-1)(x+4)=0$ $(2x-5)(x+1)=0$ $(4x+1)(x-6)=0$ $(3x-2)(2x-5)=0$
 $x=-4$ or $\frac{1}{3}$ $x=-1$ or $\frac{5}{2}$ $x=-\frac{1}{4}$ or 6 $x=\frac{2}{3}$ or $\frac{5}{2}$
q $(2x+1)^2=0$ **r** $3x^2-13x+12=0$ **s** $4x^2+20x+25=5-x$ **t** $6x^2-21x=14x+6$
 $x=-\frac{1}{2}$ $(3x-4)(x-3)=0$ $4x^2+21x+20=0$ $6x^2-35x-6=0$
 $x=\frac{4}{3}$ or 3 $(4x+5)(x+4)=0$ $(6x+1)(x-6)=0$
 $x=-4$ or $-\frac{5}{4}$ $x=-\frac{1}{6}$ or 6
- 4** **a** $=2(y^2-5y+6)$ **b** $=x(x^2+x-2)$ **c** $=p(p^2-4)$ **d** $=3m(m^2+7m+6)$
 $=2(y-3)(y-2)$ $=x(x-1)(x+2)$ $=p(p+2)(p-2)$ $=3m(m+1)(m+6)$
e $=(a^2+1)(a^2+3)$ **f** $=(t^2+5)(t^2-2)$ **g** $=4(3+5x-2x^2)$ **h** $=3(2r^2-3r-14)$
 $=4(3-x)(1+2x)$ $=3(2r-7)(r+2)$
i $=2x(3x^2-13x+4)$ **j** $=y^2(y^2+3y-18)$ **k** $=(m^2+1)(m^2-1)$ **l** $=p(p^4-4p^2+4)$
 $=2x(3x-1)(x-4)$ $=y^2(y+6)(y-3)$ $=(m^2+1)(m+1)(m-1)$ $=p(p^2-2)^2$

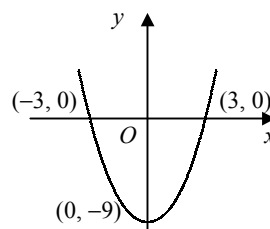
5 a $x^2 - 3x + 2 = 0$
 $(x - 1)(x - 2) = 0$
 $x = 1$ or 2



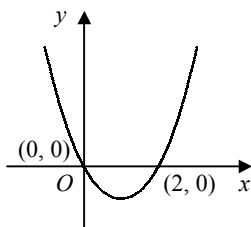
b $x^2 + 5x + 6 = 0$
 $(x + 3)(x + 2) = 0$
 $x = -3$ or -2



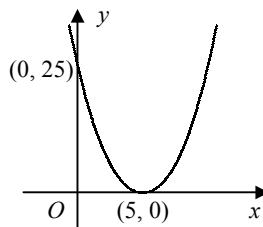
c $x^2 - 9 = 0$
 $(x + 3)(x - 3) = 0$
 $x = -3$ or 3



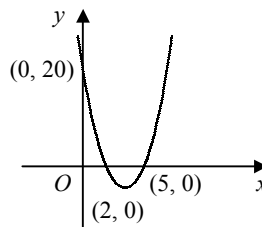
d $x^2 - 2x = 0$
 $x(x - 2) = 0$
 $x = 0$ or 2



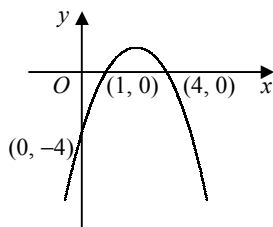
e $x^2 - 10x + 25 = 0$
 $(x - 5)^2 = 0$
 $x = 5$



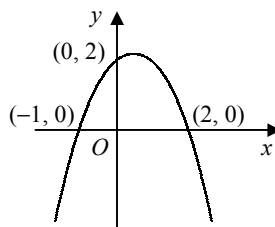
f $2x^2 - 14x + 20 = 0$
 $2(x - 2)(x - 5) = 0$
 $x = 2$ or 5



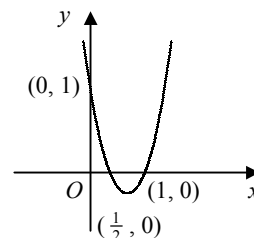
g $-x^2 + 5x - 4 = 0$
 $x^2 - 5x + 4 = 0$
 $(x - 1)(x - 4) = 0$
 $x = 1$ or 4



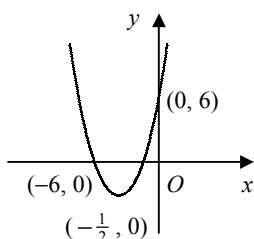
h $2 + x - x^2 = 0$
 $x^2 - x - 2 = 0$
 $(x + 1)(x - 2) = 0$
 $x = -1$ or 2



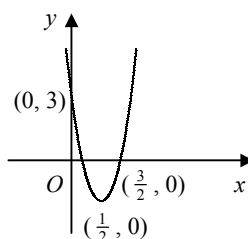
i $2x^2 - 3x + 1 = 0$
 $(2x - 1)(x - 1) = 0$
 $x = \frac{1}{2}$ or 1



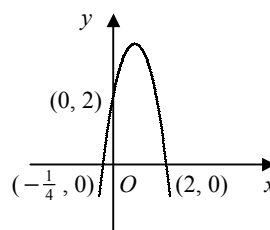
j $2x^2 + 13x + 6 = 0$
 $(2x + 1)(x + 6) = 0$
 $x = -6$ or $-\frac{1}{2}$



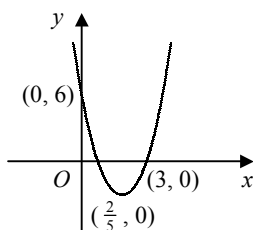
k $3 - 8x + 4x^2 = 0$
 $(2x - 1)(2x - 3) = 0$
 $x = \frac{1}{2}$ or $\frac{3}{2}$



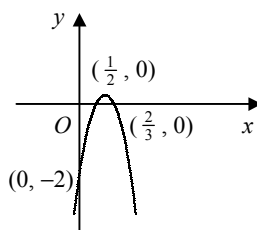
l $2 + 7x - 4x^2 = 0$
 $4x^2 - 7x - 2 = 0$
 $(4x + 1)(x - 2) = 0$
 $x = -\frac{1}{4}$ or 2



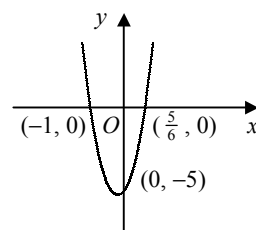
$$\begin{aligned} \text{m } 5x^2 - 17x + 6 &= 0 \\ (5x - 2)(x - 3) &= 0 \\ x &= \frac{2}{5} \text{ or } 3 \end{aligned}$$



$$\begin{aligned} \text{n } -6x^2 + 7x - 2 &= 0 \\ 6x^2 - 7x + 2 &= 0 \\ (2x - 1)(3x - 2) &= 0 \\ x &= \frac{1}{2} \text{ or } \frac{2}{3} \end{aligned}$$



$$\begin{aligned} \text{o } 6x^2 + x - 5 &= 0 \\ (6x - 5)(x + 1) &= 0 \\ x &= -1 \text{ or } \frac{5}{6} \end{aligned}$$



$$\begin{aligned} 6 \quad \text{a } x^2 - 5x + 4 &= 0 \\ (x - 1)(x - 4) &= 0 \\ x &= 1 \text{ or } 4 \end{aligned}$$

$$\begin{aligned} \text{b } x^2 - 10 &= 3x \\ x^2 - 3x - 10 &= 0 \\ (x + 2)(x - 5) &= 0 \\ x &= -2 \text{ or } 5 \end{aligned}$$

$$\begin{aligned} \text{c } x(2x^2 - x - 3) &= 0 \\ x(2x - 3)(x + 1) &= 0 \\ x &= -1, 0 \text{ or } \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{d } 10x^2 - x^4 &= 9 \\ x^4 - 10x^2 + 9 &= 0 \\ (x^2 - 1)(x^2 - 9) &= 0 \\ x^2 &= 1 \text{ or } 9 \\ x &= \pm 1 \text{ or } \pm 3 \end{aligned}$$

$$\begin{aligned} \text{e } 5 + 4x - x^2 &= 0 \\ x^2 - 4x - 5 &= 0 \\ (x + 1)(x - 5) &= 0 \\ x &= -1 \text{ or } 5 \end{aligned}$$

$$\begin{aligned} \text{f } x - 6 &= x(x - 4) \\ x - 6 &= x^2 - 4x \\ x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x &= 2 \text{ or } 3 \end{aligned}$$

$$\begin{aligned} \text{g } (x + 5)(x + 3) &= 3 \\ x^2 + 8x + 15 &= 3 \\ x^2 + 8x + 12 &= 0 \\ (x + 6)(x + 2) &= 0 \\ x &= -6 \text{ or } -2 \end{aligned}$$

$$\begin{aligned} \text{h } x^4 - 4 &= 3x^2 \\ x^4 - 3x^2 - 4 &= 0 \\ (x^2 + 1)(x^2 - 4) &= 0 \\ x^2 &= -1 \text{ (no sol's) or } 4 \\ x &= \pm 2 \end{aligned}$$

$$\begin{aligned} \text{i } 4x^4 + 7x^2 - 2 &= 0 \\ (4x^2 - 1)(x^2 + 2) &= 0 \\ x^2 &= -2 \text{ (no sol's) or } \frac{1}{4} \\ x &= \pm \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{j } 2x(x + 2) &= 3 - x \\ 2x^2 + 4x &= 3 - x \\ 2x^2 + 5x - 3 &= 0 \\ (2x - 1)(x + 3) &= 0 \\ x &= -3 \text{ or } \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{k } x(2x + 1) &= 2(x + 3) \\ 2x^2 + x &= 2x + 6 \\ 2x^2 - x - 6 &= 0 \\ (2x + 3)(x - 2) &= 0 \\ x &= -\frac{3}{2} \text{ or } 2 \end{aligned}$$

$$\begin{aligned} \text{l } 7 - 3x(x + 2) &= 2(x + 2) \\ 7 - 3x^2 - 6x &= 2x + 4 \\ 3x^2 + 8x - 3 &= 0 \\ (3x - 1)(x + 3) &= 0 \\ x &= -3 \text{ or } \frac{1}{3} \end{aligned}$$

- 1 **a** $= (x+1)^2 - 1 + 4$
 $= (x+1)^2 + 3$ **b** $= (x-1)^2 - 1 + 4$
 $= (x-1)^2 + 3$ **c** $= (x-2)^2 - 4 + 1$
 $= (x-2)^2 - 3$ **d** $= (x+3)^2 - 9$
- e** $= (x+2)^2 - 4 + 8$
 $= (x+2)^2 + 4$ **f** $= (x-4)^2 - 16 - 5$
 $= (x-4)^2 - 21$ **g** $= (x+6)^2 - 36 + 30$
 $= (x+6)^2 - 6$ **h** $= (x-5)^2 - 25 + 25$
 $= (x-5)^2$
- i** $= (x+3)^2 - 9 - 9$
 $= (x+3)^2 - 18$ **j** $= (x-2)^2 - 4 + 18$
 $= (x-2)^2 + 14$ **k** $= (x + \frac{3}{2})^2 - \frac{9}{4} + 3$
 $= (x + \frac{3}{2})^2 + \frac{3}{4}$ **l** $= (x + \frac{1}{2})^2 - \frac{1}{4} - 1$
 $= (x + \frac{1}{2})^2 - \frac{5}{4}$
- m** $= (x-9)^2 - 81 + 100$
 $= (x-9)^2 + 19$ **n** $= (x - \frac{1}{2})^2 - \frac{1}{4} - \frac{1}{2}$
 $= (x - \frac{1}{2})^2 - \frac{3}{4}$ **o** $= (x + \frac{9}{2})^2 - \frac{81}{4} + 20$
 $= (x + \frac{9}{2})^2 - \frac{1}{4}$ **p** $= (x - \frac{7}{2})^2 - \frac{49}{4} - 2$
 $= (x - \frac{7}{2})^2 - \frac{57}{4}$
- q** $= (x - \frac{3}{2})^2 - \frac{9}{4} + 5$
 $= (x - \frac{3}{2})^2 + \frac{11}{4}$ **r** $= (x - \frac{11}{2})^2 - \frac{121}{4} + 37$
 $= (x - \frac{11}{2})^2 + \frac{27}{4}$ **s** $= (x + \frac{1}{3})^2 - \frac{1}{9} + 1$
 $= (x + \frac{1}{3})^2 + \frac{8}{9}$ **t** $= (x - \frac{1}{4})^2 - \frac{1}{16} - \frac{1}{4}$
 $= (x - \frac{1}{4})^2 - \frac{5}{16}$
- 2 **a** $= 2[x^2 + 2x] + 3$
 $= 2[(x+1)^2 - 1] + 3$
 $= 2(x+1)^2 + 1$ **b** $= 2[x^2 - 4x] - 7$
 $= 2[(x-2)^2 - 4] - 7$
 $= 2(x-2)^2 - 15$ **c** $= 3[x^2 - 2x] + 3$
 $= 3[(x-1)^2 - 1] + 3$
 $= 3(x-1)^2$ **d** $= 4[x^2 + 6x] + 11$
 $= 4[(x+3)^2 - 9] + 11$
 $= 4(x+3)^2 - 25$
- e** $= -[x^2 + 2x] - 5$
 $= -[(x+1)^2 - 1] - 5$
 $= -(x+1)^2 - 4$ **f** $= -[x^2 - 10x] + 1$
 $= -[(x-5)^2 - 25] + 1$
 $= -(x-5)^2 + 26$ **g** $= 2[x^2 + x] - 1$
 $= 2[(x + \frac{1}{2})^2 - \frac{1}{4}] - 1$
 $= 2(x + \frac{1}{2})^2 - \frac{3}{2}$ **h** $= 3[x^2 - 3x] + 5$
 $= 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + 5$
 $= 3(x - \frac{3}{2})^2 - \frac{7}{4}$
- i** $= 3[x^2 - 8x] + 48$
 $= 3[(x-4)^2 - 16] + 48$
 $= 3(x-4)^2$ **j** $= 3[x^2 - 5x]$
 $= 3[(x - \frac{5}{2})^2 - \frac{25}{4}]$
 $= 3(x - \frac{5}{2})^2 - \frac{75}{4}$ **k** $= 5[x^2 + 8x] + 70$
 $= 5[(x+4)^2 - 16] + 70$
 $= 5(x+4)^2 - 10$ **l** $= 2[x^2 + \frac{5}{2}x] + 2$
 $= 2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 2$
 $= 2(x + \frac{5}{4})^2 - \frac{9}{8}$
- m** $= 4[x^2 + \frac{3}{2}x] - 7$
 $= 4[(x + \frac{3}{4})^2 - \frac{9}{16}] - 7$
 $= 4(x + \frac{3}{4})^2 - \frac{37}{4}$ **n** $= -2[x^2 - 2x] - 1$
 $= -2[(x-1)^2 - 1] - 1$
 $= -2(x-1)^2 + 1$ **o** $= -3[x^2 + \frac{2}{3}x] + 4$
 $= -3[(x + \frac{1}{3})^2 - \frac{1}{9}] + 4$
 $= -3(x + \frac{1}{3})^2 + \frac{13}{3}$ **p** $= \frac{1}{3}[x^2 + \frac{3}{2}x] - \frac{1}{4}$
 $= \frac{1}{3}[(x + \frac{3}{4})^2 - \frac{9}{16}] - \frac{1}{4}$
 $= \frac{1}{3}(x + \frac{3}{4})^2 - \frac{7}{16}$
- 3 **a** $(y-2)^2 - 4 + 2 = 0$
 $(y-2)^2 = 2$
 $y-2 = \pm\sqrt{2}$
 $y = 2 \pm \sqrt{2}$ **b** $(p+1)^2 - 1 - 2 = 0$
 $(p+1)^2 = 3$
 $p+1 = \pm\sqrt{3}$
 $p = -1 \pm \sqrt{3}$ **c** $(x-3)^2 - 9 + 4 = 0$
 $(x-3)^2 = 5$
 $x-3 = \pm\sqrt{5}$
 $x = 3 \pm \sqrt{5}$ **d** $(r+5)^2 - 25 + 7 = 0$
 $(r+5)^2 = 18$
 $r+5 = \pm\sqrt{18} = \pm 3\sqrt{2}$
 $r = -5 \pm 3\sqrt{2}$
- e** $(x-1)^2 - 1 = 11$
 $(x-1)^2 = 12$
 $x-1 = \pm\sqrt{12} = \pm 2\sqrt{3}$
 $x = 1 \pm 2\sqrt{3}$ **f** $(a-6)^2 - 36 - 18 = 0$
 $(a-6)^2 = 54$
 $a-6 = \pm\sqrt{54} = \pm 3\sqrt{6}$
 $a = 6 \pm 3\sqrt{6}$ **g** $(m - \frac{3}{2})^2 - \frac{9}{4} + 1 = 0$
 $(m - \frac{3}{2})^2 = \frac{5}{4}$
 $m - \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$
 $m = \frac{1}{2}(3 \pm \sqrt{5})$ **h** $(t - \frac{7}{2})^2 - \frac{49}{4} + 9 = 0$
 $(t - \frac{7}{2})^2 = \frac{13}{4}$
 $t - \frac{7}{2} = \pm\sqrt{\frac{13}{4}}$
 $t = \frac{1}{2}(7 \pm \sqrt{13})$

$$\begin{array}{llll}
 \text{i} & (u + \frac{7}{2})^2 - \frac{49}{4} = 44 & \text{j} & y^2 - 2y + \frac{1}{2} = 0 \\
 & (u + \frac{7}{2})^2 = \frac{225}{4} & & (y - 1)^2 - 1 + \frac{1}{2} = 0 \\
 & u + \frac{7}{2} = \pm \frac{15}{2} & & (y - 1)^2 = \frac{1}{2} \\
 & u = -\frac{7}{2} \pm \frac{15}{2} & & y - 1 = \pm \frac{1}{\sqrt{2}} = \pm \frac{1}{2}\sqrt{2} \\
 & u = -11 \text{ or } 4 & & y = 1 \pm \frac{1}{2}\sqrt{2}
 \end{array}$$

$$\begin{array}{llll}
 \text{k} & p^2 + 6p = -\frac{23}{3} & \text{l} & x^2 + 6x = \frac{9}{2} \\
 & (p + 3)^2 - 9 = -\frac{23}{3} & & (x + 3)^2 - 9 = \frac{9}{2} \\
 & (p + 3)^2 = \frac{4}{3} & & (x + 3)^2 = \frac{27}{2} \\
 & p + 3 = \pm \frac{2}{\sqrt{3}} = \pm \frac{2}{3}\sqrt{3} & & x + 3 = \pm \sqrt{\frac{27}{2}} = \pm \frac{3}{2}\sqrt{6} \\
 & p = -3 \pm \frac{2}{3}\sqrt{3} & & x = -3 \pm \frac{3}{2}\sqrt{6}
 \end{array}$$

$$\begin{array}{llll}
 \text{m} & m^2 - m = 1 & \text{n} & 4x^2 - 28x + 49 = 0 \\
 & (m - \frac{1}{2})^2 - \frac{1}{4} = 1 & & x^2 - 7x + \frac{49}{4} = 0 \\
 & (m - \frac{1}{2})^2 = \frac{5}{4} & & (x - \frac{7}{2})^2 - \frac{49}{4} + \frac{49}{4} = 0 \\
 & m - \frac{1}{2} = \pm \frac{\sqrt{5}}{2} & & (x - \frac{7}{2})^2 = 0 \\
 & m = \frac{1}{2}(1 \pm \sqrt{5}) & & x = \frac{7}{2}
 \end{array}$$

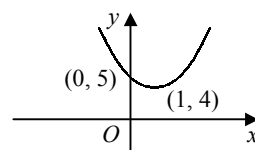
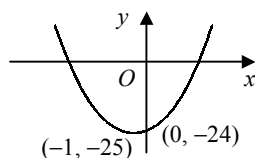
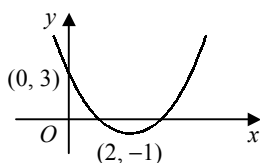
$$\begin{array}{llll}
 \text{o} & t^2 + \frac{1}{3}t = \frac{1}{3} & \text{p} & a^2 - \frac{7}{2}a + 2 = 0 \\
 & (t + \frac{1}{6})^2 - \frac{1}{36} = \frac{1}{3} & & (a - \frac{7}{4})^2 - \frac{49}{16} + 2 = 0 \\
 & (t + \frac{1}{6})^2 = \frac{13}{36} & & (a - \frac{7}{4})^2 = \frac{17}{16} \\
 & t + \frac{1}{6} = \pm \frac{\sqrt{13}}{6} & & a - \frac{7}{4} = \pm \frac{\sqrt{17}}{4} \\
 & t = \frac{1}{6}(-1 \pm \sqrt{13}) & & a = \frac{1}{4}(7 \pm \sqrt{17})
 \end{array}$$

4

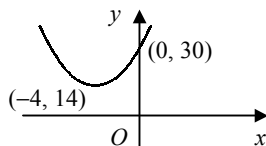
$$\begin{array}{lll}
 \text{a} & y = (x - 1)^2 - 1 + 7 \\
 & y = (x - 1)^2 + 6 \\
 & y = 6 \text{ at } x = 1, \text{ minimum} \\
 \text{b} & y = (x + 1)^2 - 1 - 3 \\
 & y = (x + 1)^2 - 4 \\
 & y = -4 \text{ at } x = -1, \text{ minimum} \\
 \text{c} & y = (x - 3)^2 - 9 + 1 \\
 & y = (x - 3)^2 - 8 \\
 & y = -8 \text{ at } x = 3, \text{ minimum} \\
 \text{d} & y = (x + 5)^2 - 25 + 35 \\
 & y = (x + 5)^2 + 10 \\
 & y = 10 \text{ at } x = -5, \text{ minimum} \\
 \text{e} & y = -[x^2 - 4x] + 4 \\
 & y = -[(x - 2)^2 - 4] + 4 \\
 & y = -(x - 2)^2 + 8 \\
 & y = 8 \text{ at } x = 2, \text{ maximum} \\
 \text{f} & y = (x + \frac{3}{2})^2 - \frac{9}{4} - 2 \\
 & y = (x + \frac{3}{2})^2 - \frac{17}{4} \\
 & y = -\frac{17}{4} \text{ at } x = -\frac{3}{2}, \text{ minimum} \\
 \text{g} & y = 2[x^2 + 4x] + 5 \\
 & y = 2[(x + 2)^2 - 4] + 5 \\
 & y = 2(x + 2)^2 - 3 \\
 & y = -3 \text{ at } x = -2, \text{ minimum} \\
 \text{h} & y = -3[x^2 - 2x] \\
 & y = -3[(x - 1)^2 - 1] \\
 & y = -3(x - 1)^2 + 3 \\
 & y = 3 \text{ at } x = 1, \text{ maximum} \\
 \text{i} & y = -[x^2 + 5x] + 7 \\
 & y = -[(x + \frac{5}{2})^2 - \frac{25}{4}] + 7 \\
 & y = -(x + \frac{5}{2})^2 + \frac{53}{4} \\
 & y = \frac{53}{4} \text{ at } x = -\frac{5}{2}, \text{ maximum} \\
 \text{j} & y = 4[x^2 - 3x] + 9 \\
 & y = 4[(x - \frac{3}{2})^2 - \frac{9}{4}] + 9 \\
 & y = 4(x - \frac{3}{2})^2 \\
 & y = 0 \text{ at } x = \frac{3}{2}, \text{ minimum} \\
 \text{k} & y = 4[x^2 + 5x] - 8 \\
 & y = 4[(x + \frac{5}{2})^2 - \frac{25}{4}] - 8 \\
 & y = 4(x + \frac{5}{2})^2 - 33 \\
 & y = -33 \text{ at } x = -\frac{5}{2}, \text{ minimum} \\
 \text{l} & y = -2[x^2 + x] + 17 \\
 & y = -2[(x + \frac{1}{2})^2 - \frac{1}{4}] + 17 \\
 & y = -2(x + \frac{1}{2})^2 + \frac{35}{2} \\
 & y = \frac{35}{2} \text{ at } x = -\frac{1}{2}, \text{ maximum}
 \end{array}$$

5

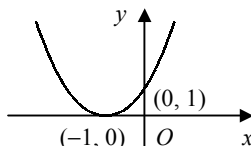
$$\begin{array}{lll}
 \text{a} & y = (x - 2)^2 - 4 + 3 \\
 & y = (x - 2)^2 - 1 \\
 & \text{minimum } (2, -1) \\
 \text{b} & y = (x + 1)^2 - 1 - 24 \\
 & y = (x + 1)^2 - 25 \\
 & \text{minimum } (-1, -25) \\
 \text{c} & y = (x - 1)^2 - 1 + 5 \\
 & y = (x - 1)^2 + 4 \\
 & \text{minimum } (1, 4)
 \end{array}$$



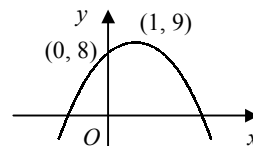
d $y = (x + 4)^2 - 16 + 30$
 $y = (x + 4)^2 + 14$
 minimum $(-4, 14)$



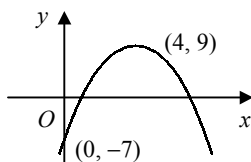
e $y = (x + 1)^2 - 1 + 1$
 $y = (x + 1)^2$
 minimum $(-1, 0)$



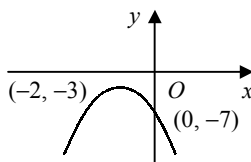
f $y = -[x^2 - 2x] + 8$
 $y = -[(x - 1)^2 - 1] + 8$
 $y = -(x - 1)^2 + 9$
 maximum $(1, 9)$



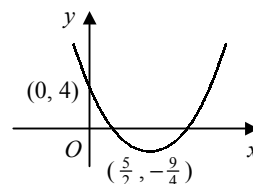
g $y = -[x^2 - 8x] - 7$
 $y = -[(x - 4)^2 - 16] - 7$
 $y = -(x - 4)^2 + 9$
 maximum $(4, 9)$



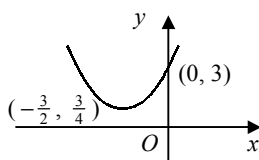
h $y = -[x^2 + 4x] - 7$
 $y = -[(x + 2)^2 - 4] - 7$
 $y = -(x + 2)^2 - 3$
 maximum $(-2, -3)$



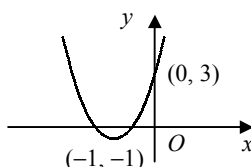
i $y = (x - \frac{5}{2})^2 - \frac{25}{4} + 4$
 $y = (x - \frac{5}{2})^2 - \frac{9}{4}$
 minimum $(\frac{5}{2}, -\frac{9}{4})$



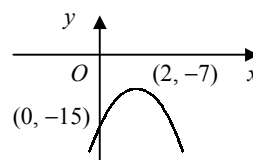
j $y = (x + \frac{3}{2})^2 - \frac{9}{4} + 3$
 $y = (x + \frac{3}{2})^2 + \frac{3}{4}$
 minimum $(-\frac{3}{2}, \frac{3}{4})$



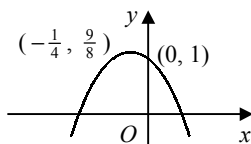
k $y = 4[x^2 + 2x] + 3$
 $y = 4[(x + 1)^2 - 1] + 3$
 $y = 4(x + 1)^2 - 1$
 minimum $(-1, -1)$



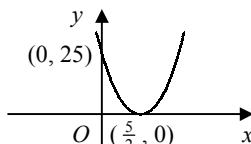
l $y = -2[x^2 - 4x] - 15$
 $y = -2[(x - 2)^2 - 4] - 15$
 $y = -2(x - 2)^2 - 7$
 maximum $(2, -7)$



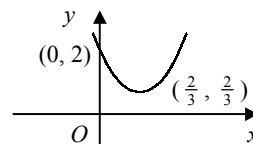
m $y = -2[x^2 + \frac{1}{2}x] + 1$
 $y = -2[(x + \frac{1}{4})^2 - \frac{1}{16}] + 1$
 $y = -2(x + \frac{1}{4})^2 + \frac{9}{8}$
 maximum $(-\frac{1}{4}, \frac{9}{8})$



n $y = 4[x^2 - 5x] + 25$
 $y = 4[(x - \frac{5}{2})^2 - \frac{25}{4}] + 25$
 $y = 4(x - \frac{5}{2})^2$
 minimum $(\frac{5}{2}, 0)$



o $y = 3[x^2 - \frac{4}{3}x] + 2$
 $y = 3[(x - \frac{2}{3})^2 - \frac{4}{9}] + 2$
 $y = 3(x - \frac{2}{3})^2 + \frac{2}{3}$
 minimum $(\frac{2}{3}, \frac{2}{3})$



6 **a** $= (x - 2\sqrt{2})^2 - 8 + 5$
 $= (x - 2\sqrt{2})^2 - 3$
b $x = 2\sqrt{2}$

7 $x^2 + 2kx - 3 = 0$
 $(x + k)^2 - k^2 - 3 = 0$
 $(x + k)^2 = k^2 + 3$
 $x + k = \pm\sqrt{k^2 + 3}$
 $x = -k \pm \sqrt{k^2 + 3}$

1 $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 **a** $x = \frac{-4 \pm \sqrt{16-4}}{2}$ **b** $t = \frac{-8 \pm \sqrt{64+16}}{-2}$ **c** $y = \frac{20 \pm \sqrt{400-364}}{2}$ **d** $r = \frac{-2 \pm \sqrt{4+28}}{2}$

$$x = \frac{-4 \pm 2\sqrt{3}}{2} \quad t = \frac{-8 \pm 4\sqrt{5}}{-2} \quad y = \frac{20 \pm 6}{2} \quad r = \frac{-2 \pm 4\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{3} \quad t = 4 \pm 2\sqrt{5} \quad y = 7 \text{ or } 13 \quad r = -1 \pm 2\sqrt{2}$$

e $a = \frac{-18 \pm \sqrt{324-24}}{2}$ **f** $m^2 - 5m - 5 = 0$ **g** $x = \frac{-11 \pm \sqrt{121-108}}{2}$ **h** $u = \frac{-6 \pm \sqrt{36-24}}{4}$

$$a = \frac{-18 \pm 10\sqrt{3}}{2} \quad m = \frac{5 \pm \sqrt{25+20}}{2} \quad x = \frac{1}{2}(-11 \pm \sqrt{13}) \quad u = \frac{-6 \pm 2\sqrt{3}}{4}$$

$$a = -9 \pm 5\sqrt{3} \quad m = \frac{1}{2}(5 \pm 3\sqrt{5}) \quad u = \frac{1}{2}(-3 \pm \sqrt{3})$$

i $y = \frac{1 \pm \sqrt{1+20}}{-2}$ **j** $2x^2 - 3x - 2 = 0$ **k** $p = \frac{-7 \pm \sqrt{49-12}}{6}$ **l** $t^2 - 14t - 14 = 0$

$$y = -\frac{1}{2}(1 \pm \sqrt{21}) \quad x = \frac{3 \pm \sqrt{9+16}}{4} \quad p = \frac{1}{6}(-7 \pm \sqrt{37}) \quad t = \frac{14 \pm \sqrt{196+56}}{2}$$

$$x = \frac{3 \pm 5}{4} \quad t = \frac{14 \pm 6\sqrt{7}}{2}$$

$$x = -\frac{1}{2} \text{ or } 2 \quad t = 7 \pm 3\sqrt{7}$$

m $r^2 + 14r - 9 = 0$ **n** $6u^2 + 4u - 1 = 0$ **o** $3y^2 - 18y - 4 = 0$ **p** $4x^2 - 8x - 11 = 0$

$$r = \frac{-14 \pm \sqrt{196+36}}{2} \quad u = \frac{-4 \pm \sqrt{16+24}}{12} \quad y = \frac{18 \pm \sqrt{324+48}}{6} \quad x = \frac{8 \pm \sqrt{64+176}}{8}$$

$$r = \frac{-14 \pm 2\sqrt{58}}{2} \quad u = \frac{-4 \pm 2\sqrt{10}}{12} \quad y = \frac{18 \pm 2\sqrt{93}}{6} \quad x = \frac{8 \pm 4\sqrt{15}}{8}$$

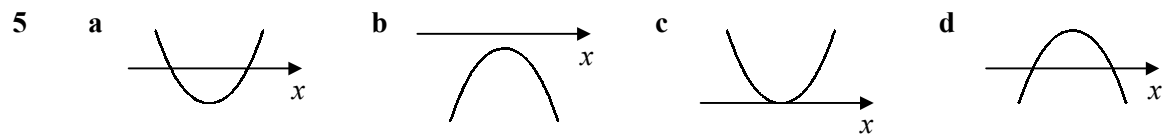
$$r = -7 \pm \sqrt{58} \quad u = \frac{1}{6}(-2 \pm \sqrt{10}) \quad y = 3 \pm \frac{1}{3}\sqrt{93} \quad x = 1 \pm \frac{1}{2}\sqrt{15}$$

3 $2x^2 - 8x + 3 = 0$

$$x = \frac{8 \pm \sqrt{64-24}}{4} = \frac{8 \pm 2\sqrt{10}}{4} = 2 \pm \frac{1}{2}\sqrt{10}$$

$$\therefore (2 - \frac{1}{2}\sqrt{10}, 0) \text{ and } (2 + \frac{1}{2}\sqrt{10}, 0)$$

4 **a** $b^2 - 4ac > 0$ **b** $b^2 - 4ac = 0$ **c** $b^2 - 4ac < 0$



6 **a** $b^2 - 4ac = 32$ **b** $b^2 - 4ac = -11$ **c** $b^2 - 4ac = -4$ **d** $b^2 - 4ac = 24$
 $\therefore$ real and distinct $\therefore$ not real $\therefore$ not real $\therefore$ real and distinct

e $b^2 - 4ac = 0$ **f** $b^2 - 4ac = 13$ **g** $b^2 - 4ac = 53$ **h** $b^2 - 4ac = -7$
 $\therefore$ real and equal $\therefore$ real and distinct $\therefore$ real and distinct $\therefore$ not real

i $b^2 - 4ac = 4$ **j** $b^2 - 4ac = -11$ **k** $b^2 - 4ac = 0$ **l** $b^2 - 4ac = -3$
 $\therefore$ real and distinct $\therefore$ not real $\therefore$ real and equal $\therefore$ not real

m $b^2 - 4ac = -7$ **n** $b^2 - 4ac = \frac{13}{9}$ **o** $b^2 - 4ac = \frac{1}{16}$ **p** $b^2 - 4ac = -\frac{13}{75}$
 $\therefore$ not real $\therefore$ real and distinct $\therefore$ real and distinct $\therefore$ not real

7 equal roots
 $\therefore b^2 - 4ac = 0$
 $1 - 4p = 0$
 $p = \frac{1}{4}$

8 repeated root
 $\therefore b^2 - 4ac = 0$
 $4q^2 + 4q = 0$
 $4q(q + 1) = 0$
 $q \neq 0 \therefore q = -1$

9 $x^2 + rx - 2x + 4 = 0$ has equal roots
 $\therefore b^2 - 4ac = 0$
 $(r - 2)^2 - 16 = 0$
 $r^2 - 4r - 12 = 0$
 $(r + 2)(r - 6) = 0$
 $r = -2$ or 6

1 a $= 2x(10 - x - 3x^2)$
 $= 2x(2 + x)(5 - 3x)$
 b $2x(2 + x)(5 - 3x) = 0$
 $x = -2, 0$ or $\frac{5}{3}$

3 a $x^2 - 5 = 4x$
 $x^2 - 4x - 5 = 0$
 $(x + 1)(x - 5) = 0$
 $x = -1$ or 5
 b $9 - (5 - x) = 2x(5 - x)$
 $2x^2 - 9x + 4 = 0$
 $(2x - 1)(x - 4) = 0$
 $x = \frac{1}{2}$ or 4

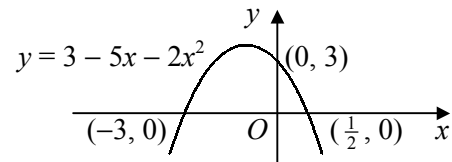
5 $x = \frac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
 $= \frac{-5\sqrt{2} \pm \sqrt{98}}{4}$
 $= \frac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
 $= -3\sqrt{2}$ or $\frac{1}{2}\sqrt{2}$

7 $y^2 - 10y + 16 = 0$
 $(y - 2)(y - 8) = 0$
 $y = 2^x = 2$ or 8
 $x = 1$ or 3

9 a $f(x) = -[x^2 - 4x] + 3$
 $= -[(x - 2)^2 - 4] + 3$
 $= -(x - 2)^2 + 7$
 b turning point is $(2, 7)$
 c $-(x - 2)^2 + 7 = 2$
 $(x - 2)^2 = 5$
 $x = 2 \pm \sqrt{5}$

2 a $AB^2 = (6 + 2)^2 + (k - 1)^2 = 64 + k^2 - 2k + 1$
 $= k^2 - 2k + 65$
 b $k^2 - 2k + 65 = 10^2 = 100$
 $k^2 - 2k - 35 = 0$
 $(k + 5)(k - 7) = 0$
 $k = -5$ or 7

4 a $y = -2[x^2 + \frac{5}{2}x] + 3$
 $= -2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3$
 $= -2(x + \frac{5}{4})^2 + \frac{49}{8}$
 $\therefore$ turning point is $(-\frac{5}{4}, \frac{49}{8})$
 b $3 - 5x - 2x^2 = 0$
 $2x^2 + 5x - 3 = 0$
 $(2x - 1)(x + 3) = 0, x = -3$ or $\frac{1}{2}$



6 a $y = 3[x^2 - 3x] + k = 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + k$
 $= 3(x - \frac{3}{2})^2 - \frac{27}{4} + k$
 $\therefore$ x-coordinate of $P = \frac{3}{2}$
 b y-coord of $P = k - \frac{27}{4} = \frac{17}{4} \therefore k = 11$
 $\therefore$ curve is $y = 3x^2 - 9x + 11$
 $\therefore$ coordinates of Q are $(0, 11)$

8 equal roots $\therefore b^2 - 4ac = 0$
 $4 - 4k(3 - 2k) = 0$
 $2k^2 - 3k + 1 = 0$
 $(2k - 1)(k - 1) = 0$
 $k = \frac{1}{2}$ or 1

10 a $x = \frac{5 \pm \sqrt{25 - 12}}{6}$
 $= \frac{1}{6}(5 \pm \sqrt{13})$
 b $x(x - 1) = 3(x + 2)$
 $x^2 - 4x - 6 = 0$
 $x = \frac{4 \pm \sqrt{16 + 24}}{2} = \frac{4 \pm 2\sqrt{10}}{2}$
 $= 2 \pm \sqrt{10}$

11 a $(x - 2k)^2 - 4k^2 + 6 = 0$

$$(x - 2k)^2 = 4k^2 - 6$$

$$x - 2k = \pm\sqrt{4k^2 - 6}$$

$$x = 2k \pm \sqrt{4k^2 - 6}$$

b $k = 3$

$$\therefore x = 6 \pm \sqrt{36 - 6}$$

$$= 6 \pm \sqrt{30}$$

12 a $x^2 - 6x - 3 = 0$

$$x = \frac{6 \pm \sqrt{36 + 12}}{2} = \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}$$

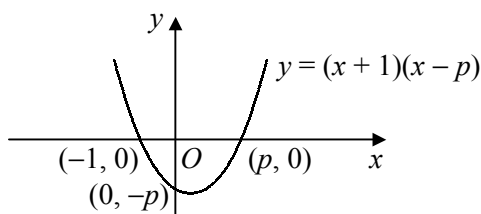
b $y(2y^2 + y - 15) = 0$

$$y(2y - 5)(y + 3) = 0$$

$$y = -3, 0 \text{ or } \frac{5}{2}$$

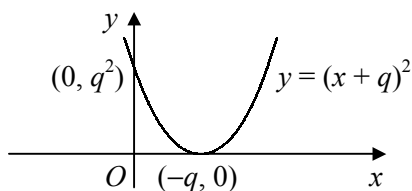
13 a $x = 0 \Rightarrow y = -p$

$$y = 0 \Rightarrow x = -1 \text{ or } p$$



b $x = 0 \Rightarrow y = q^2$

$$y = 0 \Rightarrow x = -q \quad [-q > 0]$$



15 a $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \Rightarrow 2t^2 + t - 6 = 0$
 $(2t - 3)(t + 2) = 0$

$$t = -2 \text{ or } \frac{3}{2}$$

but $x = t^3 \therefore x = -8 \text{ or } \frac{27}{8}$

16 a $= (k - 4)^2 - 16 + 20$

$$= (k - 4)^2 + 4$$

b $x^2 - kx + 2k - 5 = 0$

$$\text{discriminant} = b^2 - 4ac$$

$$= k^2 - 4(2k - 5)$$

$$= k^2 - 8k + 20$$

using a $= (k - 4)^2 + 4$

for all real k , $(k - 4)^2 \geq 0$

$$\therefore \text{discriminant} > 0$$

$$\therefore \text{real and distinct roots for all real } k$$

17 a $(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^2(x^2 - 3x - 4) + 2x(x^2 - 3x - 4) - 3(x^2 - 3x - 4)$
 $\equiv x^4 - 3x^3 - 4x^2 + 2x^3 - 6x^2 - 8x - 3x^2 + 9x + 12$
 $\equiv x^4 - x^3 - 13x^2 + x + 12$

b $(x^2 + 2x - 3)(x^2 - 3x - 4) = 0$

$$(x + 3)(x - 1)(x + 1)(x - 4) = 0$$

$$x = -3, -1, 1 \text{ or } 4$$

- 1**
- a** $3x = 2x + 1$
 $x = 1$
 $\therefore x = 1, y = 3$
- b** $x - 6 = \frac{1}{2}x - 4$
 $x = 4$
 $\therefore x = 4, y = -2$
- c** $2x + 6 = 3 - 4x$
 $x = -\frac{1}{2}$
 $\therefore x = -\frac{1}{2}, y = 5$
- d** subtracting
 $y + 4 = 0$
 $y = -4$
 $\therefore x = 7, y = -4$
- e** $2x + 4y + 22 = 0$
 $2x - 3y + 1 = 0$
subtracting
 $7y + 21 = 0$
 $y = -3$
 $\therefore x = -5, y = -3$
- f** $6x + 6y + 8 = 0$
 $15x - 6y - 15 = 0$
adding
 $21x - 7 = 0$
 $x = \frac{1}{3}$
 $\therefore x = \frac{1}{3}, y = -\frac{5}{3}$
- 2**
- a** $x + 2 = x^2 - 4$
 $x^2 - x - 6 = 0$
 $(x + 2)(x - 3) = 0$
 $x = -2$ or 3
 $\therefore (-2, 0)$ and $(3, 5)$
- b** $4x + 11 = x^2 + 3x - 1$
 $x^2 - x - 12 = 0$
 $(x + 3)(x - 4) = 0$
 $x = -3$ or 4
 $\therefore (-3, -1)$ and $(4, 27)$
- c** $2x - 1 = 2x^2 + 3x - 7$
 $2x^2 + x - 6 = 0$
 $(2x - 3)(x + 2) = 0$
 $x = -2$ or $\frac{3}{2}$
 $\therefore (-2, -5)$ and $(\frac{3}{2}, 2)$
- 3**
- a** subtracting
 $x^2 - x - 2 = 0$
 $(x + 1)(x - 2) = 0$
 $x = -1$ or 2
 $\therefore x = -1, y = 4$
or $x = 2, y = 7$
- b** adding
 $2x^2 - 7x + 3 = 0$
 $(2x - 1)(x - 3) = 0$
 $x = \frac{1}{2}$ or 3
 $\therefore x = \frac{1}{2}, y = -\frac{7}{2}$
or $x = 3, y = -6$
- c** $y = 2x - 5$
sub
 $x^2 + (2x - 5)^2 = 25$
 $x^2 - 4x = 0$
 $x(x - 4) = 0$
 $x = 0$ or 4
 $\therefore x = 0, y = -5$
or $x = 4, y = 3$
- d** $y = 2x + 10$
sub.
 $x^2 + 2x(2x + 10) + 15 = 0$
 $x^2 + 4x + 3 = 0$
 $(x + 3)(x + 1) = 0$
 $x = -3$ or -1
 $\therefore x = -3, y = 4$
or $x = -1, y = 8$
- e** $y = 1 - x$
sub.
 $x^2 - 2x(1 - x) - (1 - x)^2 = 7$
 $x^2 = 4$
 $x = \pm 2$
 $\therefore x = -2, y = 3$
or $x = 2, y = -1$
- f** $y = 1 - x$
sub.
 $3x^2 - x - (1 - x)^2 = 0$
 $2x^2 + x - 1 = 0$
 $(2x - 1)(x + 1) = 0$
 $x = -1$ or $\frac{1}{2}$
 $\therefore x = -1, y = 2$
or $x = \frac{1}{2}, y = \frac{1}{2}$
- g** $y = 4 - x$
sub.
 $2x^2 + x(4 - x) + (4 - x)^2 = 22$
 $x^2 - 2x - 3 = 0$
 $(x + 1)(x - 3) = 0$
 $x = -1$ or 3
 $\therefore x = -1, y = 5$
or $x = 3, y = 1$
- h** $x = 2y$
sub.
 $(2y)^2 - 4y - y^2 = 0$
 $3y^2 - 4y = 0$
 $y(3y - 4) = 0$
 $y = 0$ or $\frac{4}{3}$
 $\therefore x = 0, y = 0$
or $x = \frac{8}{3}, y = \frac{4}{3}$
- i** $y = 3 - \frac{3}{2}x$
sub.
 $x^2 + x(3 - \frac{3}{2}x) = 4$
 $x^2 - 6x + 8 = 0$
 $(x - 2)(x - 4) = 0$
 $x = 2$ or 4
 $\therefore x = 2, y = 0$
or $x = 4, y = -3$

j $y = 2x - 3$

sub.

$2x^2 + (2x - 3) - (2x - 3)^2 = 8$

$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x = 2 \text{ or } 5$

$\therefore x = 2, y = 1$

or $x = 5, y = 7$

k $y = 2x - 7$

sub.

$x^2 - x(2x - 7) + (2x - 7)^2 = 13$

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3 \text{ or } 4$

$\therefore x = 3, y = -1$

or $x = 4, y = 1$

l $y = 5 - 3x$

sub.

$x^2 - 5x + (5 - 3x)^2 = 0$

$2x^2 - 7x + 5 = 0$

$(2x - 5)(x - 1) = 0$

$x = 1 \text{ or } \frac{5}{2}$

$\therefore x = 1, y = 2$

or $x = \frac{5}{2}, y = -\frac{5}{2}$

m $x = 2y + 10$

sub.

$3(2y + 10)^2 - y(2y + 10) + y^2 = 36$

$y^2 + 10y + 24 = 0$

$(y + 6)(y + 4) = 0$

$y = -6 \text{ or } -4$

$\therefore x = -2, y = -6$

or $x = 2, y = -4$

n $y = \frac{3}{2}x - 2$

sub.

$2x^2 + x - 4(\frac{3}{2}x - 2) = 6$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2} \text{ or } 2$

$\therefore x = \frac{1}{2}, y = -\frac{5}{4}$

or $x = 2, y = 1$

o $x = 3y - 17$

sub.

$(3y - 17)^2 + (3y - 17) + 2y^2 - 52 = 0$

$y^2 - 9y + 20 = 0$

$(y - 4)(y - 5) = 0$

$y = 4 \text{ or } 5$

$\therefore x = -5, y = 4$

or $x = -2, y = 5$

4 a subtracting

$-\frac{1}{y} + 2y + 1 = 0$

$-1 + 2y^2 + y = 0$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = -1 \text{ or } \frac{1}{2}$

$\therefore x = -5, y = -1$

or $x = 4, y = \frac{1}{2}$

b $y = x - 5$

sub.

$x(x - 5) = 6$

$x^2 - 5x - 6 = 0$

$(x + 1)(x - 6) = 0$

$x = -1 \text{ or } 6$

$\therefore x = -1, y = -6$

or $x = 6, y = 1$

c $y = 7 - 4x$

sub.

$\frac{3}{x} - 2(7 - 4x) + 4 = 0$

$3 - 2x(7 - 4x) + 4x = 0$

$8x^2 - 10x + 3 = 0$

$(4x - 3)(2x - 1) = 0$

$x = \frac{1}{2} \text{ or } \frac{3}{4}$

$\therefore x = \frac{1}{2}, y = 5$

or $x = \frac{3}{4}, y = 4$

5 $5 - x = x^2 - 3x + 2$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1 \text{ or } 3$

 P and Q are the points $(-1, 6)$ and $(3, 2)$

$PQ^2 = (3 + 1)^2 + (2 - 6)^2$

$PQ = \sqrt{32} = 4\sqrt{2}$

6 $3^{x-1} = (3^2)^{2y} \therefore x - 1 = 4y$

$(2^3)^{x-2} = (2^2)^{1+y} \therefore 3x - 6 = 2 + 2y$

$6x - 16 = 4y$

$\Rightarrow 6x - 16 = x - 1$

$x = 3$

$\therefore x = 3, y = \frac{1}{2}$

7 $AB - A\sqrt{3} + 2B\sqrt{3} - 6 \equiv 9\sqrt{3} - 1$

A and B integers $\therefore AB - 6 = -1$ (1) and $-A + 2B = 9$ (2)

(2) $\Rightarrow A = 2B - 9$

sub. (1) $B(2B - 9) - 6 = -1 \Rightarrow 2B^2 - 9B - 5 = 0$

$(2B + 1)(B - 5) = 0$

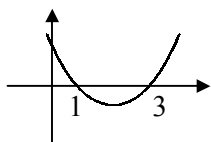
$B = -\frac{1}{2} \text{ or } 5$

B integer $\therefore B = 5 \Rightarrow A = 1, B = 5$

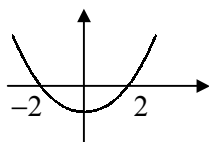
- 1 **a** $2x < 6$
 $x < 3$ **b** $3x \geq 21$
 $x \geq 7$ **c** $2x > 8$
 $x > 4$ **d** $3x \leq 36$
 $x \leq 12$
- e** $5x \geq -15$ **f** $\frac{1}{3}x < 1$ **g** $9x \geq 54$ **h** $3x < -4$
 $x \geq -3$ $x < 3$ $x \geq 6$ $x < -\frac{4}{3}$
- i** $x < 14$ **j** $4x \leq -10$ **k** $2 < 3x$ **l** $5 \geq \frac{1}{2}x$
 $x \leq -\frac{5}{2}$ $x > \frac{2}{3}$ $x \leq 10$

- 2 **a** $y > 7$ **b** $4p \leq 2$
 $p \leq \frac{1}{2}$ **c** $6 < 2x$
 $x > 3$
- d** $2a \geq 4$ **e** $15 < 3u$ **f** $2b \geq 9$
 $a \geq 2$ $u > 5$ $b \geq \frac{9}{2}$
- g** $3x < -18$ **h** $y \geq -13$ **i** $-20 \leq 4p$
 $x < -6$ $p \geq -5$
- j** $r - 2 > 6$ **k** $3 - 6t \leq t - 4$ **l** $6 + 2x \geq 24 - 4x$
 $r > 8$ $7 \leq 7t$ $6x \geq 18$
 $t \geq 1$ $x \geq 3$
- m** $7y + 21 - 6y + 2 < 0$ **n** $20 - 8x > 21 - 6x$ **o** $12u - 3 - 5u + 15 < 9$
 $y < -23$ $-1 > 2x$ $7u < -3$
 $x < -\frac{1}{2}$ $u < -\frac{3}{7}$

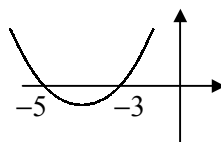
- 3 **a** $(x-1)(x-3) < 0$ **b** $(x+2)(x-2) \leq 0$ **c** $(x+5)(x+3) < 0$ **d** $x^2 + 2x - 8 \leq 0$
 $(x+4)(x-2) \leq 0$



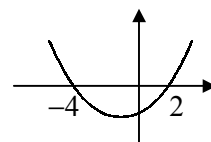
$$\therefore 1 < x < 3$$



$$\therefore -2 \leq x \leq 2$$

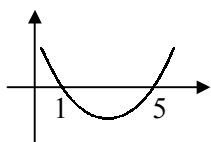


$$\therefore -5 < x < -3$$

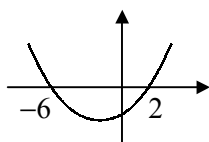


$$\therefore -4 \leq x \leq 2$$

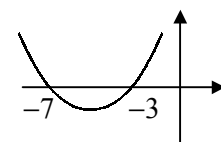
- e** $(x-1)(x-5) > 0$ **f** $x^2 + 4x - 12 > 0$
 $(x+6)(x-2) > 0$ **g** $(x+7)(x+3) \geq 0$ **h** $x^2 - 9x - 22 < 0$
 $(x+2)(x-11) < 0$



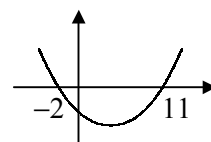
$$\therefore x < 1 \text{ or } x > 5$$



$$\therefore x < -6 \text{ or } x > 2$$

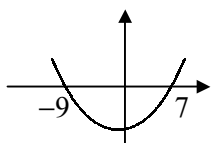


$$\therefore x \leq -7 \text{ or } x \geq -3$$

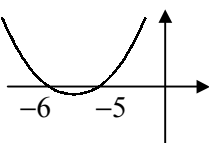


$$\therefore -2 < x < 11$$

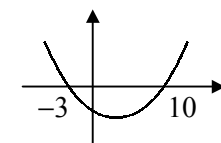
- i** $x^2 + 2x - 63 \geq 0$
 $(x+9)(x-7) \geq 0$ **j** $(x+6)(x+5) > 0$ **k** $x^2 - 7x - 30 < 0$
 $(x+3)(x-10) < 0$ **l** $x^2 - 20x + 91 \geq 0$
 $(x-7)(x-13) \geq 0$



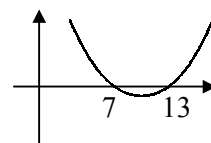
$$\therefore x \leq -9 \text{ or } x \geq 7$$



$$\therefore x < -6 \text{ or } x > -5$$

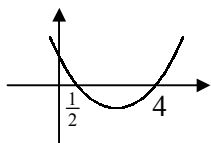


$$\therefore -3 < x < 10$$



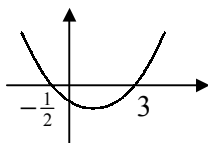
$$\therefore x \leq 7 \text{ or } x \geq 13$$

4 a $(2x - 1)(x - 4) \leq 0$



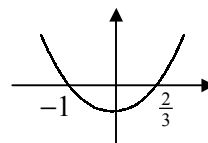
$$\therefore \frac{1}{2} \leq x \leq 4$$

b $(2r + 1)(r - 3) < 0$



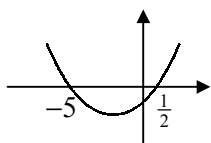
$$\therefore -\frac{1}{2} < r < 3$$

c $3p^2 + p - 2 \leq 0$
 $(3p - 2)(p + 1) \leq 0$



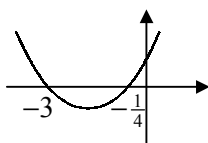
$$\therefore -1 \leq p \leq \frac{2}{3}$$

d $(2y - 1)(y + 5) > 0$



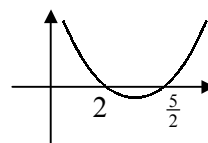
$$\therefore y < -5 \text{ or } y > \frac{1}{2}$$

e $(4m + 1)(m + 3) < 0$



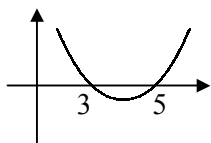
$$\therefore -3 < m < -\frac{1}{4}$$

f $2x^2 - 9x + 10 \geq 0$
 $(2x - 5)(x - 2) \geq 0$



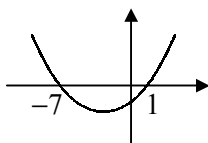
$$\therefore x \leq 2 \text{ or } x \geq \frac{5}{2}$$

g $a^2 - 8a + 15 < 0$
 $(a - 3)(a - 5) < 0$



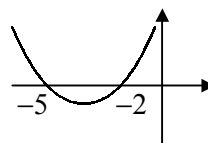
$$\therefore 3 < a < 5$$

h $x^2 + 4x \leq 7 - 2x$
 $x^2 + 6x - 7 \leq 0$
 $(x + 7)(x - 1) \leq 0$



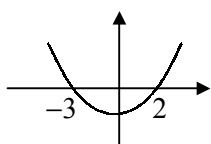
$$\therefore -7 \leq x \leq 1$$

i $y^2 + 9y > 2y - 10$
 $y^2 + 7y + 10 > 0$
 $(y + 5)(y + 2) > 0$



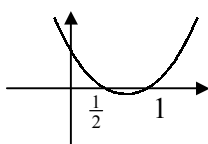
$$\therefore y < -5 \text{ or } y > -2$$

j $2x^2 + x > x^2 + 6$
 $x^2 + x - 6 > 0$
 $(x + 3)(x - 2) < 0$



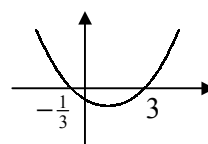
$$\therefore -3 < x < 2$$

k $5u - 6u^2 < 3 - 4u$
 $2u^2 - 3u + 1 > 0$
 $(2u - 1)(u - 1) > 0$



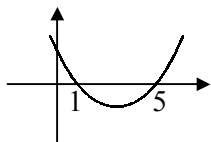
$$\therefore u < \frac{1}{2} \text{ or } u > 1$$

l $2t + 3 \geq 3t^2 - 6t$
 $3t^2 - 8t - 3 \leq 0$
 $(3t + 1)(t - 3) \leq 0$



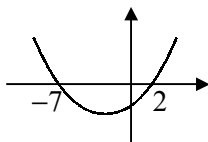
$$\therefore -\frac{1}{3} \leq t \leq 3$$

m $y^2 - 4y + 4 \leq 2y - 1$
 $y^2 - 6y + 5 \leq 0$
 $(y - 1)(y - 5) \leq 0$



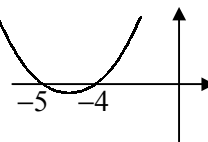
$$\therefore 1 \leq y \leq 5$$

n $p^2 + 5p + 6 \geq 20$
 $p^2 + 5p - 14 \geq 0$
 $(p + 7)(p - 2) \geq 0$



$$\therefore p \leq -7 \text{ or } p \geq 2$$

o $26 + 4x < 6 - 5x - x^2$
 $x^2 + 9x + 20 < 0$
 $(x + 5)(x + 4) < 0$

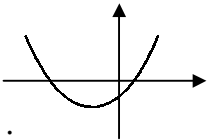


$$\therefore -5 < x < -4$$

- 5**
- a** for critical values

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

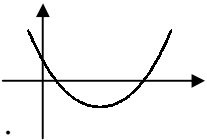
$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$


$$\therefore -1 - \sqrt{2} < x < -1 + \sqrt{2}$$
- b** for critical values

$$x = \frac{6 \pm \sqrt{36-16}}{2}$$

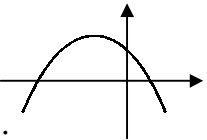
$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$


$$\therefore x < 3 - \sqrt{5} \text{ or } x > 3 + \sqrt{5}$$
- c** for critical values

$$x = \frac{6 \pm \sqrt{36+44}}{-2}$$

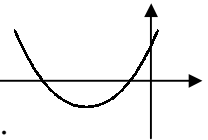
$$x = \frac{6 \pm 4\sqrt{5}}{-2}$$

$$x = -3 \pm 2\sqrt{5}$$


$$\therefore -3 - 2\sqrt{5} < x < -3 + 2\sqrt{5}$$
- d** for critical values

$$x = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$x = -2 \pm \sqrt{3}$$


$$\therefore x \leq -2 - \sqrt{3} \text{ or } x \geq -2 + \sqrt{3}$$
- 6**
- a** equal roots

$$\therefore b^2 - 4ac = 0$$

$$36 - 4k = 0$$

$$k = 9$$
- b** real and distinct roots

$$\therefore b^2 - 4ac > 0$$

$$4 - 4k > 0$$

$$4 > 4k$$

$$k < 1$$
- c** no real roots

$$\therefore b^2 - 4ac < 0$$

$$9 - 4k < 0$$

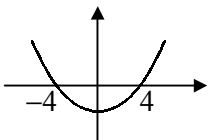
$$9 < 4k$$

$$k > \frac{9}{4}$$
- d** real roots

$$\therefore b^2 - 4ac \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k+4)(k-4) \geq 0$$

$$k \leq -4 \text{ or } k \geq 4$$

- e** equal roots

$$\therefore b^2 - 4ac = 0$$

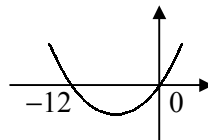
$$1 + 4k = 0$$

$$k = -\frac{1}{4}$$
- f** no real roots

$$\therefore b^2 - 4ac < 0$$

$$k^2 + 12k < 0$$

$$k(k+12) < 0$$

$$-12 < k < 0$$

- g** real and distinct roots

$$\therefore b^2 - 4ac > 0$$

$$4 - 4(k-2) > 0$$

$$12 > 4k$$

$$k < 3$$
- h** equal roots

$$\therefore b^2 - 4ac = 0$$

$$k^2 - 8k = 0$$

$$k(k-8) = 0$$

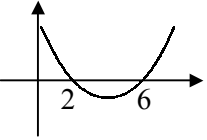
$$k = 0 \text{ or } 8$$
- i** no real roots

$$\therefore b^2 - 4ac < 0$$

$$k^2 - 4(2k-3) < 0$$

$$k^2 - 8k + 12 < 0$$

$$(k-2)(k-6) < 0$$

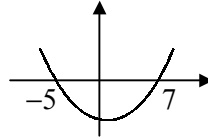
$$2 < k < 6$$

- j** real roots

$$\therefore b^2 - 4ac \geq 0$$

$$(k-1)^2 - 36 \geq 0$$

$$k^2 - 2k - 35 \geq 0$$

$$(k+5)(k-7) \geq 0$$

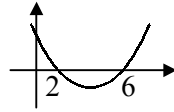
$$k \leq -5 \text{ or } k \geq 7$$


1 a $4 > \frac{3}{2}y$

$y < \frac{8}{3}$

b $(x-2)(x-6) \geq 0$

$\therefore x \leq 2 \text{ or } x \geq 6$

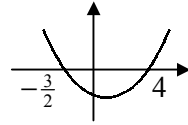


2 $2n^2 - 5n - 12 < 0$

$(2n+3)(n-4) < 0$

$-\frac{3}{2} < n < 4$

n integer $\therefore n = -1, 0, 1, 2, 3$



3 a $(x+8) \geq 1.5 \times x$

$8 \geq 0.5x$

$x \leq 16$

b $x(x+8) \geq 180$

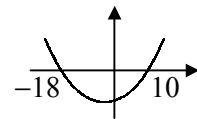
$x^2 + 8x - 180 \geq 0$

$(x+18)(x-10) \geq 0$

$x \leq -18 \text{ or } x \geq 10$

but $x > 0$ (width > 0)

and $x \leq 16 \therefore 10 \leq x \leq 16$

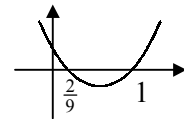


4 $9x^2 - 6x + 1 < 5x - 1$

$9x^2 - 11x + 2 < 0$

$(9x-2)(x-1) < 0$

$\frac{2}{9} < x < 1$



5 $x = y + 8$

sub. $y(y+8) \leq 240$

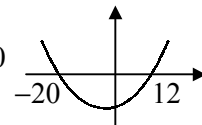
$y^2 + 8y - 240 \leq 0$

$(y+20)(y-12) \leq 0$

$-20 \leq y \leq 12$

$x + y = y + 8 + y = 2y + 8$

$\therefore \text{max value of } (x+y) = 2(12) + 8 = 32$

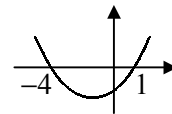


6 $3t^2 - 11t - 4 \geq 2t^2 - 14t$

$t^2 + 3t - 4 \geq 0$

$(t+4)(t-1) \geq 0$

$t \leq -4 \text{ or } t \geq 1$



7 a $2x^2 + 2x - kx + 8 = 0$

real and distinct roots

$\therefore b^2 - 4ac > 0$

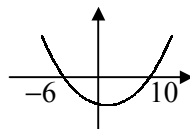
$(2-k)^2 - 64 > 0$

$4 - 4k + k^2 - 64 > 0$

$k^2 - 4k - 60 > 0$

b $(k+6)(k-10) > 0$

$k < -6 \text{ or } k > 10$



8 let height be $h \therefore h^2 = (3r-4)^2 - r^2$

but $h \leq 24$

$\therefore h^2 \leq 24^2$

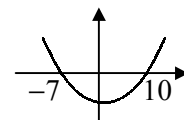
$(3r-4)^2 - r^2 \leq 576$

$r^2 - 3r - 70 \leq 0$

$(r+7)(r-10) \leq 0$

$-7 \leq r \leq 10$

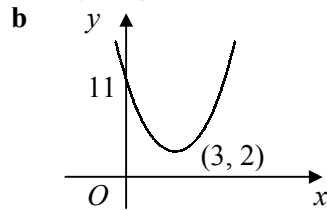
$\therefore \text{maximum value of } r = 10$



1 a $2^{x-1} = 2^4$
 $x - 1 = 4$
 $x = 5$

b $3^y - 10 = 17$
 $3^y = 27$
 $y = 3$

2 a $= (x - 3)^2 - 9 + 11$
 $= (x - 3)^2 + 2$



3 a $= \left(\frac{49}{4}\right)^{-\frac{1}{2}} = \sqrt{\frac{4}{49}} = \frac{2}{7}$

b $3x^{-3} = \frac{64}{9}$

$$x^3 = \frac{27}{64}$$

$$x = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

4 $2x\sqrt{3} + 9 = x\sqrt{3}$

$$x\sqrt{3} = -9$$

$$x = \frac{-9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -3\sqrt{3}$$

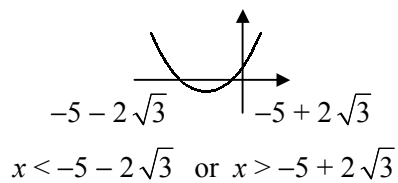
5 a $x = \frac{-10 \pm \sqrt{100 - 52}}{2}$

$$= \frac{-10 \pm \sqrt{48}}{2}$$

$$= \frac{-10 \pm 4\sqrt{3}}{2}$$

$$= -5 \pm 2\sqrt{3}$$

b



6 a $42x - 49 = 9x^2$

$$9x^2 - 42x + 49 = 0$$

$$(3x - 7)^2 = 0$$

$$x = \frac{7}{3}$$

b $2 + (y + 1) = 2y(y + 1)$

$$2y^2 + y - 3 = 0$$

$$(2y + 3)(y - 1) = 0$$

$$y = -\frac{3}{2} \text{ or } 1$$

7 $y = x + 3$

sub.

$$3x^2 - 2x(x + 3) + (x + 3)^2 - 17 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\therefore x = -2, y = 1 \text{ or } x = 2, y = 5$$

8 a $x^{\frac{1}{3}} = \sqrt[3]{64} = 4$

$$x = 4^2 = 16$$

b $\frac{\sqrt{3}+1}{2\sqrt{3}-3} = \frac{\sqrt{3}+1}{2\sqrt{3}-3} \times \frac{2\sqrt{3}+3}{2\sqrt{3}+3} = \frac{(\sqrt{3}+1)(2\sqrt{3}+3)}{12-9}$

$$= \frac{1}{3}(6 + 3\sqrt{3} + 2\sqrt{3} + 3)$$

$$= 3 + \frac{5}{3}\sqrt{3}$$

$$\therefore a = 3, b = \frac{5}{3}$$

9 a let A be (2, 4)

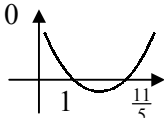
$$\therefore AP^2 = (2k - 2)^2 + (k - 4)^2$$

$$AP < 3 \therefore (2k - 2)^2 + (k - 4)^2 < 9$$

$$5k^2 - 16k + 11 < 0$$

b $(5k - 11)(k - 1) < 0$

$$1 < k < \frac{11}{5}$$



10 a $2x \leq 7$

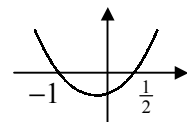
$$x \leq \frac{7}{2}$$

b $2x^2 + x < 1$

$$2x^2 + x - 1 < 0$$

$$(2x - 1)(x + 1) < 0$$

$$-1 < x < \frac{1}{2}$$



$$\begin{aligned}
 11 \quad \mathbf{a} \quad f(x) &= 2[x^2 - 4x] + 5 \\
 &= 2[(x-2)^2 - 4] + 5 \\
 &= 2(x-2)^2 - 3
 \end{aligned}$$

$$\mathbf{b} \quad (2, -3)$$

$$\begin{aligned}
 \mathbf{c} \quad 2(x-2)^2 - 3 &= 0 \\
 x-2 &= \pm \sqrt{\frac{3}{2}} \\
 x &= 2 \pm \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 x &= 2 \pm \frac{1}{2}\sqrt{6}
 \end{aligned}$$

13 no real roots

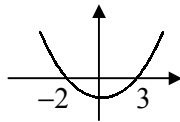
$$\therefore b^2 - 4ac < 0$$

$$4k^2 - 4(k+6) < 0$$

$$k^2 - k - 6 < 0$$

$$(k+2)(k-3) < 0$$

$$-2 < k < 3$$



$$\begin{aligned}
 12 \quad \mathbf{a} \quad &= 2\sqrt{3} - \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= 2\sqrt{3} - \frac{5}{3}\sqrt{3} \\
 &= \frac{1}{3}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &= \frac{64x\sqrt{x}}{16x} \\
 &= 4\sqrt{x}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \mathbf{a} \quad AM &= \frac{1}{2}AC = 2 + 2\sqrt{3} \\
 BM^2 &= AB^2 - AM^2 \\
 &= (4 + \sqrt{3})^2 - (2 + 2\sqrt{3})^2 \\
 &= 16 + 8\sqrt{3} + 3 - (4 + 8\sqrt{3} + 12) = 3 \\
 \therefore BM &= \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &= \frac{1}{2} \times AC \times BM \\
 &= \frac{1}{2} \times (4 + 4\sqrt{3}) \times \sqrt{3} \\
 &= \frac{1}{2}(4\sqrt{3} + 12) = 6 + 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad (2^2)^{2y+7} &= (2^3)^{y+3} \\
 4y + 14 &= 3y + 9 \\
 y &= -5
 \end{aligned}$$

$$\begin{aligned}
 16 \quad \text{LHS} &= x^2(2x^2 - 3x - 9) - x(2x^2 - 3x - 9) \\
 &\quad + 3(2x^2 - 3x - 9) \\
 &= 2x^4 - 3x^3 - 9x^2 - 2x^3 + 3x^2 + 9x \\
 &\quad + 6x^2 - 9x - 27 \\
 &= 2x^4 - 5x^3 - 27 \\
 \therefore A &= 2, B = -5 \text{ and } C = -27
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \mathbf{a} \quad x^2 + 4x + k &= 0 \\
 (x+2)^2 - 4 + k &= 0 \\
 x+2 &= \pm \sqrt{4-k} \\
 x &= -2 \pm \sqrt{4-k}
 \end{aligned}$$

$\mathbf{b}$ real roots only if $4 - k \geq 0$

$$\therefore k \leq 4$$

$$\mathbf{c} \quad k = -4$$

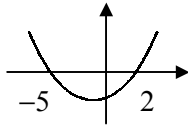
$$\begin{aligned}
 \therefore x &= -2 \pm \sqrt{8} \\
 x &= -2 \pm 2\sqrt{2}
 \end{aligned}$$

1 $x^2 + 3x + 2 \leq 12$

$$x^2 + 3x - 10 \leq 0$$

$$(x + 5)(x - 2) \leq 0$$

$$-5 \leq x \leq 2$$



2 a $= 8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$

b $= x + 12\sqrt{x} + 36 + 4x - 12\sqrt{x} + 9$
 $= 5x + 45$

3 a $(-2, 0) \Rightarrow 0 = 8 - 2p + q$ (1)

$(3, 0) \Rightarrow 0 = 18 + 3p + q$ (2)

$(2) - (1) \quad 0 = 10 + 5p \Rightarrow p = -2$

sub. $\Rightarrow q = -12$

b x-coord $= \frac{-2+3}{2} = \frac{1}{2}$

$\therefore y = -\frac{25}{2} \Rightarrow (\frac{1}{2}, -\frac{25}{2})$

4 $2x - 2\sqrt{32} = \sqrt{98} - x$

$$3x = 2\sqrt{32} + \sqrt{98}$$

$$3x = 8\sqrt{2} + 7\sqrt{2}$$

$$3x = 15\sqrt{2}$$

$$x = 5\sqrt{2}$$

5 a real and distinct roots

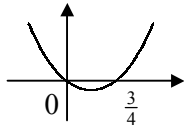
$$\therefore b^2 - 4ac > 0$$

$$16k^2 - 12k > 0$$

$$4k^2 - 3k > 0$$

$$k(4k - 3) > 0$$

b



$$k < 0 \text{ or } k > \frac{3}{4}$$

6 $(2^2)^{2x} = 2^{y-1}$

$$4x = y - 1 \quad (1)$$

$$(3^2)^{4x} = 3^{y+1}$$

$$8x = y + 1 \quad (2)$$

(1) and (2) $\Rightarrow y = 4x + 1 = 8x - 1$
 $4x = 2$
 $x = \frac{1}{2}, y = 3$

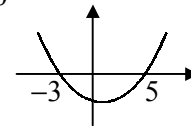
7 a LHS $= (x - \frac{7}{2})^2 - \frac{49}{4} + 9$

$$= (x - \frac{7}{2})^2 - \frac{13}{4}$$

$$\therefore a = -\frac{7}{2}, b = -\frac{13}{4}$$

b $x = \frac{7}{2}$

8 a $(y + 3)(y - 5) < 0$



$$-3 < y < 5$$

b $x(2 - x) = 4(x - 3)$

$$x^2 + 2x - 12 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 48}}{2} = \frac{-2 \pm 2\sqrt{13}}{2}$$

$$x = -1 \pm \sqrt{13}$$

9 $2^{x^2+2} = (2^3)^x$

$$x^2 + 2 = 3x$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1 \text{ or } 2$$

10 a $t - 2t^2 = 3t - 15$

$$2t^2 + 2t - 15 = 0$$

$$t = \frac{-2 \pm \sqrt{4 + 120}}{4} = \frac{-2 \pm \sqrt{124}}{4} = \frac{-2 \pm 2\sqrt{31}}{4}$$

$$t = \frac{1}{2}(-1 \pm \sqrt{31})$$

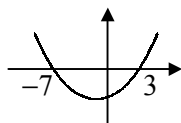
b $(x^2 + 2)(x^2 - 3) = 0$

$$x^2 = -2 \text{ [no solutions] or } 3$$

$$x = \pm\sqrt{3}$$

11 $x^2 + 4x - 21 \geq 0$
 $(x + 7)(x - 3) \geq 0$

$x \leq -7$ or $x \geq 3$



12 a $3^{2x+2} = 3^2(3^x)^2 = 9y^2$

b $9y^2 - 10y + 1 = 0$
 $(9y - 1)(y - 1) = 0$
 $y = 3^x = \frac{1}{9}, 1$
 $\therefore x = -2, 0$

13 a $= \sqrt{25 \times 3} = \sqrt{75}$

b $\sqrt{64} < \sqrt{75} < \sqrt{81}$

$\therefore 8 < 5\sqrt{3} < 9$

$\therefore n = 8$

14 $y = \frac{2x+7}{3}$

sub. $2x^2 - \left(\frac{2x+7}{3}\right)^2 - 7 = 0$

$18x^2 - (2x+7)^2 - 63 = 0$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = -2$ or 4

$\therefore x = -2, y = 1$ or $x = 4, y = 5$

15 a $= \sqrt{\frac{48}{12}} - \sqrt{\frac{600}{12}}$
 $= \sqrt{4} - \sqrt{50}$
 $= 2 - 5\sqrt{2}$

b $= \frac{\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{\sqrt{2}(4-3\sqrt{2})}{16-18}$
 $= -\frac{1}{2}(4\sqrt{2} - 6)$
 $= 3 - 2\sqrt{2}$

16 a $5^{x+1} = (5^2)^{y-3}$

$x + 1 = 2y - 6$

$y = \frac{x+7}{2}$

b $(4^2)^{x-1} = 4^z$

$2x - 2 = z$

$x = 2y - 7 \quad \therefore z = 2(2y - 7) - 2$

$z = 4y - 16$

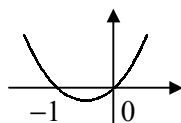
17 a $(x-k)^2 - k^2 - k = 0$

$x - k = \pm \sqrt{k^2 + k}$

$x = k \pm \sqrt{k^2 + k}$

b real roots $\therefore k^2 + k \geq 0$

$k(k+1) \geq 0$



$k \leq -1$ or $k \geq 0$

18 a $\frac{1}{y} - y = \frac{3}{2}$

$2 - 2y^2 = 3y$

$2y^2 + 3y - 2 = 0$

b $(2y-1)(y+2) = 0$

$y = -2, \frac{1}{2}$

$x = y^5 = -32, \frac{1}{32}$

1 a $= (\frac{3}{2})^2 = \frac{9}{4}$ or $2\frac{1}{4}$

b $x^{\frac{3}{2}} = 27$

$x^{\frac{1}{2}} = \sqrt[3]{27} = 3$

$x = 3^2 = 9$

2 $x = 16 - 3y$

sub. $(16 - 3y)^2 - y(16 - 3y) + 2y^2 = 46$

$y^2 - 8y + 15 = 0$

$(y - 3)(y - 5) = 0$

$y = 3$ or 5

$\therefore x = 1, y = 5$ or $x = 7, y = 3$

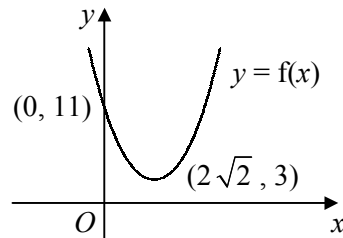
3 a $= 8\sqrt{3} - 4\sqrt{3} + 5\sqrt{3}$
 $= 9\sqrt{3}$

b $= 10 - 4\sqrt{3} + 5\sqrt{3} - 6$
 $= 4 + \sqrt{3}$

4 a $f(x) = (x - 2\sqrt{2})^2 - 8 + 11$
 $= (x - 2\sqrt{2})^2 + 3$

$\therefore a = 1, b = -2\sqrt{2}$ and $c = 3$

b turning point is $(2\sqrt{2}, 3)$

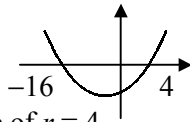


5 a $S.A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$
 $S.A \leq 128\pi \therefore 2\pi r^2 + 24\pi r \leq 128\pi$
 $r^2 + 12r \leq 64$
 $r^2 + 12r - 64 \leq 0$

b $(r + 16)(r - 4) \leq 0$

$-16 \leq r \leq 4$

$\therefore$ maximum value of $r = 4$



6 $8x\sqrt{x} = 4x$

$4x(2\sqrt{x} - 1) = 0$

$x \neq 0 \therefore \sqrt{x} = \frac{1}{2}$

$x = (\frac{1}{2})^2 = \frac{1}{4}$

7 a $x = 5$

b $(2^5)^{y+1} = (2^2)^y$

$5y + 5 = 2y$

$y = -\frac{5}{3}$

8 a $t^2 - 5t$

b $t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t = 2, 3$

$x = t^2 = 4, 9$

9 $x^2 + kx + 3 + k^2 = 0$

$\Rightarrow (x + \frac{1}{2}k)^2 - \frac{1}{4}k^2 + 3 + k^2 = 0$

$\Rightarrow x + \frac{1}{2}k = \pm \sqrt{-\frac{3}{4}k^2 - 3}$

$\Rightarrow x = -\frac{1}{2}k \pm \sqrt{-\frac{3}{4}k^2 - 3}$

real $k \Rightarrow k^2 \geq 0$

$\Rightarrow -\frac{3}{4}k^2 - 3 < 0$

$\therefore$ no real roots

10 a $(2^3)^{2x-1} = 2^5$

$6x - 3 = 5$

$x = \frac{4}{3}$

b $(3^{-1})^{y-2} = 3^4$

$-y + 2 = 4$

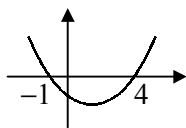
$y = -2$

11 $2x^2 - 7x < x^2 - 4x + 4$

$$x^2 - 3x - 4 < 0$$

$$(x+1)(x-4) < 0$$

$$-1 < x < 4$$



12
$$\frac{2}{3\sqrt{2}-4} = \frac{2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{2(3\sqrt{2}+4)}{18-16} = 3\sqrt{2} + 4$$

$$\frac{3-\sqrt{2}}{\sqrt{2}+1} = \frac{3-\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(3-\sqrt{2})(\sqrt{2}-1)}{2-1}$$

$$= (3-\sqrt{2})(\sqrt{2}-1)$$

$$= 3\sqrt{2} - 3 - 2 + \sqrt{2} = 4\sqrt{2} - 5$$

$$\therefore \frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1} = 3\sqrt{2} + 4 - (4\sqrt{2} - 5)$$

$$= 9 - \sqrt{2}$$

13 a $(3y-1)(2y+9) = 0$

$$y = -\frac{9}{2} \text{ or } \frac{1}{3}$$

b equal roots

$$\therefore b^2 - 4ac = 0$$

$$k^2 - 64 = 0$$

$$k = \pm 8$$

14 a i $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

ii $2^{x-1} = 2^{-1} \times 2^x = \frac{1}{2}y$

b let $y = 2^x \Rightarrow y^2 - 9(\frac{1}{2}y) + 2 = 0$

$$2y^2 - 9y + 4 = 0$$

$$(2y-1)(y-4) = 0$$

$$y = 2^x = \frac{1}{2} \text{ or } 4$$

$$x = -1 \text{ or } 2$$

15 $x = 3y + 1$

sub.

$$(3y+1)^2 + 2y(3y+1) + y^2 = 9$$

$$2y^2 + y - 1 = 0$$

$$(2y-1)(y+1) = 0$$

$$y = -1 \text{ or } \frac{1}{2}$$

$$\therefore (-2, -1) \text{ and } (\frac{5}{2}, \frac{1}{2})$$

16 a $(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + b = 0$

$$(x + \frac{1}{2}a)^2 = \frac{1}{4}a^2 - b = \frac{a^2 - 4b}{4}$$

$$x + \frac{1}{2}a = \pm \sqrt{\frac{a^2 - 4b}{4}} = \pm \frac{\sqrt{a^2 - 4b}}{2}$$

$$x = -\frac{1}{2}a \pm \frac{\sqrt{a^2 - 4b}}{2}$$

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

b for repeated root, $a^2 - 4b = 0$

$$\Rightarrow b = \frac{1}{4}a^2$$

17 a $f(x) = 2[x^2 - 6x] + 19$

$$= 2[(x-3)^2 - 9] + 19$$

$$= 2(x-3)^2 + 1$$

$$\text{real } x \Rightarrow (x-3)^2 \geq 0$$

$$\Rightarrow 2(x-3)^2 + 1 \geq 1$$

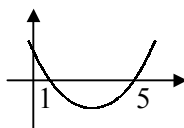
$$\Rightarrow f(x) \geq 1$$

b $2x^2 - 12x + 19 < 9$

$$x^2 - 6x + 5 < 0$$

$$(x-1)(x-5) < 0$$

$$1 < x < 5$$



18 a $= 1 - 2\sqrt{5} + 5$

$$= 6 - 2\sqrt{5}$$

b $y^2 = \frac{1}{2}(6 - 2\sqrt{5}) = \frac{1}{2}(1 - \sqrt{5})^2$

$$y = \pm \frac{1}{\sqrt{2}}(1 - \sqrt{5})$$

$$y = \pm \frac{1}{2}\sqrt{2}(1 - \sqrt{5})$$

$$y = \pm \frac{1}{2}(\sqrt{2} - \sqrt{10})$$

$$y = \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{10} \text{ or } -\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{10}$$