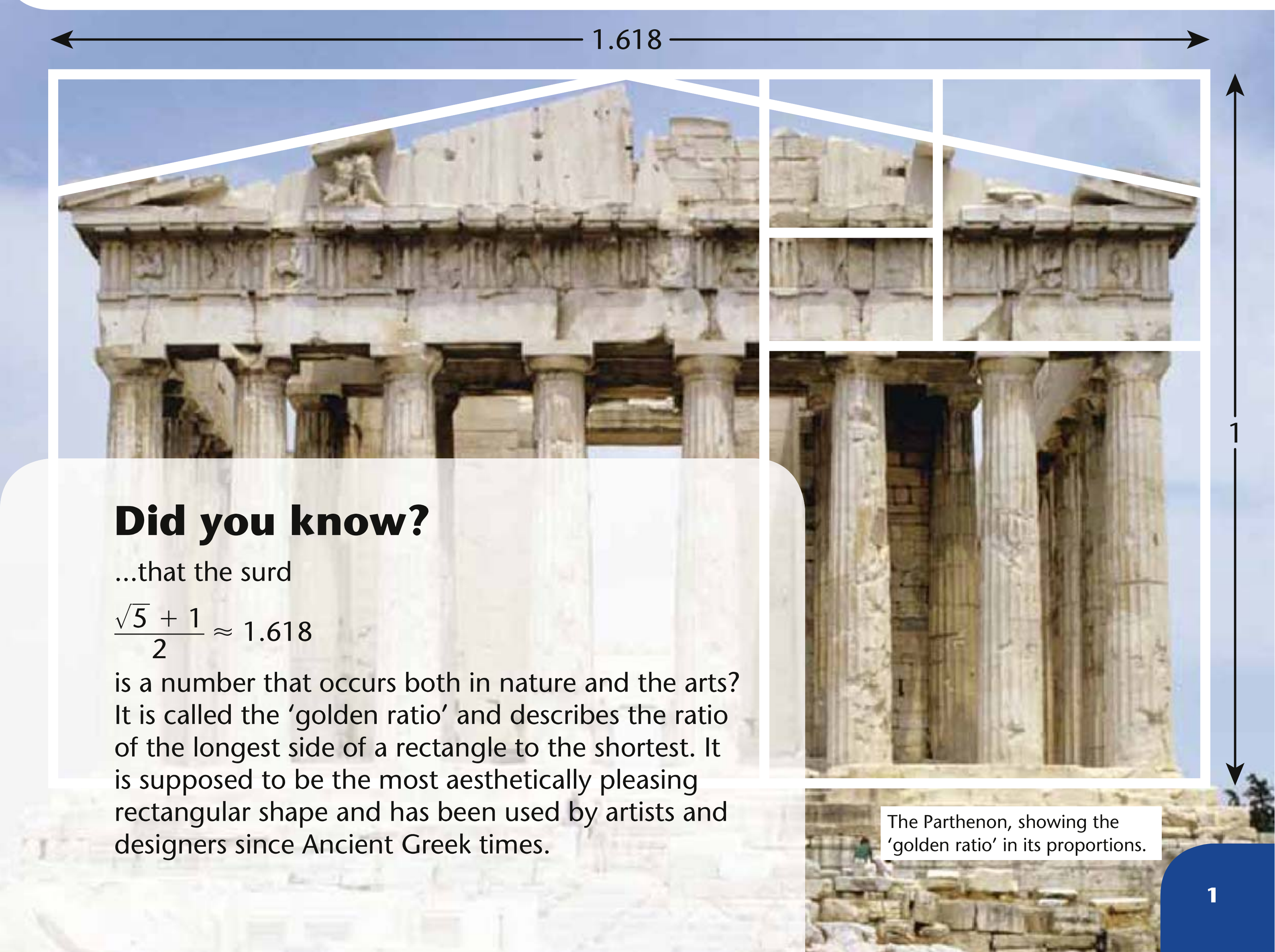
After completing this chapter you should be able to

1 simplify expressions and collect like terms
2 apply the rules of indices
3 multiply out brackets
4 factorise expressions including quadratics
5 manipulate surds.

This chapter provides the foundations for many aspects of A level Mathematics. Factorising expressions will enable you to solve equations; it could help sketch the graph of a function. A knowledge of indices is very important when differentiating and integrating. Surds are an important way of giving exact answers to problems and you will meet them again when solving quadratic equations.

1 Algebra and functions



The Parthenon, showing the 'golden ratio' in its proportions.

Did you know?

...that the surd

$$\frac{\sqrt{5}+1}{2} \approx 1.618$$

is a number that occurs both in nature and the arts? It is called the 'golden ratio' and describes the ratio of the longest side of a rectangle to the shortest. It is supposed to be the most aesthetically pleasing rectangular shape and has been used by artists and designers since Ancient Greek times.

1.1 You can simplify expressions by collecting like terms.

Example 1

Simplify these expressions:

a $3x + 2xy + 7 - x + 3xy - 9$ **b** $3x^2 - 6x + 4 - 2x^2 + 3x - 3$

c $3(a + b^2) - 2(3a - 4b^2)$

a $3x + 2xy + 7 - x + 3xy - 9$

$= 3x - x + 2xy + 3xy + 7 - 9$

$= 2x + 5xy - 2$

Rewrite the expression with the like terms next to each other.
$+7 - 9 = -2$

b $3x^2 - 6x + 4 - 2x^2 + 3x - 3$

$= 3x^2 - 2x^2 - 6x + 3x + 4 - 3$

$= x^2 - 3x + 1$

$3x^2$ and $3x$ are not like terms:
$3x^2 = 3 \times x \times x$ $3x = 3 \times x$
$1x^2$ is written as x^2.

c $3(a + b^2) - 2(3a - 4b^2)$

$= 3a + 3b^2 - 6a + 8b^2$

$= -3a + 11b^2$

Multiply the term outside the bracket by both terms inside the bracket:
$-2 \times 3a = -6a$
$-2 \times -4b^2 = 8b^2$
So $-2(3a - 4b^2) = -6a + 8b^2$

Exercise 1A

Simplify these expressions:

1 $4x - 5y + 3x + 6y$

2 $3r + 7t - 5r + 3t$

3 $3m - 2n - p + 5m + 3n - 6p$

4 $3ab - 3ac + 3a - 7ab + 5ac$

5 $7x^2 - 2x^2 + 5x^2 - 4x^2$

6 $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

7 $5x^2 + 4x + 1 - 3x^2 + 2x + 7$

8 $6x^2 + 5x - 12 + 3x^2 - 7x + 11$

9 $3x^2 - 5x + 2 + 3x^2 - 7x - 12$

10 $4c^2d + 5cd^2 - c^2d + 3cd^2 + 7c^2d$

11 $2x^2 + 3x + 1 + 2(3x^2 + 6)$

12 $4(a + a^2b) - 3(2a + a^2b)$

13 $2(3x^2 + 4x + 5) - 3(x^2 - 2x - 3)$

14 $7(1 - x^2) + 3(2 - 3x + 5x^2)$

15 $4(a + b + 3c) - 3a + 2c$

16 $4(c + 3d^2) - 3(2c + d^2)$

17 $5 - 3(x^2 + 2x - 5) + 3x^2$

18 $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

1.2 You can simplify expressions and functions by using rules of indices (powers).

- $a^m \times a^n = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $a^{-m} = \dfrac{1}{a^m}$
  $a^{\frac{1}{m}} = \sqrt[m]{a}$ — The mth root of a.
  $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Example 2

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $b^4 \div b^4$
d $6x^{-3} \div 3x^{-5}$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

a $x^2 \times x^5$
$= x^{2+5}$ — Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.
$= x^7$

b $2r^2 \times 3r^3$
$= 2 \times 3 \times r^2 \times r^3$
$= 6 \times r^{2+3}$
$= 6r^5$

Rewrite the expression with the numbers together and the r terms together.
$2 \times 3 = 6$
$r^2 \times r^3 = r^{2+3}$

c $b^4 \div b^4$ — Use the rule $a^m \div a^n = a^{m-n}$
$= b^{4-4}$
$= b^0 = 1$ — Any term raised to the power of zero = 1.

d $6x^{-3} \div 3x^{-5}$
$= 6 \div 3 \times x^{-3} \div x^{-5}$ — $x^{-3} \div x^{-5} = x^{-3--5} = x^2$
$= 2 \times x^2$
$= 2x^2$

e $(a^3)^2 \times 2a^2$
$= a^6 \times 2a^2$
$= 2 \times a^6 \times a^2$
$= 2 \times a^{6+2}$
$= 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.
$a^6 \times 2a^2 = 1 \times 2 \times a^6 \times a^2$
$= 2 \times a^{6+2}$

f $(3x^2)^3 \div x^4$ — Use the rule $(a^m)^n = a^{mn}$ to simplify the index.
$= 27x^6 \div x^4$
$= 27 \div 1 \times x^6 \div x^4$
$= 27 \times x^{6-4}$
$= 27x^2$

Exercise 1B

Simplify these expressions:

1 $x^3 \times x^4$

2 $2x^3 \times 3x^2$

3 $4p^3 \div 2p$

4 $3x^{-4} \div x^{-2}$

5 $k^3 \div k^{-2}$

6 $(y^2)^5$

7 $10x^5 \div 2x^{-3}$

8 $(p^3)^2 \div p^4$

9 $(2a^3)^2 \div 2a^3$

10 $8p^{-4} \div 4p^3$

11 $2a^{-4} \times 3a^{-5}$

12 $21a^3b^2 \div 7ab^4$

13 $9x^2 \times 3(x^2)^3$

14 $3x^3 \times 2x^2 \times 4x^6$

15 $7a^4 \times (3a^4)^2$

16 $(4y^3)^3 \div 2y^3$

17 $2a^3 \div 3a^2 \times 6a^5$

18 $3a^4 \times 2a^5 \times a^3$

1.3 You can expand an expression by multiplying each term inside the bracket by the term outside.

Example 3

Expand these expressions, simplify if possible:

a $5(2x + 3)$

b $-3x(7x - 4)$

c $y^2(3 - 2y^3)$

d $4x(3x - 2x^2 + 5x^3)$

e $2x(5x + 3) - 5(2x + 3)$

Hint: A − sign outside a bracket changes the sign of every term inside the brackets.

a $5(2x + 3)$

$= 10x + 15$

$5 \times 2x + 5 \times 3$

b $-3x(7x - 4)$

$= -21x^2 + 12x$

$-3x \times 7x = -21x^{1+1} = -21x^2$

$-3x \times -4 = +12x$

c $y^2(3 - 2y^3)$

$= 3y^2 - 2y^5$

$y^2 \times -2y^3 = -2y^{2+3} = -2y^5$

d $4x(3x - 2x^2 + 5x^3)$

$= 12x^2 - 8x^3 + 20x^4$

e $2x(5x + 3) - 5(2x + 3)$

$= 10x^2 + 6x - 10x - 15$

$= 10x^2 - 4x - 15$

Remember a minus sign outside the brackets changes the signs within the brackets. Simplify $6x - 10x$ to give $-4x$.

Exercise 1C

Expand and simplify if possible:

1 $9(x-2)$
2 $x(x+9)$
3 $-3y(4-3y)$
4 $x(y+5)$
5 $-x(3x+5)$
6 $-5x(4x+1)$
7 $(4x+5)x$
8 $-3y(5-2y^2)$
9 $-2x(5x-4)$
10 $(3x-5)x^2$
11 $3(x+2)+(x-7)$
12 $5x-6-(3x-2)$
13 $x(3x^2-2x+5)$
14 $7y^2(2-5y+3y^2)$
15 $-2y^2(5-7y+3y^2)$
16 $7(x-2)+3(x+4)-6(x-2)$
17 $5x-3(4-2x)+6$
18 $3x^2-x(3-4x)+7$
19 $4x(x+3)-2x(3x-7)$
20 $3x^2(2x+1)-5x^2(3x-4)$

1.4 You can factorise expressions.

■ **Factorising is the opposite of expanding expressions.**

When you have completely factorised an expression, the terms inside do not have a common factor.

Example 4

Factorise these expressions completely:

a $3x+9$ **b** x^2-5x **c** $8x^2+20x$
d $9x^2y+15xy^2$ **e** $3x^2-9xy$

a $3x+9$
$=3(x+3)$

3 is a common factor of $3x$ and 9.

b x^2-5x
$=x(x-5)$

x is a common factor of x^2 and $-5x$.

c $8x^2+20x$
$=4x(2x+5)$

4 and x are common factors of $8x^2$ and $20x$. So take $4x$ outside the bracket.

d $9x^2y+15xy^2$
$=3xy(3x+5y)$

3, x and y are common factors of $9x^2y$ and $15xy^2$. So take $3xy$ outside the bracket.

e $3x^2-9xy$
$=3x(x-3y)$

Exercise 1D

Factorise these expressions completely:

1	$4x + 8$	**2**	$6x - 24$
3	$20x + 15$	**4**	$2x^2 + 4$
5	$4x^2 + 20$	**6**	$6x^2 - 18x$
7	$x^2 - 7x$	**8**	$2x^2 + 4x$
9	$3x^2 - x$	**10**	$6x^2 - 2x$
11	$10y^2 - 5y$	**12**	$35x^2 - 28x$
13	$x^2 + 2x$	**14**	$3y^2 + 2y$
15	$4x^2 + 12x$	**16**	$5y^2 - 20y$
17	$9xy^2 + 12x^2y$	**18**	$6ab - 2ab^2$
19	$5x^2 - 25xy$	**20**	$12x^2y + 8xy^2$
21	$15y - 20yz^2$	**22**	$12x^2 - 30$
23	$xy^2 - x^2y$	**24**	$12y^2 - 4yx$

1.5 You can factorise quadratic expressions.

■ **A quadratic expression has the form $ax^2 + bx + c$, where a, b, c are constants and $a \neq 0$.**

Example 5

Factorise:

a $6x^2 + 9x$ **b** $x^2 - 5x - 6$
c $x^2 + 6x + 8$ **d** $6x^2 - 11x - 10$
e $x^2 - 25$ **f** $4x^2 - 9y^2$

a $6x^2 + 9x$
$= 3x(2x + 3)$

3 and x are common factors of $6x^2$ and $9x$. So take $3x$ outside the bracket.

b $x^2 - 5x - 6$
$ac = -6$
So $x^2 - 5x + 6 = x^2 + x - 6x - 6$
$= x(x + 1) - 6(x + 1)$
$= (x + 1)(x - 6)$

Here $a = 1$, $b = -5$ and $c = -6$.
You need to find two brackets that multiply together to give $x^2 - 5x - 6$. So:

① Work out ac.
② Work out the two factors of ac which add that give you b.
-6 and $+1 = -5$
③ Rewrite the bx term using these two factors.
④ Factorise first two terms and last two terms.
⑤ $x + 1$ is a factor of both terms, so take that outside the bracket. This is now completely factorised.

c $x^2 + 6x + 8$
$= x^2 + 2x + 4x + 8$
$= x(x + 2) + 4(x + 2)$
$= (x + 2)(x + 4)$

Since $ac = 8$ and $2 + 4 = 6 = b$, factorise.
$x + 2$ is a factor so you can factorise again.

d $6x^2 - 11x - 10$
$= 6x^2 - 15x + 4x - 10$
$= 3x(2x - 5) + 2(2x - 5)$
$= (2x - 5)(3x + 2)$

$ac = -60$ and $4 - 15 = -11 = b$.
Factorise.
Factorise $(2x - 5)$.

e $x^2 - 25$
$= x^2 - 5^2$
$= (x + 5)(x - 5)$

This is called the difference of two squares as the two terms are x^2 and 5^2.
The two x terms, $5x$ and $-5x$, cancel each other out.

f $4x^2 - 9y^2$
$= 2^2x^2 - 3^2y^2$
$= (2x + 3y)(2x - 3y)$

This is the same as $(2x)^2 - (3y)^2$.

■ $x^2 - y^2 = (x + y)(x - y)$
This is called the difference of two squares.

Exercise 1E

Factorise:

1 $x^2 + 4x$
2 $2x^2 + 6x$
3 $x^2 + 11x + 24$
4 $x^2 + 8x + 12$
5 $x^2 + 3x - 40$
6 $x^2 - 8x + 12$
7 $x^2 + 5x + 6$
8 $x^2 - 2x - 24$
9 $x^2 - 3x - 10$
10 $x^2 + x - 20$
11 $2x^2 + 5x + 2$
12 $3x^2 + 10x - 8$
13 $5x^2 - 16x + 3$
14 $6x^2 - 8x - 8$
15 $2x^2 + 7x - 15$
16 $2x^4 + 14x^2 + 24$
17 $x^2 - 4$
18 $x^2 - 49$
19 $4x^2 - 25$
20 $9x^2 - 25y^2$
21 $36x^2 - 4$
22 $2x^2 - 50$
23 $6x^2 - 10x + 4$
24 $15x^2 + 42x - 9$

Hints:
Question 14 – Take 2 out as a common factor first.
Question 16 – let $y = x^2$.

1.6 You can extend the rules of indices to all rational exponents.

- $a^m \times a^n = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $a^{\frac{1}{m}} = \sqrt[m]{a}$
  $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
  $a^{-m} = \dfrac{1}{a^m}$
  $a^0 = 1$

Hint: Rational numbers can be written as $\frac{a}{b}$ where a and b are both integers, e.g. -3.5, $1\frac{1}{4}$, 0.9, 7, $0.1\dot{3}$

Example 6

Simplify:

a $x^4 \div x^{-3}$

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$

c $(x^3)^{\frac{2}{3}}$

d $2x^{1.5} \div 4x^{-0.25}$

a $x^4 \div x^{-3}$

$= x^{4--3}$

Use the rule $a^m \div a^n = a^{m-n}$.
Remember $- + - = +$.

$= x^7$

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$

This could also be written as $\sqrt{x}$.

$= x^{\frac{1}{2}+\frac{3}{2}}$

Use the rule $a^m \times a^n = a^{m+n}$.

$= x^2$

c $(x^3)^{\frac{2}{3}}$

Use the rule $(a^m)^n = a^{mn}$.

$= x^{3\times\frac{2}{3}}$

$= x^2$

d $2x^{1.5} \div 4x^{-0.25}$

$= \frac{1}{2}x^{1.5--0.25}$

Use the rule $a^m \div a^n = a^{m-n}$.
$2 \div 4 = \frac{1}{2}$
$1.5 - -0.25 = 1.75$

$= \frac{1}{2}x^{1.75}$

Example 7

Evaluate:

a $9^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $49^{\frac{3}{2}}$

d $25^{-\frac{3}{2}}$

a $9^{\frac{1}{2}}$

$= \sqrt{9}$ — Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

$= \pm 3$ — When you take a square root, the answer can be positive or negative as $+ \times + = +$ and $- \times - = +$.

b $64^{\frac{1}{3}}$

$= \sqrt[3]{64}$ — This means the cube root of 64.

$= 4$ — As $4 \times 4 \times 4 = 64$.

c $49^{\frac{3}{2}}$

$= (\sqrt{49})^3$ — Using $a^{\frac{n}{m}} = \sqrt[m]{a^n}$. This means the square root of 49, cubed.

$= \pm 343$

d $25^{-\frac{3}{2}}$

$= \dfrac{1}{25^{\frac{3}{2}}}$ — Using $a^{-m} = \dfrac{1}{a^m}$.

$= \dfrac{1}{(\pm\sqrt{25})^3}$ — $\sqrt{25} = \pm 5$

$= \dfrac{1}{(\pm 5)^3}$

$= \pm \dfrac{1}{125}$

Exercise 1F

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{1\frac{2}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{1}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $(\frac{3}{4})^0$ **h** $1296^{\frac{1}{4}}$ **i** $(1\frac{9}{16})^{\frac{3}{2}}$

j $(\frac{27}{8})^{\frac{2}{3}}$ **k** $(\frac{6}{5})^{-1}$ **l** $(\frac{343}{512})^{-\frac{2}{3}}$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

1.7 You can write a number exactly using surds, e.g. $\sqrt{2}$, $\sqrt{3} - 5$, $\sqrt{19}$.
You cannot evaluate surds exactly because they give never-ending, non-repeating decimal fractions, e.g. $\sqrt{2} = 1.414\,213\,562\ldots$
The square root of a prime number is a surd.

- **You can manipulate surds using these rules:**

$$\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example 8

Simplify:

a $\sqrt{12}$ **b** $\frac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

a $\sqrt{12}$

$= \sqrt{(4 \times 3)}$

$= \sqrt{4} \times \sqrt{3}$

$= 2\sqrt{3}$

Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
$\sqrt{4} = 2$

b $\frac{\sqrt{20}}{2}$

$= \frac{\sqrt{4} \times \sqrt{5}}{2}$

$= \frac{2 \times \sqrt{5}}{2}$

$= \sqrt{5}$

$\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$\sqrt{4} = 2$

Cancel by 2.

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$= 5\sqrt{6} - 2\sqrt{6}\sqrt{4} + \sqrt{6} \times \sqrt{49}$

$= \sqrt{6}(5 - 2\sqrt{4} + \sqrt{49})$

$= \sqrt{6}(5 - 2 \times 2 + 7)$

$= \sqrt{6}(8)$

$= 8\sqrt{6}$

$\sqrt{6}$ is a common factor.
Work out the square roots $\sqrt{4}$ and $\sqrt{49}$.
$5 - 4 + 7 = 8$

Exercise 1G

Simplify:

1 $\sqrt{28}$ **2** $\sqrt{72}$ **3** $\sqrt{50}$

4 $\sqrt{32}$ **5** $\sqrt{90}$ **6** $\frac{\sqrt{12}}{2}$

7 $\frac{\sqrt{27}}{3}$ **8** $\sqrt{20} + \sqrt{80}$ **9** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

10 $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **11** $1\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **12** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

13 $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **14** $\frac{\sqrt{44}}{\sqrt{11}}$ **15** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

1.8 You rationalise the denominator of a fraction when it is a surd.

- **The rules to rationalise surds are:**
  - **Fractions in the form $\sqrt{\frac{1}{a}}$, multiply the top and bottom by $\sqrt{a}$.**
  - **Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the top and bottom by $a - \sqrt{b}$.**
  - **Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the top and bottom by $a + \sqrt{b}$.**

Example 9

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3 + \sqrt{2}}$ **c** $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$

a $\frac{1}{\sqrt{3}}$

$= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

Multiply the top and bottom by $\sqrt{3}$.
$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

b $\frac{1}{3 + \sqrt{2}}$

$= \frac{1 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

$= \frac{3 - \sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - 2}$

$= \frac{3 - \sqrt{2}}{7}$

Multiply top and bottom by $(3 - \sqrt{2})$.
$\sqrt{2} \times \sqrt{2} = 2$
$9 - 2 = 7$, $-3\sqrt{2} + 3\sqrt{2} = 0$

c $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \dfrac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$= \dfrac{5+\sqrt{5}\sqrt{2}+\sqrt{2}\sqrt{5}+2}{5-2}$

$= \dfrac{7+2\sqrt{10}}{3}$

Multiply top and bottom by $\sqrt{5}+\sqrt{2}$.

$-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.

$\sqrt{5}\sqrt{2} = \sqrt{10}$

Exercise 1H

Rationalise the denominators:

1 $\dfrac{1}{\sqrt{5}}$ **2** $\dfrac{1}{\sqrt{11}}$ **3** $\dfrac{1}{\sqrt{2}}$

4 $\dfrac{\sqrt{3}}{\sqrt{15}}$ **5** $\dfrac{\sqrt{12}}{\sqrt{48}}$ **6** $\dfrac{\sqrt{5}}{\sqrt{80}}$

7 $\dfrac{\sqrt{12}}{\sqrt{156}}$ **8** $\dfrac{\sqrt{7}}{\sqrt{63}}$ **9** $\dfrac{1}{1+\sqrt{3}}$

10 $\dfrac{1}{2+\sqrt{5}}$ **11** $\dfrac{1}{3-\sqrt{7}}$ **12** $\dfrac{4}{3-\sqrt{5}}$

13 $\dfrac{1}{\sqrt{5}-\sqrt{3}}$ **14** $\dfrac{3-\sqrt{2}}{4-\sqrt{5}}$ **15** $\dfrac{5}{2+\sqrt{5}}$

16 $\dfrac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **17** $\dfrac{11}{3+\sqrt{11}}$ **18** $\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

19 $\dfrac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **20** $\dfrac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **21** $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

Mixed exercise 1I

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$

c $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$

c $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

3 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$

c $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

4 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$

c $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

5 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$

c $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

6 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

7 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

8 Rationalise:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$

c $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$

Summary of key points

1 You can simplify expressions by collecting like terms.

2 You can simplify expressions by using rules of indices (powers).

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

3 You can expand an expression by multiplying each term inside the bracket by the term outside.

4 Factorising expressions is the opposite of expanding expressions.

5 A quadratic expression has the form $ax^2 + bx + c$, where a, b, c are constants and $a \neq 0$.

6 $x^2 - y^2 = (x + y)(x - y)$
This is called a difference of squares.

7 You can write a number exactly using surds.

8 The square root of a prime number is a surd.

9 You can manipulate surds using the rules:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

10 The rules to rationalise surds are:

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by $\sqrt{a}$.
- Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the top and bottom by $a - \sqrt{b}$.
- Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the top and bottom by $a + \sqrt{b}$.