

1 a

x -coordinate of B	y -coordinate of B	gradient of AB
2	4	$\frac{4-1}{2-1} = 3$
1.1	1.21	$\frac{1.21-1}{1.1-1} = 2.1$
1.01	1.0201	$\frac{1.0201-1}{1.01-1} = 2.01$
1.001	1.002001	$\frac{1.002001-1}{1.001-1} = 2.001$

b gradient = 2

c

x -coordinate of B	y -coordinate of B	gradient of AB
0	0	$\frac{1-0}{1-0} = 1$
0.9	0.81	$\frac{1-0.81}{1-0.9} = 1.9$
0.99	0.9801	$\frac{1-0.9801}{1-0.99} = 1.99$
0.999	0.998001	$\frac{1-0.998001}{1-0.999} = 1.999$

this table supports the answer to part **b** as the gradient of the chord AB again gets closer to 2 as B gets closer to A

2 possible tables of values are:

a

x -coordinate of B	y -coordinate of B	gradient of AB
3	9	$\frac{9-4}{3-2} = 5$
2.1	4.41	$\frac{4.41-4}{2.1-2} = 4.1$
2.01	4.0401	$\frac{4.0401-4}{2.01-2} = 4.01$
2.001	4.004001	$\frac{4.004001-4}{2.001-2} = 4.001$

$\therefore$ gradient = 4

b

x -coordinate of B	y -coordinate of B	gradient of AB
5	25	$\frac{25-16}{5-4} = 9$
4.1	16.81	$\frac{16.81-16}{4.1-4} = 8.1$
4.01	16.0801	$\frac{16.0801-16}{4.01-4} = 8.01$
4.001	16.008001	$\frac{16.008001-16}{4.001-4} = 8.001$

$\therefore$ gradient = 8

c

x -coordinate of B	y -coordinate of B	gradient of AB
2.5	6.25	$\frac{6.25-2.25}{2.5-1.5} = 4$
1.6	2.56	$\frac{2.56-2.25}{1.6-1.5} = 3.1$
1.51	2.2801	$\frac{2.2801-2.25}{1.51-1.5} = 3.01$
1.501	2.253001	$\frac{2.253001-2.25}{1.501-1.5} = 3.001$

$\therefore$ gradient = 3

d

x -coordinate of B	y -coordinate of B	gradient of AB
-2	4	$\frac{4-9}{-2-(-3)} = -5$
-2.9	8.41	$\frac{8.41-9}{-2.9-(-3)} = -5.9$
-2.99	8.9401	$\frac{8.9401-9}{-2.99-(-3)} = -5.99$
-2.999	8.994001	$\frac{8.994001-9}{-2.999-(-3)} = -5.999$

$\therefore$ gradient = -6

3 a gradient = $2x$

b i 12 ii 4.8 iii -6.4

4 possible answers are:

a let A be $(1, 1)$

x -coordinate of B	y -coordinate of B	gradient of AB
2	16	$\frac{16-1}{2-1} = 15$
1.1	1.4641	$\frac{1.4641-1}{1.1-1} = 4.641$
1.01	1.04060401	$\frac{1.04060401-1}{1.01-1} = 4.060401$
1.001	1.004006004	$\frac{1.004006004-1}{1.001-1} = 4.006004$

$\therefore$ gradient = 4

b let A be $(2, -3)$

x -coordinate of B	y -coordinate of B	gradient of AB
3	-3	$\frac{-3-(-3)}{3-2} = 0$
2.1	-3.09	$\frac{-3.09-(-3)}{2.1-2} = -0.9$
2.01	-3.0099	$\frac{-3.0099-(-3)}{2.01-2} = -0.99$
2.001	-3.000999	$\frac{-3.000999-(-3)}{2.001-2} = -0.999$

$\therefore$ gradient = -1

c let A be $(4, 2)$

x -coordinate of B	y -coordinate of B	gradient of AB
5	2.236067977	$\frac{2.236067977-2}{5-4} = 0.236068$
4.1	2.024845673	$\frac{2.024845673-2}{4.1-4} = 0.248457$
4.01	2.002498439	$\frac{2.002498439-2}{4.01-4} = 0.249844$
4.001	2.000249984	$\frac{2.000249984-2}{4.001-4} = 0.249984$

$\therefore$ gradient = 0.25

d let A be $(2, 1)$

x -coordinate of B	y -coordinate of B	gradient of AB
3	0.666666667	$\frac{0.666666667-1}{3-2} = -0.333333$
2.1	0.952380952	$\frac{0.952380952-1}{2.1-2} = -0.476190$
2.01	0.995024876	$\frac{0.995024876-1}{2.01-2} = -0.497512$
2.001	0.999500250	$\frac{0.999500250-1}{2.001-2} = -0.499750$

$\therefore$ gradient = -0.5

5 a possible answers are:

i let A be $(1, 1)$

x -coordinate of B	y -coordinate of B	gradient of AB
2	8	$\frac{8-1}{2-1} = 7$
1.1	1.331	$\frac{1.331-1}{1.1-1} = 3.31$
1.01	1.030301	$\frac{1.030301-1}{1.01-1} = 3.0301$
1.001	1.003003001	$\frac{1.003003001-1}{1.001-1} = 3.003001$

$\therefore$ gradient = 3

ii let A be $(2, 8)$

x -coordinate of B	y -coordinate of B	gradient of AB
3	27	$\frac{27-8}{3-2} = 19$
2.1	9.261	$\frac{9.261-8}{2.1-2} = 12.61$
2.01	8.120601	$\frac{8.120601-8}{2.01-2} = 12.0601$
2.001	8.012006001	$\frac{8.012006001-8}{2.001-2} = 12.006001$

$\therefore$ gradient = 12

iii let A be $(3, 27)$

x -coordinate of B	y -coordinate of B	gradient of AB
4	64	$\frac{64-27}{4-3} = 37$
3.1	29.791	$\frac{29.791-27}{3.1-3} = 27.91$
3.01	27.270901	$\frac{27.270901-27}{3.01-3} = 27.0901$
3.001	27.027009	$\frac{27.027009-27}{3.001-3} = 27.009$

$\therefore$ gradient = 27

b gradient = $3x^2$

c i 48 ii 12 iii 6.75

- 1 a $2x$ b $4x^3$ c 1 d $9x^8$ e $-3x^{-4}$ f $-x^{-2}$
 g $8x$ h 7 i $10x^4$ j 0 k $-16x^{-3}$ l $-44x^{-5}$
- 2 a $5x^4 + 2x$ b $1 + 3x^2$ c $4x^3$ d $6x^5 - 2$
 e $18x^2 - 10x^{-3}$ f $2x - 4$ g $-x^{-2} + 5x^{-6}$ h $12x^2 - 12x^{-5}$
- 3 a $6t^5$ b $-15t^{-4}$ c $\frac{1}{2}t^{-\frac{1}{2}}$ d $\frac{2}{3}t^{-\frac{1}{3}}$ e $\frac{3}{2}t$ f $2t^{-\frac{3}{4}}$
 g $7t^{\frac{5}{2}}$ h $-\frac{1}{5}t^{-\frac{6}{5}}$ i $\frac{3}{5}t^{\frac{1}{5}}$ j $-\frac{3}{2}t^{-\frac{5}{2}}$ k $-15t^{-\frac{9}{4}}$ l $\frac{2}{9}t^{\frac{1}{3}}$
- 4 a $2 + 2x^5$ b $\frac{3}{2}x^{\frac{1}{2}}$ c $1 + 2x^{-\frac{1}{2}}$ d $10x^{\frac{2}{3}} + 4x^{-5}$
 e $-\frac{4}{5}x^{-\frac{9}{5}}$ f $\frac{1}{3}x^{-\frac{5}{6}} + \frac{3}{4}x^{-\frac{1}{4}}$ g $-3x^{-2} + \frac{15}{2}x^{-\frac{5}{2}}$ h $7x^{-2} - \frac{8}{3}x^{-\frac{11}{3}}$
- 5 a $y = x^{\frac{1}{2}}$ b $y = 4 - x^{-1}$ c $y = 3x^2 + x^{\frac{1}{3}}$ d $y = 9x + 3x^{-1}$
 $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx} = x^{-2}$ $\frac{dy}{dx} = 6x + \frac{1}{3}x^{-\frac{2}{3}}$ $\frac{dy}{dx} = 9 - 3x^{-2}$
 e $y = \frac{1}{4}x^{-1} - x^{-2}$ f $y = 6x^{-\frac{1}{4}}$ g $y = x^{\frac{5}{2}}$ h $y = 8x^{\frac{1}{2}} + \frac{4}{3}x^{-2}$
 $\frac{dy}{dx} = -\frac{1}{4}x^{-2} + 2x^{-3}$ $\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{5}{4}}$ $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ $\frac{dy}{dx} = 4x^{-\frac{1}{2}} - \frac{8}{3}x^{-3}$
- 6 a $s = t^2 + 3t$ b $s = t^2 - 4t + 4$ c $s = 5t^4 + 20t^2$ d $s = 7t^3 - t$
 $\frac{ds}{dt} = 2t + 3$ $\frac{ds}{dt} = 2t - 4$ $\frac{ds}{dt} = 20t^3 + 40t$ $\frac{ds}{dt} = 21t^2 - 1$
 e $s = t^2 + 7t + 6$ f $s = t^2 - 2t - 8$ g $s = t^5 + 3t^3 + 9t$ h $s = 2t^3 - 5t^2 + 3t$
 $\frac{ds}{dt} = 2t + 7$ $\frac{ds}{dt} = 2t - 2$ $\frac{ds}{dt} = 5t^4 + 9t^2 + 9$ $\frac{ds}{dt} = 6t^2 - 10t + 3$
- 7 a $y = x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$ b $y = x^2 - 2$ c $y = 4x + x^{-1}$ d $y = x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$
 $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x$ $\frac{dy}{dx} = 4 - x^{-2}$ $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$
 e $y = 2x^{-1} - \frac{1}{2}x^2$ f $y = 5x^{-2} + x^{-\frac{3}{2}}$ g $y = 3 - \frac{2}{3}x^{-1}$ h $y = 2x^{\frac{1}{2}} + \frac{1}{4}x^{\frac{5}{2}}$
 $\frac{dy}{dx} = -2x^{-2} - x$ $\frac{dy}{dx} = -10x^{-3} - \frac{3}{2}x^{-\frac{5}{2}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-2}$ $\frac{dy}{dx} = x^{-\frac{1}{2}} + \frac{5}{8}x^{\frac{3}{2}}$
- 8 a $\frac{dy}{dx} = 8x - 1$ b $\frac{dy}{dx} = 3x^2 + 10x + 2$ c $\frac{dy}{dx} = 2x^{-2}$
 $\frac{d^2y}{dx^2} = 8$ $\frac{d^2y}{dx^2} = 6x + 10$ $\frac{d^2y}{dx^2} = -4x^{-3}$
 d $\frac{dy}{dx} = 8x^3 + 6x$ e $y = 3x^4 - 4x^{-2}$ f $\frac{dy}{dx} = 3x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$
 $\frac{d^2y}{dx^2} = 24x^2 + 6$ $\frac{dy}{dx} = 12x^3 + 8x^{-3}$ $\frac{d^2y}{dx^2} = -\frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$
 $\frac{d^2y}{dx^2} = 36x^2 - 24x^{-4}$

- 1 **a** $\frac{dy}{dx} = 3x^2$ **b** $\frac{dy}{dx} = 4 - 2x$ **c** $\frac{dy}{dx} = 4x - 8$ **d** $\frac{dy}{dx} = -3x^{-2}$
 grad = 27 grad = -2 grad = 4 grad = $-\frac{1}{3}$
- 2 **a** $\frac{dy}{dx} = 6x + 1$
 at (1, -1) grad = 7 **b** $\frac{dy}{dx} = 4x^3 + 6x^2$
 at (-2, 0) grad = -8
- c** $y = 2x^2 - 3x$, $\frac{dy}{dx} = 4x - 3$
 at (2, 2) grad = 5 **d** $\frac{dy}{dx} = 2x + 2x^{-2}$
 at (2, 3) grad = $\frac{9}{2}$
- e** $\frac{dy}{dx} = 2x + 6$
 at (-3, -1) grad = 0 **f** $\frac{dy}{dx} = 4 - 2x^{-3}$
 at ($\frac{1}{2}$, 6) grad = -12
- 3 **a** $f(x) = x^2 + 2x + 1$ **b** $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ **c** $f'(x) = 1 + 8x^{-3}$ **d** $f'(x) = -9x^{\frac{1}{2}}$
 $f'(x) = 2x + 2$ $f'(4) = \frac{1}{4}$ $f'(4) = \frac{9}{8}$ $f'(4) = -18$
 $f'(4) = 10$
- 4 **a** $x(x-1)(x-3) = 0$, $x = 0, 1, 3$
 $\therefore (0, 0), (1, 0)$ and $(3, 0)$
 b $\frac{dy}{dx} = 3x^2 - 8x + 3$
 at (0, 0) grad = 3
 at (1, 0) grad = -2
 at (3, 0) grad = 6
- 5 **a** $\frac{dy}{dx} = 4x - 5$
 b $4x - 5 = 7$
 $x = 3$
- 6 $\frac{dy}{dx} = 3x^2 - 8$
 $\therefore 3x^2 - 8 = 4$
 $x^2 = 4$
 $x = \pm 2$
 $\therefore (-2, 8)$ and $(2, -8)$
- 7 **a** $\frac{dy}{dx} = 3x^2 + 2x - 4$
 grad at $P = -3$
 b grad at $Q = -3$
 $\therefore 3x^2 + 2x - 4 = -3$
 $3x^2 + 2x - 1 = 0$
 $(3x-1)(x+1) = 0$
 $x = -1$ (at P) or $\frac{1}{3}$
 $\therefore Q(\frac{1}{3}, -\frac{5}{27})$
- 8 **a** $\frac{dy}{dx} = 2x$, grad = 4
 $\therefore y - 4 = 4(x - 2)$ $[y = 4x - 4]$ **b** $\frac{dy}{dx} = 2x + 3$, grad = 1
 $\therefore y - 2 = x + 1$ $[y = x + 3]$
- c** $\frac{dy}{dx} = 4x - 6$, grad = -2
 $\therefore y - 4 = -2(x - 1)$ $[y = -2x + 6]$ **d** $\frac{dy}{dx} = 3x^2 - 8x$, grad = 3
 $\therefore y + 7 = 3(x - 3)$ $[y = 3x - 16]$

9 a $\frac{dy}{dx} = -2x$, grad = 6

$$\therefore y + 6 = 6(x + 3)$$

$$y + 6 = 6x + 18$$

$$6x - y + 12 = 0$$

c $\frac{dy}{dx} = 4x + 5$, grad = 7

$$\therefore y - 2 = 7(x - \frac{1}{2})$$

$$2y - 4 = 14x - 7$$

$$14x - 2y - 3 = 0$$

b $\frac{dy}{dx} = -2x^{-2}$, grad = $-\frac{1}{2}$

$$\therefore y - 1 = -\frac{1}{2}(x - 2)$$

$$2y - 2 = -x + 2$$

$$x + 2y - 4 = 0$$

d $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$, grad = $\frac{1}{4}$

$$\therefore y + 2 = \frac{1}{4}(x - 4)$$

$$4y + 8 = x - 4$$

$$x - 4y - 12 = 0$$

10 a $\frac{dy}{dx} = 2x$, grad = 2

$$\therefore \text{grad of normal} = -\frac{1}{2}$$

$$\therefore y + 3 = -\frac{1}{2}(x - 1)$$

$$2y + 6 = -x + 1$$

$$x + 2y + 5 = 0$$

b $\frac{dy}{dx} = 6x + 7$, grad = -5

$$\therefore \text{grad of normal} = \frac{1}{5}$$

$$\therefore y - 5 = \frac{1}{5}(x + 2)$$

$$5y - 25 = x + 2$$

$$x - 5y + 27 = 0$$

c $\frac{dy}{dx} = 3x^2 - 8$, grad = 4

$$\therefore \text{grad of normal} = -\frac{1}{4}$$

$$\therefore y + 4 = -\frac{1}{4}(x - 2)$$

$$4y + 16 = -x + 2$$

$$x + 4y + 14 = 0$$

d $\frac{dy}{dx} = 1 + 6x^{-2}$, grad = $\frac{5}{3}$

$$\therefore \text{grad of normal} = -\frac{3}{5}$$

$$\therefore y - 1 = -\frac{3}{5}(x - 3)$$

$$5y - 5 = -3x + 9$$

$$3x + 5y - 14 = 0$$

11 a $x = 2 \therefore y = 4$

$$\frac{dy}{dx} = 6x - 5, \text{ grad} = 7$$

$$\therefore y - 4 = 7(x - 2)$$

$$y = 7x - 10$$

b $x = -3 \therefore y = 6$

$$\frac{dy}{dx} = 3x^2 + 10x, \text{ grad} = -3$$

$$\therefore \text{grad of normal} = \frac{1}{3}$$

$$\therefore y - 6 = \frac{1}{3}(x + 3)$$

$$y = \frac{1}{3}x + 7$$

12 a $\frac{dy}{dx} = 3x^2 + 6x - 16$, grad = 8

$$\therefore y + 10 = 8(x - 2) \quad [y = 8x - 26]$$

b $3x^2 + 6x - 16 = 8$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$x = 2 \text{ (at } P) \text{ or } -4$$

$$\therefore Q(-4, 50)$$

13 a $\frac{dy}{dx} = 2x - 3$, grad = 1

$$\therefore \text{grad of normal} = -1$$

$$\therefore y - 2 = -(x - 2) \quad [y = 4 - x]$$

b $x^2 - 3x + 4 = 4 - x$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 2 \text{ (at } A) \text{ or } 0$$

$$\therefore B(0, 4)$$

14 a $f'(x) = 3x^2 + 8x$

b $x = -3 \therefore y = -9$

$$\text{grad} = 3$$

$$\therefore y + 9 = 3(x + 3)$$

$$y = 3x \text{ which passes through } (0, 0)$$

$$15 \quad \text{a} \quad y = 0 \Rightarrow 6 + x - x^2 = 0$$

$$(2 + x)(3 - x) = 0$$

$$x = -2, 3$$

$$\text{+ve } x\text{-axis} \therefore P(3, 0)$$

$$x = 0 \Rightarrow y = 6 \therefore Q(0, 6)$$

$$\text{b} \quad \frac{dy}{dx} = 1 - 2x$$

$$\text{grad at } P = -5$$

$$y = -5(x - 3) \quad [y = 15 - 5x]$$

$$\text{c} \quad \text{grad at } Q = 1$$

$$\text{tangent at } Q: y = x + 6$$

$$\therefore 15 - 5x = x + 6$$

$$x = \frac{3}{2}$$

$$\therefore \left(\frac{3}{2}, \frac{15}{2}\right)$$

$$16 \quad \text{a} \quad \text{grad of } l = -3$$

$$\text{for curve, } \frac{dy}{dx} = 2x - 5$$

$$\therefore \text{at } A, \quad 2x - 5 = -3$$

$$x = 1$$

$$\therefore A(1, -1)$$

$$\text{b} \quad y + 1 = -3(x - 1)$$

$$y = -3x + 2$$

$$17 \quad \text{grad of normal} = 2$$

$$\therefore \text{grad of curve} = -\frac{1}{2}$$

$$\text{for curve, } \frac{dy}{dx} = -32x^{-3}$$

$$\therefore -\frac{32}{x^3} = -\frac{1}{2}$$

$$x^3 = 64$$

$$x = 4 \therefore (4, 1)$$

$$\text{sub. } 1 = 8 + k$$

$$k = -7$$

$$18 \quad \text{a} \quad \frac{ds}{dt} = 3 + 10t$$

$$t = 0.6 \Rightarrow \frac{ds}{dt} = 9 \text{ metres per second}$$

$$\text{b} \quad 54 = 3t + 5t^2$$

$$5t^2 + 3t - 54 = 0$$

$$(5t + 18)(t - 3) = 0$$

$$t > 0 \therefore t = 3$$

$$\therefore \frac{ds}{dt} = 33 \text{ metres per second}$$

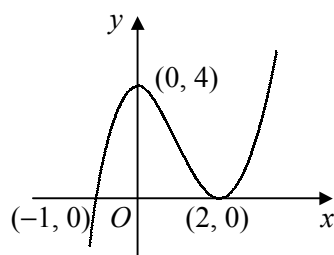
$$19 \quad \text{a} \quad \frac{dh}{dt} = \frac{1}{3}kt^{-\frac{2}{3}}$$

$$\text{when } t = 1, \frac{dh}{dt} = 3$$

$$\therefore \frac{1}{3}k = 3$$

$$k = 9$$

$$\text{b} \quad \frac{dh}{dt} = 3 \times 8^{-\frac{2}{3}} = 0.75 \text{ cm per second}$$

1 a


$$\begin{aligned} \text{b } f(x) &= (x+1)(x^2 - 4x + 4) \\ &= x^3 - 4x^2 + 4x + x^2 - 4x + 4 \\ &= x^3 - 3x^2 + 4 \end{aligned}$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ \text{c } x = 1 \quad \therefore y &= 2 \times (-1)^2 = 2 \\ \text{grad} &= 3 - 6 = -3 \end{aligned}$$

$$\begin{aligned} \therefore y - 2 &= -3(x - 1) \\ y - 2 &= -3x + 3 \\ y &= 5 - 3x \end{aligned}$$

$$\text{3 a } x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1 \quad a < b \quad \therefore a = -2, b = 1$$

$$\text{b } \frac{dy}{dx} = 2x + 1$$

$$\text{grad at } A = -3$$

$$\therefore \text{grad of normal} = \frac{1}{3}$$

$$\therefore y - 0 = \frac{1}{3}(x + 2)$$

$$3y = x + 2$$

$$x - 3y + 2 = 0$$

$$\text{c grad at } B = 3$$

$$\text{tangent at } B: y - 0 = 3(x - 1)$$

$$y = 3x - 3$$

$$\text{at } C, x - 3(3x - 3) + 2 = 0$$

$$x = \frac{11}{8}$$

$$\therefore C\left(\frac{11}{8}, \frac{9}{8}\right)$$

$$\text{5 a } \frac{dy}{dx} = -24x^{-3}$$

$$\text{at } A, y = 3, \text{ grad} = -3$$

$$\therefore y - 3 = -3(x - 2)$$

$$3x + y - 9 = 0$$

$$\text{b tangent:}$$

$$x = -1 \Rightarrow -3 + y - 9 = 0 \Rightarrow y = 12$$

$$\text{curve:}$$

$$x = -1 \Rightarrow y = \frac{12}{1} \Rightarrow y = 12$$

$$\therefore \text{tangent intersects curve at } (-1, 12)$$

$$\text{2 a } \frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$$

$$\text{grad at } P = \frac{1}{4}$$

$$\therefore y - 1 = \frac{1}{4}(x - 4)$$

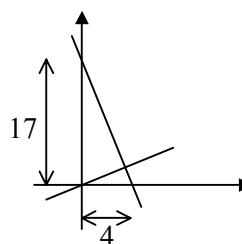
$$y = \frac{1}{4}x \text{ which passes through } (0, 0)$$

$$\text{b grad of normal} = -4$$

$$\therefore y - 1 = -4(x - 4) \quad [y = 17 - 4x]$$

$$\text{at } Q, x = 0 \Rightarrow y = 17$$

$$\therefore \text{area} = \frac{1}{2} \times 17 \times 4 = 34$$



$$\text{4 } y = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

$$= \frac{x^2 - 2x + 1}{2x^{\frac{3}{2}}}$$

$$= \frac{(x-1)^2}{2x^{\frac{3}{2}}} \quad [a = -1, b = 2]$$

$$\text{6 a } \frac{dy}{dx} = 3 + 2kx - 3x^2$$

$$\text{at } P, 3 - 2k - 3 = -6$$

$$k = 3$$

$$\text{b } y = 2 + 3x + 3x^2 - x^3 \quad \therefore P(-1, 3)$$

$$\text{at } Q, 3 + 6x - 3x^2 = -6$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1 \text{ (at } P) \text{ or } 3 \quad \therefore Q(3, 11)$$

$$PQ = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$$

$$7 \quad = \frac{d}{dx} (x^2 + \frac{1}{2}x^{-1})$$

$$= 2x - \frac{1}{2}x^{-2}$$

$$8 \quad a \quad \frac{dy}{dx} = 4x - 7$$

$$\text{at } A, y = -5, \text{ grad} = 1$$

$$\therefore y + 5 = 1(x - 2)$$

$$[y = x - 7]$$

$$b \quad \text{grad of normal at } B = 1$$

$$\therefore \text{grad of curve at } B = -1$$

$$\therefore 4x - 7 = -1$$

$$x = \frac{3}{2}, y = 2(\frac{3}{2}) - 7(\frac{3}{2}) + 1 = -5$$

$$\therefore B(\frac{3}{2}, -5)$$

$$9 \quad a \quad \frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$$

$$b \quad \frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$$

$$\therefore 2x \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x$$

$$= 2x(2 - \frac{3}{4}x^{-\frac{3}{2}}) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$= 4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$= 0$$

$$10 \quad a \quad \frac{dy}{dx} = -4x^{-2}$$

$$\text{grad at } M = -\frac{1}{4}$$

$$\therefore \text{grad of normal} = 4$$

$$\therefore y - 3 = 4(x - 4) \quad [y = 4x - 13]$$

$$b \quad 4x - 13 = 2 + \frac{4}{x}$$

$$4x^2 - 15x - 4 = 0$$

$$(4x + 1)(x - 4) = 0$$

$$x = 4 \text{ (at } M) \text{ or } -\frac{1}{4}$$

$$\therefore N(-\frac{1}{4}, -14)$$

$$11 \quad a \quad \frac{dy}{dx} = 3x^2 - 6x - 8$$

$$\text{grad at } P = 1$$

$$\therefore y - 8 = 1(x + 1) \quad [y = x + 9]$$

$$b \quad \text{at } Q, 3x^2 - 6x - 8 = 1$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \text{ at } P \therefore Q(3, -20)$$

$$\therefore y + 20 = 1(x - 3) \quad [y = x - 23]$$

$$c \quad \text{grad normal} = -1$$

$$\therefore y - 8 = -(x + 1) \quad [y = 7 - x]$$

$$d \quad \text{normal at } P \text{ meets } m \text{ when}$$

$$7 - x = x - 23$$

$$x = 15 \therefore (15, -8)$$

$$\text{dist between lines} = \text{dist } P \text{ to } (15, -8)$$

$$= \sqrt{16^2 + 16^2} = \sqrt{16^2 \times 2} = 16\sqrt{2}$$

$$12 \quad a \quad y = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$\text{at } P, \frac{1}{2}k(\frac{1}{\sqrt{2}}) - \frac{3}{2}(\sqrt{2}) = \sqrt{2}$$

$$k - 6 = 4$$

$$k = 10$$

$$b \quad y = \sqrt{x}(10 - x)$$

$$\text{at } P, y = \sqrt{2}(10 - 2) = 8\sqrt{2}$$

$$\text{grad of normal} = -\frac{1}{\sqrt{2}}$$

$$\therefore y - 8\sqrt{2} = -\frac{1}{\sqrt{2}}(x - 2)$$

$$\sqrt{2}y - 16 = -x + 2$$

$$x + \sqrt{2}y = 18 \quad [c = 18]$$