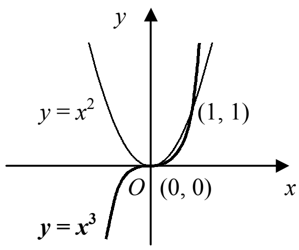
C1 GRAPHS OF FUNCTIONS — Answers - Worksheet A

1 **a**



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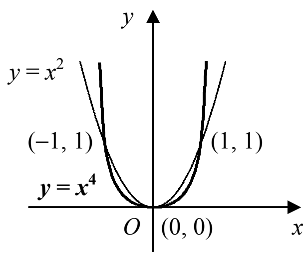

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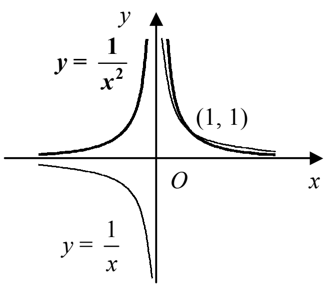
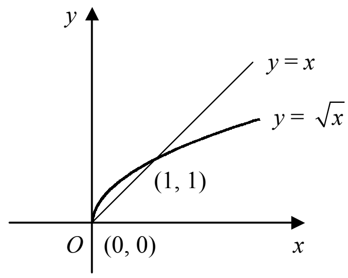


asymptotes: $y = 0$ and $x = 0$

d



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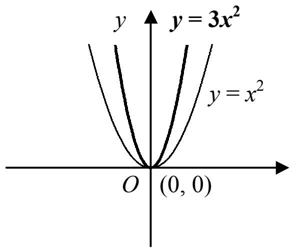

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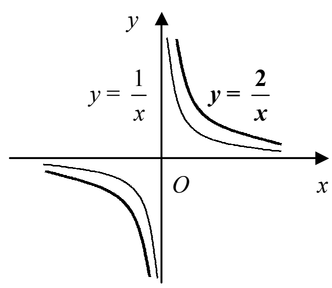
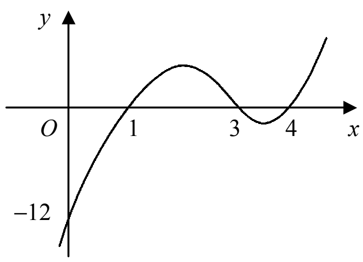


asymptotes: $y = 0$ and $x = 0$

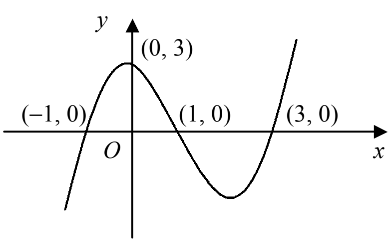
2 **a** $= (-1) \times (-3) \times (-4) = -12$

b $x = 1, 3, 4$

c



3 **a**



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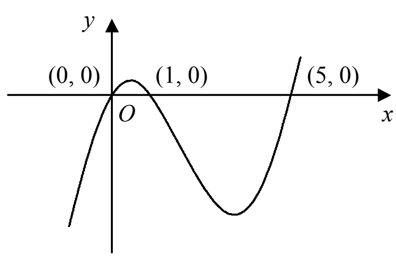

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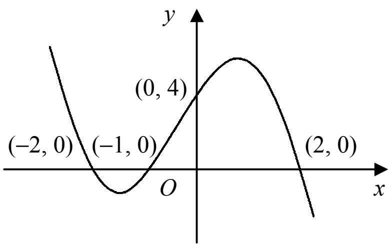

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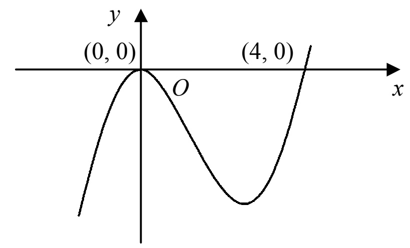

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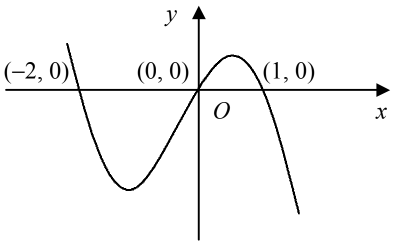

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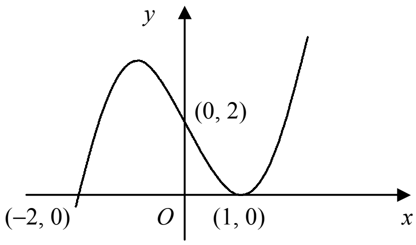
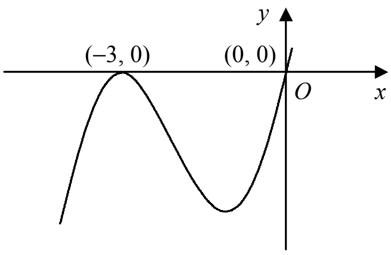
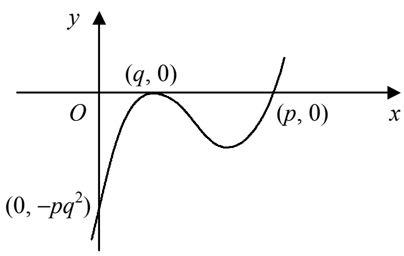


4 **a** $= x(x^2 + 6x + 9) = x(x + 3)^2$

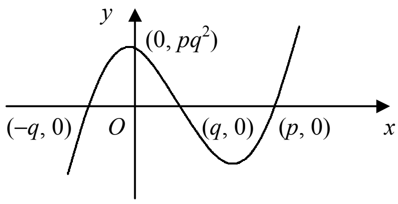
b



5 **a**



b $y = (x - p)(x + q)(x - q)$



6 TP at $(1, -2)$

$\therefore\ f(x) = k(x - 1)^2 - 2$

crosses y-axis at $(0, -5)$

$\therefore\ -5 = k - 2$

$k = -3$

$\therefore\ f(x) = -3(x - 1)^2 - 2$

$[\ f(x) = -3x^2 + 6x - 5\ ]$

7 crosses x-axis at $(-2, 0)$, $(1, 0)$ and $(2, 0)$

$\therefore\ y = k(x + 2)(x - 1)(x - 2)$

crosses y-axis at $(0, -8)$

$\therefore\ -8 = 4k$

$k = -2$

$\therefore\ y = -2(x + 2)(x - 1)(x - 2)$

$= -2(x + 2)(x^2 - 3x + 2)$

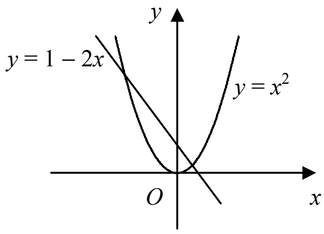
$= -2(x^3 - 3x^2 + 2x + 2x^2 - 6x + 4)$

$= -2x^3 + 2x^2 + 8x - 8$

$\therefore\ a = -2,\ b = 2,\ c = 8,\ d = -8$

8 **a** 4 **b** 0 **c** 2 **d** 3

9 **a**



b 2 roots as $x^2 + 2x - 1 = 0 \Rightarrow x^2 = 1 - 2x$ and the graphs of $y = x^2$ and $y = 1 - 2x$ intersect at 2 points

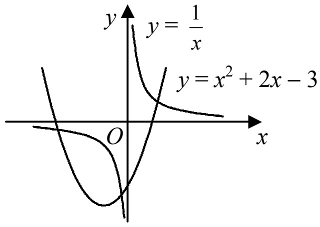
10 **a** $x^2 + 2x - 3 = (x + 1)^2 - 1 - 3 = (x + 1)^2 - 4 \;\therefore$ turning point is $(-1, -4)$

b $x^2 + 2x - 3 - \frac{1}{x} = 0 \Rightarrow x^2 + 2x - 3 = \frac{1}{x}$

$\therefore$ roots where $y = x^2 + 2x - 3$ and $y = \frac{1}{x}$ intersect

graphs intersect at 1 point for $x > 0$ and 2 points for $x < 0$

$\therefore$ one positive and two negative real roots



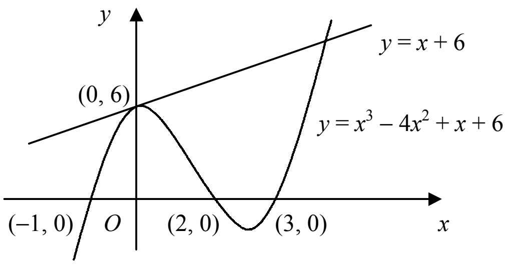
11 $x - 3 = x^2 - 5x + 6$
$x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$
repeated root
$\therefore\; y = x - 3$ is tangent to $y = x^2 - 5x + 6$

12 **a** $x^2 + 5x + 8 = 3x + 7$
$x^2 + 2x + 1 = 0$
$(x + 1)^2 = 0$
$x = -1 \;\therefore\; x = -1, y = 4$

b repeated root
$\therefore\; y = 3x + 7$ is tangent to $y = x^2 + 5x + 8$ at the point $(-1, 4)$

13 **a** $x^3 - 4x^2 + x + 6 = x + 6$
$x^3 - 4x^2 = 0$
$x^2(x - 4) = 0$
$x = 0, 4 \;\therefore\; (0, 6)$ and $(4, 10)$

b



14 $2x^2 - 5x + 1 = 3x + k$
$2x^2 - 8x + 1 - k = 0$
for tangent, repeated root $\therefore\; b^2 - 4ac = 0$
$\therefore\; 64 - 8(1 - k) = 0$
$k = -7$

15 $x^2 + ax + 18 = 2 - 5x$
$x^2 + (a + 5)x + 16 = 0$
intersect at 2 points $\therefore\; b^2 - 4ac > 0$
$\therefore\; (a + 5)^2 - 64 > 0$
$a^2 + 10a - 39 > 0$
$(a + 13)(a - 3) > 0$
$a < -13$ or $a > 3$

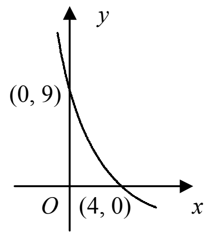
16 **a** $x^2 - 2x + 6 = px + p$
$x^2 - (p + 2)x + 6 - p = 0$
for tangent, repeated root $\therefore\; b^2 - 4ac = 0$
$\therefore\; (p + 2)^2 - 4(6 - p) = 0$
$p^2 + 8p - 20 = 0$
$(p + 10)(p - 2) = 0$
$p = -10, 2$

b $x^2 - 2x + 6 = qx + 7$
$x^2 - (q + 2)x - 1 = 0$
for tangent, repeated root $\therefore\; b^2 - 4ac = 0$
$\Rightarrow\; (q + 2)^2 + 4 = 0$
but for real q, $(q + 2)^2 \geq 0 \;\therefore$ no solutions

C1 GRAPHS OF FUNCTIONS — Answers - Worksheet B

1 **a** translated 1 unit in positive x-direction
b translated 3 units in negative y-direction
c stretched by a factor of 2 in y-direction
d stretched by a factor of $\frac{1}{4}$ in x-direction
e reflected in the x-axis
f stretched by a factor of $\frac{1}{5}$ in y-direction
g reflected in the y-axis
h stretched by a factor of $\frac{3}{2}$ in x-direction

2 **a**



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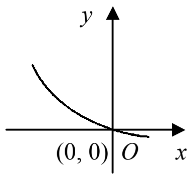
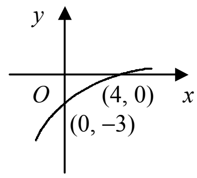
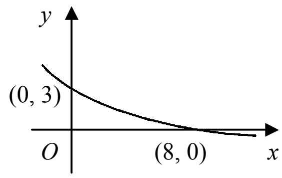


c

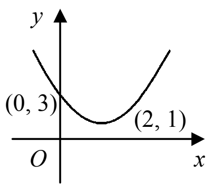


d



3 **a** $y = 2x + 5 + 1 \Rightarrow y = 2x + 6$
b $y = 3(1 - 4x) \Rightarrow y = 3 - 12x$
c $y = 3(x + 4) + 1 \Rightarrow y = 3x + 13$
d $y = -(4x - 7) \Rightarrow y = 7 - 4x$

4 **a**



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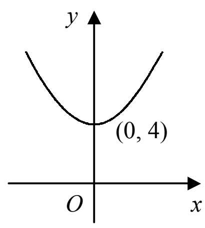

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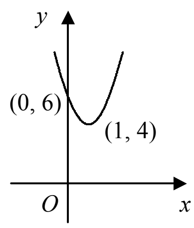
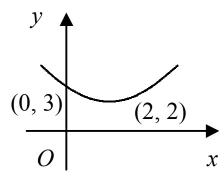


d



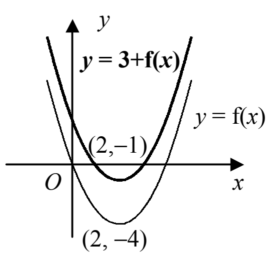
5 **a** stretch by a factor of 4 in y-direction
b translation by 2 units in positive x-direction
c reflection in the x-axis
d translation by 5 units in positive y-direction

6 **a** $y = 2(x^2 + 2)$
stretch by a factor of 2 in y-direction
b $y = (x^2 + 2) - 7$
translation by 7 units in negative y-direction
c $y = (\frac{1}{3}x)^2 + 2$
stretch by a factor of 3 in x-direction
d $y = (x + 2)^2 + 2$
translation by 2 units in negative x-direction

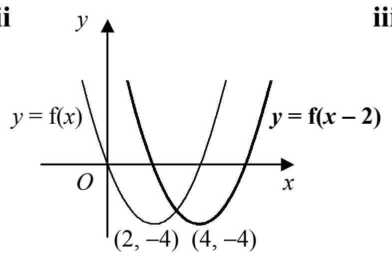
7 **a** $y = (x - 1)^2 + 2(x - 1) \Rightarrow y = x^2 - 1$
b $y = (3x)^2 - 4(3x) + 5 \Rightarrow y = 9x^2 - 12x + 5$
c $y = (-x)^2 + (-x) - 6 \Rightarrow y = x^2 - x - 6$
d $y = 2(\frac{1}{2}x)^2 - 3(\frac{1}{2}x) \Rightarrow y = \frac{1}{2}x^2 - \frac{3}{2}x$

8 **a** $f(x) = (x - 2)^2 - 4 \quad \therefore$ turning point $(2, -4)$

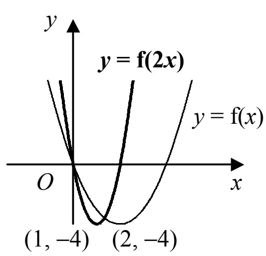
b **i**



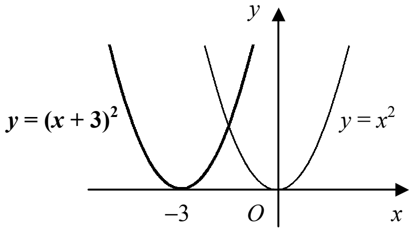
ii



iii



9 **a**



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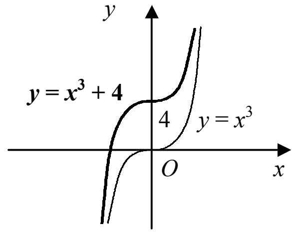
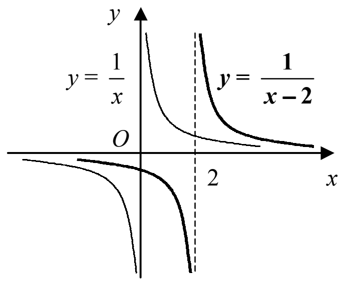
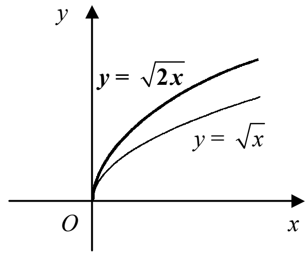


c



d



10 **a** let $f(x) = \frac{1}{x}$ $\therefore$ $\frac{1}{3x} = \frac{1}{3}f(x)$ or $f(3x)$

$\therefore$ stretch by a factor of $\frac{1}{3}$ in y-direction

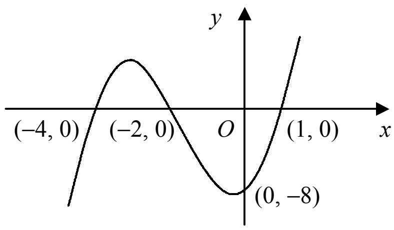
or stretch by a factor of $\frac{1}{3}$ in x-direction

b let $g(x) = x^2$ $\therefore$ $4x^2 = 4g(x)$ or $g(2x)$

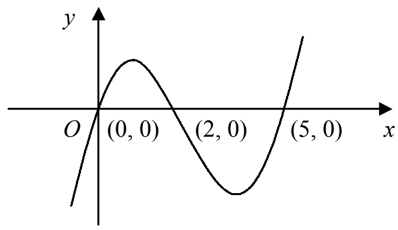
$\therefore$ stretch by a factor of 4 in y-direction

or stretch by a factor of $\frac{1}{2}$ in x-direction

11 **a**



b



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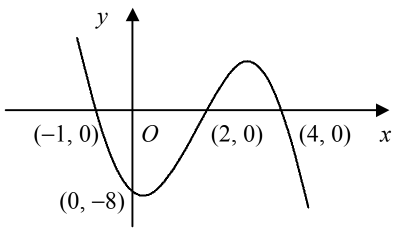

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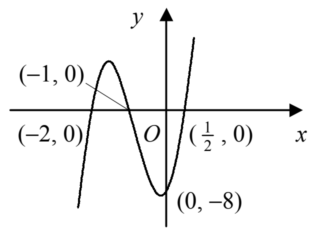
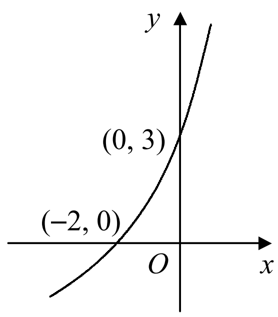


12 **a** $(a, 3b)$ **b** $(a, b + 4)$ **c** $(a - 1, b)$ **d** $(3a, b)$

13 **a**



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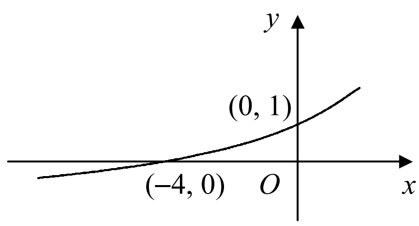
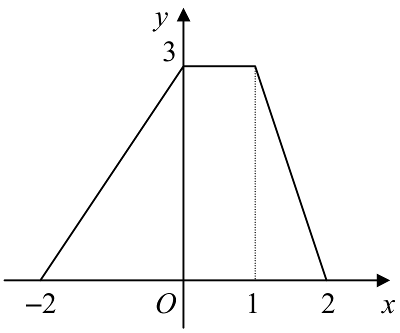


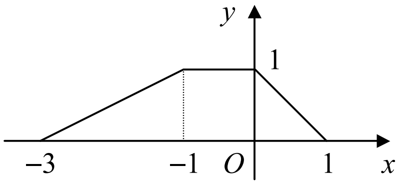
C1 GRAPHS OF FUNCTIONS — Answers - Worksheet C

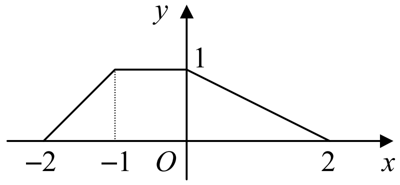
1 **a** $4x^2 - 9x + 5 = 3x - 4$
$4x^2 - 12x + 9 = 0$
$(2x - 3)^2 = 0$
$x = \frac{3}{2}$
$\therefore\ x = \frac{3}{2}, y = \frac{1}{2}$

b $y = 3x - 4$ is a tangent to the curve
$y = 4x^2 - 9x + 5$ at the point $(\frac{3}{2}, \frac{1}{2})$

2 **a**



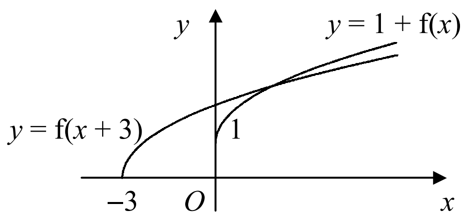




3 **a** $x^2 + 5x + 2 = 4x + 1$
$x^2 + x + 1 = 0$
$b^2 - 4ac = 1 - 4 = -3$
$b^2 - 4ac < 0\ \therefore$ no real roots
$\therefore$ does not intersect

b $x^2 + 5x + 2 = mx + 1$
$x^2 + (5 - m)x + 1 = 0$
only one root $\therefore\ b^2 - 4ac = 0$
$(5 - m)^2 - 4 = 0$
$5 - m = \pm 2$
$m = 3$ or 7

4 **a**

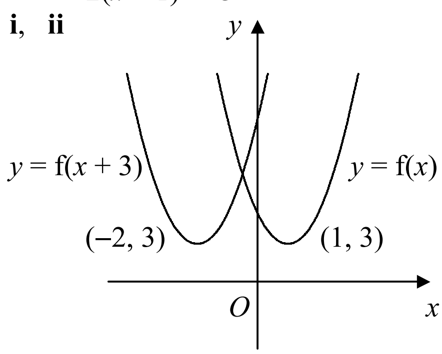


b $1 + \sqrt{x} = \sqrt{x+3}$
$(1 + \sqrt{x})^2 = x + 3$
$1 + 2\sqrt{x} + x = x + 3$
$\sqrt{x} = 1$
$x = 1\ \therefore\ (1, 2)$

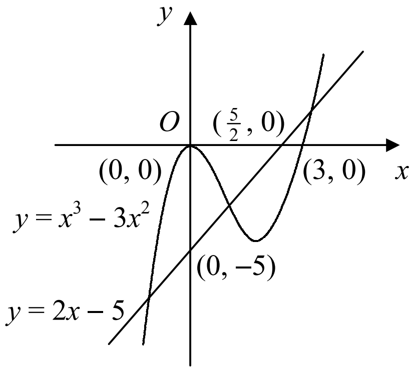
5 $x^2 + kx - 3 = k - x$
$x^2 + (k + 1)x - (k + 3) = 0$
$b^2 - 4ac = (k + 1)^2 + 4(k + 3)$
$= k^2 + 6k + 13$
$= (k + 3)^2 - 9 + 13$
$= (k + 3)^2 + 4$
real $k \Rightarrow\ (k + 3)^2 \geq 0$
$\Rightarrow\ (k + 3)^2 + 4 \geq 4$
$\therefore\ b^2 - 4ac > 0$
$\Rightarrow$ real and distinct roots
$\therefore$ l intersects C at exactly two points

6 **a** $f(x) = 2[x^2 - 2x] + 5$
$= 2[(x - 1)^2 - 1] + 5$
$= 2(x - 1)^2 + 3$

b **i**, **ii**



7 **a** $y = x^3 - 3x^2 = x^2(x - 3)$



b 3 real roots

$x^3 - 3x^2 - 2x + 5 = 0 \Rightarrow x^3 - 3x^2 = 2x - 5$

the graphs of $y = x^3 - 3x^2$ and $y = 2x - 5$

intersect at three points

8 touches x-axis at (2, 0)

$\therefore\ y = k(x - 2)^2$

crosses y-axis at (0, –6)

$\therefore\ -6 = 4k$

$k = -\frac{3}{2}$

$\therefore\ y = -\frac{3}{2}(x - 2)^2$

$y = -\frac{3}{2}x^2 + 6x - 6$

$\therefore\ a = -\frac{3}{2}$, $b = 6$ and $c = -6$

9 **a** LHS $= (1 - x)(2 + x)^2$

$= (1 - x)(4 + 4x + x^2)$

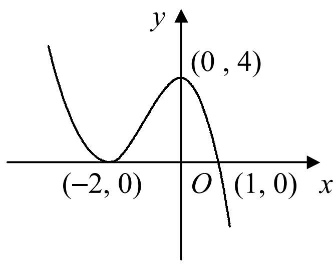
$= (4 + 4x + x^2) - x(4 + 4x + x^2)$

$= 4 + 4x + x^2 - 4x - 4x^2 - x^3$

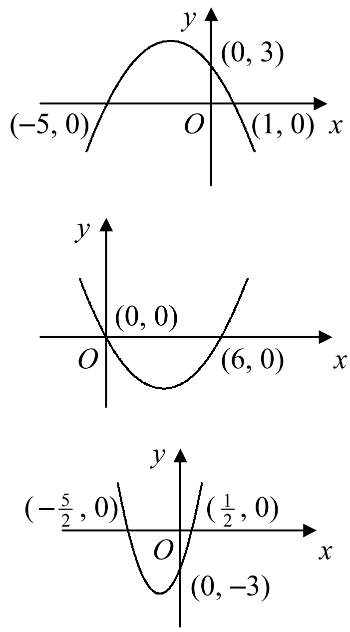
$= 4 - 3x^2 - x^3$

= RHS

b

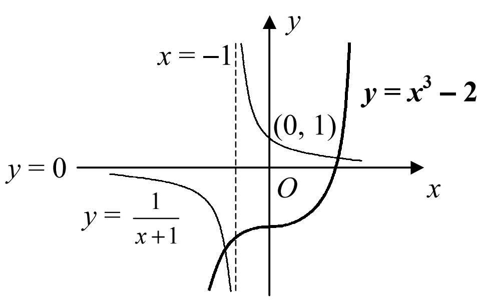


10 **a**



11 **a** translation by 1 unit in the negative x-direction

b



c $x^3 - \frac{1}{x+1} = 2 \Rightarrow x^3 - 2 = \frac{1}{x+1}$

the graphs $y = x^3 - 2$ and $y = \frac{1}{x+1}$ intersect

at one point for $x > 0$ and at one point for $x < 0$

$\therefore$ one positive and one negative real root