

1 **a** $\frac{1}{3}x^3 + c$ **b** $\frac{1}{7}x^7 + c$ **c** $\frac{1}{2}x^2 + c$ **d** $-\frac{1}{3}x^{-3} + c$ **e** $5x + c$ **f** $x^3 + c$

g $\frac{1}{2}x^8 + c$ **h** $-6x^{-1} + c$ **i** $\frac{4}{3}x^6 + c$ **j** $\frac{1}{6}x^2 + c$ **k** $-\frac{1}{4}x^{-8} + c$ **l** $-\frac{3}{8}x^{-2} + c$

2 **a** $= x^2 + 3x + c$ **b** $= 3x^4 - 2x^2 + c$ **c** $= 7x - \frac{1}{3}x^3 + c$ **d** $= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$

$$\begin{aligned} \mathbf{e} &= \frac{1}{5}x^5 + \frac{5}{3}x^3 + c & \mathbf{f} &= \int (x^3 - 3x) \, dx & \mathbf{g} &= \int (x^2 - 4x + 4) \, dx & \mathbf{h} &= \frac{3}{5}x^5 + \frac{1}{3}x^3 - 6x + c \\ & & &= \frac{1}{4}x^4 - \frac{3}{2}x^2 + c & &= \frac{1}{3}x^3 - 2x^2 + 4x + c & \end{aligned}$$

$$\begin{array}{llll} \mathbf{i} = \int (2 + x^{-2}) \, dx & \mathbf{j} = \int (x - x^{-3}) \, dx & \mathbf{k} = \int (2x^{-2} - 3x^2) \, dx & \mathbf{l} = \int (x^2 - 8 + 16x^{-2}) \, dx \\ = 2x - x^{-1} + c & = \frac{1}{2}x^2 + \frac{1}{2}x^{-2} + c & = -2x^{-1} - x^3 + c & = \frac{1}{3}x^3 - 8x - 16x^{-1} + c \end{array}$$

$$\mathbf{3} \quad \mathbf{a} = \frac{2}{3}y^{\frac{3}{2}} + c \qquad \mathbf{b} = \frac{2}{7}y^{\frac{7}{2}} + c \qquad \mathbf{c} = 2y^{\frac{1}{2}} + c$$

$$\mathbf{d} = 3y^{\frac{4}{3}} + c \qquad \mathbf{e} = \frac{4}{7}y^{\frac{7}{4}} + c \qquad \mathbf{f} = 15y^{\frac{1}{3}} + c$$

$$\begin{array}{lll} \mathbf{g} = \int y^{\frac{1}{4}} \, dx & \mathbf{h} = \int 7y^{-\frac{1}{2}} \, dx & \mathbf{i} = \int \frac{1}{2}y^{-2} \, dx \\ = \frac{4}{5}y^{\frac{5}{4}} + c & = 14y^{\frac{1}{2}} + c & = -\frac{1}{2}y^{-1} + c \end{array}$$

$$\begin{array}{lll} \mathbf{j} & = \int y^{\frac{3}{2}} \, dx & \mathbf{k} = \int \frac{5}{2} y^{-4} \, dx & \mathbf{l} = \int \frac{1}{3} y^{-\frac{1}{2}} \, dx \\ & = \frac{2}{5} y^{\frac{5}{2}} + c & = -\frac{5}{6} y^{-3} + c & = \frac{2}{3} y^{\frac{1}{2}} + c \end{array}$$

$$\begin{array}{lll} \mathbf{4} & \mathbf{a} = 2t^{\frac{3}{2}} - t + c & \mathbf{b} = \int (2r + r^{\frac{1}{2}}) \, dr = r^2 + \frac{2}{3}r^{\frac{3}{2}} + c \\ & & \mathbf{c} = \int (9p^2 - 6p + 1) \, dp = 3p^3 - 3p^2 + p + c \end{array}$$

$$\begin{aligned} \mathbf{e} &= \int (y^{-3} + y) \, dy & \mathbf{f} &= \frac{1}{6}x^3 - \frac{2}{5}x^{\frac{5}{2}} + c & \mathbf{g} &= \int (t^2 + 2) \, dt & \mathbf{h} &= \frac{3}{8}r^{\frac{8}{3}} - \frac{3}{5}r^{\frac{5}{3}} + c \\ &= -\frac{1}{2}y^{-2} + \frac{1}{2}y^2 + c & & & &= \frac{1}{3}t^3 + 2t + c & & \end{aligned}$$

$$\begin{aligned} \mathbf{i} &= \int (2p^3 - \tfrac{1}{2}p) \, dp & \mathbf{j} &= 4y - \tfrac{4}{11}y^{\frac{11}{4}} + c & \mathbf{k} &= \int (\tfrac{1}{3}x^{-2} + 2) \, dx & \mathbf{l} &= \int (2t^{\frac{1}{2}} + 3t^{-\frac{1}{2}}) \, dt \\ &= \tfrac{1}{2}p^4 - \tfrac{1}{4}p^2 + c & & & &= -\tfrac{1}{3}x^{-1} + 2x + c & &= \tfrac{4}{3}t^{\frac{3}{2}} + 6t^{\frac{1}{2}} + c \end{aligned}$$

- 5 **a** $= x^3 - \frac{1}{2}x^2 + 6x + c$ **b** $= \frac{1}{7}x^7 - \frac{1}{4}x^4 + x^2 - 5x + c$ **c** $= \int (x^3 - x^2 - 2x) \, dx$
 $= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + c$
- d** $= \int (x + 4x^{\frac{1}{2}} + 4) \, dx$ **e** $= \int (2x^3 + 3x^2 - 8x - 12) \, dx$ **f** $= \int (x^3 - 2x^{\frac{4}{3}} + 7x^{-2}) \, dx$
 $= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$ $= \frac{1}{2}x^4 + x^3 - 4x^2 - 12x + c$ $= \frac{1}{4}x^4 - \frac{6}{7}x^{\frac{7}{3}} - 7x^{-1} + c$
- g** $= \int (\frac{1}{4}x^{-3} - \frac{2}{3}x^{-2}) \, dx$ **h** $= \int (1 - 4x^{-2} + 4x^{-4}) \, dx$ **i** $= \int (x^4 + x^{\frac{5}{2}} - x^{\frac{3}{2}} - 1) \, dx$
 $= -\frac{1}{8}x^{-2} + \frac{2}{3}x^{-1} + c$ $= x + 4x^{-1} - \frac{4}{3}x^{-3} + c$ $= \frac{1}{5}x^5 + \frac{2}{7}x^{\frac{7}{2}} - \frac{2}{5}x^{\frac{5}{2}} - x + c$
- 6 **a** $y = \int (8x + 3) \, dx$ **b** $y = \int (\frac{1}{2}x^3 - x^2) \, dx$ **c** $y = \int \frac{4}{3}x^{-3} \, dx$
 $y = 4x^2 + 3x + c$ $y = \frac{1}{8}x^4 - \frac{1}{3}x^3 + c$ $y = -\frac{2}{3}x^{-2} + c$
- d** $y = \int (x^3 + 3x^2 + 3x + 1) \, dx$ **e** $y = \int (2x - 3x^{-\frac{1}{2}}) \, dx$ **f** $y = \int (x^{\frac{3}{2}} - 2x^{-\frac{3}{2}}) \, dx$
 $y = \frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$ $y = x^2 - 6x^{\frac{1}{2}} + c$ $y = \frac{2}{5}x^{\frac{5}{2}} + 4x^{-\frac{1}{2}} + c$
- g** $y = \int (\frac{3}{2}x^{-2} - \frac{1}{2}) \, dx$ **h** $y = \int (10x^{-3} - 2x^{-2}) \, dx$ **i** $y = \int (3x^{\frac{1}{2}} - \frac{2}{3}x^{-\frac{1}{2}}) \, dx$
 $y = -\frac{3}{2}x^{-1} - \frac{1}{2}x + c$ $y = -5x^{-2} + 2x^{-1} + c$ $y = 2x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{1}{2}} + c$

1 a $x^2 + x + c$

b $y = x^2 + x + c$

$(1, 5) \Rightarrow 5 = 1 + 1 + c$

$\therefore c = 3$

$y = x^2 + x + 3$

2 a $y = \int (3 - 6x) \, dx$

$y = 3x - 3x^2 + c$

$(2, 1) \Rightarrow 1 = 6 - 12 + c$

$\therefore c = 7$

$y = 3x - 3x^2 + 7$

c $y = \int (x^2 + 4x + 1) \, dx$

$y = \frac{1}{3}x^3 + 2x^2 + x + c$

$(-3, 4) \Rightarrow 4 = -9 + 18 - 3 + c$

$\therefore c = -2$

$y = \frac{1}{3}x^3 + 2x^2 + x - 2$

e $y = \int (8x - 2x^{-2}) \, dx$

$y = 4x^2 + 2x^{-1} + c$

$(\frac{1}{2}, -1) \Rightarrow -1 = 1 + 4 + c$

$\therefore c = -6$

$y = 4x^2 + 2x^{-1} - 6$

3 $f(x) = \int (3 + 2x - x^2) \, dx$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$

$(3, 5) \Rightarrow 5 = 9 + 9 - 9 + c$

$\therefore c = -4$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$

b $y = \int (3x^2 - x) \, dx$

$y = x^3 - \frac{1}{2}x^2 + c$

$(4, 41) \Rightarrow 41 = 64 - 8 + c$

$\therefore c = -15$

$y = x^3 - \frac{1}{2}x^2 - 15$

d $y = \int (7 - 5x - x^3) \, dx$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4 + c$

$(2, 0) \Rightarrow 0 = 14 - 10 - 4 + c$

$\therefore c = 0$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4$

f $y = \int (3 - x^{\frac{1}{2}}) \, dx$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + c$

$(4, 8) \Rightarrow 8 = 12 - \frac{16}{3} + c$

$\therefore c = \frac{4}{3}$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$

4 $y = \int (10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}) \, dx$

$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$

$y = 0$ when $x = 7$

$\therefore 7 = 0 + 0 + c$

$c = 7$

$\therefore y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$

when $x = 4$

$y = 4(32) - 4(2) + 7$

$y = 127$

$$\begin{aligned}
 5 \quad \mathbf{a} \quad f(x) &= \int (2x^3 - x - 8) \, dx \\
 f(x) &= \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x + c \\
 (-1, 4) &\Rightarrow 4 = \frac{1}{2} - \frac{1}{2} + 8 + c \\
 \therefore c &= -4
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x - 4 \\
 \mathbf{b} \quad \text{at } x = 2, \quad y &= 8 - 2 - 16 - 4 = -14 \\
 \text{grad} &= 16 - 2 - 8 = 6 \\
 \therefore y + 14 &= 6(x - 2) \\
 [y &= 6x - 26]
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad y &= \int (3x + 2x^{-2}) \, dx \\
 y &= \frac{3}{2}x^2 - 2x^{-1} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad y &= 8 \text{ when } x = 2 \\
 \therefore 8 &= 6 - 1 + c \\
 c &= 3 \\
 \therefore y &= \frac{3}{2}x^2 - 2x^{-1} + 3 \\
 \text{when } x &= \frac{1}{2} \\
 y &= \frac{3}{8} - 4 + 3 \\
 y &= -\frac{5}{8}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad f(x) &= \int (3x^2 - 8x - 5) \, dx \\
 f(x) &= x^3 - 4x^2 - 5x + c \\
 (0, 0) &\Rightarrow 0 = 0 + c \\
 \therefore c &= 0
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= x^3 - 4x^2 - 5x \\
 &= x(x^2 - 4x - 5) \\
 &= x(x + 1)(x - 5) \\
 \text{crosses } x\text{-axis when } f(x) &= 0 \\
 \therefore (-1, 0) \text{ and } (5, 0)
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} \quad y &= \int (3x^2 + kx) \, dx \\
 y &= x^3 + \frac{1}{2}kx^2 + c \\
 (1, 6) &\Rightarrow 6 = 1 + \frac{1}{2}k + c \\
 5 &= \frac{1}{2}k + c \quad (1) \\
 (2, 1) &\Rightarrow 1 = 8 + 2k + c \\
 -7 &= 2k + c \quad (2) \\
 (2) - (1) \quad -12 &= \frac{3}{2}k \\
 k &= -8
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \text{sub. } -7 &= -16 + c \\
 c &= 9 \\
 \therefore y &= x^3 - 4x^2 + 9
 \end{aligned}$$

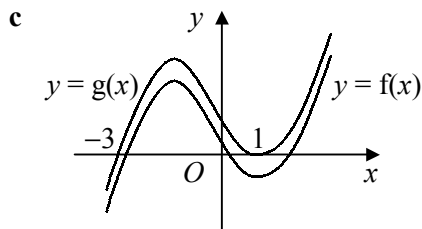
$$1 \quad = \frac{1}{3}x^3 + 4x^{\frac{3}{2}} - 3x + c$$

$$2 \quad \begin{aligned} \mathbf{a} \quad f(x) &= \int (1 - 6x^{-3}) \, dx \\ &= x + 3x^{-2} + c \\ (1, -2) &\Rightarrow -2 = 1 + 3 + c \\ c &= -6 \\ \therefore f(x) &= x - 6 + \frac{3}{x^2} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad x = 2 &\Rightarrow y = -\frac{13}{4}, \text{ grad} = \frac{1}{4} \\ \therefore \text{grad of normal} &= -4 \\ \therefore y + \frac{13}{4} &= -4(x - 2) \\ 4y + 13 &= -16x + 32 \\ 16x + 4y - 19 &= 0 \end{aligned}$$

$$3 \quad \begin{aligned} \mathbf{a} \quad f(x) &= \int (3x^2 + 2x - 5) \, dx \\ &= x^3 + x^2 - 5x + c \\ (3, 22) &\Rightarrow 22 = 27 + 9 - 15 + c \\ c &= 1 \\ \therefore f(x) &= x^3 + x^2 - 5x + 1 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad g(x) &= (x+3)(x^2-2x+1) \\ &= x^3 - 2x^2 + x + 3x^2 - 6x + 3 \\ &= x^3 + x^2 - 5x + 3 \\ &= f(x) + 2 \end{aligned}$$



$$5 \quad \text{grad of tangent} = 12 - 8 - 1 = 3$$

tangent passes through (0, 0)

$$\therefore \text{tangent: } y = 3x$$

$$\text{when } x = 2, y = 6$$

$$\therefore \text{curve passes through } (2, 6)$$

$$\text{curve: } y = \int (3x^2 - 4x - 1) \, dx$$

$$y = x^3 - 2x^2 - x + c$$

$$(2, 6) \Rightarrow 6 = 8 - 8 - 2 + c$$

$$c = 8$$

$$\therefore y = x^3 - 2x^2 - x + 8$$

$$6 \quad \begin{aligned} \mathbf{a} \quad &= 3\sqrt{2} - \frac{2}{\sqrt{2}} \\ &= 3\sqrt{2} - \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad y &= \int (3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}) \, dx \\ &= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c \\ (4, 7) &\Rightarrow 7 = 2(8) - 4(2) + c \\ 7 &= 16 - 8 + c \\ c &= -1 \\ \therefore y &= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 1 \\ \text{when } x &= 3 \\ y &= 6\sqrt{3} - 4\sqrt{3} - 1 \\ y &= 2\sqrt{3} - 1 \end{aligned}$$

$$7 \quad \mathbf{a} \quad = \int (x^2 + 4x + 4) \, dx \\ = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$$\mathbf{b} \quad = \int \frac{1}{4}x^{-\frac{1}{2}} \, dx \\ = \frac{1}{2}x^{\frac{1}{2}} + c$$

$$8 \quad \mathbf{a} \quad f(x) = \int (3x^2 - 2x - 3) \, dx \\ = x^3 - x^2 - 3x + c$$

$$(-2, 0) \Rightarrow 0 = -8 - 4 + 6 + c$$

$$c = 6$$

$$\therefore f(x) = x^3 - x^2 - 3x + 6$$

$$\mathbf{b} \quad x = 1 \Rightarrow y = 1 - 1 - 3 + 6 = 3 \\ \text{grad} = 3 - 2 - 3 = -2$$

$$\therefore y - 3 = -2(x - 1)$$

$$y - 3 = -2x + 2$$

$$y = 5 - 2x$$

$$9 \quad \mathbf{a} \quad y = \int (2x - 3x^{-2}) \, dx \\ = x^2 + 3x^{-1} + c$$

$$y = 0 \text{ at } x = 1$$

$$\therefore 0 = 1 + 3 + c$$

$$c = -4$$

$$\therefore y = x^2 - 4 + \frac{3}{x}$$

$$\mathbf{b} \quad \frac{d^2y}{dx^2} = 2 + 6x^{-3}$$

$$\therefore x^2 \frac{d^2y}{dx^2} - 2y \\ = x^2(2 + 6x^{-3}) - 2(x^2 - 4 + 3x^{-1}) \\ = 2x^2 + 6x^{-1} - 2x^2 + 8 - 6x^{-1} \\ = 8 \quad [k = 8]$$

$$10 \quad \mathbf{a} \quad = -\frac{1}{2}x^{-2} + c$$

$$\mathbf{b} \quad = \int \frac{x^2 - 2x + 1}{x^{\frac{1}{2}}} \, dx \\ = \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \, dx \\ = \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

$$11 \quad \mathbf{a} \quad f(x) = \int (4x^3 - 8x) \, dx \\ = x^4 - 4x^2 + c \\ (2, -5) \Rightarrow -5 = 16 - 16 + c \\ c = -5$$

$$\therefore f(x) = x^4 - 4x^2 - 5$$

$$\mathbf{b} \quad x^4 - 4x^2 - 5 = 0 \\ (x^2 + 1)(x^2 - 5) = 0 \\ x^2 = -1 \text{ [no sols] or } 5 \\ x = \pm\sqrt{5} \\ \therefore (-\sqrt{5}, 0), (\sqrt{5}, 0)$$

$$12 \quad \mathbf{a} \quad y = \int (k - x^{-\frac{1}{2}}) \, dx$$

$$y = kx - 2x^{\frac{1}{2}} + c$$

$$(1, -2) \Rightarrow -2 = k - 2 + c$$

$$0 = k + c \quad (1)$$

$$(4, 5) \Rightarrow 5 = 4k - 4 + c$$

$$9 = 4k + c \quad (2)$$

$$(2) - (1) \quad 9 = 3k$$

$$k = 3$$

$$\mathbf{b} \quad \text{grad} = 3 - 1 = 2$$

$$\therefore \text{grad of normal} = -\frac{1}{2}$$

$$\therefore y + 2 = -\frac{1}{2}(x - 1)$$

$$2y + 4 = -x + 1$$

$$x + 2y + 3 = 0$$