Practice paper

You may not use a calculator when answering this paper.

You must show sufficient working to make your methods clear.

Answers without working may gain no credit.

- **1 a** Write down the value of $16^{\frac{1}{2}}$. (1)
 - **b** Hence find the value of $16^{\frac{3}{2}}$. (2)
- $\mathbf{2} \quad \text{Find } \int (6x^2 + \sqrt{x}) \mathrm{d}x. \tag{4}$
- **3** A sequence a_1 , a_2 , a_3 , ... a_n is defined by

$$a_1 = 2$$
, $a_{n+1} = 2a_n - 1$.

- **a** Write down the value of a_2 and the value of a_3 . (2)
- **b** Calculate $\sum_{r=1}^{5} a_r$. (2)
- **4 a** Express $(5 + \sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers. (3)
 - **b** Hence, or otherwise, simplify $(5 + \sqrt{2})^2 (5 \sqrt{2})^2$. (2)
- **5** Solve the simultaneous equations:

$$x - 3y = 6$$

$$3xy + x = 24$$
(7)

- **6** The points A and B have coordinates (-3, 8) and (5, 4) respectively. The straight line l_1 passes through A and B.
 - **a** Find an equation for l_1 , giving your answer in the form ax + by + c = 0, where a, b and c are integers. (4)
 - **b** Another straight line l_2 is perpendicular to l_1 and passes through the origin. Find an equation for l_2 . (2)
 - **c** The lines l_1 and l_2 intersect at the point P. Use algebra to find the coordinates of P. (3)
- **7** On separate diagrams, sketch the curves with equations:

a
$$y = \frac{2}{x'}$$
, $-2 \le x \le 2, x \ne 0$ (2)

b
$$y = \frac{2}{x} - 4, -2 \le x \le 2, x \ne 0$$
 (3)

$$\mathbf{c} \ \ y = \frac{2}{x+1}, \ -2 \le x \le 2, \ x \ne -1$$
 (3)

In each part, show clearly the coordinates of any point at which the curve meets the x-axis or the y-axis.

8 In the year 2007, a car dealer sold 400 new cars. A model for future sales assumes that sales will increase by x cars per year for the next 10 years, so that (400 + x) cars are sold in 2008, (400 + 2x) cars are sold in 2009, and so on.

Using this model with x = 30, calculate:

- **a** The number of cars sold in the year 2016. (2)
- **b** The total number of cars sold over the 10 years from 2007 to 2016. (3)

The dealer wants to sell at least 6000 cars over the 10-year period.

Using the same model:

- **c** Find the least value of x required to achieve this target. (4)
- 9 a Given that

$$x^2 + 4x + c = (x + a)^2 + b$$

where a, b and c are constants:

- **i** Find the value of a. (1)
- **ii** Find b in terms of c. (2)

Given also that the equation $x^2 + 4x + c = 0$ has unequal real roots:

- **iii** Find the range of possible values of c. (2)
- **b** Find the set of values of *x* for which:
 - $\mathbf{i} \quad 3x < 20 x,\tag{2}$
 - ii $x^2 + 4x 21 > 0$, (4)
 - **iii** both 3x < 20 x and $x^2 + 4x 21 > 0$. (2)
- **10 a** Show that $\frac{(3x-4)^2}{x^2}$ may be written as $P + \frac{Q}{x} + \frac{R}{x^2}$, where P, Q and R are constants to be found.
 - **b** The curve *C* has equation $y = \frac{(3x-4)^2}{x^2}$, $x \ne 0$. Find the gradient of the tangent to *C* at the point on *C* where x = -2.
 - **c** Find the equation of the normal to C at the point on C where x = -2, giving your answer in the form ax + by + c = 0, where a, b and c are integers. (5)