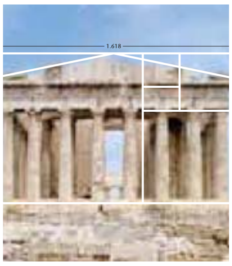



1 Review Exercise

1 Factorise completely:

a $2x^3 - 13x^2 - 7x$

b $9x^2 - 16$

c $x^4 + 7x^2 - 8$.

2 Find the value of:

a $81^{\frac{1}{2}}$

b $81^{\frac{3}{4}}$

c $81^{-\frac{3}{4}}$. *E*

3 **a** Write down the value of $8^{\frac{1}{3}}$.

b Find the value of $8^{-\frac{2}{3}}$. *E*

4 **a** Find the value of $125^{\frac{4}{3}}$.

b Simplify $24x^2 \div 18x^{\frac{4}{3}}$.

5 **a** Express $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer.

b Express $(4 - \sqrt{5})^2$ in the form $b + c\sqrt{5}$, where b and c are integers. *E*

6 **a** Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

b Express $\frac{26}{4 + \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. *E*

7 **a** Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

b Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found. *E*

8 **a** Express $(2\sqrt{7})^3$ in the form $a\sqrt{7}$, where a is an integer.

b Express $(8 + \sqrt{7})(3 - 2\sqrt{7})$ in the form $b + c\sqrt{7}$, where b and c are integers.

c Express $\frac{6 + 2\sqrt{7}}{3 - \sqrt{7}}$ in the form $d + e\sqrt{7}$, where d and e are integers.

9 Solve the equations:

a $x^2 - x - 72 = 0$

b $2x^2 + 7x = 0$

c $10x^2 + 9x - 9 = 0$.

10 Solve the equations, giving your answers to 3 significant figures:

a $x^2 + 10x + 17 = 0$

b $2x^2 - 5x - 1 = 0$

c $(2x - 3)^2 = 7$.

11 $x^2 - 8x - 29 \equiv (x + a)^2 + b,$
where a and b are constants.

a Find the value of a and the value of b.

b Hence, or otherwise, show that the roots of
$$x^2 - 8x - 29 = 0$$
are $c \pm d\sqrt{5}$, where c and d are integers. *E*

12 Given that
$$f(x) = x^2 - 6x + 18, x \geqslant 0,$$

a express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

The curve C with equation $y = f(x)$, $x \geqslant 0$, meets the y-axis at P and has a minimum point at Q.

b Sketch the graph of C, showing the coordinates of P and Q.

The line $y = 41$ meets C at the point R.

c Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. *E*

13 Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

14 Given that
$$x^2 + 10x + 36 \equiv (x + a)^2 + b,$$
where a and b are constants,

a find the value of a and the value of b.

b Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots.

The equation $x^2 + 10x + k = 0$ has equal roots.

c Find the value of k.

d For this value of k, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinate axes. *E*

15 $x^2 + 2x + 3 \equiv (x + a)^2 + b.$

a Find the values of the constants a and b.

b Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

c Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part **b**.

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

d Find the set of possible values of k, giving your answer in surd form. *E*

16 Solve the simultaneous equations:
$$x + y = 2$$
$$x^2 + 2y = 12$$

17 **a** By eliminating y from the equations:
$$y = x - 4,$$
$$2x^2 - xy = 8,$$
show that
$$x^2 + 4x - 8 = 0.$$

b Hence, or otherwise, solve the simultaneous equations:
$$y = x - 4,$$
$$2x^2 - xy = 8,$$
giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. *E*

18 Solve the simultaneous equations:
$$2x - y - 5 = 0$$
$$x^2 + xy - 2 = 0$$

19 Find the set of values of x for which:

a $3(2x + 1) > 5 - 2x,$

b $2x^2 - 7x + 3 > 0,$

c **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0.$ *E*

20 Find the set of values of x for which:

a $x(x - 5) < 7x - x^2$

b $x(3x + 7) > 20$

21 **a** Solve the simultaneous equations:

$$y + 2x = 5$$
$$2x^2 - 3x - y = 16.$$

b Hence, or otherwise, find the set of values of x for which:

$$2x^2 - 3x - 16 > 5 - 2x.$$ *E*

22 The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

a Show that $k^2 - 4k - 12 > 0$.

b Find the set of possible values of k. *E*

23 Given that the equation $kx^2 + 3kx + 2 = 0$, where k is a constant, has no real roots, find the set of possible values of k.

24 The equation $(2p + 5)x^2 + px + 1 = 0$, where p is a constant, has different real roots.

a Show that $p^2 - 8p - 20 > 0$.

b Find the set of possible values of p.

Given that $p = -3$,

c find the exact roots of $(2p + 5)x^2 + px + 1 = 0$.

25 **a** Factorise completely $x^3 - 4x$.

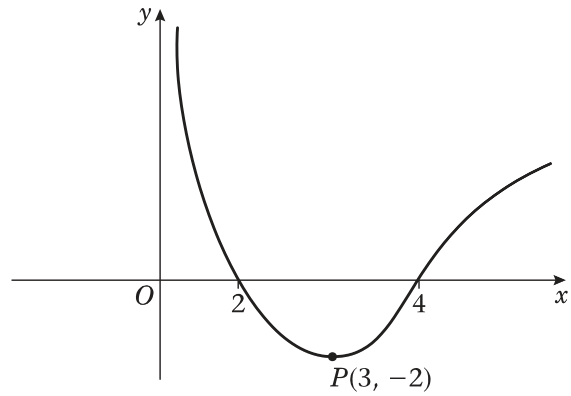
b Sketch the curve with equation $y = x^3 - 4x$, showing the coordinates of the points where the curve crosses the x-axis.

c On a separate diagram, sketch the curve with equation

$$y = (x - 1)^3 - 4(x - 1)$$

showing the coordinates of the points where the curve crosses the x-axis. *E*

26



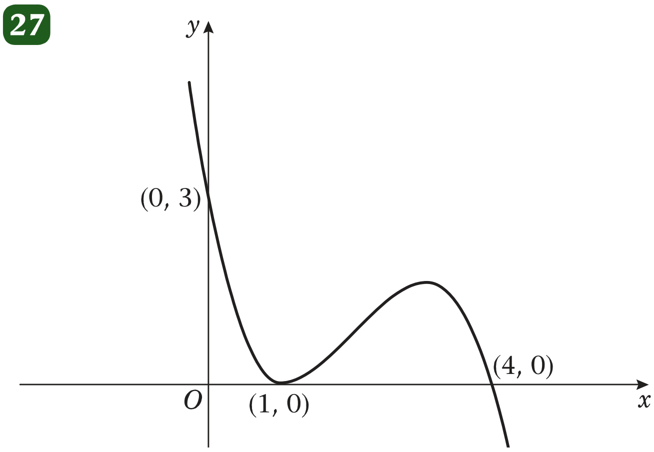
The figure shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams, sketch the curve with equation

a $y = -f(x)$ **b** $y = f(2x)$

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation. *E*

27



The figure shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

a $y = f(x + 1)$ **b** $y = 2f(x)$

c $y = f(\frac{1}{2}x)$

On each diagram, show clearly the coordinates of all the points where the curve meets the axes. *E*

28 Given that $f(x) = \frac{1}{x}, x \neq 0$,

a sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes

b find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis. *E*

29 Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

a express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants

b hence factorise $f(x)$ completely

c sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes.

30 **a** Sketch on the same diagram the graph of $y = x(x + 2)(x - 4)$ and the graph of $y = 3x - x^2$, showing the coordinates of the points at which each graph meets the x-axis.

b Find the exact coordinates of each of the intersection points of $y = x(x + 2)(x - 4)$ and $y = 3x - x^2$.