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Review Exercise

1 The line L has equation $y = 5 - 2x$.

a Show that the point $P(3, -1)$ lies on L.

b Find an equation of the line, perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. *E*

2 The points A and B have coordinates $(-2, 1)$ and $(5, 2)$ respectively.

a Find, in its simplest surd form, the length AB.

b Find an equation of the line through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The line through A and B meets the y-axis at the point C.

c Find the coordinates of C.

3 The line l_1 passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The line l_2 passes through the origin O and has gradient -2. The lines l_1 and l_2 intersect at the point P.

b Calculate the coordinates of P.

Given that l_1 crosses the y-axis at the point C,

c calculate the exact area of ΔOCP. *E*

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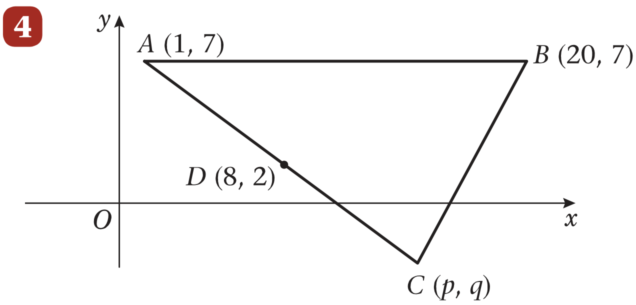

The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in the figure. The point $D(8, 2)$ is the mid-point of AC.

a Find the value of p and the value of q.

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

b Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers.

c Find the exact x-coordinate of E. *E*

5 The straight line l_1 has equation $y = 3x - 6$. The straight line l_2 is perpendicular to l_1 and passes through the point (6, 2).

a Find an equation for l_2 in the form $y = mx + c$, where m and c are constants.

The lines l_1 and l_2 intersect at the point C.

b Use algebra to find the coordinates of C.

The lines l_1 and l_2 cross the x-axis at the points A and B respectively.

c Calculate the exact area of triangle ABC. *E*

6 The line l_1 has equation $6x - 4y - 5 = 0$. The line l_2 has equation $x + 2y - 3 = 0$.

a Find the coordinates of P, the point of intersection of l_1 and l_2.

The line l_1 crosses the y-axis at the point M and the line l_2 crosses the y-axis at the point N.

b Find the area of ΔMNP.

7 The 5th term of an arithmetic series is 4 and the 15th term of the series is 39.

a Find the common difference of the series.

b Find the first term of the series.

c Find the sum of the first 15 terms of the series.

8 An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.
He runs 9 km on the 11^{th} day, and he runs a total of 77 km over the 11 day period.
Find the value of a and the value of d. *E*

9 The rth term of an arithmetic series is $(2r - 5)$.

a Write down the first three terms of this series.

b State the value of the common difference.

c Show that $\sum_{r=1}^{n} (2r - 5) = n(n - 4)$ *E*

10 Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

a Find the amount he plans to save in the year 2011.

b Calculate his total planned savings over the 20 year period from 2001 to 2020.

Ben also plans to save money over the same 20 year period. He saves £A in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben's total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

c calculate the value of A. *E*

11 A sequence $a_1, a_2, a_3, \ldots$ is defined by
$a_1 = 3$,
$a_{n+1} = 3a_n - 5$, n $\geqslant$ 1.

a Find the value of a_2 and the value of a_3.

b Calculate the value of $\sum_{r=1}^{5} a_r$. *E*

12 A sequence $a_1, a_2, a_3, \ldots$ is defined by
$a_1 = k$,
$a_{n+1} = 3a_n + 5, n \geqslant 1$
where k is a positive integer.

a Write down an expression for a_2 in terms of k.

b Show that $a_3 = 9k + 20$.

c **i** Find $\sum_{r=1,}^{4} a_r$ in terms of k.

ii Show that $\sum_{r=1}^{4} a_r$ is divisible by 10. *E*

13 A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k$$

$$a_{n+1} = 2a_n - 3, \qquad n \geqslant 1$$

a Show that $a_5 = 16k - 45$.

Given that $a_5 = 19$, find the value of

b k

c $\sum_{r=1}^{6} a_r$.

14 An arithmetic sequence has first term a and common difference d.

a Prove that the sum of the first n terms of the series are $\frac{1}{2}n[2a + (n - 1)d]$

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

b Find the amount Sean repays in the 21st month.

Over the n months, he repays a total of £5000.

c Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0$$

d Solve the equation in part **c**.

e State, with a reason, which of the solutions to the equation in part **c** is **not** a sensible solution to the repayment problem. *E*

15 A sequence is given by

$a_1 = 2$

$a_{n+1} = a_n^2 - ka_n, \quad n \geq 1,$

where k is a constant.

a Show that $a_3 = 6k^2 - 20k + 16$

Given that $a_3 = 2$,

b find the possible values of k.

For the larger of the possible values of k, find the value of:

c a_2

d a_5

e a_{100}.

16 Given that $y = 4x^3 - 1 + 2x^{1/2}$, $x > 0$, find $\frac{dy}{dx}$. *E*

17 Given that $y = 2x^2 - 6/x^3$, $x \neq 0$,

a find $\frac{dy}{dx}$,

b find $\int y\,dx$. *E*

18 Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

a $\frac{dy}{dx}$,

b $\frac{d^2y}{dx^2}$,

c $\int y\,dx$. *E*

19 **a** Given that $y = 5x^3 + 7x + 3$, find

i $\frac{dy}{dx}$,

ii $\frac{d^2y}{dx^2}$,

b Find $\int (1 + 3\sqrt{x} - 1/x^2)\,dx$. *E*

20 The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

a Find an expression for $\frac{dy}{dx}$.

b Show that the point $P(4, 8)$ lies on C.

c Show that an equation of the normal to C at point P is $3y = x + 20$.

The normal to C at P cuts the x-axis at point Q.

d Find the length PQ, giving your answer in a simplified surd form. *E*

21 The curve C has equation $y = 4x^2 + \frac{5 - x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

a Show that the value of $\frac{dy}{dx}$ at P is 3.

b Find an equation of the tangent to C at P.

This tangent meets the x-axis at the point $(k, 0)$.

c Find the value of k. *E*

22 The curve C has equation
$y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.
The point P has coordinates (3, 0).

a Show that P lies on C.

b Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

c Find the coordinates of Q. *E*

23 $f(x) = \frac{(2x + 1)(x + 4)}{\sqrt{x}}, \quad x > 0.$

a Show that f(x) can be written in the form $Px^{\frac{3}{2}} + Qx^{\frac{1}{2}} + Rx^{-\frac{1}{2}}$, stating the values of the constants P, Q and R.

b Find f′(x).

c Show that the tangent to the curve with equation $y = f(x)$ at the point where $x = 1$ is parallel to the line with equation $2y = 11x + 3$. *E*

24 The curve C with equation $y = f(x)$ passes through the point (3, 5).
Given that $f'(x) = x^2 + 4x - 3$, find f(x).

25 The curve with equation $y = f(x)$ passes through the point (1, 6). Given that

$f'(x) = 3 + (5x^2 + 2)/x^{1/2}, x > 0,$

find f(x) and simplify your answer. *E*

26 For the curve C with equation $y = f(x)$,

$$\frac{dy}{dx} = x^3 + 2x - 7$$

a find $\frac{d^2y}{dx^2}$

b show that $\frac{d^2y}{dx^2} \geqslant 2$ for all values of x.

Given that the point $P(2, 4)$ lies on C,

c find y in terms of x

d find an equation for the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers. *E*

27 For the curve C with equation $y = f(x)$,

$$\frac{dy}{dx} = \frac{1 - x^2}{x^4}$$

Given that C passes through the point $\left(\frac{1}{2}, \frac{2}{3}\right)$,

a find y in terms of x

b find the coordinates of the points on C at which $\frac{dy}{dx} = 0$.

28 The curve C with equation $y = f(x)$ passes through the point (5, 65).
Given that $f'(x) = 6x^2 - 10x - 12$,

a use integration to find f(x)

b hence show that
$f(x) = x(2x + 3)(x - 4)$

c sketch C, showing the coordinates of the points where C crosses the x-axis. *E*

29 The curve C has equation

$y = x^2(x - 6) + \frac{4}{x}, \quad x > 0.$

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

a Show that the length of PQ is $\sqrt{170}$.

b Show that the tangents to C at P and Q are parallel.

c Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *E*

30 **a** Factorise completely $x^3 - 7x^2 + 12x$.

b Sketch the graph of $y = x^3 - 7x^2 + 12x$, showing the coordinates of the points at which the graph crosses the x-axis.

The graph of $y = x^3 - 7x^2 + 12x$ crosses the positive x-axis at the points A and B.

The tangents to the graph at A and B meet at the point P.

c Find the coordinates of P.