C1 SEQUENCES AND SERIES — Answers - Worksheet A

1 **a** 9, 13, 17, 21, 25 **b** 4, 9, 16, 25, 36 **c** 2, 4, 8, 16, 32 **d** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

e −1, 4, 21, 56, 115 **f** $\frac{2}{3}, \frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}$ **g** $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$ **h** 16, 8, 4, 2, 1

2 **a** $u_n = 3n + 1$
$a = 3, b = 1$

b $u_n = 7n - 7$
$a = 7, b = -7$

c $u_n = 18 - 2n$
$a = -2, b = 18$

d $u_n = 1.3n - 0.9$
$a = 1.3, b = -0.9$

e $u_n = 117 - 17n$
$a = -17, b = 117$

f $u_n = 8n - 21$
$a = 8, b = -21$

3 possible answers are

a $5n - 4$ **b** 3^n **c** $2n^2$

d $\frac{1}{4} \times 2^n$ **e** $33 - 11n$ **f** $(n - 1)^3$

g $n^2 + 3$ **h** $\frac{n}{2n+1}$ **i** $2^n - 1$

4 **a** $u_3 = c + 3 = 11 \quad \therefore c = 8$

b $u_6 = 8 + 3^4 = 89$

5 **a** $u_4 = 4(8 + k) = 32 + 4k$
$u_6 = 6(12 + k) = 72 + 6k$
$\therefore\ 72 + 6k = 2(32 + 4k) - 2$
$72 + 6k = 62 + 8k$
$k = 5$

b $u_n = n(2n + 5) = 2n^2 + 5n$
$u_{n-1} = (n - 1)[2(n - 1) + 5] = (n - 1)(2n + 3) = 2n^2 + n - 3$
$\therefore\ u_n - u_{n-1} = (2n^2 + 5n) - (2n^2 + n - 3) = 4n + 3$

6 **a** $u_1 = k - 3$
$u_2 = k^2 - 3$
$\therefore\ k - 3 + k^2 - 3 = 0$
$k^2 + k - 6 = 0$
$(k + 3)(k - 2) = 0$
$k = -3$ or 2

b $k = -3 \quad \Rightarrow \quad u_5 = (-3)^5 - 3 = -243 - 3 = -246$
$k = 2 \quad \Rightarrow \quad u_5 = 2^5 - 3 = 32 - 3 = 29$

7 **a** 3, 7, 11, 15 **b** 2, 7, 22, 67

c −2, 1, 7, 19 **d** 5, 2, 5, 2

e −1, 14, −46, 194 **f** 10, 3, 2.3, 2.23

g $6, -1, 1\frac{1}{3}, \frac{5}{9}$ **h** $0, \frac{1}{2}, \frac{2}{5}, \frac{5}{12}$

8 possible answers are

a $u_{n+1} = u_n + 4,\ u_1 = 5$ **b** $u_{n+1} = 3u_n,\ u_1 = 1$ **c** $u_{n+1} = u_n - 18,\ u_1 = 62$

d $u_{n+1} = \frac{1}{2}u_n,\ u_1 = 120$ **e** $u_{n+1} = 2u_n + 1,\ u_1 = 4$ **f** $u_{n+1} = 4u_n - 1,\ u_1 = 1$

9 **a** $-3 = -4a + b$
$-1 = -3a + b$
subtracting, $2 = a$
$a = 2,\ b = 5$

b $8 = b$
$4 = 8a + b$
$a = -\frac{1}{2},\ b = 8$

c $4 = \frac{11}{2}a + b$
$3 = 4a + b$
subtracting, $1 = \frac{3}{2}a$
$a = \frac{2}{3},\ b = \frac{1}{3}$

10 **a** $u_2 = 4 + 3k$
$u_3 = 4(4 + 3k) + 3k = 16 + 15k$

b $u_2 = 2k + 5$
$u_3 = k(2k + 5) + 5 = 2k^2 + 5k + 5$

c $u_2 = 4k - k = 3k$
$u_3 = 4(3k) - k = 11k$

d $u_2 = 2 + k$
$u_3 = 2 - k(2 + k) = 2 - 2k - k^2$

e $u_2 = \frac{4}{k}$
$u_3 = \frac{4}{k} \div k = \frac{4}{k^2}$

f $u_2 = \sqrt[3]{61k^3 + 3k^3} = \sqrt[3]{64k^3} = 4k$
$u_3 = \sqrt[3]{61k^3 + 64k^3} = \sqrt[3]{125k^3} = 5k$

11 **a** $u_2 = \frac{1}{2}(k + 6)$
$u_3 = \frac{1}{2}[k + \frac{3}{2}(k + 6)] = \frac{1}{4}(5k + 18)$

b $\frac{1}{4}(5k + 18) = 7$
$k = 2$
$u_4 = \frac{1}{2}(2 + 21) = 11\frac{1}{2}$

12 **a** $u_4 = 30 - 2 = 28$
$10 = 3u_2 - 2 \ \therefore\ u_2 = 4$
$4 = 3u_1 - 2 \ \therefore\ u_1 = 2$

b $u_4 = \frac{15}{4} + 2 = 5\frac{3}{4}$
$5 = \frac{3}{4}u_2 + 2 \ \therefore\ u_2 = 4$
$4 = \frac{3}{4}u_1 + 2 \ \therefore\ u_1 = 2\frac{2}{3}$

c $u_4 = 0.2 \times 1.2 = 0.24$
$-0.2 = 0.2(1 - u_2) \ \therefore\ u_2 = 2$
$2 = 0.2(1 - u_1) \ \therefore\ u_1 = -9$

d $u_4 = \frac{1}{2}$
$1 = \frac{1}{2}\sqrt{u_2} \ \therefore\ u_2 = 4$
$4 = \frac{1}{2}\sqrt{u_1} \ \therefore\ u_1 = 64$

13 **a** $u_5 = 2 + 4c = 30 \ \therefore\ c = 7$

b sequence is 2, 9, 16, 23, 30, …
$\therefore\ u_n = 7n - 5$

14 **a** $u_2 = 3(-4 - k) = -12 - 3k$
$u_3 = 3[(-12 - 3k) - k] = -36 - 12k$

b $-36 - 12k = 7(-12 - 3k) + 3$
$9k = -45$
$k = -5$

c $u_3 = -36 + 60 = 24$
$\therefore\ u_4 = 3(24 + 5) = 87$

15 **a** $t_2 = 1.5k + 2$
$t_3 = k(1.5k + 2) + 2 = 1.5k^2 + 2k + 2$

b $1.5k^2 + 2k + 2 = 12$
$3k^2 + 4k - 20 = 0$
$(3k + 10)(k - 2) = 0$
$k = -3\frac{1}{3},\ 2$

C1 SEQUENCES AND SERIES — Answers - Worksheet B

1 **a** $d = 6$
$u_{40} = 4 + (39 \times 6) = 238$

b $d = -3$
$u_{40} = 30 + (39 \times -3) = -87$

c $d = 2.3$
$u_{40} = 8.9 + (39 \times 2.3) = 98.6$

2 **a** $a = 7, d = 2$
$u_n = 7 + 2(n - 1) = 5 + 2n$

b $a = \frac{1}{6}, d = \frac{4}{3}$
$u_n = \frac{1}{6} + \frac{4}{3}(n - 1) = -\frac{7}{6} + \frac{4}{3}n$

c $a = 17, d = -8$
$u_n = 17 - 8(n - 1) = 25 - 8n$

3 **a** $a = 8, d = 4, n = 30$
$S_{30} = \frac{30}{2}[16 + (29 \times 4)]$
$= 1980$

b $a = 60, d = -7, n = 30$
$S_{30} = \frac{30}{2}[120 + (29 \times -7)]$
$= -1245$

c $a = 7\frac{1}{4}, d = 1\frac{1}{2}, n = 30$
$S_{30} = \frac{30}{2}[14\frac{1}{2} + (29 \times 1\frac{1}{2})]$
$= 870$

4 **a** $S_{20} = \frac{20}{2}(60 + 136)$
$= 1960$

b $S_{32} = \frac{32}{2}(100 + 84.5)$
$= 2952$

c $S_{17} = \frac{17}{2}[28 + (-20)]$
$= 68$

5 **a** $S_{48} = \frac{48}{2}[4 + (47 \times 9)]$
$= 10\,248$

b $S_{36} = \frac{36}{2}[200 + (35 \times -5)]$
$= 450$

c $S_{55} = \frac{55}{2}[38 + (54 \times 13)]$
$= 20\,350$

6 **a** $8 + 3(n - 1) = 65$
$n = 20$
$S_{20} = \frac{20}{2}(8 + 65)$
$= 730$

b $3.4 + 1.2(n - 1) = 23.8$
$n = 18$
$S_{18} = \frac{18}{2}(3.4 + 23.8)$
$= 244.8$

c $22 - 8(n - 1) = -226$
$n = 32$
$S_{32} = \frac{32}{2}[22 + (-226)]$
$= -3264$

7 **a** $a = 21$
$21 + 2d = 27$
$\therefore d = 3$

b $S_{40} = \frac{40}{2}[42 + (39 \times 3)] = 3180$

8 $n = 1$, first term $= 7 + 16 = 23$
$d = 7$
$S_{35} = \frac{35}{2}[46 + (34 \times 7)] = 4970$

9 **a** $a + d = 13$
$a + 4d = 46$

b subtracting, $3d = 33$
$d = 11$
sub. $a = 2$

c $u_{40} = 2 + (39 \times 11) = 431$

10 **a** $a + 2d = 72$
$a + 7d = 37$
subtracting, $5d = -35$
$d = -7$
sub. $a = 86$

b $S_{25} = \frac{25}{2}[172 + (24 \times -7)] = 50$

11 **a** $a + 4d = 23$ (1)
$\frac{10}{2}(2a + 9d) = 240 \Rightarrow 2a + 9d = 48$
$2 \times (1) \Rightarrow 2a + 8d = 46$
subtracting, $d = 2$
sub. $a = 15$

b $S_{60} = \frac{60}{2}[30 + (59 \times 2)] = 4440$

12 **a** $S_n = 1 + 2 + 3 + \ldots + (n - 1) + n$
write in reverse
$S_n = n + (n - 1) + \ldots + 3 + 2 + 1$
adding, $2S_n = n \times (n + 1)$
$S_n = \frac{1}{2}n(n + 1)$

b $= S_{100} - S_{29}$
$= \frac{1}{2} \times 100 \times 101 - \frac{1}{2} \times 29 \times 30$
$= 5050 - 435 = 4615$

13 **a** $5 + 7 + 9 + 11 + 13$

b $15 + 12 + 9 + 6 + 3 + 0 - 3 - 6 - 9$

c $15 + 19 + 23 + 27 + 31 + 35 + 39$

d $4\frac{1}{2} + 4 + 3\frac{1}{2} + 3 + 2\frac{1}{2} + 2 + 1\frac{1}{2} + 1$

14 **a** AP: $a = 4$, $l = 61$, $n = 20$
$S_{20} = \frac{20}{2}(4 + 61)$
$= 650$

b AP: $a = 88$, $l = 0$, $n = 45$
$S_{45} = \frac{45}{2}(88 + 0)$
$= 1980$

c AP: $a = 19$, $l = 127$, $n = 28$
$S_{28} = \frac{28}{2}(19 + 127)$
$= 2044$

d AP: $a = 3$, $l = 13$, $n = 41$
$S_{41} = \frac{41}{2}(3 + 13)$
$= 328$

15 AP: $a = -2$, $l = 4n - 6$
$S_n = \frac{n}{2}[-2 + (4n - 6)] = 720$
$\therefore\ n(4n - 8) = 1440$
$n^2 - 2n - 360 = 0$
$(n + 18)(n - 20) = 0$
$n > 0\ \therefore\ n = 20$

16 **a** AP: $a = 2$, $l = 160$, $n = 80$
$S_{80} = \frac{80}{2}(2 + 160) = 6480$

b AP: $a = 3$, $l = 198$, $n = 66$
$S_{66} = \frac{66}{2}(3 + 198) = 6633$

c AP: $a = 30$, $l = 300$, $d = 6$
$30 + 6(n - 1) = 300\ \therefore\ n = 46$
$S_{46} = \frac{46}{2}(30 + 300) = 7590$

17 **a** $a + (9 \times -11) = 101$
$a = 200$

b $S_{30} = \frac{30}{2}[400 + (29 \times -11)] = 1215$

18 **a** $a = 17$, $17 + 4d = 27\ \therefore\ d = 2.5$

b $17 + 2.5(r - 1) = 132$
$r = 47$

c $S_{47} = \frac{47}{2}(17 + 132) = 3501.5$

19 **a** $\frac{6}{2}(2a + 5d) = 213 \ \Rightarrow\ 2a + 5d = 71$
$\frac{10}{2}(2a + 9d) = 295 \ \Rightarrow\ 2a + 9d = 59$
subtracting, $4d = -12$
$d = -3$
sub. $a = 43$

b $43 - 3(n - 1) > 0$
$n < \frac{46}{3}\ \therefore$ 15 positive terms

c max S_n when $n = 15$
$S_{15} = \frac{15}{2}[86 + (14 \times -3)] = 330$

20 **a** $S_8 = (2 \times 8^2) + (5 \times 8) = 168$

b $S_7 = (2 \times 7^2) + (5 \times 7) = 133$
$u_8 = S_8 - S_7 = 35$

c $S_{n-1} = 2(n - 1)^2 + 5(n - 1)$
$= 2n^2 + n - 3$
$u_n = S_n - S_{n-1}$
$= (2n^2 + 5n) - (2n^2 + n - 3)$
$= 4n + 3$

21 **a** $(2k + 3) - (k + 2) = (4k - 2) - (2k + 3)$
$k + 1 = 2k - 5$
$k = 6$

b $a = 8$, $a + d = 15\ \therefore\ d = 7$
$S_{25} = \frac{25}{2}[16 + (24 \times 7)] = 2300$

22 **a** $2t - (5 - t) = (6t - 3) - 2t$
$3t - 5 = 4t - 3$
$t = -2$

b $u_5 = 7$, $u_6 = -4\ \therefore\ d = -11$
$a + (4 \times -11) = 7\ \therefore\ a = 51$
$S_{18} = \frac{18}{2}[102 + (17 \times -11)] = -765$

C1 SEQUENCES AND SERIES — Answers - Worksheet C

1 **a** $a + 2d = -10$ (1)

$\frac{8}{2}(2a + 7d) = 16 \Rightarrow 2a + 7d = 4$

$2 \times (1) \Rightarrow 2a + 4d = -20$

subtracting, $3d = 24$

$d = 8$

sub. $a = -26$

b $-26 + 8(n - 1) > 300$

$n > 41\frac{3}{4}$ $\therefore$ smallest $n = 42$

2 **a** $a + 2d = \frac{5}{6}$

$a + 6d = 2\frac{1}{3}$

subtracting, $4d = 1\frac{1}{2}$

$d = \frac{3}{8}$

sub. $a = \frac{1}{12}$

b $S_n = \frac{n}{2}[\frac{1}{6} + \frac{3}{8}(n - 1)]$

$= \frac{1}{48}n[4 + 9(n - 1)]$

$= \frac{1}{48}n(9n - 5)$ $[\, k = \frac{1}{48}\,]$

3 **a** $\frac{9}{2}(2a + 8d) = 126$

$9(a + 4d) = 126$

$a + 4d = 14$

b $\frac{15}{2}(2a + 14d) = 277.5$

$a + 7d = 18.5$

subtracting, $3d = 4.5$

$d = 1.5$

sub. $a = 8$

c $S_{32} = \frac{32}{2}[16 + (31 \times 1.5)] = 1000$

4 **a** $(5k + 3) - (7k - 1) = (4k + 1) - (5k + 3)$

$-2k + 4 = -k - 2$

$k = 6$

b given terms = 41, 33, 25

$d = -8$

smallest +ve term $= 25 + (3 \times -8) = 1$

c consider series of +ve terms in reverse

$a = 1, d = 8$

$S_r = \frac{r}{2}[2 + 8(r - 1)] = r(4r - 3)$

5 **a** AP: $a = 4, l = 120, n = 30$

$S_{30} = \frac{30}{2}(4 + 120) = 1860$

b **i** $= \sum_{r=1}^{30} 4r + 30 = 1890$

ii $= 2 \times \sum_{r=1}^{30} 4r - (30 \times 5)$

$= (2 \times 1860) - 150 = 3570$

6 **a** $500 + (7 \times 40) = £780$

b AP: $a = 500, d = 40$

$S_n = \frac{n}{2}[1000 + 40(n - 1)] = 20n(n + 24)$

c AP: $a = 400, d = 60$

$S_n = \frac{n}{2}[800 + 60(n - 1)] = 10n(3n + 37)$

$\therefore 20n(n + 24) = 10n(3n + 37)$

$n \neq 0 \therefore 2(n + 24) = (3n + 37)$

$n = 11 \therefore$ 11 years

7 **a** $S_n = 2 + 4 + 6 + \ldots + (2n - 2) + 2n$

write in reverse

$S_n = 2n + (2n - 2) + \ldots + 6 + 4 + 2$

adding, $2S_n = n \times (2n + 2)$

$S_n = n(n + 1)$

b integers 200 to 800, AP: $n = 601$

$S_{601} = \frac{601}{2}(200 + 800) = 300\,500$

integers 200 to 800 divisible by 4

AP: $a = 200, l = 800$

$200 + 4(n - 1) = 800 \Rightarrow n = 151$

$S_{151} = \frac{151}{2}(200 + 800) = 75\,500$

required sum $= 300\,500 - 75\,500$

$= 225\,000$

8 **a** $S_n = \frac{1}{2}n[2a + (n - 1)d]$

b $S_2 = \frac{2}{2}(2a + d) = 2a + d$

$S_6 = \frac{6}{2}(2a + 5d) = 6a + 15d$

$S_8 = \frac{8}{2}(2a + 7d) = 8a + 28d$

$2(S_6 - S_2) = 2[(6a + 15d) - (2a + d)]$

$= 2(4a + 14d)$

$= 8a + 28d = S_8$

c for +ve terms $40 - 3(n - 1) > 0$

$n < \frac{43}{3} \therefore$ 14 terms

$\therefore S_{14} = \frac{14}{2}[80 + (13 \times -3)] = 287$

9 **a** **i** $u_4 - u_1 = x + 3$

$u_7 = u_4 + (x + 3) = 3x + 6$

ii $3d = x + 3$

$d = \frac{1}{3}x + 1$

iii $S_{10} = \frac{10}{2}[2x + 9(\frac{1}{3}x + 1)]$

$= 5[2x + 3x + 9] = 25x + 45$

b $x + 19(\frac{1}{3}x + 1) = 52$

$3x + 19x + 57 = 156$

$x = \frac{99}{22} = \frac{9}{2}$ or $4\frac{1}{2}$

10 $S_{20} = \frac{20}{2}(2a + 19d) = 20a + 190d$

$S_{30} = \frac{30}{2}(2a + 29d) = 30a + 435d$

$S_{30} - S_{20} = 10a + 245d$

$\therefore\ 20a + 190d = 10a + 245d$

$10a = 55d$

$2a = 11d$

$\therefore\ a : d = 11 : 2$

11 **a** $S_6 = 12(16 - 6) = 120$

$S_5 = 10(16 - 5) = 110$

$u_6 = S_6 - S_5 = 10$

b $S_n = 2n(16 - n) = 32n - 2n^2$

$S_{n-1} = 2(n - 1)[16 - (n - 1)]$

$= 2(n - 1)(17 - n)$

$= -2n^2 + 36n - 34$

$u_n = S_n - S_{n-1}$

$= (32n - 2n^2) - (-2n^2 + 36n - 34)$

$= 34 - 4n$

c $u_{n-1} = 34 - 4(n - 1) = 38 - 4n$

$u_n - u_{n-1} = (34 - 4n) - (38 - 4n) = -4$

$u_n - u_{n-1}$ constant $\therefore$ arithmetic series

12 **a** **i** $2400 + (5 \times 250) = 3650$

ii AP: $a = 2400,\ d = 250$

$S_{10} = \frac{10}{2}[4800 + (9 \times 250)]$

$= 35\,250$

b AP: $a = 2400,\ d = C$

$\frac{10}{2}[4800 + (9 \times C)] = 40\,000$

$C = \frac{3200}{9} = 356$ (nearest unit)

13 **a** let common difference be d

$S_r = a + (a + d) + (a + 2d) + \ldots + (l - 2d) + (l - d) + l$

write in reverse

$S_r = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a$

adding, $2S_r = r \times (a + l)$

$S_r = \frac{1}{2}r(a + l)$

b $n = 18,\ l = 68,\ S_{18} = 153$

$\therefore\ 153 = \frac{18}{2}(a + 68)$

$a = 17 - 68 = -51$

C1 SEQUENCES AND SERIES — Answers - Worksheet D

1 **a** $a + d = 40$, $a + 4d = 121$
subtracting, $3d = 81$
$d = 27$
sub. $a = 13$
b $S_{25} = \frac{25}{2}[26 + (24 \times 27)] = 8425$

2 **a** 3, 7, 11, 15, 19
b AP: $a = 3$, $d = 4$, $n = 20$
$S_{20} = \frac{20}{2}[6 + (19 \times 4)]$
$= 820$

3 **a** $(2t - 5) - t = 8.6 - (2t - 5)$
$t = 6.2$
b $u_1 = 6.2$, $u_2 = 12.4 - 5 = 7.4$
$a = 6.2$, $d = 7.4 - 6.2 = 1.2$
$u_{16} = 6.2 + (15 \times 1.2) = 24.2$
c $S_{20} = \frac{20}{2}[12.4 + (19 \times 1.2)] = 352$

4 **a** $S_n = \frac{1}{2}n(n + 1)$
b $= S_{400} - S_{199}$
$= \frac{1}{2} \times 400 \times 401 - \frac{1}{2} \times 199 \times 200$
$= 80\,200 - 19\,900 = 60\,300$
c $\frac{1}{2}N(N + 1) = 4950$
$N^2 + N - 9900 = 0$
$(N + 100)(N - 99) = 0$
$N > 0 \;\therefore\; N = 99$

5 **a** $u_2 = k + 1$
$u_3 = k + (k + 1)^2 = k^2 + 3k + 1$
b $k^2 + 3k + 1 = 1$
$k(k + 3) = 0$
$k \neq 0 \;\therefore\; k = -3$
c $u_{25} = 1$
$u_1 = 1 \Rightarrow u_3 = 1$
$\therefore\; u_n = 1$ for all odd values of n

6 **a** AP: $a = 3$, $d = 3$
$500 \div 3 = 166\frac{2}{3} \;\therefore\; n = 166$
$S_{166} = \frac{166}{2}[6 + (165 \times 3)]$
$= 41\,583$
b AP: $a = 14$, $l = 99$, $n = 18$
$S_{18} = \frac{18}{2}(14 + 99)$
$= 1017$

7 **a** $S_n = a + (a + d) + (a + 2d) + \ldots + [a + (n - 2)d] + [a + (n - 1)d]$
write in reverse
$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + (a + 2d) + (a + d) + a$
adding $2S_n = n \times \{a + [a + (n - 1)d]\}$
$S_n = \frac{1}{2}n[2a + (n - 1)d]$
b $S_{26} = \frac{26}{2}[-2 + (25 \times 6)] = 1924$
$S_{27} = \frac{27}{2}[-2 + (26 \times 6)] = 2079$
$\therefore$ largest $n = 26$

8 $t_2 = 4 + 2k$
$t_3 = 4 - k(4 + 2k)$
$\therefore\; 4 - 4k - 2k^2 = 3$
$2k^2 + 4k - 1 = 0$
$k = \frac{-4 \pm \sqrt{16 + 8}}{4} = \frac{-4 \pm 2\sqrt{6}}{4}$
$k > 0 \;\therefore\; k = -1 + \frac{1}{2}\sqrt{6}$

9 **a** $= 6 + (19 \times 3) = 63$
b $S_n = \frac{n}{2}[12 + 3(n - 1)] = 270$
$\therefore\; n(3n + 9) = 540$
$n^2 + 3n - 180 = 0$
$(n + 15)(n - 12) = 0$
$n > 0 \;\therefore\; n = 12$

10 **a** $= 3 \times 570 = 1710$
b $= 570 + (2 \times 30) = 630$
c $= 570 + (\frac{1}{2} \times 30 \times 31) = 1035$

11 **a** 2 years = 8×3 months
total = $3 \times S_8$ [AP: $a = 40$, $d = 2$]
$= 3 \times \frac{8}{2}[80 + (7 \times 2)]$
$= 3 \times 376 = £1128$

b n years = $4n \times 3$ months
total = $3 \times S_{4n}$
$= 3 \times \frac{4n}{2}\{80 + [(4n - 1) \times 2]\}$
$= 6n(80 + 8n - 2)$
$= 12n(4n + 39)$

12 AP: $a = 80$, $d = -3$, $n = 45$
$S_{45} = \frac{45}{2}[160 + (44 \times -3)] = 630$

13 **a** $a + 2d = 298$, $a + 7d = 263$
subtracting, $5d = -35$
$d = -7$

b sub. $a = 312$
$312 - 7(n - 1) > 0$
$n < \frac{319}{7}$ $\therefore$ 45 positive terms

c max S_n when $n = 45$
$S_{45} = \frac{45}{2}[624 + (44 \times -7)] = 7110$

14 **a** AP: $a = 10$, $d = 6$
$S_n = \frac{n}{2}[20 + 6(n - 1)]$
$= n(3n + 7)$

b $S_{2n} = 2n[(3 \times 2n) + 7]$
$= 12n^2 + 14n$
required sum = $S_{2n} - S_n$
$= (12n^2 + 14n) - (3n^2 + 7n)$
$= 9n^2 + 7n = n(9n + 7)$

15 **a** $u_2 = k^2 - 2$, $u_4 = k^4 - 4$
$\therefore$ $k^2 - 2 + k^4 - 4 = 6$
$k^4 + k^2 - 12 = 0$
$(k^2 + 4)(k^2 - 3) = 0$
$k^2 = -4$ [no solutions] or 3
$k > 0$ $\therefore$ $k = \sqrt{3}$

b $u_1 = \sqrt{3} - 1$
$u_3 = (\sqrt{3})^3 - 3 = 3(\sqrt{3} - 1) = 3u_1$

16 **a** $(4k - 2) - (k + 4) = (k^2 - 2) - (4k - 2)$
$3k - 6 = k^2 - 4k$
$k^2 - 7k + 6 = 0$

b $(k - 1)(k - 6) = 0$
$k = 1$ or 6
$d = 3k - 6$
$d > 0$ $\therefore$ $k = 6$
$a = 10$, $d = 12$
$u_{15} = 10 + (14 \times 12) = 178$