C2 ALGEBRA *Worksheet A*

1 Find the quotient obtained in dividing

a $(x^3 + 2x^2 - x - 2)$ by $(x + 1)$ **b** $(x^3 + 2x^2 - 9x + 2)$ by $(x - 2)$

c $(20 + x + 3x^2 + x^3)$ by $(x + 4)$ **d** $(2x^3 - x^2 - 4x + 3)$ by $(x - 1)$

e $(6x^3 - 19x^2 - 73x + 90)$ by $(x - 5)$ **f** $(-x^3 + 5x^2 + 10x - 8)$ by $(x + 2)$

g $(x^3 - 2x + 21)$ by $(x + 3)$ **h** $(3x^3 + 16x^2 + 72)$ by $(x + 6)$

2 Find the quotient and remainder obtained in dividing

a $(x^3 + 8x^2 + 17x + 16)$ by $(x + 5)$ **b** $(x^3 - 15x^2 + 61x - 48)$ by $(x - 7)$

c $(3x^3 + 4x^2 + 7)$ by $(2 + x)$ **d** $(-x^3 - 5x^2 + 15x - 50)$ by $(x + 8)$

e $(4x^3 + 2x^2 - 16x + 3)$ by $(x - 3)$ **f** $(1 - 22x^2 - 6x^3)$ by $(x + 2)$

3 Use the factor theorem to determine whether or not

a $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$ **b** $(x + 2)$ is a factor of $(x^3 - 5x^2 - 9x + 2)$

c $(x - 3)$ is a factor of $(x^3 - x^2 - 14x + 27)$ **d** $(x + 6)$ is a factor of $(2x^3 + 13x^2 + 2x - 24)$

e $(2x + 1)$ is a factor of $(2x^3 - 5x^2 + 7x - 14)$ **f** $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$

4 $\mathrm{f}(x) \equiv x^3 - 2x^2 - 11x + 12.$

a Show that $(x - 1)$ is a factor of $\mathrm{f}(x)$.

b Hence, express $\mathrm{f}(x)$ as the product of three linear factors.

5 $\mathrm{g}(x) \equiv 2x^3 + x^2 - 13x + 6.$

Show that $(x + 3)$ is a factor of $\mathrm{g}(x)$ and solve the equation $\mathrm{g}(x) = 0$.

6 $\mathrm{f}(x) \equiv 6x^3 - 7x^2 - 71x + 12.$

Given that $\mathrm{f}(4) = 0$, find all solutions to the equation $\mathrm{f}(x) = 0$.

7 $\mathrm{g}(x) \equiv x^3 + 7x^2 + 7x - 6.$

Given that $x = -2$ is a solution to the equation $\mathrm{g}(x) = 0$,

a express $\mathrm{g}(x)$ as the product of a linear factor and a quadratic factor,

b find, to 2 decimal places, the other two solutions to the equation $\mathrm{g}(x) = 0$.

8 $\mathrm{f}(x) \equiv x^3 + 2x^2 - 11x - 12.$

a Evaluate $\mathrm{f}(1)$, $\mathrm{f}(2)$, $\mathrm{f}(-1)$ and $\mathrm{f}(-2)$.

b Hence, state a linear factor of $\mathrm{f}(x)$ and fully factorise $\mathrm{f}(x)$.

9 By first finding a linear factor, fully factorise

a $x^3 - 2x^2 - 5x + 6$ **b** $x^3 + x^2 - 5x - 2$ **c** $20 + 11x - 8x^2 + x^3$

d $3x^3 - 4x^2 - 35x + 12$ **e** $x^3 + 8$ **f** $12 + 29x + 8x^2 - 4x^3$

10 Solve each equation, giving your answers in exact form.

a $x^3 - x^2 - 10x - 8 = 0$ **b** $x^3 + 2x^2 - 9x - 18 = 0$ **c** $4x^3 - 12x^2 + 9x = 2$

d $x^3 - 5x^2 + 3x + 1 = 0$ **e** $x^2(x + 4) = 3(3x + 2)$ **f** $x^3 - 14x + 15 = 0$

11 $f(x) \equiv 2x^3 - x^2 - 15x + c$.

Given that $(x - 2)$ is a factor of $f(x)$,

a find the value of the constant c,

b fully factorise $f(x)$.

12 $g(x) \equiv x^3 + px^2 - 13x + q$.

Given that $(x + 1)$ and $(x - 3)$ are factors of $g(x)$,

a show that $p = 3$ and find the value of q,

b solve the equation $g(x) = 0$.

13 Use the remainder theorem to find the remainder obtained in dividing

a $(x^3 + 4x^2 - x + 6)$ by $(x - 2)$

b $(x^3 - 2x^2 + 7x + 1)$ by $(x + 1)$

c $(2x^3 + x^2 - 9x + 17)$ by $(x + 5)$

d $(8x^3 + 4x^2 - 6x - 3)$ by $(2x - 1)$

e $(2x^3 - 3x^2 - 20x - 7)$ by $(2x + 1)$

f $(3x^3 - 6x^2 + 2x - 7)$ by $(3x - 2)$

14 Given that when $(x^3 - 4x^2 + 5x + c)$ is divided by $(x - 2)$ the remainder is 5, find the value of the constant c.

15 Given that when $(2x^3 - 9x^2 + kx + 5)$ is divided by $(2x - 1)$ the remainder is -2, find the value of the constant k.

16 Given that when $(2x^3 + ax^2 + 13)$ is divided by $(x + 3)$ the remainder is 22,

a find the value of the constant a,

b find the remainder when $(2x^3 + ax^2 + 13)$ is divided by $(x - 4)$.

17 $f(x) \equiv px^3 + qx^2 + qx + 3$.

Given that $(x + 1)$ is a factor of $f(x)$,

a find the value of the constant p.

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is 15,

b find the value of the constant q.

18 $p(x) \equiv x^3 + ax^2 + 9x + b$.

Given that $(x - 3)$ is a factor of $p(x)$,

a find a linear relationship between the constants a and b.

Given also that when $p(x)$ is divided by $(x + 2)$ the remainder is -30,

b find the values of the constants a and b.

19 $f(x) \equiv 4x^3 - 6x^2 + mx + n$.

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 3 and that when $f(x)$ is divided by $(2x - 1)$ the remainder is 15, find the values of the constants m and n.

20 $g(x) \equiv x^3 + cx + 3$.

Given that when $g(x)$ is divided by $(x - 4)$ the remainder is 39,

a find the value of the constant c,

b find the quotient and remainder when $g(x)$ is divided by $(x + 2)$.

C2 ALGEBRA — Worksheet B

1 $f(x) \equiv x^3 - 5x^2 + ax + b$.

Given that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$,

a show that $a = -2$ and find the value of b.

b Hence, express $f(x)$ as the product of three linear factors.

2 $f(x) \equiv 8x^3 - x^2 + 7$.

The remainder when $f(x)$ is divided by $(x - k)$ is eight times the remainder when $f(x)$ is divided by $(2x - k)$.

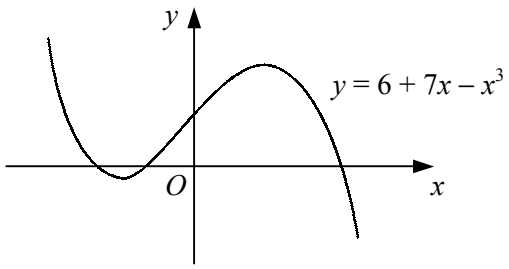
Find the two possible values of the constant k.

3 $f(x) \equiv 3x^3 - x^2 - 12x + 4$.

a Show that $(x - 2)$ is a factor of $f(x)$.

b Solve the equation $f(x) = 0$.

4



The diagram shows the curve with the equation $y = 6 + 7x - x^3$.

Find the coordinates of the points where the curve crosses the x-axis.

5 $f(x) \equiv 3x^3 + px^2 + 8x + q$.

When $f(x)$ is divided by $(x + 1)$ there is a remainder of -4.

When $f(x)$ is divided by $(x - 2)$ there is a remainder of 80.

a Find the values of the constants p and q.

b Show that $(x + 2)$ is a factor of $f(x)$.

c Solve the equation $f(x) = 0$.

6 **a** Solve the equation

$$x^3 - 4x^2 - 7x + 10 = 0.$$

b Hence, solve the equation

$$y^6 - 4y^4 - 7y^2 + 10 = 0.$$

7 $f(n) \equiv n^3 + 7n^2 + 14n + 3$.

a Find the remainder when $f(n)$ is divided by $(n + 1)$.

b Express $f(n)$ in the form

$$f(n) \equiv (n + 1)(n + a)(n + b) + c,$$

where a, b and c are integers.

c Hence, show that $f(n)$ is odd for all positive integer values of n.

C2 ALGEBRA — Worksheet C

1 $f(x) \equiv x^3 + x^2 - 22x - 40.$

a Show that $(x + 2)$ is a factor of $f(x)$. **(2)**

b Express $f(x)$ as the product of three linear factors. **(4)**

c Solve the equation $f(x) = 0$. **(1)**

2 $f(x) \equiv x^3 - 2x^2 + kx + 1.$

Given that the remainder when $f(x)$ is divided by $(x - 2)$ and the remainder when $f(x)$ is divided by $(x + 3)$ are equal,

a find the value of the constant k, **(4)**

b find the remainder when $f(x)$ is divided by $(x + 2)$. **(2)**

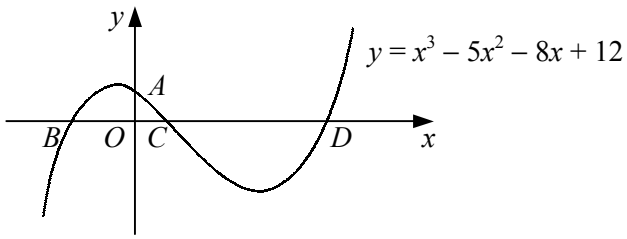
3 The polynomial $p(x)$ is defined by

$$p(x) \equiv 2x^3 - 9x^2 - 2x + 11.$$

a Find the remainder when $p(x)$ is divided by $(x + 2)$. **(2)**

b Find the quotient and remainder when $p(x)$ is divided by $(x - 4)$. **(3)**

4



The diagram shows the curve with the equation $y = x^3 - 5x^2 - 8x + 12$.

a State the coordinates of the point A where the curve crosses the y-axis. **(1)**

The curve crosses the x-axis at the points B, C and D. Given that C has coordinates $(1, 0)$,

b find the coordinates of the points B and D. **(6)**

5 $f(x) \equiv x^3 - 3x^2 + kx + 8.$

Given that $(x - 1)$ is a factor of $f(x)$,

a find the value of k, **(2)**

b solve the equation $f(x) = 0$. **(5)**

6 Solve the equation

$$2x^3 + x^2 - 13x + 6 = 0. \quad \textbf{(7)}$$

7 The polynomial $p(x)$ is defined by

$$p(x) \equiv bx^3 + ax^2 - 10x + b,$$

where a and b are constants.

Given that when $p(x)$ is divided by $(x + 1)$ the remainder is 3,

a find the value of a. **(2)**

Given also that when $p(x)$ is divided by $(3x - 1)$ the remainder is -1,

b find the value of b. **(3)**

8 $f(x) \equiv x^3 - 7x^2 + x + 10.$

a Find the remainder when $f(x)$ is divided by $(x + 1)$. **(2)**

b Hence, or otherwise, solve the equation $f(x) = 1$, giving your answers in exact form. **(6)**

9 $f(x) \equiv 3x^3 + kx^2 - 7x + 2k.$

When $f(x)$ is divided by $(3x - 2)$ the remainder is 6.

Find the value of the constant k. **(3)**

10 $f(x) \equiv 2x^3 - 7x^2 + 4x - 3.$

a Show that $(x - 3)$ is a factor of $f(x)$. **(2)**

b Hence, express $f(x)$ as the product of a linear factor and a quadratic factor. **(3)**

c Show that there is only one real solution to the equation $f(x) = 0$. **(3)**

11 The polynomial $f(x)$ is defined by

$$f(x) \equiv x^3 + px + q,$$

where p and q are constants.

Given that $(x - 2)$ is a factor of $f(x)$,

a find an expression for q in terms of p. **(2)**

Given also that when $f(x)$ is divided by $(x + 1)$ the remainder is -15,

b find the values of p and q. **(4)**

12 $f(x) \equiv x^3 + 4x^2 - 9.$

Given that $x = -3$ is a solution to the equation $f(x) = 0$, find the other two solutions correct to 2 decimal places. **(6)**

13 $f(x) \equiv (x + k)^3 - 8.$

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is -7,

a find the value of the constant k, **(3)**

b show that $(x + 1)$ is a factor of $f(x)$. **(2)**

14 $f(x) \equiv x^3 - 4x^2 - 7x + 8.$

a Find the remainder when $f(x)$ is divided by $(x + 2)$. **(2)**

Given that

$$g(x) \equiv f(x) + c,$$

and that $(x + 2)$ is a factor of $g(x)$,

b state the value of the constant c, **(1)**

c solve the equation $g(x) = 0$. **(4)**

15 $f(x) \equiv x^3 - 4x + 1.$

Given that when $f(x)$ is divded by $(2x - k)$, where k is a constant, the remainder is 4,

a show that $k^3 - 16k - 24 = 0$. **(3)**

Given also that when $f(x)$ is divided by $(x + k)$ the remainder is 1,

b find the value of k. **(3)**