

- 1 **a** $x^2 + y^2 = 25$ **b** $(x - 1)^2 + (y - 3)^2 = 4$ **c** $(x - 4)^2 + (y + 6)^2 = 1$
 d $(x + 1)^2 + (y + 8)^2 = 9$ **e** $(x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$ **f** $(x + 3)^2 + (y - 9)^2 = 12$
- 2 **a** centre (0, 0) radius 4 **b** centre (6, 1) radius 9 **c** centre (-1, 4) radius 11
 d centre (7, 0) radius 0.3 **e** centre (-2, -5) radius $4\sqrt{2}$ **f** centre (8, -9) radius $6\sqrt{3}$
- 3 **a** $x^2 + (y - 2)^2 - 4 + 3 = 0$
 $x^2 + (y - 2)^2 = 1$
 centre (0, 2) radius 1
 c $(x + 6)^2 - 36 + (y - 4)^2 - 16 + 36 = 0$
 $(x + 6)^2 + (y - 4)^2 = 16$
 centre (-6, 4) radius 4
 e $(x - 4)^2 - 16 + (y + 3)^2 - 9 = 0$
 $(x - 4)^2 + (y + 3)^2 = 25$
 centre (4, -3) radius 5
 g $x^2 + y^2 - x - 6y + \frac{1}{4} = 0$
 $(x - \frac{1}{2})^2 - \frac{1}{4} + (y - 3)^2 - 9 + \frac{1}{4} = 0$
 $(x - \frac{1}{2})^2 + (y - 3)^2 = 9$
 centre $(\frac{1}{2}, 3)$ radius 3
 b $(x - 1)^2 - 1 + (y - 5)^2 - 25 - 23 = 0$
 $(x - 1)^2 + (y - 5)^2 = 49$
 centre (1, 5) radius 7
 d $(x - 1)^2 - 1 + (y + 8)^2 - 64 = 35$
 $(x - 1)^2 + (y + 8)^2 = 100$
 centre (1, -8) radius 10
 f $(x + 5)^2 - 25 + (y - 1)^2 - 1 - 19 = 0$
 $(x + 5)^2 + (y - 1)^2 = 45$
 centre (-5, 1) radius $3\sqrt{5}$
 h $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$
 $(x + \frac{1}{3})^2 - \frac{1}{9} + (y - \frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$
 $(x + \frac{1}{3})^2 + (y - \frac{4}{3})^2 = 1$
 centre $(-\frac{1}{3}, \frac{4}{3})$ radius 1
- 4 **a** radius = $\sqrt{9+16} = 5$ $\therefore (x - 1)^2 + (y + 2)^2 = 25$
 b radius = $\sqrt{25+4} = \sqrt{29}$ $\therefore (x + 5)^2 + (y - 7)^2 = 29$
- 5 **a** centre $(\frac{1+3}{2}, -2) = (2, -2)$ **b** centre $(\frac{-7+1}{2}, \frac{2+8}{2}) = (-3, 5)$ **c** centre $(\frac{1+4}{2}, \frac{1+0}{2}) = (\frac{5}{2}, \frac{1}{2})$
 radius = 1 radius = $\sqrt{16+9} = 5$ radius = $\sqrt{\frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{5}{2}}$
 $\therefore (x - 2)^2 + (y + 2)^2 = 1$ $\therefore (x + 3)^2 + (y - 5)^2 = 25$ $\therefore (x - \frac{5}{2})^2 + (y - \frac{1}{2})^2 = \frac{5}{2}$
- 6 **a** grad $PQ = \frac{10-1}{3-0} = 3$, grad $QR = \frac{9-10}{6-3} = -\frac{1}{3}$
 grad $PQ \times$ grad $QR = 3 \times (-\frac{1}{3}) = -1$
 $\therefore PQ$ and QR are perpendicular
 $\therefore \angle PQR$ is a right-angle
 b $\angle PQR$ is a right-angle $\therefore PR$ is a diameter of C
 $\therefore$ centre is $(\frac{0+6}{2}, \frac{1+9}{2}) = (3, 5)$
 radius = 5
 $\therefore (x - 3)^2 + (y - 5)^2 = 25$
 $x^2 - 6x + 9 + y^2 - 10y + 25 - 25 = 0$
 $x^2 + y^2 - 6x - 10y + 9 = 0$

- 7 a centre (0, 0) radius 8
dist. pt to centre = 9
 $\therefore$ outside circle
- c $(x+5)^2 - 25 + (y-2)^2 - 4 = 140$
 $(x+5)^2 + (y-2)^2 = 169$
centre (-5, 2) radius 13
dist. pt to centre = $\sqrt{144+25} = 13$
 $\therefore$ on circle
- 8 $(x+6)^2 - 36 + (y-3)^2 - 9 + 27 = 0$
 $(x+6)^2 + (y-3)^2 = 18$
centre (-6, 3) radius $3\sqrt{2}$
dist. Q to centre = $\sqrt{196+4} = 10\sqrt{2}$
min. PQ = $10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$
- 10 $(x+4)^2 - 16 + (y-6)^2 - 36 + k = 0$
 $(x+4)^2 + (y-6)^2 = 52 - k$
centre (-4, 6) $r^2 = 52 - k$
 $r > 0 \therefore k < 52$
also require $r < 4$
 $\therefore 52 - k < 16$
 $k > 36$
 $\therefore 36 < k < 52$
- 12 a $(x-2)^2 - 4 + (y-2)^2 - 4 - 28 = 0$
 $(x-2)^2 + (y-2)^2 = 36$
centre (2, 2) radius 6
dist. = $\sqrt{64+36} = 10$
- b tangent perp to radius
 $\therefore AB^2 = 10^2 - 6^2 = 64$
 $AB = 8$
- b $(x-1)^2 - 1 + (y-3)^2 - 9 - 26 = 0$
 $(x-1)^2 + (y-3)^2 = 36$
centre (1, 3) radius 6
dist. pt to centre = $\sqrt{9+16} = 5$
 $\therefore$ inside circle
- d $(x+1)^2 - 1 + (y+4)^2 - 16 - 13 = 0$
 $(x+1)^2 + (y+4)^2 = 30$
centre (-1, -4) radius $\sqrt{30}$
dist. pt to centre = $\sqrt{9+25} = \sqrt{34}$
 $\therefore$ outside circle
- 9 x-coord of centre = $\frac{-2+8}{2} = 5$
y-coord of centre = 4 $\therefore$ centre (5, 4)
radius = dist. (0, 4) to (5, 4) = 5
 $\therefore (x-5)^2 + (y-4)^2 = 25$
- 11 a mid-point PQ = $(\frac{-2+2}{2}, \frac{-2+(-4)}{2}) = (0, -3)$
grad PQ = $\frac{-4+2}{2+2} = -\frac{1}{2}$
perp. grad = 2
 $\therefore y = 2x - 3$
- b mid-point PR = $(\frac{-2+7}{2}, \frac{-2+1}{2}) = (\frac{5}{2}, -\frac{1}{2})$
grad PR = $\frac{1+2}{7+2} = \frac{1}{3}$
perp. grad = -3
perp. bisector $y + \frac{1}{2} = -3(x - \frac{5}{2})$
 $y = 7 - 3x$
centre where intersect $2x - 3 = 7 - 3x$
 $x = 2 \therefore (2, 1)$
- c radius = dist. (2, 1) to (7, 1) = 5
 $\therefore (x-2)^2 + (y-1)^2 = 25$
- 13 $(x+3)^2 - 9 + (y-1)^2 - 1 = 0$
 $(x+3)^2 + (y-1)^2 = 10$
centre (-3, 1) radius $\sqrt{10}$
dist. centre to (2, 6) = $\sqrt{25+25} = \sqrt{50}$
 $PQ^2 = (\sqrt{50})^2 - (\sqrt{10})^2 = 40$
 $PQ = \sqrt{40} = 2\sqrt{10}$

14 a $(x-3)^2 - 9 + (y-5)^2 - 25 + 16 = 0$
 $\therefore$ centre (3, 5)

b $\text{grad} = \frac{5-2}{3-6} = -1$

c $y-2 = -(x-6) \quad [y = 8-x]$

15 a $(x+2)^2 - 4 + y^2 = 13$
 $\therefore$ centre (-2, 0)

$\text{grad} = \frac{0-4}{-2+1} = 4$

$\therefore y-4 = 4(x+1) \quad [y = 4x+8]$

b $(x+1)^2 - 1 + (y+2)^2 - 4 - 40 = 0$
 $\therefore$ centre (-1, -2)

$\text{grad normal} = \frac{-2-1}{-1-5} = \frac{1}{2}$

$\therefore$ grad tangent = -2

$\therefore y-1 = -2(x-5) \quad [y = 11-2x]$

c $(x-5)^2 - 25 + (y+2)^2 - 4 + 4 = 0$
 $\therefore$ centre (5, -2)

$\text{grad normal} = \frac{-2-2}{5-2} = -\frac{4}{3}$

$\therefore$ grad tangent = $\frac{3}{4}$

$\therefore y-2 = \frac{3}{4}(x-2) \quad [3x-4y+2=0]$

16 $x=0 \Rightarrow y^2 + 6y - 16 = 0$
 $(y+8)(y-2) = 0$
 $y = -8, 2$

$y=0 \Rightarrow x^2 - 6x - 16 = 0$
 $(x+2)(x-8) = 0$
 $x = -2, 8$

$\therefore (0, -8), (0, 2), (-2, 0) \text{ and } (8, 0)$

17 a sub. $x^2 + (x-4)^2 = 10$
 $x^2 - 4x + 3 = 0$
 $(x-1)(x-3) = 0$
 $x = 1, 3$

$\therefore (1, -3) \text{ and } (3, -1)$

b sub. $y = 17 - 3x$
 $x^2 + (17-3x)^2 - 4x - 2(17-3x) - 15 = 0$
 $x^2 - 10x + 24 = 0$
 $(x-4)(x-6) = 0$
 $x = 4, 6$

$\therefore (4, 5) \text{ and } (6, -1)$

c sub.
 $4x^2 + 4(2x+2)^2 + 4x - 8(2x+2) - 15 = 0$
 $4x^2 + 4x - 3 = 0$
 $(2x+3)(2x-1) = 0$
 $x = -\frac{3}{2}, \frac{1}{2}$

$\therefore (-\frac{3}{2}, -1) \text{ and } (\frac{1}{2}, 3)$

18 sub.
 $x^2 + (1-x)^2 + 6x + 2(1-x) = 27$
 $x^2 + x - 12 = 0$
 $(x+4)(x-3) = 0$
 $x = -4, 3$
 $\therefore (-4, 5) \text{ and } (3, -2)$
 $AB = \sqrt{49+49} = 7\sqrt{2}$

19 sub.
 $x^2 + (2x+1)^2 - 8x - 8(2x+1) + 27 = 0$
 $x^2 - 4x + 4 = 0$
 $(x-2)^2 = 0$
 repeated root $\therefore$ tangent
 touch when $x = 2 \therefore$ at (2, 5)

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sub.

$$x^2 + (x+k)^2 + 6x - 8(x+k) + 17 = 0$$

$$2x^2 + (2k-2)x + k^2 - 8k + 17 = 0$$

tangent $\therefore$ repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (2k-2)^2 - 8(k^2 - 8k + 17) = 0$$

$$k^2 - 14k + 33 = 0$$

$$(k-3)(k-11) = 0$$

$$\therefore k = 3 \text{ or } 11$$

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$$\text{sub. } x = \frac{k-3y}{2}$$

$$\left(\frac{k-3y}{2}\right)^2 + y^2 + 6\left(\frac{k-3y}{2}\right) + 4y = 0$$

$$(k-3y)^2 + 4y^2 + 12(k-3y) + 16y = 0$$

$$13y^2 - (6k+20)y + k^2 + 12k = 0$$

tangent $\therefore$ repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (6k+20)^2 - 52(k^2 + 12k) = 0$$

$$k^2 + 24k - 25 = 0$$

$$(k+25)(k-1) = 0$$

$$\therefore k = -25, 1$$

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sub.

$$x^2 + m^2x^2 - 8x - 16mx + 72 = 0$$

$$(1+m^2)x^2 - (8+16m)x + 72 = 0$$

tangent $\therefore$ repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (8+16m)^2 - 288(1+m^2) = 0$$

$$m^2 - 8m + 7 = 0$$

$$(m-1)(m-7) = 0$$

$$\therefore m = 1, 7$$

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$$\text{a } x = 0 \Rightarrow y^2 - 6y - 7 = 0$$

$$(y+1)(y-7) = 0$$

$$y = -1, 7$$

$$\therefore (0, -1) \text{ and } (0, 7)$$

$$\text{b } (x-2)^2 - 4 + (y-3)^2 - 9 = 7$$

$$\therefore \text{centre } (2, 3)$$

$$\text{grad normal at } (0, -1) = \frac{3+1}{2-0} = 2$$

$$\therefore \text{grad tangent at } (0, -1) = -\frac{1}{2}$$

$$\therefore y = -\frac{1}{2}x - 1$$

$$\text{grad normal at } (0, 7) = \frac{3-7}{2-0} = -2$$

$$\therefore \text{grad tangent at } (0, 7) = \frac{1}{2}$$

$$\therefore y = \frac{1}{2}x + 7$$

$$\text{intersect when } -\frac{1}{2}x - 1 = \frac{1}{2}x + 7$$

$$x = -8$$

$$\therefore (-8, 3)$$

- 1 a $(x-3)^2 + (y+2)^2 = 25$
 b sub. $(x-3)^2 + [(2x-3)+2]^2 = 25$
 $(x-3)^2 + (2x-1)^2 = 25$
 $x^2 - 2x - 3 = 0$
 $(x+1)(x-3) = 0$
 $x = -1, 3$
 $\therefore (-1, -5) \text{ and } (3, 3)$
 $AB^2 = 4^2 + 8^2 = 80$
 $AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$
- 2 a $= (\frac{-5+3}{2}, \frac{6+8}{2}) = (-1, 7)$
 b radius $= \sqrt{16+1} = \sqrt{17}$
 $\therefore (x+1)^2 + (y-7)^2 = 17$
 c grad of radius $= \frac{7-6}{-1-(-5)} = \frac{1}{4}$
 $\therefore$ grad of tangent $= -4$
 $\therefore y-6 = -4(x+5)$
 $[y = -4x - 14]$
- 3 a $(x+4)^2 - 16 + (y-8)^2 - 64 + 62 = 0$
 $(x+4)^2 + (y-8)^2 = 18$
 $\therefore$ centre $(-4, 8)$ radius $3\sqrt{2}$
 b grad of $l = 2 \therefore$ grad of perp. $= -\frac{1}{2}$
 eqn. of line perp to l through centre:
 $y-8 = -\frac{1}{2}(x+4)$
 $y = 6 - \frac{1}{2}x$
 intersects l when:
 $2x+1 = 6 - \frac{1}{2}x$
 $x = 2 \therefore (2, 5)$ is closest point
 dist. $(2, 5)$ to centre
 $= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$
 min. dist. $= 3\sqrt{5} - 3\sqrt{2} = 3(\sqrt{5} - \sqrt{2})$
- 4 a $PQ = \sqrt{1+9} = \sqrt{10}$
 radius $= \frac{1}{2}PQ = \frac{1}{2}\sqrt{10}$
 b $=$ midpoint of PR
 $= (\frac{0+7}{2}, \frac{4+3}{2}) = (\frac{7}{2}, \frac{7}{2})$
 c midpoint of $PQ = (\frac{0+1}{2}, \frac{4+1}{2}) = (\frac{1}{2}, \frac{5}{2})$
 centre of $C_1 =$ midpoint of $(\frac{1}{2}, \frac{5}{2})$ and $(\frac{7}{2}, \frac{7}{2})$
 $= (\frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2}) = (2, 3)$
 $\therefore$ eqn. of C_1 :
 $(x-2)^2 + (y-3)^2 = (\frac{1}{2}\sqrt{10})^2$
 $x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{5}{2}$
 $2x^2 - 8x + 8 + 2y^2 - 12y + 18 = 5$
 $2x^2 + 2y^2 - 8x - 12y + 21 = 0$
- 5 a midpoint $AB = (\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$
 grad $AB = \frac{7-3}{2-0} = 2$
 $\therefore$ perp. grad $= -\frac{1}{2}$
 $\therefore y-5 = -\frac{1}{2}(x-1)$
 $[y = \frac{11}{2} - \frac{1}{2}x]$
 b circle touches y -axis at $(0, 3)$
 $\therefore y$ -coord of centre $= 3$
 sub. $3 = \frac{11}{2} - \frac{1}{2}x$
 $x = 5$
 $\therefore$ centre $(5, 3)$ radius 5
 $\therefore (x-5)^2 + (y-3)^2 = 25$
 c grad of radius $= \frac{7-3}{2-5} = -\frac{4}{3}$
 $\therefore$ grad of tangent $= \frac{3}{4}$
 $\therefore y-7 = \frac{3}{4}(x-2)$
 $4y-28 = 3x-6$
 $3x-4y+22 = 0$
- 6 $AP^2 = (x+3)^2 + (y-4)^2$
 $BP^2 = x^2 + (y+2)^2$
 $AP = 2BP \therefore AP^2 = 4BP^2$
 $\therefore (x+3)^2 + (y-4)^2 = 4[x^2 + (y+2)^2]$
 $x^2 + 6x + 9 + y^2 - 8y + 16 = 4x^2 + 4y^2 + 16y + 16$
 $x^2 - 2x + y^2 + 8y - 3 = 0$
 $(x-1)^2 - 1 + (y+4)^2 - 16 - 3 = 0$
 $(x-1)^2 + (y+4)^2 = 20$
 in form $(x-a)^2 + (y-b)^2 = r^2 \therefore$ circle
 centre $(1, -4)$ radius $2\sqrt{5}$

7 a $= \left(\frac{-4+(-2)}{2}, \frac{9+(-5)}{2} \right) = (-3, 2)$

b radius $= \sqrt{1+49} = \sqrt{50}$
 $\therefore (x+3)^2 + (y-2)^2 = 50$

c sub. (2, 7) into eqn of C:

$$(2+3)^2 + (7-2)^2 = 50$$

$$25 + 25 = 50$$

true $\therefore R$ lies on C

d 90°

PQ is a diameter

$\therefore \angle PRQ$ is the angle in a semicircle

8 a $x^2 + (y-2)^2 - 4 - 16 = 0$

$\therefore$ centre (0, 2)

b $C_2: (x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$

$\therefore$ centre (1, 4)

$$\text{grad} = \frac{4-2}{1-0} = 2$$

$$\therefore y = 2x + 2$$

c sub. into eqn of C_1 :

$$x^2 + [(2x+2)-2]^2 - 20 = 0$$

$$x^2 + (2x)^2 - 20 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

from diagram, $x = -2$ at P

$\therefore P(-2, -2)$

l perp to line through centres

$$\therefore \text{grad} = -\frac{1}{2}$$

$$\therefore y + 2 = -\frac{1}{2}(x + 2)$$

$$[y = -\frac{1}{2}x - 3]$$

9 a $(x-4)^2 - 16 + (y+2)^2 - 4 + 12 = 0$

$$(x-4)^2 + (y+2)^2 = 8$$

centre (4, -2) radius $2\sqrt{2}$

b dist. P to centre

$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$\therefore \text{max. } PQ = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

$$\text{min. } PQ = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$$

c tangent perp. to radius

$$PQ^2 = (5\sqrt{2})^2 - (2\sqrt{2})^2 = 50 - 8 = 42$$

$$PQ = \sqrt{42} = 6.48$$

10 a radius $= b$

$$\therefore (x-a)^2 + (y-b)^2 = b^2$$

b sub. $y = x$ into eqn

$$(x-a)^2 + (x-b)^2 = b^2$$

$$x^2 - 2ax + a^2 + x^2 - 2bx + b^2 = b^2$$

$$2x^2 - 2(a+b)x + a^2 = 0$$

tangent $\therefore$ repeated root

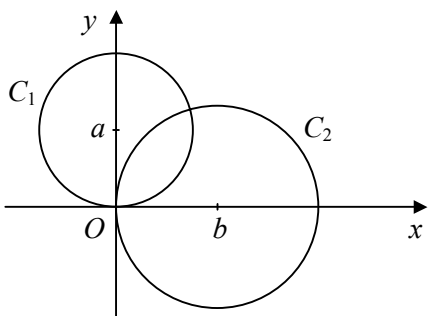
$$\therefore "b^2 - 4ac" = 0$$

$$4(a+b)^2 - 8a^2 = 0$$

$$a^2 - 2ab - b^2 = 0$$

$$a = \frac{2b \pm \sqrt{4b^2 + 4b^2}}{2} = b \pm \sqrt{2} b$$

$$a > 0, b > 0 \therefore a = (1 + \sqrt{2})b$$

- 1 a $(x-4)^2 - 16 + y^2 + 7 = 0$
 $\therefore$ centre $(4, 0)$
 b $(x-4)^2 + y^2 = 9$
 $\therefore$ radius $= 3$
- 2 a $(x-3)^2 - 9 + (y+1)^2 - 1 - 15 = 0$
 $\therefore$ centre $(3, -1)$
 b $(x-3)^2 + (y+1)^2 = 25$
 $\therefore$ radius $= 5$
 c grad of radius $= \frac{2-(-1)}{7-3} = \frac{3}{4}$
 $\therefore$ grad of tangent $= -\frac{4}{3}$
 $\therefore y-2 = -\frac{4}{3}(x-7)$
 $3y-6 = -4x+28$
 $4x+3y-34 = 0$
- 3 a $(x+3)^2 - 9 + (y-4)^2 - 16 + 21 = 0$
 $(x+3)^2 + (y-4)^2 = 4$
 $\therefore$ centre $(-3, 4)$ radius 2
 b dist. of centre from $O = \sqrt{9+16} = 5$
 $\therefore$ max. dist. of P from O
 $= 5 + 2 = 7$
- 4 a centre $(0, 0) \therefore$ grad of radius $= 1$
 $\therefore$ grad of tangent $= -1$
 $\therefore y-5 = -(x-5)$ [$y = 10 - x$]
 b grad of radius $= -7$
 $\therefore$ grad of tangent $= \frac{1}{7}$
 $\therefore y+7 = \frac{1}{7}(x-1)$
 $7y+49 = x-1$
 $x-7y-50 = 0$
 c sub. $x-7(10-x)-50 = 0$
 $x = 15$
 $\therefore (15, -5)$
- 5 a $x^2 + (y-a)^2 - a^2 = 0$
 $x^2 + (y-a)^2 = a^2$
 $\therefore$ centre $(0, a)$ radius a
 b $C_2: (x-b)^2 - b^2 + y^2 = 0$
 $(x-b)^2 + y^2 = b^2$, centre $(b, 0)$ radius b
- 
- 6 a $(x+1)^2 - 1 + (y-7)^2 - 49 + 30 = 0$
 $\therefore$ centre $(-1, 7)$
 b $(x+1)^2 + (y-7)^2 = 20$
 $\therefore$ radius $= \sqrt{20} = 2\sqrt{5}$
 c sub. $y = 2x - 1$ into eqn. of circle
 $x^2 + (2x-1)^2 + 2x - 14(2x-1) + 30 = 0$
 $x^2 - 6x + 9 = 0$
 $(x-3)^2 = 0$
 repeated root $\therefore$ tangent
 point of contact $(3, 5)$
- 7 a $(x-3)^2 - 9 + (y-6)^2 - 36 + 28 = 0$
 $\therefore$ centre $(3, 6)$
 b sub.
 $x^2 + (x-2)^2 - 6x - 12(x-2) + 28 = 0$
 $x^2 - 11x + 28 = 0$
 $(x-4)(x-7) = 0$
 $x = 4, 7$
 $\therefore A(4, 2), B(7, 5)$
 $\therefore AB = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$
- 8 a radius $= \sqrt{16+4} = \sqrt{20}$
 $\therefore (x-8)^2 + (y+1)^2 = 20$
 b sub. $x = -2y - 4$ into eqn. of circle:
 $(-2y-12)^2 + (y+1)^2 = 20$
 $4y^2 + 48y + 144 + y^2 + 2y + 1 = 20$
 $y^2 + 10y + 25 = 0$
 $(y+5)^2 = 0$
 repeated root $\therefore$ tangent

- 9 a $\text{grad } PQ = \frac{14-2}{8+10} = \frac{2}{3}$
 $\text{grad } PR = \frac{-10-2}{-2+10} = -\frac{3}{2}$
 $\text{grad } PR \times \text{grad } PQ = -\frac{3}{2} \times \frac{2}{3} = -1$
 $\therefore PR$ is perpendicular to PQ
- b $\angle QPR = 90^\circ \therefore QR$ is a diameter of the circle
 $\therefore$ centre of circle is mid-point of QR
 $= (\frac{8-2}{2}, \frac{14-10}{2}) = (3, 2)$
radius $= \sqrt{25+144} = 13$
 $\therefore (x-3)^2 + (y-2)^2 = 169$
 $x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$
 $x^2 + y^2 - 6x - 4y - 156 = 0$
- 10 a $(x-1)^2 - 1 + (y-\frac{7}{2})^2 - \frac{49}{4} - 16 = 0$
 $\therefore$ centre $(1, \frac{7}{2})$
- b $(x-1)^2 + (y-\frac{7}{2})^2 = \frac{117}{4}$
 $\therefore$ radius $= \sqrt{\frac{117}{4}} = \sqrt{\frac{9 \times 13}{4}} = \frac{3}{2}\sqrt{13}$ [$k = \frac{3}{2}$]
- c $\text{grad of radius} = \frac{8-\frac{7}{2}}{4-1} = \frac{3}{2}$
 $\therefore$ grad of tangent $= -\frac{2}{3}$
 $y-8 = -\frac{2}{3}(x-4)$
 $3y-24 = -2x+8$
 $2x+3y-32=0$
- 11 a grad of $x-2y+3=0$ is $\frac{1}{2}$
 $\therefore$ grad of perp bisector $= -2$
passes through centre of circle
 $\therefore y-7 = -2(x-6)$
 $y = -2x+19$
mid-point of chord where intersect
 $x-2(-2x+19)+3=0$
 $x=7 \therefore (7, 5)$
- b $3-2y+3=0$
 $\therefore y=3 \therefore A(3, 3)$
let B be (p, q)
 $\therefore (\frac{3+p}{2}, \frac{3+q}{2}) = (7, 5)$
 $p=11, q=7 \therefore B(11, 7)$
- c radius $= \sqrt{9+16} = 5$
 $\therefore (x-6)^2 + (y-7)^2 = 25$
- 12 a $(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$
 $(x-4)^2 + (y-8)^2 = 8$
 $\therefore$ centre $(4, 8)$ radius $2\sqrt{2}$
- b $= \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$
- c tangent perp. to radius
 $\therefore OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$
 $OA = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$
- 13 a $C: (x-2)^2 - 4 + y^2 - 6 = 0$
 $\therefore$ centre $(2, 0)$
 l : when $x=2, y=3(2)-6=0$
 $\therefore l$ passes through centre of C
- b eqn. of tangent: $y=3x+k$
sub. into eqn. of circle:
 $x^2 + (3x+k)^2 - 4x - 6 = 0$
 $10x^2 + (6k-4)x + k^2 - 6 = 0$
tangent $\therefore$ repeated root $\therefore b^2 - 4ac = 0$
 $(6k-4)^2 - 40(k^2-6) = 0$
 $k^2 + 12k - 64 = 0$
 $(k+16)(k-4) = 0$
 $k = -16, 4$
 $\therefore y = 3x - 16$ and $y = 3x + 4$