

- 1 Write down an equation of the circle with the given centre and radius in each case.

a centre (0, 0) radius 5 b centre (1, 3) radius 2 c centre (4, -6) radius 1
 d centre (-1, -8) radius 3 e centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ f centre (-3, 9) radius $2\sqrt{3}$
- 2 Write down the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 = 16$ b $(x - 6)^2 + (y - 1)^2 = 81$ c $(x + 1)^2 + (y - 4)^2 = 121$
 d $(x - 7)^2 + y^2 = 0.09$ e $(x + 2)^2 + (y + 5)^2 = 32$ f $(x - 8)^2 + (y + 9)^2 = 108$
- 3 Find the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 - 4y + 3 = 0$ b $x^2 + y^2 - 2x - 10y - 23 = 0$
 c $x^2 + y^2 + 12x - 8y + 36 = 0$ d $x^2 + y^2 - 2x + 16y = 35$
 e $x^2 + y^2 = 8x - 6y$ f $x^2 + y^2 + 10x - 2y - 19 = 0$
 g $4x^2 + 4y^2 - 4x - 24y + 1 = 0$ h $9x^2 + 9y^2 + 6x - 24y + 8 = 0$
- 4 Find an equation of the circle

a with centre (1, -2) which passes through the point (4, 2),
 b with centre (-5, 7) which passes through the point (0, 5).
- 5 Find an equation of the circle in which AB is a diameter in each case.

a $A(1, -2)$ $B(3, -2)$ b $A(-7, 2)$ $B(1, 8)$ c $A(1, 1)$ $B(4, 0)$
- 6 The points $P(0, 1)$, $Q(3, 10)$ and $R(6, 9)$ all lie on circle C .

a Show that $\angle PQR$ is a right-angle.
 b Hence, show that C has the equation $x^2 + y^2 - 6x - 10y + 9 = 0$.
- 7 Find in each case whether the given point lies inside, outside or on the given circle.

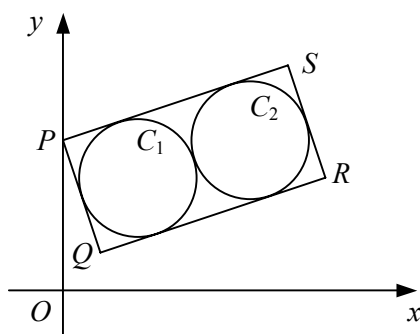
a $(0, -9)$ $x^2 + y^2 = 64$ b $(4, 7)$ $x^2 + y^2 - 2x - 6y - 26 = 0$
 c $(7, -3)$ $x^2 + y^2 + 10x - 4y = 140$ d $(-4, 1)$ $x^2 + y^2 + 2x + 8y - 13 = 0$
- 8 The point P lies on the circle with equation $x^2 + y^2 + 12x - 6y + 27 = 0$ and the point Q has coordinates (8, 1). Find the minimum length of PQ giving your answer in the form $k\sqrt{2}$.
- 9 Find an equation of the circle which crosses the x -axis at the points (2, 0) and (8, 0) and touches the y -axis at the point (0, 4).
- 10 Given that the circle with equation $x^2 + y^2 + 8x - 12y + k = 0$ does not touch or cross either of the coordinate axes, find the set of possible values of the constant k .
- 11 The circle C passes through the points P , Q and R with coordinates $(-2, -2)$, $(2, -4)$ and $(7, 1)$ respectively.

a Find an equation of the perpendicular bisector of the points P and Q .
 b Find the coordinates of the centre of C .
 c Find an equation of C .

- 12 The circle C has the equation $x^2 + y^2 - 4x - 4y - 28 = 0$.
a Find the distance of the point $A(10, 8)$ from the centre of C .
The tangent to C at the point B passes through A .
b Find the length AB .
- 13 A circle has the equation $x^2 + y^2 + 6x - 2y = 0$ and passes through the point P .
Given that the tangent to the circle at P passes through the point $Q(2, 6)$, find the exact length PQ in its simplest form.
- 14 The circle C has the equation $x^2 + y^2 - 6x - 10y + 16 = 0$ and passes through the point $A(6, 2)$.
a Find the coordinates of the centre of C .
b Find the gradient of the normal to the circle at A .
c Find an equation of the normal to the circle at A .
- 15 Find an equation of
a the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point $(-1, 4)$,
b the tangent to the circle with equation $x^2 + y^2 + 2x + 4y - 40 = 0$ at the point $(5, 1)$,
c the tangent to the circle with equation $x^2 + y^2 - 10x + 4y + 4 = 0$ at the point $(2, 2)$.
- 16 Find the coordinates of the points where the circle with equation $x^2 + y^2 - 6x + 6y - 16 = 0$ intersects the coordinate axes.
- 17 Find in each case the coordinates of the points where the line l intersects the circle C .
a $l: y = x - 4$ $C: x^2 + y^2 = 10$
b $l: 3x + y = 17$ $C: x^2 + y^2 - 4x - 2y - 15 = 0$
c $l: y = 2x + 2$ $C: 4x^2 + 4y^2 + 4x - 8y - 15 = 0$
- 18 The line with equation $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B .
Find the length of the chord AB , giving your answer in the form $k\sqrt{2}$.
- 19 Show that the line with equation $y = 2x + 1$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 8y + 27 = 0$ and find the coordinates of the point where they touch.
- 20 The line with equation $y = x + k$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 8y + 17 = 0$.
Find the two possible values of k .
- 21 The line with equation $y = mx$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 16y + 72 = 0$.
Find the two possible values of m .
- 22 The line with equation $2x + 3y = k$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y = 0$.
Find the two possible values of k .
- 23 The circle with equation $x^2 + y^2 - 4x - 6y = 7$ crosses the y -axis at the points A and B .
a Find the coordinates of the points A and B .
b Find the coordinates of the point where the tangent to the circle at A intersects the tangent to the circle at B .

- 1 The circle C has centre $(3, -2)$ and radius 5.
 a Write down an equation of C in cartesian form.
 The line $y = 2x - 3$ intersects C at the points A and B .
 b Show that $AB = 4\sqrt{5}$.
- 2 The line AB is a diameter of circle C .
 Given that A has coordinates $(-5, 6)$ and B has coordinates $(3, 8)$, find
 a the coordinates of the centre of C ,
 b a cartesian equation for C ,
 c an equation of the tangent to C at A .
- 3 The circle C has equation $x^2 + y^2 + 8x - 16y + 62 = 0$.
 a Find the coordinates of the centre of C and the exact radius of C .
 The line l has equation $y = 2x + 1$.
 b Show that the minimum distance between l and C is $3(\sqrt{5} - \sqrt{2})$.

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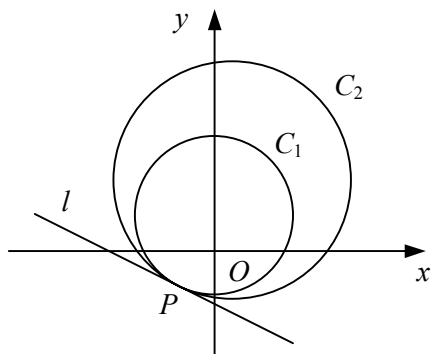


- The diagram shows rectangle $PQRS$ and circles C_1 and C_2 .
 Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are $P(0, 4)$, $Q(1, 1)$, $R(7, 3)$ and $S(6, 6)$.
 a Find the radius of C_1 .
 b Find the coordinates of the point where the two circles touch.
 c Show that C_1 has equation $2x^2 + 2y^2 - 8x - 12y + 21 = 0$.
- 5 The circle C touches the y -axis at the point $A(0, 3)$ and passes through the point $B(2, 7)$.
 a Find an equation of the perpendicular bisector of AB .
 b Find an equation for C .
 c Show that the tangent to C at B has equation $3x - 4y + 22 = 0$.
- 6 The point $P(x, y)$ moves such that its distance from the point $A(-3, 4)$ is twice its distance from the point $B(0, -2)$.
 Show that the locus of P is a circle and find the coordinates of the centre and the exact radius of this circle.

- 7 The points $P(-4, 9)$ and $Q(-2, -5)$ are such that PQ is a diameter of circle C .

- Find the coordinates of the centre of C .
- Find an equation for C .
- Show that the point $R(2, 7)$ lies on C .
- Hence, state the size of $\angle PRQ$, giving a reason for your answer.

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The diagram shows circles C_1 and C_2 , which both pass through the point P , and the common tangent to the circles at P , the line l .

Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.

- Find the coordinates of the centre of C_1 .

Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$.

- Find an equation of the straight line passing through the centre of C_1 and the centre of C_2 .
- Find an equation of line l .

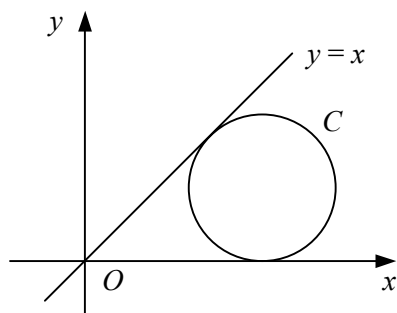
- 9 The circle C has equation $x^2 + y^2 - 8x + 4y + 12 = 0$.

- Find the coordinates of the centre of C and the radius of C .

The point P has coordinates $(3, 5)$ and the point Q lies on C .

- Find the largest and smallest values of the length PQ , giving your answers in the form $k\sqrt{2}$.
- Find the length of PQ correct to 3 significant figures when the line PQ is a tangent to C .

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The diagram shows the circle C and the line $y = x$.

Given that circle C has centre (a, b) , where a and b are positive constants, and that C touches the x -axis,

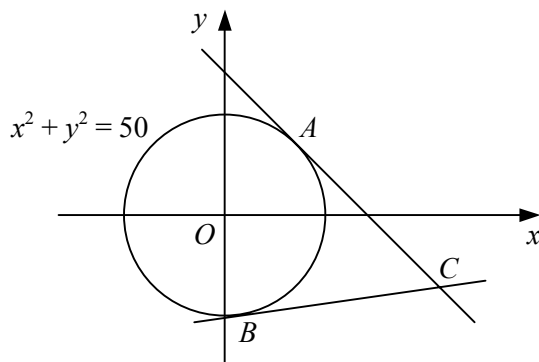
- find a cartesian equation for C in terms of a and b .

Given also that the line $y = x$ is a tangent to C ,

- show that $a = (1 + \sqrt{2})b$.

- 1 A circle has the equation $x^2 + y^2 - 8x + 7 = 0$.
- Find the coordinates of the centre of the circle. (2)
 - Find the radius of the circle. (2)
- 2 A circle has the equation $x^2 + y^2 - 6x + 2y - 15 = 0$.
- Find the coordinates of the centre of the circle. (2)
 - Find the radius of the circle. (1)
 - Show that the tangent to the circle at the point $(7, 2)$ has equation $4x + 3y - 34 = 0$. (4)
- 3 A circle has the equation $x^2 + y^2 + 6x - 8y + 21 = 0$.
- Find the coordinates of the centre and the radius of the circle. (3)
The point P lies on the circle.
 - Find the greatest distance of P from the origin. (2)

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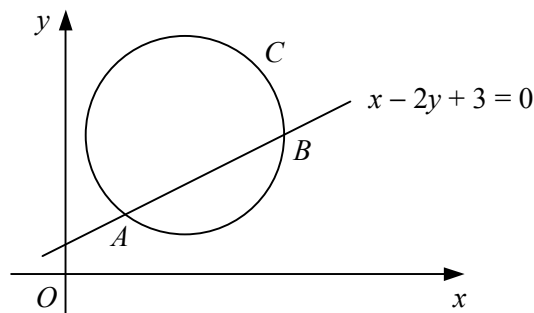


The diagram shows the circle with equation $x^2 + y^2 = 50$ and the tangents to the circle at the points $A(5, 5)$ and $B(1, -7)$.

- Find an equation of the tangent to the circle at A . (3)
 - Show that the tangent to the circle at B has the equation $x - 7y - 50 = 0$. (3)
 - Find the coordinates of the point C where the tangents to the circle at A and B intersect. (2)
- 5 Circle C_1 has the equation $x^2 + y^2 - 2ay = 0$, where a is a positive constant.
- Find the coordinates of the centre and the radius of C_1 . (4)
- Circle C_2 has the equation $x^2 + y^2 - 2bx = 0$, where b is a constant and $b > a$.
- Sketch C_1 and C_2 on the same diagram. (4)
- 6 The circle C has the equation $x^2 + y^2 + 2x - 14y + 30 = 0$.
- Find the coordinates of the centre of C . (2)
 - Find the radius of C , giving your answer in the form $k\sqrt{5}$. (2)
 - Show that the line $y = 2x - 1$ is a tangent to C and find the coordinates of the point of contact. (4)

- 7 The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.
a Find the coordinates of the centre of C . (2)
The line $y = x - 2$ intersects C at the points A and B .
b Find the length AB in the form $k\sqrt{2}$. (6)
- 8 The circle C has centre $(8, -1)$ and passes through the point $(4, 1)$.
a Find an equation for C . (3)
b Show that the line with equation $x + 2y + 4 = 0$ is a tangent to C . (3)
- 9 The points $P(-10, 2)$, $Q(8, 14)$ and $R(-2, -10)$ all lie on circle C .
a Show that PR is perpendicular to PQ . (2)
b Hence, show that C has the equation $x^2 + y^2 - 6x - 4y - 156 = 0$. (5)
- 10 A circle has the equation $x^2 + y^2 - 2x - 7y - 16 = 0$.
a Find the coordinates of the centre of the circle. (2)
b Show that the radius of the circle is $k\sqrt{13}$, where k is an exact fraction to be found. (2)
c Find an equation of the tangent to the circle at the point $(4, 8)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

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The line with equation $x - 2y + 3 = 0$ intersects the circle C at the points A and B as shown in the diagram above. Given that the centre of C has coordinates $(6, 7)$,

- a find the coordinates of the mid-point of the chord AB . (6)
Given also that the x -coordinate of the point A is 3,
b find the coordinates of the point B , (3)
c find an equation for C . (2)
- 12 The circle C has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.
a Find the coordinates of the centre and the radius of C . (3)
b Find the distance of the centre of C from the origin in the form $k\sqrt{5}$. (2)
The point A lies on C and the tangent to C at A passes through the origin O .
c Show that $OA = 6\sqrt{2}$. (3)
- 13 The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.
a Show that l passes through the centre of C . (3)
b Find an equation for each tangent to C that is parallel to l . (6)