

- 1 **a** $\frac{dy}{dx} = 2x + 6$
 $2x + 6 = 0$
 $x = -3$
- b** $\frac{dy}{dx} = 8x + 2$
 $8x + 2 = 0$
 $x = -\frac{1}{4}$
- c** $\frac{dy}{dx} = 3x^2 - 12$
 $3x^2 - 12 = 0$
 $x^2 = 4$
 $x = \pm 2$
- d** $\frac{dy}{dx} = 18x - 3x^2$
 $18x - 3x^2 = 0$
 $3x(6 - x) = 0$
 $x = 0, 6$
- e** $\frac{dy}{dx} = 3x^2 - 10x + 3$ **f** $\frac{dy}{dx} = 1 - 9x^{-2}$
 $3x^2 - 10x + 3 = 0$ $1 - 9x^{-2} = 0$
 $(3x - 1)(x - 3) = 0$ $x^2 = 9$
 $x = \frac{1}{3}, 3$ $x = \pm 3$
- g** $y = x^3 - 3x^2 + 3x - 9$ **h** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2$
 $\frac{dy}{dx} = 3x^2 - 6x + 3$ $\frac{1}{2}x^{-\frac{1}{2}} - 2 = 0$
 $3x^2 - 6x + 3 = 0$ $x^{-\frac{1}{2}} = 4$
 $3(x - 1)^2 = 0$ $x = \frac{1}{16}$
 $x = 1$
- 2 **a** $f'(x) = 4x + 2$
 $\therefore 4x + 2 \geq 0$
 $x \geq -\frac{1}{2}$
- b** $f'(x) = 6x - 6x^2$
 $\therefore 6x - 6x^2 \geq 0$
 $6x(1 - x) \geq 0$
 $0 \leq x \leq 1$
- c** $f'(x) = 9x^2 - 1$
 $\therefore 9x^2 - 1 \geq 0$
 $x^2 \geq \frac{1}{9}$
 $x \leq -\frac{1}{3}$ and $x \geq \frac{1}{3}$
- d** $f'(x) = 3x^2 + 12x - 15$
 $\therefore 3x^2 + 12x - 15 \geq 0$
 $3(x + 5)(x - 1) \geq 0$
 $x \leq -5$ and $x \geq 1$
- e** $f(x) = x^3 - 12x^2 + 36x$
 $f'(x) = 3x^2 - 24x + 36$
 $\therefore 3x^2 - 24x + 36 \geq 0$
 $3(x - 2)(x - 6) \geq 0$
 $x \leq 2$ and $x \geq 6$
- f** $f'(x) = 2 - 8x^{-2}$
 $\therefore 2 - 8x^{-2} \geq 0$
 $x^2 \geq 4$
 $x \leq -2$ and $x \geq 2$
- 3 **a** $f'(x) = 3x^2 + 4x$
 $\therefore 3x^2 + 4x \leq 0$
 $x(3x + 4) \leq 0$
 $-\frac{4}{3} \leq x \leq 0$
- b** $f'(x) = 27 - 3x^2$
 $\therefore 27 - 3x^2 \leq 0$
 $x^2 \geq 9$
 $x \leq -3$ and $x \geq 3$
- c** $f(x) = 2x^3 - x^2 - 4x + 2$
 $f'(x) = 6x^2 - 2x - 4$
 $\therefore 6x^2 - 2x - 4 \leq 0$
 $2(3x + 2)(x - 1) \leq 0$
 $-\frac{2}{3} \leq x \leq 1$
- 4 **a** $(x + 1)$ factor $\therefore f(-1) = 0$
 $\therefore -1 + k + 3 = 0$
 $k = -2$
- b** $f'(x) = 3x^2 - 4x$
 $\therefore 3x^2 - 4x \geq 0$
 $x(3x - 4) \geq 0$
 $x \leq 0$ and $x \geq \frac{4}{3}$

- 5**
- a** $\frac{dy}{dx} = 2x + 2$
 SP: $2x + 2 = 0$
 $x = -1$
 $\therefore (-1, -1)$
- b** $\frac{dy}{dx} = 10x - 4$
 SP: $10x - 4 = 0$
 $x = \frac{2}{5}$
 $\therefore (\frac{2}{5}, \frac{1}{5})$
- c** $\frac{dy}{dx} = 3x^2 - 3$
 SP: $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \pm 1$
 $\therefore (-1, 6), (1, 2)$
- d** $\frac{dy}{dx} = 12x^2 + 6x$
 SP: $12x^2 + 6x = 0$
 $6x(2x + 1) = 0$
 $x = -\frac{1}{2}, 0$
 $\therefore (-\frac{1}{2}, \frac{9}{4}), (0, 2)$
- e** $\frac{dy}{dx} = 2 - 8x^{-2}$
 SP: $2 - 8x^{-2} = 0$
 $x^2 = 4$
 $x = \pm 2$
 $\therefore (-2, -5), (2, 11)$
- f** $\frac{dy}{dx} = 3x^2 - 18x - 21$
 SP: $3x^2 - 18x - 21 = 0$
 $3(x + 1)(x - 7) = 0$
 $x = -1, 7$
 $\therefore (-1, 22), (7, -234)$
- g** $\frac{dy}{dx} = -x^{-2} - 8x$
 SP: $-x^{-2} - 8x = 0$
 $x^3 = -\frac{1}{8}$
 $x = -\frac{1}{2}$
 $\therefore (-\frac{1}{2}, -3)$
- h** $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$
 SP: $3x^{\frac{1}{2}} - 6 = 0$
 $x^{\frac{1}{2}} = 2$
 $x = 4$
 $\therefore (4, -8)$
- i** $\frac{dy}{dx} = 6x^{-\frac{1}{3}} - 2$
 SP: $6x^{-\frac{1}{3}} - 2 = 0$
 $x^{-\frac{1}{3}} = \frac{1}{3}$
 $x = \frac{1}{27}$
 $\therefore (\frac{1}{27}, 5\frac{25}{27})$
- 6**
- a** $\frac{dy}{dx} = 4 - 2x$
 SP: $4 - 2x = 0$
 $x = 2$
 $\frac{d^2y}{dx^2} = -2$
 $(2, 9)$: max
- b** $\frac{dy}{dx} = 3x^2 - 3$
 SP: $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \pm 1$
 $\frac{d^2y}{dx^2} = 6x$
 $(-1, 2)$: $\frac{d^2y}{dx^2} = -6$, max
 $(1, -2)$: $\frac{d^2y}{dx^2} = 6$, min
- c** $\frac{dy}{dx} = 3x^2 + 18x$
 SP: $3x^2 + 18x = 0$
 $3x(x + 6) = 0$
 $x = -6, 0$
 $\frac{d^2y}{dx^2} = 6x + 18$
 $(-6, 100)$: $\frac{d^2y}{dx^2} = -18$, max
 $(0, -8)$: $\frac{d^2y}{dx^2} = 18$, min
- d** $\frac{dy}{dx} = 3x^2 - 12x - 36$
 SP: $3x^2 - 12x - 36 = 0$
 $3(x + 2)(x - 6) = 0$
 $x = -2, 6$
 $\frac{d^2y}{dx^2} = 6x - 12$
 $(-2, 55)$: $\frac{d^2y}{dx^2} = -24$, max
 $(6, -201)$: $\frac{d^2y}{dx^2} = 24$, min
- e** $\frac{dy}{dx} = 4x^3 - 16x$
 SP: $4x^3 - 16x = 0$
 $4x(x^2 - 4) = 0$
 $x = 0, \pm 2$
 $\frac{d^2y}{dx^2} = 12x^2 - 16$
 $(-2, -18)$: $\frac{d^2y}{dx^2} = 32$, min
 $(0, -2)$: $\frac{d^2y}{dx^2} = -16$, max
 $(2, -18)$: $\frac{d^2y}{dx^2} = 32$, min
- f** $\frac{dy}{dx} = 9 - 4x^{-2}$
 SP: $9 - 4x^{-2} = 0$
 $x^2 = \frac{4}{9}$
 $x = \pm \frac{2}{3}$
 $\frac{d^2y}{dx^2} = 8x^{-3}$
 $(-\frac{2}{3}, -12)$: $\frac{d^2y}{dx^2} = -27$, max
 $(\frac{2}{3}, 12)$: $\frac{d^2y}{dx^2} = 27$, min

$$\mathbf{g} \quad \frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}}$$

$$\text{SP: } 1 - 3x^{-\frac{1}{2}} = 0$$

$$x^{-\frac{1}{2}} = \frac{1}{3}$$

$$x = 9$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{3}{2}}$$

$$(9, -9): \frac{d^2y}{dx^2} = \frac{1}{18}, \text{ min}$$

$$\mathbf{h} \quad \frac{dy}{dx} = -8 + 14x - 6x^2$$

$$\text{SP: } -8 + 14x - 6x^2 = 0$$

$$-2(3x - 4)(x - 1) = 0$$

$$x = 1, \frac{4}{3}$$

$$\frac{d^2y}{dx^2} = 14 - 12x$$

$$(1, 0): \frac{d^2y}{dx^2} = 2, \text{ min}$$

$$(\frac{4}{3}, \frac{1}{27}): \frac{d^2y}{dx^2} = -2, \text{ max}$$

$$\mathbf{i} \quad y = \frac{1}{2}x^2 + 8x^{-2}$$

$$\frac{dy}{dx} = x - 16x^{-3}$$

$$\text{SP: } x - 16x^{-3} = 0$$

$$x^4 = 16$$

$$x = \pm 2$$

$$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$$

$$(-2, 4): \frac{d^2y}{dx^2} = 4, \text{ min}$$

$$(2, 4): \frac{d^2y}{dx^2} = 4, \text{ min}$$

$$\mathbf{7} \quad \mathbf{a} \quad \frac{dy}{dx} = 2x - 3x^2$$

$$\text{SP: } 2x - 3x^2 = 0$$

$$x(2 - 3x) = 0$$

$$x = 0, \frac{2}{3}$$

$$\frac{d^2y}{dx^2} = 2 - 6x$$

$$(0, 0): \frac{d^2y}{dx^2} = 2, \text{ min}$$

$$(\frac{2}{3}, \frac{4}{27}): \frac{d^2y}{dx^2} = -2, \text{ max}$$

$$\mathbf{b} \quad \frac{dy}{dx} = 3x^2 + 6x + 3$$

$$\text{SP: } 3x^2 + 6x + 3 = 0$$

$$3(x + 1)^2 = 0$$

$$x = -1$$

$$\frac{d^2y}{dx^2} = 6x + 6$$

$$(-1, -1): \frac{d^2y}{dx^2} = 0$$

x	< -1	-1	> -1
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore (-1, -1): \text{point of inflexion}$

$$\mathbf{c} \quad \frac{dy}{dx} = 4x^3$$

$$\text{SP: } 4x^3 = 0$$

$$x = 0$$

$$\frac{d^2y}{dx^2} = 12x^2$$

$$(0, -2): \frac{d^2y}{dx^2} = 0$$

x	< 0	0	> 0
$\frac{dy}{dx}$	$-$	0	$+$

$\therefore (0, -2): \text{min}$

$$\mathbf{d} \quad \frac{dy}{dx} = -12 + 12x - 3x^2$$

$$\text{SP: } -12 + 12x - 3x^2 = 0$$

$$-3(x - 2)^2 = 0$$

$$x = 2$$

$$\frac{d^2y}{dx^2} = 12 - 6x$$

$$(2, -4): \frac{d^2y}{dx^2} = 0$$

x	< 2	2	> 2
$\frac{dy}{dx}$	$-$	0	$-$

$\therefore (2, -4): \text{point of inflexion}$

$$\mathbf{e} \quad \frac{dy}{dx} = 2x - 16x^{-2}$$

$$\text{SP: } 2x - 16x^{-2} = 0$$

$$x^3 = 8$$

$$x = 2$$

$$\frac{d^2y}{dx^2} = 2 + 32x^{-3}$$

$$(2, 12): \frac{d^2y}{dx^2} = 6, \text{ min}$$

$$\mathbf{f} \quad \frac{dy}{dx} = 4x^3 + 12x^2$$

$$\text{SP: } 4x^3 + 12x^2 = 0$$

$$4x^2(x + 3) = 0$$

$$x = -3, 0$$

$$\frac{d^2y}{dx^2} = 12x^2 + 24x$$

$$(-3, -28): \frac{d^2y}{dx^2} = 36, \text{ min}$$

$$(0, -1): \frac{d^2y}{dx^2} = 0$$

x	$-3 < x < 0$	0	> 0
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore (0, -1): \text{point of inflexion}$

8 a $\frac{dy}{dx} = 3x^2 + 6x$

SP: $3x^2 + 6x = 0$

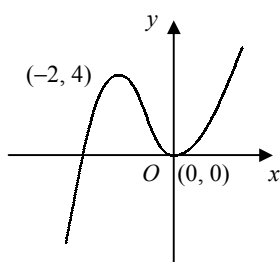
$3x(x + 2) = 0$

$x = -2, 0$

$\frac{d^2y}{dx^2} = 6x + 6$

$(-2, 4): \frac{d^2y}{dx^2} = -6, \text{ max}$

$(0, 0): \frac{d^2y}{dx^2} = 6, \text{ min}$



b $\frac{dy}{dx} = 1 - x^{-2}$

SP: $1 - x^{-2} = 0$

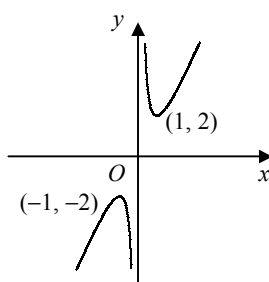
$x^2 = 1$

$x = \pm 1$

$\frac{d^2y}{dx^2} = 2x^{-3}$

$(-1, -2): \frac{d^2y}{dx^2} = -2, \text{ max}$

$(1, 2): \frac{d^2y}{dx^2} = 2, \text{ min}$



c $\frac{dy}{dx} = 3x^2 - 6x + 3$

SP: $3x^2 - 6x + 3 = 0$

$3(x - 1)^2 = 0$

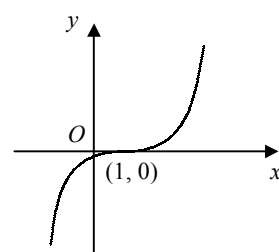
$x = 1$

$\frac{d^2y}{dx^2} = 6x - 6$

$(1, 0): \frac{d^2y}{dx^2} = 0$

x	< 1	1	> 1
$\frac{d^2y}{dx^2}$	+	0	+

$\therefore (1, 0): \text{ point of inflexion}$



d $\frac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$

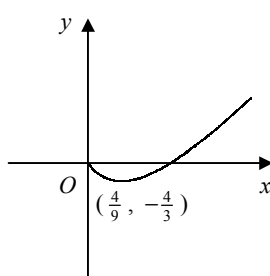
SP: $3 - 2x^{-\frac{1}{2}} = 0$

$x^{-\frac{1}{2}} = \frac{3}{2}$

$x = \frac{4}{9}$

$\frac{d^2y}{dx^2} = x^{-\frac{3}{2}}$

$(\frac{4}{9}, -\frac{4}{3}): \frac{d^2y}{dx^2} = \frac{27}{8}, \text{ min}$



e $\frac{dy}{dx} = 3x^2 + 8x - 3$

SP: $3x^2 + 8x - 3 = 0$

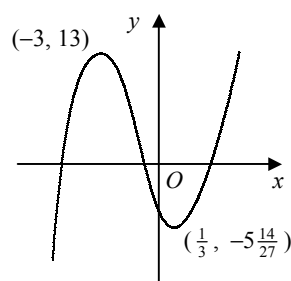
$(3x - 1)(x + 3) = 0$

$x = -3, \frac{1}{3}$

$\frac{d^2y}{dx^2} = 6x + 8$

$(-3, 13): \frac{d^2y}{dx^2} = -10, \text{ max}$

$(\frac{1}{3}, -5\frac{14}{27}): \frac{d^2y}{dx^2} = 10, \text{ min}$



f $y = x^4 - 8x^2 + 12$

$\frac{dy}{dx} = 4x^3 - 16x$

SP: $4x^3 - 16x = 0$

$4x(x + 2)(x - 2) = 0$

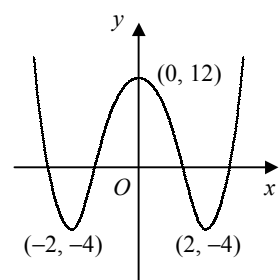
$x = -2, 0, 2$

$\frac{d^2y}{dx^2} = 12x^2 - 16$

$(-2, -4): \frac{d^2y}{dx^2} = 32, \text{ min}$

$(0, 12): \frac{d^2y}{dx^2} = -16, \text{ max}$

$(2, -4): \frac{d^2y}{dx^2} = 32, \text{ min}$



1 a volume = $2x^2h = 4000$

$$\therefore h = \frac{2000}{x^2}$$

b $A = 2x^2 + 2(2xh) + 2(xh)$

$$= 2x^2 + 6xh$$

$$= 2x^2 + (6x \times \frac{2000}{x^2})$$

$$= 2x^2 + \frac{12000}{x}$$

c $\frac{dA}{dx} = 4x - 12000x^{-2}$

SP: $4x - 12000x^{-2} = 0$

$$x^3 = 3000$$

$$x = \sqrt[3]{3000} = 14.4 \text{ (3sf)}$$

d min $A = 1250$ (3sf)

e $\frac{d^2A}{dx^2} = 4 + 24000x^{-3}$

when $x = \sqrt[3]{3000}$, $\frac{d^2A}{dx^2} = 12$

$$\frac{d^2A}{dx^2} > 0 \therefore \text{minimum}$$

3 a S.A. = $2x^2 + 4xl = k$

$$\therefore 4xl = k - 2x^2$$

$$l = \frac{k - 2x^2}{4x}$$

b $V = x^2l$

$$= x^2 \times \frac{k - 2x^2}{4x}$$

$$= \frac{1}{4}kx - \frac{1}{2}x^3$$

$$\frac{dV}{dx} = \frac{1}{4}k - \frac{3}{2}x^2$$

SP: $\frac{1}{4}k - \frac{3}{2}x^2 = 0$

$$x^2 = \frac{1}{6}k$$

$$x = \sqrt{\frac{k}{6}}$$

$$\frac{d^2V}{dx^2} = -3x$$

when $x = \sqrt{\frac{k}{6}}$, $\frac{d^2V}{dx^2} < 0 \therefore \text{maximum}$

$$l = \frac{k - \frac{1}{3}k}{4\sqrt{\frac{k}{6}}} = \frac{2}{3}k \times \frac{1}{4} \times \sqrt{\frac{6}{k}}$$

$$= \frac{k}{6} \times \sqrt{\frac{6}{k}} = \sqrt{\frac{k}{6}}$$

$\therefore$ maximum V when $l = x \therefore$ prism is a cube

2 a S.A. = $2\pi r^2 + 2\pi rh = 30\,000$

$$\therefore \pi rh = 15\,000 - \pi r^2$$

$$h = \frac{15000}{\pi r} - r$$

$$V = \pi r^2 h$$

$$= \pi r^2 \left(\frac{15000}{\pi r} - r \right)$$

$$= 15\,000r - \pi r^3$$

b $\frac{dV}{dr} = 15\,000 - 3\pi r^2$

SP: $15\,000 - 3\pi r^2 = 0$

$$r^2 = \frac{5000}{\pi}$$

$$r = \sqrt{\frac{5000}{\pi}} \quad [= 39.9 \text{ (3sf)}]$$

max volume = $399\,000 \text{ cm}^3$ (3sf)

$$\frac{d^2V}{dr^2} = -6\pi r$$

when $r = \sqrt{\frac{5000}{\pi}}$, $\frac{d^2V}{dr^2} = -752$

$$\frac{d^2V}{dr^2} < 0 \therefore \text{maximum}$$

1 a $f'(x) = 6x^2 + 10x$

b $6x^2 + 10x \geq 0$
 $2x(3x + 5) \geq 0$
 $x \leq -\frac{5}{3}$ and $x \geq 0$

3 a $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2}$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} + 8x^{-3}$$

b SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2} = 0$
 $\frac{1}{2}x^{-2}(x^{\frac{3}{2}} - 8) = 0$
 $x^{\frac{3}{2}} = 8$
 $x = 4$

$\therefore (4, 3)$

when $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{32}$

$\frac{d^2y}{dx^2} > 0 \therefore$ minimum

5 a $\frac{dh}{dt} = 8t^3 - 24t^2 + 16t$

b when $t = 0.25$,
 $\frac{dh}{dt} = 2.625$ cm per second

c SP: $8t^3 - 24t^2 + 16t = 0$
 $8t(t - 1)(t - 2) = 0$
 $t = 0, 1, 2$
 from graph, max when $t = 1$
 $\therefore$ max height = 3 cm

2 a $\frac{dy}{dx} = 3x^2 - 2x + 2$

at $(1, -2)$, grad = 3

$\therefore y + 2 = 3(x - 1)$

$3x - y - 5 = 0$

b SP when $3x^2 - 2x + 2 = 0$

$b^2 - 4ac = 4 - 24 = -20$

$b^2 - 4ac < 0 \therefore$ no real roots

$\therefore$ no stationary points

4 a $y = 0 \Rightarrow x(x + 3)^2 = 0$

$x = -3, 0$

$\therefore (-3, 0), (0, 0)$

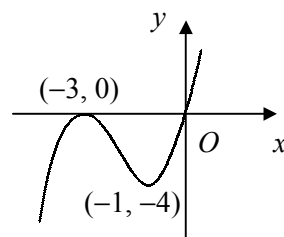
b $f'(x) = 3x^2 + 12x + 9$

decreasing when $3x^2 + 12x + 9 \leq 0$

$3(x + 3)(x + 1) \leq 0$

$\therefore -3 \leq x \leq -1$

c



6 a $\frac{dy}{dx} = 3x^2 + 6kx - 9k^2$

stationary when $3x^2 + 6kx - 9k^2 = 0$

$\Rightarrow x^2 + 2kx - 3k^2 = 0$

b $(x + 3k)(x - k) = 0$

$x = -3k, k$

when $x = k$, $y = k^3 + 3k^3 - 9k^3 = -5k^3$

$\therefore$ stationary at $(k, -5k^3)$

c when $x = -3k$,

$y = -27k^3 + 27k^3 + 27k^3 = 27k^3$

$\therefore (-3k, 27k^3)$

$$7 \quad \mathbf{a} \quad V = \frac{1}{2}x^2 \sin 60^\circ \times l$$

$$= \frac{1}{2}x^2 l \times \frac{\sqrt{3}}{2} = 250$$

$$\therefore l = \frac{1000}{\sqrt{3}x^2} \text{ or } \frac{1000\sqrt{3}}{3x^2}$$

$$\mathbf{b} \quad A = (2 \times \frac{\sqrt{3}}{4}x^2) + 3xl$$

$$= \frac{\sqrt{3}}{2}x^2 + (3x \times \frac{1000\sqrt{3}}{3x^2})$$

$$= \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x})$$

$$\mathbf{c} \quad \frac{dA}{dx} = \frac{\sqrt{3}}{2}(2x - 2000x^{-2})$$

$$\text{SP: } \frac{\sqrt{3}}{2}(2x - 2000x^{-2}) = 0$$

$$x^3 = 1000$$

$$x = 10$$

$$\mathbf{d} \quad \min A = 150\sqrt{3}$$

$$\mathbf{e} \quad \frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}(2 + 4000x^{-3})$$

$$\text{when } x = 10, \frac{d^2A}{dx^2} = 3\sqrt{3}$$

$$\frac{d^2A}{dx^2} > 0 \therefore \text{minimum}$$

$$9 \quad \mathbf{a} \quad x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}} = 0$$

$$x - 4x^{\frac{1}{2}} + 3 = 0$$

$$(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 3) = 0$$

$$x^{\frac{1}{2}} = 1, 3$$

$$x = 1, 9$$

$$\therefore (1, 0) \text{ and } (9, 0)$$

$$\mathbf{b} \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

$$\text{SP: } \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} = 0$$

$$\frac{1}{2}x^{-\frac{3}{2}}(x - 3) = 0$$

$$x = 3$$

$$y = \sqrt{3} - 4 + \frac{3}{\sqrt{3}} = 2\sqrt{3} - 4$$

$$\therefore (3, 2\sqrt{3} - 4)$$

$$8 \quad \mathbf{a} \quad f'(x) = 3x^2 + 8x + k$$

for 2 SPs, $f'(x) = 0$ has 2 distinct roots

$$\therefore b^2 - 4ac > 0$$

$$64 - 12k > 0$$

$$k < \frac{16}{3}$$

$$\mathbf{b} \quad \text{SP: } 3x^2 + 8x - 3 = 0$$

$$(3x - 1)(x + 3) = 0$$

$$x = -3, \frac{1}{3}$$

$$\therefore (-3, 19) \text{ and } (\frac{1}{3}, \frac{13}{27})$$

$$10 \quad \mathbf{a} \quad f(-1) = -1 - 3 + 4 = 0$$

$$\therefore (x + 1) \text{ is a factor}$$

$$\mathbf{b} \quad \begin{array}{r} x^2 - 4x + 4 \\ x+1 \overline{) x^3 - 3x^2 + 0x + 4} \\ \underline{x^3 + x^2} \\ -4x^2 + 0x \\ \underline{-4x^2 - 4x} \\ 4x + 4 \\ \underline{4x + 4} \\ 0 \end{array}$$

$$\therefore f(x) \equiv (x + 1)(x^2 - 4x + 4)$$

$$f(x) \equiv (x + 1)(x - 2)^2$$

$$\mathbf{c} \quad (2, 0), \text{ as } (x - 2) \text{ is a repeated factor}$$

of $f(x)$ so x -axis is a tangent at $(2, 0)$

$$\mathbf{d} \quad f'(x) = 3x^2 - 6x$$

$$\text{SP: } 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0, 2$$

$$\therefore (0, 4) \text{ is other turning point}$$

1 a $f'(x) = 24 + 6x - 3x^2$

b $24 + 6x - 3x^2 \geq 0$

$$x^2 - 2x - 8 \leq 0$$

$$(x+2)(x-4) \leq 0$$

$$-2 \leq x \leq 4$$

2 a $(-2, 30) \Rightarrow 30 = -8 + 4a + 48 + b$

$$\therefore 4a + b + 10 = 0$$

b $\frac{dy}{dx} = 3x^2 + 2ax - 24$

SP at P $\therefore \frac{dy}{dx} = 0$

$$\Rightarrow 12 - 4a - 24 = 0$$

$$a = -3, b = 2$$

c $3x^2 - 6x - 24 = 0$

$$3(x+2)(x-4) = 0$$

$$x = -2 \text{ (at P) or } 4$$

$$\text{other SP } (4, -78)$$

3 a $f'(x) = 2x - 16x^{-2}$

b SP: $2x - 16x^{-2} = 0$

$$x^3 = 8$$

$$x = 2$$

$$\therefore (2, 12)$$

$$f''(x) = 2 + 32x^{-3}$$

$$f''(2) = 6$$

$$f''(x) > 0 \therefore \text{minimum}$$

4 a $\text{area} = (2 \times \frac{1}{2}r^2\theta) + \frac{1}{2}r^2(3\theta) = 25$

$$\therefore \frac{5}{2}r^2\theta = 25, \theta = \frac{10}{r^2}$$

b $P = 2r + (2 \times r\theta) + r(3\theta) = 2r + 5r\theta$

$$= 2r + 5r(\frac{10}{r^2}) = 2r + \frac{50}{r}$$

c $\frac{dP}{dr} = 2 - 50r^{-2}$

SP: $2 - 50r^{-2} = 0$

$$r^2 = 25$$

$$r = 5$$

d $\min P = 20$

e $\frac{d^2P}{dr^2} = 100r^{-3}$, when $r = 5$, $\frac{d^2P}{dr^2} = 0.8$

$$\frac{d^2P}{dr^2} > 0 \therefore \text{minimum}$$

5 a $2x - x^{\frac{3}{2}} = 0$

$$x(2 - x^{\frac{1}{2}}) = 0$$

$$x = 0 \text{ or } x^{\frac{1}{2}} = 2 \Rightarrow x = 4$$

$$\therefore (0, 0) \text{ and } (4, 0)$$

b $\frac{dy}{dx} = 2 - \frac{3}{2}x^{\frac{1}{2}}$

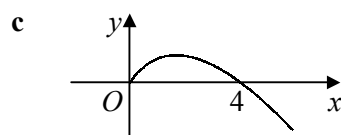
SP: $2 - \frac{3}{2}x^{\frac{1}{2}} = 0$

$$x^{\frac{1}{2}} = \frac{4}{3}$$

$$x = \frac{16}{9}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}, \text{ when } x = \frac{16}{9}, \frac{d^2y}{dx^2} = -\frac{9}{16}$$

$$\frac{d^2y}{dx^2} < 0 \therefore \text{maximum}$$



6 a $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$$x^2 = 1$$

$$x = \pm 1$$

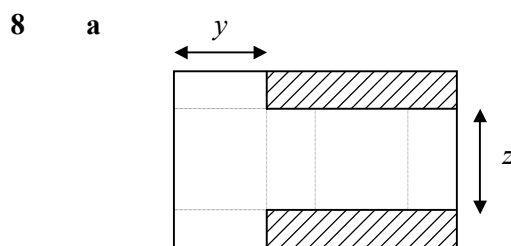
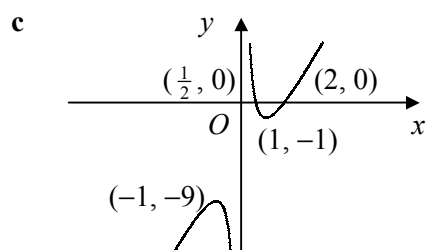
$$\therefore (-1, 3) \text{ and } (1, -1)$$

b $PQ^2 = 2^2 + 4^2 = 20$

$$\therefore PQ = \sqrt{20} = 2\sqrt{5}$$

7 a $2x - 5 + \frac{2}{x} = 0$
 $2x^2 - 5x + 2 = 0$
 $(2x - 1)(x - 2) = 0$
 $x = \frac{1}{2}, 2$

b $f'(x) = 2 - 2x^{-2}$
 $\therefore 2 - 2x^{-2} = 0$
 $x^2 = 1$
 $x = \pm 1$



$$2x + z = 25$$

$$2x + 2y = 40$$

$\therefore$ length and width $(25 - 2x)$ and $(20 - x)$

b volume $= x(25 - 2x)(20 - x)$
 $= x(500 - 65x + 2x^2)$
 $= 2x^3 - 65x^2 + 500x$

c $\frac{dV}{dx} = 6x^2 - 130x + 500$
 SP: $6x^2 - 130x + 500 = 0$
 $2(3x - 50)(x - 5) = 0$
 $x = 5, \frac{50}{3}$

$$2x < 25 \quad \therefore x < 12.5$$

$$\therefore x = 5$$

d max volume $= 1125 \text{ cm}^3$
 $\frac{d^2V}{dx^2} = 12x - 130$
 when $x = 5, \frac{d^2V}{dx^2} = -70$
 $\frac{d^2V}{dx^2} < 0 \quad \therefore$ maximum

9 a $\frac{dy}{dx} = 9 + 6x - 3x^2$
 SP: $9 + 6x - 3x^2 = 0$
 $-3(x + 1)(x - 3) = 0$
 $x = -1, 3$
 $\therefore (-1, -3)$ and $(3, 29)$

b $\frac{d^2y}{dx^2} = 6 - 6x$
 $(-1, -3): \frac{d^2y}{dx^2} = 12 \quad \therefore$ minimum
 $(3, 29): \frac{d^2y}{dx^2} = -12 \quad \therefore$ maximum

c $-3 < k < 29$

10 a $f(-1) = 15$
 $\therefore -4 + a + 12 + b = 15$
 $a + b = 7 \quad (1)$

b $f(2) = 42$
 $\therefore 32 + 4a - 24 + b = 42$
 $4a + b = 34 \quad (2)$

$$(2) - (1) \quad 3a = 27$$

$$\therefore a = 9, b = -2$$

c $f(x) = 4x^3 + 9x^2 - 12x - 2$
 $f'(x) = 12x^2 + 18x - 12$
 SP: $12x^2 + 18x - 12 = 0$
 $2x^2 + 3x - 2 = 0$
 $(2x - 1)(x + 2) = 0$
 $x = -2, \frac{1}{2}$
 $\therefore (-2, 26)$ and $(\frac{1}{2}, -\frac{21}{4})$