C2 SEQUENCES AND SERIES — Answers - Worksheet A

1 **a** $r = 3$
$u_8 = 3 \times 3^7 = 6561$

b $r = \frac{1}{4}$
$u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$

c $r = -2$
$u_8 = 1 \times (-2)^7 = -128$

2 **a** $a = 1, r = 5$
$u_n = 5^{n-1}$

b $a = 3, r = -4$
$u_n = 3 \times (-4)^{n-1}$

c $a = 81, r = \frac{2}{3}$
$u_n = 81 \times (\frac{2}{3})^{n-1}$

3 **a** $a = 2, r = 2, n = 12$
$S_{12} = \frac{2(2^{12} - 1)}{2 - 1} = 8190$

b $a = 640, r = \frac{1}{2}, n = 12$
$S_{12} = \frac{640[1 - (\frac{1}{2})^{12}]}{1 - \frac{1}{2}} = 1279\frac{11}{16}$

c $a = \frac{1}{6}, r = -3, n = 12$
$S_{12} = \frac{\frac{1}{6}[1 - (-3)^{12}]}{1 - (-3)} = -22\,143\frac{1}{3}$

4 **a** $S_8 = \frac{4(3^8 - 1)}{3 - 1} = 13\,120$

b $S_{14} = \frac{48[1 - (\frac{1}{2})^{14}]}{1 - \frac{1}{2}} = 95.994$

c $S_{12} = \frac{-[1 - (-4)^{12}]}{1 - (-4)} = 3\,355\,443$

d $S_{20} = \frac{200[1 - (0.7)^{20}]}{1 - 0.7}$
$= 666.135$

e $S_{15} = \frac{120[1 - (-\frac{3}{4})^{15}]}{1 - (-\frac{3}{4})}$
$= 69.488$

f $S_{30} = \frac{-25[(1.2)^{30} - 1]}{1.2 - 1}$
$= -29\,547.039$

5 **a** GP: $a = 3$
$r = 3, n = 9$
$S_9 = \frac{3(3^9 - 1)}{3 - 1}$
$= 29\,523$

b GP: $a = 64$
$r = 8, n = 6$
$S_6 = \frac{64(8^6 - 1)}{8 - 1}$
$= 2\,396\,736$

c GP: $a = 20$
$r = 2, n = 10$
$S_{10} = \frac{20(2^{10} - 1)}{2 - 1}$
$= 20\,460$

d GP: $a = 0.8$
$r = 0.8, n = 8$
$S_8 = \frac{0.8[1 - (0.8)^8]}{1 - 0.8}$
$= 3.329$ (3dp)

e GP: $a = 2$
$r = \frac{1}{6}, n = 10$
$S_{10} = \frac{2[1 - (\frac{1}{6})^{10}]}{1 - \frac{1}{6}}$
$= 2.400$ (3dp)

f GP: $a = -4$
$r = -4, n = 9$
$S_9 = \frac{-4[1 - (-4)^9]}{1 - (-4)}$
$= -209\,716$

g GP: $a = \frac{1}{16}$
$r = \frac{1}{2}, n = 17$
$S_{17} = \frac{\frac{1}{16}[1 - (\frac{1}{2})^{17}]}{1 - \frac{1}{2}}$
$= 0.125$ (3dp)

h GP: $a = -54$
$r = -3, n = 7$
$S_7 = \frac{-54[1 - (-3)^7]}{1 - (-3)}$
$= -29\,538$

6 **a** $r = 10 \div 2 = 5$

b $a \times 5 = 2 \;\therefore\; a = 0.4$

c $S_8 = \frac{0.4(5^8 - 1)}{5 - 1} = 39\,062.4$

7 **a** $a = 2, ar^3 = 54 \;\therefore\; r^3 = 54 \div 2 = 27$
$r = \sqrt[3]{27} = 3$

b $u_9 = 2 \times 3^8 = 13\,122$

8 **a** $r = 8 \div 24 = \frac{1}{3}$

b $a \times (\frac{1}{3})^2 = 24 \;\therefore\; a = 216$

c $S_{11} = \frac{216[1 - (\frac{1}{3})^{11}]}{1 - \frac{1}{3}} = 323.998$

9 **a** $a = 6, ar^2 = 24 \;\therefore\; r^2 = 24 \div 6 = 4$
$r = \pm 2$

b $r = 2,\; S_{15} = \frac{6(2^{15} - 1)}{2 - 1} = 196\,602$

10 **a** $a = 768, ar^3 = -96$
$r^3 = -96 \div 768 = -\frac{1}{8}$
$r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

b $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$

11 **a** $ar = 0.5,\; ar^4 = 32 \;\therefore\; r^3 = 32 \div 0.5 = 64$
$r = \sqrt[3]{64} = 4,\; a \times 4 = 0.5 \;\therefore\; a = 0.125$

b $0.125 \times 4^{n-1} < 10\,000 \;\therefore\; 4^{n-1} < 80\,000$
$(n - 1) \lg 4 < \lg 80\,000$
$n < \frac{\lg 80000}{\lg 4} + 1$
$n < 9.14 \;\therefore\;$ 9 terms

12 **a** $\dfrac{a[(\frac{3}{2})^4-1]}{\frac{3}{2}-1}=130$

$a = 130 \div \frac{65}{8} = 16$

b $u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$

c $\dfrac{16[(\frac{3}{2})^n-1]}{\frac{3}{2}-1} > 30\,000$

$(\frac{3}{2})^n > 938.5$

$n \lg \frac{3}{2} > \lg 938.5$

$n > \dfrac{\lg 938.5}{\lg 1.5}$

$n > 16.9 \;\therefore$ least $n = 17$

13 **a** $a + ar = a(1 + r) = 10.8$

$ar^2 + ar^3 = ar^2(1 + r) = 43.2$

$\therefore\; r^2 = 43.2 \div 10.8 = 4$

all terms +ve $\therefore$ r +ve $\therefore$ $r = 2$

sub. $a = 10.8 \div 3 = 3.6$

b $S_{16} = \dfrac{3.6(2^{16}-1)}{2-1} = 235\,926$

14 **a** $a = 12,\ r = 0.5$

$S_\infty = \dfrac{12}{1-0.5} = 24$

b $a = 270,\ r = \frac{1}{3}$

$S_\infty = \dfrac{270}{1-\frac{1}{3}} = 405$

c $a = 25,\ r = -1.2$

no S_∞ as $r < -1$ $\therefore$ diverges

d $a = 216,\ r = \frac{2}{3}$

$S_\infty = \dfrac{216}{1-\frac{2}{3}} = 648$

e $a = \frac{8}{25},\ r = \frac{5}{4}$

no S_∞ as $r > 1$ $\therefore$ diverges

f $a = 500,\ r = -0.6$

$S_\infty = \dfrac{500}{1-(-0.6)} = 312.5$

15 **a** $a = 0.9,\ r = 0.9$

$S_\infty = \dfrac{0.9}{1-0.9} = 9$

b $a = 3,\ r = \frac{1}{2}$

$S_\infty = \dfrac{3}{1-\frac{1}{2}} = 6$

c $a = 1,\ r = -\frac{3}{4}$

$S_\infty = \dfrac{1}{1-(-\frac{3}{4})} = \frac{4}{7}$

d $a = 32,\ r = 0.8$

$S_\infty = \dfrac{32}{1-0.8} = 160$

16 **a** $S_\infty = \dfrac{80}{1-0.2} = 100$

b $S_6 = \dfrac{80[1-(0.2)^6]}{1-0.2} = 99.9936$

$S_\infty - S_6 = 0.0064$

17 **a** $1,\ \frac{1}{3},\ \frac{1}{9},\ \frac{1}{27}$

b GP: $a = 1,\ r = \frac{1}{3}$

$S_\infty = \dfrac{1}{1-\frac{1}{3}} = \frac{3}{2}$

18 **a** $\dfrac{a}{1-0.55} = 40$

$a = 0.45 \times 40 = 18$

b $18 \times (0.55)^{n-1} < 0.001$

$(n-1) \lg 0.55 < \lg 0.0000556$

$n > \dfrac{\lg 0.0000556}{\lg 0.55} + 1$

$n > 17.4 \;\therefore$ smallest $n = 18$

19 **a** $u_1 = S_1 = 2^1 - 1 = 1$

$S_5 = 2^5 - 1 = 31,\; S_4 = 2^4 - 1 = 15$

$u_5 = S_5 - S_4 = 31 - 15 = 16$

b $S_{n-1} = 2^{n-1} - 1$

$u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$

$= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$

20 **a** $\dfrac{k}{k+10} = \dfrac{k-6}{k}$

$k^2 = (k + 10)(k - 6)$

$4k - 60 = 0$

$k = 15$

b $u_1 = 25,\ u_2 = 15 \;\therefore\; a = 25,\ r = 0.6$

$S_\infty = \dfrac{25}{1-0.6} = 62.5$

C2 SEQUENCES AND SERIES — Answers - Worksheet B

1 **a** $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$a \times (\frac{3}{4})^2 = 27$

$a = \frac{16}{9} \times 27 = 48$

b $S_\infty = \dfrac{48}{1-\frac{3}{4}} = 192$

2 **a** $\dfrac{k+4}{k-8} = \dfrac{3k+2}{k+4}$

$(k + 4)^2 = (3k + 2)(k - 8)$

$k^2 - 15k - 16 = 0$

$(k + 1)(k - 16) = 0$

$k > 0 \;\therefore\; k = 16$

b $u_1 = 8,\ u_2 = 20 \;\therefore\; a = 8,\ r = \frac{5}{2}$

$u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$

c $S_{10} = \dfrac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\ 857.3$

3 **a** $ar = 75,\ ar^4 = 129.6$

$r^3 = 129.6 \div 75 = 1.728$

$r = \sqrt[3]{1.728} = 1.2$

$a = 75 \div 1.2 = 62.5$

b $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c $S_{12} = \dfrac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 **a** $S_n = a + ar + ar^2 + \ldots\ ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots\ ar^{n-1} + ar^n$

subtracting,

$S_n - rS_n = a - ar^n$

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1-r^n)}{1-r}$

b $\dfrac{2[1-(\sqrt{2})^n]}{1-\sqrt{2}} = 126(\sqrt{2} + 1)$

$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$

$1 - (\sqrt{2})^n = 63(1 - 2)$

$(\sqrt{2})^n = 64$

$2^{\frac{1}{2}n} = 2^6$

$n = 12$

5 **a** $\dfrac{18}{1-r} = 15$

$\therefore\ 1 - r = \frac{18}{15} = 1.2$

$r = -0.2$

b $u_3 = 18 \times (-0.2)^2 = 0.72$

c $S_8 = \dfrac{18[1-(-0.2)^8]}{1-(-0.2)} = 14.9999616$

$S_\infty - S_8 = 0.000\,0384$

6 **a** $S_3 = 5(3^3 - 1) = 130$

$S_2 = 5(3^2 - 1) = 40$

$u_3 = S_3 - S_2 = 90$

b $S_{n-1} = 5(3^{n-1} - 1)$

$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$

$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$

7 **a** $4 \times (1.25)^7 = 19.1$ mm (3sf)

b GP: $a = 4,\ r = 1.25$

$S_{20} = \dfrac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8$ mm

$\therefore$ length = 1.37 m (3sf)

8 **a** $ar = 30,\ ar^3 = 2.7 \;\therefore\; r^2 = 2.7 \div 30 = 0.09$

$r > 0 \;\therefore\; r = \sqrt{0.09} = 0.3$

$a = 30 \div 0.3 = 100$

b $S_\infty = \dfrac{100}{1-0.3} = 142.9$ (1dp)

9 **a** GP: $a = 27$, $r = 3$

$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$

b $\sum_{r=1}^{15} 2^r$: GP, $a = 2$, $r = 2$

$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$

$\sum_{r=1}^{15} 12r$: AP, $a = 12$, $d = 12$

$S_{15} = \frac{15}{2}[24 + (14 \times 12)] = 1440$

$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$

10 **a** $a = 64$, $ar^2 - ar = 20$

$\therefore\ 64r^2 - 64r = 20$

$16r^2 - 16r - 5 = 0$

b $(4r + 1)(4r - 5) = 0$

$r = -\frac{1}{4}$ or $\frac{5}{4}$

c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$

$r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$

d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

11 **a** $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$

b $u_n = 4 \times (\frac{1}{2})^{n-1}$

$= 2^2 \times 2^{1-n}$

$= 2^{3-n}$

c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$= 8(1 - 2^{-n})$

$= 8 - (2^3 \times 2^{-n})$

$= 8 - 2^{3-n}$

12 **a** $u_6 = 4 \times 3^6 = 2916$

b GP: $a = 12$, $r = 3$

$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$

$\therefore\ 6(3^t - 1) > 10^{25}$

$3^t > \frac{10^{25}}{6} + 1$

$t \lg 3 > \lg(\frac{10^{25}}{6} + 1)$

$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$

$t > 50.8\ \therefore$ smallest $t = 51$

13 **a** $a + ar^2 = a(1 + r^2) = 150$

$ar + ar^3 = ar(1 + r^2) = -75$

$\therefore\ r = -75 \div 150 = -\frac{1}{2}$

$a = 150 \div \frac{5}{4} = 120$

b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

14 **a** $b - a = (3a + 4) - b$

$2b = 4a + 4$

$b = 2a + 2$

b $\frac{2a + 2}{a} = \frac{6a + 1}{2a + 2}$

$(2a + 2)^2 = a(6a + 1)$

$2a^2 - 7a - 4 = 0$

$(2a + 1)(a - 4) = 0$

a integer $\therefore\ a = 4$

sub. $b = 10$

15 **a** after 4th bounce,

reaches $3 \times (0.6)^4 = 0.3888$ m

b total distance

$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \ldots]$

$= h + 2 \times S_\infty$ of GP, $a = 0.6h$, $r = 0.6$

$= h + \frac{2 \times 0.6h}{1 - 0.6}$

$= h + 3h = 4h$ metres

C2 SEQUENCES AND SERIES — Answers - Worksheet C

1 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

c $= 1 + 3(4x) + 3(4x)^2 + (4x)^3$
$= 1 + 12x + 48x^2 + 64x^3$

d $= 1 + 3(-2y) + 3(-2y)^2 + (-2y)^3$
$= 1 - 6y + 12y^2 - 8y^3$

e $= 1 + 4(\tfrac{1}{2}x) + 6(\tfrac{1}{2}x)^2 + 4(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 1 + 2x + \tfrac{3}{2}x^2 + \tfrac{1}{2}x^3 + \tfrac{1}{16}x^4$

f $= 1 + 3(\tfrac{1}{3}y) + 3(\tfrac{1}{3}y)^2 + (\tfrac{1}{3}y)^3$
$= 1 + y + \tfrac{1}{3}y^2 + \tfrac{1}{27}y^3$

g $= 1 + 5(x^2) + 10(x^2)^2 + 10(x^2)^3 + 5(x^2)^4 + (x^2)^5$
$= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$

h $= 1 + 4(-\tfrac{3}{2}x) + 6(-\tfrac{3}{2}x)^2 + 4(-\tfrac{3}{2}x)^3 + (-\tfrac{3}{2}x)^4$
$= 1 - 6x + \tfrac{27}{2}x^2 - \tfrac{27}{2}x^3 + \tfrac{81}{16}x^4$

2 **a** $= x^3 + 3x^2y + 3xy^2 + y^3$

b $= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

c $= x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$
$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$

d $= 2^3 + 3(2^2)y + 3(2)y^2 + y^3$
$= 8 + 12y + 6y^2 + y^3$

e $= 3^3 + 3(3^2)(-x) + 3(3)(-x)^2 + (-x)^3$
$= 27 - 27x + 9x^2 - x^3$

f $= 5^4 + 4(5^3)(2x) + 6(5^2)(2x)^2 + 4(5)(2x)^3 + (2x)^4$
$= 625 + 1000x + 600x^2 + 160x^3 + 16x^4$

g $= 3^5 + 5(3^4)(-4y) + 10(3^3)(-4y)^2 + 10(3^2)(-4y)^3 + 5(3)(-4y)^4 + (-4y)^5$
$= 243 - 1620y + 4320y^2 - 5760y^3 + 3840y^4 - 1024y^5$

h $= 3^4 + 4(3^3)(\tfrac{1}{2}x) + 6(3^2)(\tfrac{1}{2}x)^2 + 4(3)(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 81 + 54x + \tfrac{27}{2}x^2 + \tfrac{3}{2}x^3 + \tfrac{1}{16}x^4$

3 **a** $= 1 + 10x + \tfrac{10\times 9}{2}x^2 + \tfrac{10\times 9\times 8}{3\times 2}x^3 + \ldots$
$= 1 + 10x + 45x^2 + 120x^3 + \ldots$

b $= 1 + 6(-x) + \tfrac{6\times 5}{2}(-x)^2 + \tfrac{6\times 5\times 4}{3\times 2}(-x)^3 + \ldots$
$= 1 - 6x + 15x^2 - 20x^3 + \ldots$

c $= 1 + 8(2x) + \tfrac{8\times 7}{2}(2x)^2 + \tfrac{8\times 7\times 6}{3\times 2}(2x)^3 + \ldots$
$= 1 + 16x + 112x^2 + 448x^3 + \ldots$

d $= 1 + 7(-\tfrac{1}{2}x) + \tfrac{7\times 6}{2}(-\tfrac{1}{2}x)^2 + \tfrac{7\times 6\times 5}{3\times 2}(-\tfrac{1}{2}x)^3 + \ldots$
$= 1 - \tfrac{7}{2}x + \tfrac{21}{4}x^2 - \tfrac{35}{8}x^3 + \ldots$

e $= 1 + 6(x^3) + \tfrac{6\times 5}{2}(x^3)^2 + \tfrac{6\times 5\times 4}{3\times 2}(x^3)^3 + \ldots$
$= 1 + 6x^3 + 15x^6 + 20x^9 + \ldots$

f $= 2^9 + 9(2^8)x + \tfrac{9\times 8}{2}(2^7)x^2 + \tfrac{9\times 8\times 7}{3\times 2}(2^6)x^3 + \ldots$
$= 512 + 2304x + 4608x^2 + 5376x^3 + \ldots$

g $= 3^7 + 7(3^6)(-x) + \tfrac{7\times 6}{2}(3^5)(-x)^2 + \tfrac{7\times 6\times 5}{3\times 2}(3^4)(-x)^3 + \ldots$
$= 2187 - 5103x + 5103x^2 - 2835x^3 + \ldots$

h $= 2^{10} + 10(2^9)(5x) + \tfrac{10\times 9}{2}(2^8)(5x)^2 + \tfrac{10\times 9\times 8}{3\times 2}(2^7)(5x)^3 + \ldots$
$= 1024 + 25\,600x + 288\,000x^2 + 1\,920\,000x^3 + \ldots$

4 **a** $= \binom{20}{3} = 1140$

b $= \binom{14}{4} \times (-1)^4 = 1001$

c $= \binom{9}{2} \times 4^2 = 576$

d $= \binom{14}{3} \times (-3)^3 = -9828$

e $= \binom{12}{4} \times (-\tfrac{1}{3})^4 = \tfrac{55}{9}$ or $6\tfrac{1}{9}$

f $= \binom{16}{5} \times (-\tfrac{1}{2})^5 = -136.5$

g $= \binom{15}{2} \times (\tfrac{2}{5})^2 = \tfrac{84}{5}$ or 16.8

h $= \binom{8}{3} = 56$

5 **a** $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3$
$= 1 + 3\sqrt{5} + 15 + 5\sqrt{5}$
$= 16 + 8\sqrt{5}$

b $= 1 + 4(-\sqrt{3}) + 6(-\sqrt{3})^2 + 4(-\sqrt{3})^3 + (-\sqrt{3})^4$
$= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$
$= 28 - 16\sqrt{3}$

c $= 2^3 + 3(2^2)(\sqrt{2}) + 3(2)(\sqrt{2})^2 + (\sqrt{2})^3$
$= 8 + 12\sqrt{2} + 12 + 2\sqrt{2}$
$= 20 + 14\sqrt{2}$

d $= 1 + 4(2\sqrt{3}) + 6(2\sqrt{3})^2 + 4(2\sqrt{3})^3 + (2\sqrt{3})^4$
$= 1 + 8\sqrt{3} + 72 + 96\sqrt{3} + 144$
$= 217 + 104\sqrt{3}$

6 **a** $= 1 + 6x + \frac{6\times5}{2}x^2 + \frac{6\times5\times4}{3\times2}x^3 + \ldots$
$= 1 + 6x + 15x^2 + 20x^3 + \ldots$

b **i** let $x = 0.02$
$1.02^6 \approx 1 + 6(0.02) + 15(0.02)^2 + 20(0.02)^3$
$= 1 + 0.12 + 0.0060 + 0.000\,160$
$= 1.1262$ (4dp)

ii let $x = -0.01$
$0.99^6 \approx 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3$
$= 1 - 0.06 + 0.0015 - 0.000\,020$
$= 0.9415$ (4dp)

7 **a** $= 1 + 8(2y) + \frac{8\times7}{2}(2y)^2 + \frac{8\times7\times6}{3\times2}(2y)^3 + \ldots$
$= 1 + 16y + 112y^2 + 448y^3 + \ldots$

b **i** let $y = -0.01$
$0.98^8 \approx 1 + 16(-0.01) + 112(-0.01)^2 + 448(-0.01)^3$
$= 1 - 0.16 + 0.0112 - 0.000\,448$
$= 0.8508$ (4dp)

ii let $y = 0.005$
$1.01^8 \approx 1 + 16(0.005) + 112(0.005)^2 + 448(0.005)^3$
$= 1 + 0.080 + 0.002\,800 + 0.000\,056\,000$
$= 1.0829$ (4dp)

8 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4 + (1 - 4x + 6x^2 - 4x^3 + x^4)$
$= 2 + 12x^2 + 2x^4$

b $= 1 + 3(-\frac{1}{3}x) + 3(-\frac{1}{3}x)^2 + (-\frac{1}{3}x)^3 - [1 + 3(\frac{1}{3}x) + 3(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3]$
$= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 - (1 + x + \frac{1}{3}x^2 + \frac{1}{27}x^3)$
$= -2x - \frac{2}{27}x^3$

9 **a** $6(ax)^2 = 24x^2$
$a^2 = 4$
$a < 0 \;\therefore\; a = -2$

b $= 4a^3 = -32$

C2 SEQUENCES AND SERIES — Answers - Worksheet D

1 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$
$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$
$= 27 + 270x^2 + 900x^4 + 1000x^6$

d $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$
$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$

e $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$
$= x^6 - 3x^4y + 3x^2y^2 - y^3$

f $= 5^4 + 4(5^3)(\tfrac{1}{2}x) + 6(5^2)(\tfrac{1}{2}x)^2 + 4(5)(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 625 + 250x + \tfrac{75}{2}x^2 + \tfrac{5}{2}x^3 + \tfrac{1}{16}x^4$

g $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$
$= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$

h $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$
$= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$

2 **a** $= 1 + 6(3x) + \tfrac{6\times5}{2}(3x)^2 + \tfrac{6\times5\times4}{3\times2}(3x)^3 + \ldots$
$= 1 + 18x + 135x^2 + 540x^3 + \ldots$

b $= 1 + 8(-\tfrac{1}{4}x) + \tfrac{8\times7}{2}(-\tfrac{1}{4}x)^2 + \tfrac{8\times7\times6}{3\times2}(-\tfrac{1}{4}x)^3 + \ldots$
$= 1 - 2x + \tfrac{7}{4}x^2 - \tfrac{7}{8}x^3 + \ldots$

c $= 5^7 + 7(5^6)(-x) + \tfrac{7\times6}{2}(5^5)(-x)^2 + \tfrac{7\times6\times5}{3\times2}(5^4)(-x)^3 + \ldots$
$= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \ldots$

d $= 3^{10} + 10(3^9)(2x^2) + \tfrac{10\times9}{2}(3^8)(2x^2)^2 + \tfrac{10\times9\times8}{3\times2}(3^7)(2x^2)^3 + \ldots$
$= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \ldots$

3 **a** $= \binom{15}{3} = 455$

b $= \binom{12}{4} \times (-2)^4 = 7920$

c $= \binom{7}{2} \times 3^5 = 5103$

d $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$

e $= \binom{8}{5} \times 2^3 = 448$

f $= \binom{9}{3} \times (-1)^3 = -84$

4 **a** $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$
$= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$
$= 89 - 28\sqrt{10}$

b $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$
$= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \tfrac{1}{9}\sqrt{3}$
$= 3\sqrt{2} + \tfrac{19}{9}\sqrt{3}$

c $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3]$
$= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}]$
$= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}]$
$= 16\sqrt{5}$

5 **a** $= 1 + 10(\frac{x}{2}) + \frac{10\times9}{2}(\frac{x}{2})^2 + \frac{10\times9\times8}{3\times2}(\frac{x}{2})^3 + \ldots$
$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

b **i** let $x = 0.01$
$1.005^{10} \approx 1 + 0.05 + 0.001\,125 + 0.000\,015$
$= 1.051\,14$ (5dp)

ii let $x = -0.008$
$0.996^{10} \approx 1 - 0.040 + 0.000\,720 - 0.000\,007\,680$
$= 0.960\,71$ (5dp)

6 **a** $= 3^8 + 8(3^7)x + \frac{8\times7}{2}(3^6)x^2 + \frac{8\times7\times6}{3\times2}(3^5)x^3 + \ldots$
$= 6561 + 17\,496x + 20\,412x^2 + 13\,608x^3 + \ldots$

b **i** let $x = 0.001$
$3.001^8 \approx 6561 + 17.496 + 0.020\,412 + 0.000\,013\,608$
$= 6578.516$ (7sf)

ii let $x = -0.005$
$2.995^8 \approx 6561 - 87.480 + 0.510\,300 - 0.001\,701\,000$
$= 6474.029$ (7sf)

7 **a** $(1 + 10x)^4 = 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4$
$\therefore (1 + 10x)^4 + (1 - 10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10\,000x^4)$
$= 2 + 1200x^2 + 20\,000x^4$

b $(2 + \frac{1}{3}x)^3 = 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3$
$= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3$
$\therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$
$= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3)$
$= -8x - \frac{2}{27}x^3$

c $= (1 + 4y)(1 + 3y + 3y^2 + y^3)$
$= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4$
$= 1 + 7y + 15y^2 + 13y^3 + 4y^4$

d $= (1 - x)(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3})$
$= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2}$
$= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}$

8 **a** $= (1 + x^2)[1 + 10(-3x) + \frac{10\times9}{2}(-3x)^2 + \frac{10\times9\times8}{3\times2}(-3x)^3 + \ldots]$
$= (1 + x^2)[1 - 30x + 405x^2 - 3240x^3 + \ldots]$
$= 1 - 30x + 405x^2 - 3240x^3 + x^2 - 30x^3 + \ldots$
$= 1 - 30x + 406x^2 - 3270x^3 + \ldots$

b $= (1 - 2x)[1 + 8x + \frac{8\times7}{2}x^2 + \frac{8\times7\times6}{3\times2}x^3 + \ldots]$
$= (1 - 2x)[1 + 8x + 28x^2 + 56x^3 + \ldots]$
$= 1 + 8x + 28x^2 + 56x^3 - 2x - 16x^2 - 56x^3 + \ldots$
$= 1 + 6x + 12x^2 + \ldots$

c $= (1 + x + x^2)[1 + 6(-x) + \frac{6\times5}{2}(-x)^2 + \frac{6\times5\times4}{3\times2}(-x)^3 + \ldots]$
$= (1 + x + x^2)[1 - 6x + 15x^2 - 20x^3 + \ldots]$
$= 1 - 6x + 15x^2 - 20x^3 + x - 6x^2 + 15x^3 + x^2 - 6x^3 + \ldots$
$= 1 - 5x + 10x^2 - 11x^3 + \ldots$

d $= (1 + 3x - x^2)[1 + 7(2x) + \frac{7\times6}{2}(2x)^2 + \frac{7\times6\times5}{3\times2}(2x)^3 + \ldots]$
$= (1 + 3x - x^2)[1 + 14x + 84x^2 + 280x^3 + \ldots]$
$= 1 + 14x + 84x^2 + 280x^3 + 3x + 42x^2 + 252x^3 - x^2 - 14x^3 + \ldots$
$= 1 + 17x + 125x^2 + 518x^3 + \ldots$

9 **a** $= \binom{8}{4} \times y^4 \times (\frac{1}{y})^4 = 70$

b $= \binom{12}{6} \times (2y)^6 \times (-\frac{1}{2y})^6 = 924$

c $= \binom{6}{2} \times (\frac{1}{y})^4 \times (y^2)^2 = 15$

d $= \binom{9}{3} \times (3y)^6 \times (-\frac{1}{y^2})^3 = -61\,236$

10 **a** $\frac{n(n-1)}{2} \times (\frac{2}{5})^2 = 1.6$
$n(n - 1) = \frac{25}{2} \times 1.6 = 20$
$n^2 - n - 20 = 0$
$(n + 4)(n - 5) = 0$
$n > 0 \;\therefore\; n = 5$

b $= 5 \times (\frac{2}{5})^4 = \frac{16}{125}$ or 0.128

11 **a** $y_1 = (1 - 2x)[1 + 10x + \frac{10\times9}{2}x^2 + \ldots]$
$= 1 + 10x + 45x^2 - 2x - 20x^2 + \ldots$
$= 1 + 8x + 25x^2 + \ldots$
$\therefore\; a = 25,\ b = 8,\ c = 1$

b $x = 0.2$: $y_1 = 0.6 \times (1.2)^{10} = 3.71504$
$y_2 = (25\times0.04) + (8\times0.2) + 1 = 3.6$
$\%\text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\%$ (2sf)

12 **a** $(1 + px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \ldots$
$\therefore\; pq = -12$ and $\frac{1}{2}p^2q(q - 1) = 60$
sub. $p = -\frac{12}{q}$
$\Rightarrow\; \frac{72}{q}(q - 1) = 60$
$72(q - 1) = 60q$
$q = 6,\ p = -2$

b $= \frac{6\times5\times4}{3\times2} \times (-2)^3 = -160$

13 **a** $= 3^{12} + 12(3^{11})(-\frac{x}{3}) + \frac{12\times11}{2}(3^{10})(-\frac{x}{3})^2 + \frac{12\times11\times10}{3\times2}(3^9)(-\frac{x}{3})^3 + \dots$

$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \dots$

b let $\frac{x}{3} = 0.002 \;\therefore\; x = 0.006$

$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$

$= 527\,205.03$ (2dp)

14 **a** $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

b $= 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$

c **i** $= [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$

$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$

$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$

$= 76 - 44\sqrt{3}$

ii $= (\sqrt{3} + 1)(76 - 44\sqrt{3})$

$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$

$= -56 + 32\sqrt{3}$

15 **a** $= 1 + 9(\frac{x}{2}) + \frac{9\times8}{2}(\frac{x}{2})^2 + \frac{9\times8\times7}{3\times2}(\frac{x}{2})^3 + \frac{9\times8\times7\times6}{4\times3\times2}(\frac{x}{2})^4 + \dots$

$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \dots$

b $= \frac{21}{2} - (-\frac{21}{2}) = 21$

c $= (1 \times \frac{63}{8}) + (2 \times \frac{21}{2}) = 28\frac{7}{8}$

16 $10(x^3)^2(\frac{a}{x^2})^3 = -80$

$a^3 = -8$

$a = -2$

17 **a** $(1 + \frac{x}{k})^n = 1 + n(\frac{x}{k}) + \frac{n(n-1)}{2}(\frac{x}{k})^2 + \frac{n(n-1)(n-2)}{3\times2}(\frac{x}{k})^3 + \dots$

$\therefore\; \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$

$kn(n-1) = n(n-1)(n-2)$

$n(n-1)[k - (n-2)] = 0$

$n > 1 \;\therefore\; k - (n-2) = 0$

$k = n - 2$

b $k = 7 - 2 = 5$

$(1 + \frac{x}{5})^7 = 1 + 7(\frac{x}{5}) + \frac{7\times6}{2}(\frac{x}{5})^2 + \frac{7\times6\times5}{3\times2}(\frac{x}{5})^3 + \frac{7\times6\times5\times4}{4\times3\times2}(\frac{x}{5})^4 + \dots$

$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \dots$

C2 SEQUENCES AND SERIES — Answers - Worksheet E

1 $= 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$

$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$

2 **a** $u_5 = 3 \times (-2)^4 = 48$

b $S_{10} = \frac{3[1-(-2)^{10}]}{1-(-2)} = -1023$

c positive terms form GP:
$a = 3$, $r = (-2)^2 = 4$

$S_8 = \frac{3(4^8-1)}{4-1} = 65\,535$

3 **a** $= 1 + 7(3x) + \frac{7\times6}{2}(3x)^2$

$+ \frac{7\times6\times5}{3\times2}(3x)^3 + \frac{7\times6\times5\times4}{4\times3\times2}(3x)^4 + \ldots$

$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \ldots$

b let $x = 0.01$

$1.03^7 \approx 1 + 0.21 + 0.0189$
$+ 0.000\,945 + 0.000\,028\,35$
$= 1.229\,87$ (5dp)

4 GP: $a = 8$, $r = 2$, $n = 10$

$S_{10} = \frac{8(2^{10}-1)}{2-1} = 8184$

5 **a** $= 2^5 + 5(2^4)x + 10(2^3)x^2$
$+ 10(2^2)x^3 + 5(2)x^4 + x^5$
$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$

b $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $(2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2$
$+ 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5$

$= 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5}$

$= 682 + 305\sqrt{5}$

$\therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5$

$= (682 + 305\sqrt{5}) - (682 - 305\sqrt{5})$

$= 610\sqrt{5}$, $k = 610$

6 **a** amount in account after 3rd payment in
$= 200 + (1.005 \times 200) + (1.005^2 \times 200)$
$= 603.005$

interest paid at end of 3rd month
$= 0.005\times603.005 = £3.02$ (nearest penny)

b amount paid in $= 12 \times 200 = £2400$

amount in account after 12 months

$= 200(1.005 + 1.005^2 + \ldots + 1.005^{12})$

$= 200 \times S_{12}$ [GP: $a = 1.005$, $r = 1.005$]

$= 200 \times \frac{1.005(1.005^{12}-1)}{1.005-1} = 2479.45$

total interest $= 2479.45 - 2400 = £79.45$

7 $= 1 + 8(-3x) + \frac{8\times7}{2}(-3x)^2$
$+ \frac{8\times7\times6}{3\times2}(-3x)^3 + \ldots$
$= 1 - 24x + 252x^2 - 1512x^3 + \ldots$

8 **a** $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

subtracting, $S_n - rS_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $r = 6 \div 3 = 2$

$a \times 2^3 = 3 \quad \therefore a = \frac{3}{8}$

$S_{16} = \frac{\frac{3}{8}(2^{16}-1)}{2-1} = 24\,575\frac{5}{8}$

9 **a** $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$

$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$

b $\frac{1}{2}a^2n(n-1) = 3an$

$a^2n(n-1) = 6an$

$an[a(n-1) - 6] = 0$

$n \neq 0 \ \therefore \ a(n-1) - 6 = 0$

$an - a = 6$

$n = \frac{6+a}{a}$

c $n = 10 \ \therefore$ coeff. of $x^3 = \frac{10\times9\times8}{3\times2} \times (\frac{2}{3})^3 = 35\frac{5}{9}$

10 $= 2^6 + 6(2^5)(5x) + \frac{6\times5}{2}(2^4)(5x)^2 + \ldots$

$= 64 + 960x + 6000x^2 + \ldots$

11 **a** $\frac{162}{1-r} = 486$

$1 - r = \frac{162}{486} = \frac{1}{3} \quad \therefore r = \frac{2}{3}$

b $u_6 = 162 \times (\frac{2}{3})^5 = \frac{64}{3}$ or $21\frac{1}{3}$

c $S_{10} = \frac{162[1-(\frac{2}{3})^{10}]}{1-\frac{2}{3}} = 477.572$

12 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$

coefficient of $x^2 = 54 + 48 - 1 = 101$

13 **a** time $= 120 \times (0.9)^3 = 87.48$ seconds

b GP: $a = 120$, $r = 0.9$, $n = 12$

$S_{12} = \frac{120[1-(0.9)^{12}]}{1-0.9}$

$= 861.08$ seconds

$= 14$ mins 21 secs (nearest sec.)

14 $= [1+8(\frac{x}{2})+\frac{8\times7}{2}(\frac{x}{2})^2+\ldots][1+6(-x)+\frac{6\times5}{2}(-x)^2+\ldots]$

$= [1 + 4x + 7x^2 + \ldots][1 - 6x + 15x^2 + \ldots]$

$= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \ldots$

$= 1 - 2x - 2x^2 + \ldots$

$\therefore \ A = -2, B = -2$

15 **a** 6, 12, 24, 48

b GP: $a = 6$, $r = 2$, $n = 10$

$S_{10} = \frac{6(2^{10}-1)}{2-1} = 6138$

16 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 4x + 6x^2 - 4x^3 + x^4$

c $(1 + 4x + 6x^2 + 4x^3 + x^4)$

$+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$

$2 + 12x^2 + 2x^4 = 82$

$x^4 + 6x^2 - 40 = 0$

$(x^2 + 10)(x^2 - 4) = 0$

$x^2 = -10$ [no real solutions] or $x^2 = 4$

$x = \pm 2$

17 **a** $a \times (1.5)^2 = 18$

$a = 18 \div 2.25 = 8$

b $S_6 = \frac{8[(1.5)^6 - 1]}{1.5-1} = 166.25$

c $8 \times (1.5)^{k-1} > 8000$

$(k-1)\lg 1.5 > \lg 1000$

$k > \frac{\lg 1000}{\lg 1.5} + 1$

$k > 18.04 \ \therefore$ smallest $k = 19$

18 $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$

$= 1 + 10(\frac{ax}{2}) + \frac{10\times9}{2}(\frac{ax}{2})^2 + \ldots$

$+ 1 + 10(bx) + \frac{10\times9}{2}(bx)^2 + \ldots$

$= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \ldots$

$\therefore \quad 5a + 10b = 0 \quad \Rightarrow \quad a = -2b$

and $\frac{45}{4}a^2 + 45b^2 = 90 \quad \Rightarrow \quad a^2 + 4b^2 = 8$

sub. $(-2b)^2 + 4b^2 = 8$

$b^2 = 1$

$a < b \ \therefore \ b = 1, a = -2$

C2 SEQUENCES AND SERIES — *Answers - Worksheet F*

1 **a** $a = 108,\ ar^3 = 32$

$\therefore\ r^3 = 32 \div 108 = \frac{8}{27}$

$r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

$u_3 = 108 \times (\frac{2}{3})^2 = 48$

b $S_\infty = \dfrac{108}{1-\frac{2}{3}} = 324$

2 $= 1 + 5(-2x) + 10(-2x)^2$

$+ 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

3 **a** new subscribers in 4th week

$= 200 \times (1.15)^3 = 304.175$

$= 304$ (nearest unit)

b new subscribers: GP, $a = 200$, $r = 1.15$

$S_{10} = \dfrac{200[(1.15)^{10} - 1]}{1.15 - 1} = 4060.74$

total no. of subscribers $= 3600 + S_{10}$

$= 7661$ (nearest unit)

4 **a** $= 1 + 7(4x) + \frac{7\times6}{2}(4x)^2 + \ldots$

$= 1 + 28x + 336x^2 + \ldots$

b $(1 + 2x)^2(1 + 4x)^7$

$= (1 + 4x + 4x^2)(1 + 28x + 336x^2 + \ldots)$

term in x^2

$= (1)(336x^2) + (4x)(28x) + (4x^2)(1)$

coefficient of $x^2 = 336 + 112 + 4 = 452$

5 **a** $= 1 + 2n(\frac{x}{k}) + \frac{2n(2n-1)}{2}(\frac{x}{k})^2$

$+ \frac{2n(2n-1)(2n-2)}{3\times2}(\frac{x}{k})^3 + \ldots$

$= 1 + \dfrac{2n}{k}x + \dfrac{n(2n-1)}{k^2}x^2 + \dfrac{2n(n-1)(2n-1)}{3k^3}x^3 + \ldots$

b $\dfrac{2n(n-1)(2n-1)}{3k^3} = \dfrac{1}{2} \times \dfrac{n(2n-1)}{k^2}$

$4n(n - 1)(2n - 1) = 3kn(2n - 1)$

$n(2n - 1)[4(n - 1) - 3k] = 0$

$n > 1\ \therefore\ 4(n - 1) - 3k = 0$

$3k = 4(n - 1)$

c $\dfrac{2n}{k} = 2\ \therefore\ n = k$

$\therefore\ 3k = 4k - 4$

$k = 4,\ n = 4$

6 **a** $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$

$a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$

b $S_8 = \dfrac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3} - 1}$

$= \dfrac{80\sqrt{2}}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \dfrac{80\sqrt{2}(\sqrt{3} + 1)}{3 - 1}$

$= 40\sqrt{2}(\sqrt{3} + 1)$

7 $\sum_{r=1}^{9} 3^r$: GP, $a = 3$, $r = 3$

$S_9 = \dfrac{3(3^9 - 1)}{3 - 1} = 29523$

$\therefore\ \sum_{r=1}^{9} (3^r - 1) = 29523 - 9$

$= 29\,514$

8 **a** $= 1 + 9(2x) + \frac{9\times8}{2}(2x)^2 + \frac{9\times8\times7}{3\times2}(2x)^3 + \ldots$

$= 1 + 18x + 144x^2 + 672x^3 + \ldots$

b $(1 - 2x)^9 = 1 - 18x + 144x^2 - 672x^3 + \ldots$

$\therefore\ (1 + 2x)^9 + (1 - 2x)^9$

$= (1 + 18x + 144x^2 + 672x^3 + \ldots)$

$+ (1 - 18x + 144x^2 - 672x^3 + \ldots)$

$= 2 + 288x^2$ (ignoring terms in x^4 and higher)

c let $x = 0.001$

$\therefore\ 1.002^9 + 0.998^9 \approx 2 + 0.000\,288$

$= 2.000\,288$ (7sf)

9
$(k - x)^9 = k^9 + 9(k^8)(-x) + \frac{9\times 8}{2}(k^7)(-x)^2 + \ldots$
$= k^9 - 9k^8x + 36k^7x^2 + \ldots$
$\therefore\ -b = -9k^8$ and $b = 36k^7$
$9k^8 = 36k^7$
$9k^7(k - 4) = 0$
$k \neq 0\ \therefore\ k = 4$
$a = k^9 = 262\,144$
$b = 9k^8 = 589\,824$

10
$= 3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2$
$+ 4(3)(2x)^3 + (2x)^4$
$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$

11 **a** $\frac{t}{1-r} = 3t$
$1 - r = \frac{t}{3t} = \frac{1}{3}\ \therefore\ r = \frac{2}{3}$

b $\frac{t[1-(\frac{2}{3})^4]}{1-\frac{2}{3}} = 130$
$t = (\frac{1}{3} \times 80) \div \frac{65}{81} = 54$

12 **a** $= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + (-2x)^4$
$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$

b let $x = y^2 - 2y$
$(1 + 4y - 2y^2)^4$
$= 1 - 8(y^2 - 2y) + 24(y^2 - 2y)^2 + \ldots$
term in $y^2 = -8y^2 + 24(-2y)^2$
coefficient of $y^2 = -8 + 96 = 88$

13 **a** $= 12000 \times (0.75)^4$
$= 3796.875$
$= £3800$ (3sf)

b GP: $a = 12000$, $r = 0.75$
$S_8 = \frac{12000[1-(0.75)^8]}{1-0.75}$
$= £43\,200$ (3sf)

14 **a** $p(-2) = 1^4 - (-1)^4 = 1 - 1 = 0$
$\therefore\ (x + 2)$ is a factor of $p(x)$

b $p(x) = [x^4 + 4(x^3)(3) + 6(x^2)(3^2) + 4(x)(3^3) + 3^4]$
$- [x^4 + 4x^3 + 6x^2 + 4x + 1]$
$= 8x^3 + 48x^2 + 104x + 80$
$= 8(x^3 + 6x^2 + 13x + 10)$

$$
\begin{array}{r|l}
 & x^2 + 4x + 5 \\
\hline
x + 2 & x^3 + 6x^2 + 13x + 10 \\
 & x^3 + 2x^2 \\
 & \quad 4x^2 + 13x \\
 & \quad 4x^2 + 8x \\
 & \qquad 5x + 10 \\
 & \qquad 5x + 10
\end{array}
$$

$p(x) = 8(x + 2)(x^2 + 4x + 5)$

c $8(x + 2)(x^2 + 4x + 5) = 0$
$x = -2$ or $(x^2 + 4x + 5) = 0$
$b^2 - 4ac = 16 - 20 = -4$
$b^2 - 4ac < 0\ \therefore$ no real sols to $(x^2 + 4x + 5) = 0$
$\therefore$ only one real solution to $p(x) = 0$

15 **a** $(1-x)(1+2x)^n$

$= (1-x)[1 + n(2x) + \frac{n(n-1)}{2}(2x)^2 + \ldots]$

$= (1-x)[1 + 2nx + 2n(n-1)x^2 + \ldots]$

$\therefore\ 2n(n-1) - 2n = 198$

$n^2 - 2n - 99 = 0$

$(n+9)(n-11) = 0$

$n \geq 0\ \therefore\ n = 11$

b $(1-x)(1+2x)^{11}$

$= (1-x)[\ldots + \frac{11\times10}{2}(2x)^2 + \frac{11\times10\times9}{3\times2}(2x)^3 + \ldots]$

$= (1-x)[\ldots + 220x^2 + 1320x^3 + \ldots]$

$\therefore$ coefficient of $x^3 = 1320 - 220 = 1100$

16 $= (\frac{3}{x})^4 + 4(\frac{3}{x})^3(-x) + 6(\frac{3}{x})^2(-x)^2$

$+ 4(\frac{3}{x})(-x)^3 + (-x)^4$

$= x^4 - 12x^2 + 54 - \frac{108}{x^2} + \frac{81}{x^4}$

17 **a** $S_4 = 3^4 - 1 = 80$

$S_3 = 3^3 - 1 = 26$

$u_4 = S_4 - S_3 = 80 - 26 = 54$

b $S_{n-1} = 3^{n-1} - 1$

$u_n = S_n - S_{n-1}$

$= (3^n - 1) - (3^{n-1} - 1)$

$= 3^n - 3^{n-1}$

$= 3^n(1 - \frac{1}{3}) = \frac{2}{3}(3^n)$ $[k = \frac{2}{3}]$

c $u_{n-1} = \frac{2}{3}(3^{n-1})$

$u_n \div u_{n-1} = \frac{2}{3}(3^n) \div \frac{2}{3}(3^{n-1}) = 3$

$u_n \div u_{n-1}$ is constant $\therefore$ geometric

18 **a** $3(x-3) = y - 3$

$y = 3x - 6$

b $\left(\frac{x}{3}\right)^3 = \frac{y}{3}$

$x^3 = 9y = 9(3x-6)$

$x^3 - 27x + 54 = 0$

c trying $x = 1, 2$ etc. $\Rightarrow$ $x = 3$ is a solution

$\therefore$ $(x-3)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 3x - 18 \\
x-3 & x^3 + 0x^2 - 27x + 54 \\
 & x^3 - 3x^2 \\
 & 3x^2 - 27x \\
 & 3x^2 - 9x \\
 & -18x + 54 \\
 & -18x + 54
\end{array}
$$

$(x-3)(x^2 + 3x - 18) = 0$

$(x-3)(x+6)(x-3) = 0$

$x = -6$ or 3