

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise A, Question 1

Question:

Simplify these fractions:

(a) $\frac{4x^4 + 5x^2 - 7x}{x}$

(b) $\frac{7x^8 - 5x^5 + 9x^3 + x^2}{x}$

(c) $\frac{-2x^3 + x}{x}$

(d) $\frac{-x^4 + 4x^2 + 6}{x}$

(e) $\frac{7x^5 - x^3 - 4}{x}$

(f) $\frac{8x^4 - 4x^3 + 6x}{2x}$

(g) $\frac{9x^2 - 12x^3 - 3x}{3x}$

(h) $\frac{8x^5 - 2x^3}{4x}$

(i) $\frac{7x^3 - x^4 - 2}{5x}$

(j) $\frac{-4x^2 + 6x^4 - 2x}{-2x}$

(k) $\frac{-x^8 + 9x^4 + 6}{-2x}$

(l) $\frac{-9x^9 - 6x^4 - 2}{-3x}$

Solution:

$$(a) \frac{4x^4 + 5x^2 - 7x}{x} = \frac{4x^4}{x} + \frac{5x^2}{x} - \frac{7x}{x} = 4x^3 + 5x - 7$$

$$(b) \frac{7x^8 - 5x^5 + 9x^3 + x^2}{x} = \frac{7x^8}{x} - \frac{5x^5}{x} + \frac{9x^3}{x} + \frac{x^2}{x} = 7x^7 - 5x^4 + 9x^2 + x$$

$$(c) \frac{-2x^3 + x}{x} = \frac{-2x^3}{x} + \frac{x}{x} = -2x^2 + 1$$

$$(d) \frac{-x^4 + 4x^2 + 6}{x} = \frac{-x^4}{x} + \frac{4x^2}{x} + \frac{6}{x} = -x^3 + 4x + \frac{6}{x}$$

$$(e) \frac{7x^5 - x^3 - 4}{x} = \frac{7x^5}{x} - \frac{x^3}{x} - \frac{4}{x} = 7x^4 - x^2 - \frac{4}{x}$$

$$(f) \frac{8x^4 - 4x^3 + 6x}{2x} = \frac{8x^4}{2x} - \frac{4x^3}{2x} + \frac{6x}{2x} = 4x^3 - 2x^2 + 3$$

$$(g) \frac{9x^2 - 12x^3 - 3x}{3x} = \frac{9x^2}{3x} - \frac{12x^3}{3x} - \frac{3x}{3x} = 3x - 4x^2 - 1$$

$$(h) \frac{8x^5 - 2x^3}{4x} = \frac{8x^5}{4x} - \frac{2x^3}{4x} = 2x^4 - \frac{x^2}{2}$$

$$(i) \frac{7x^3 - x^4 - 2}{5x} = \frac{7x^3}{5x} - \frac{x^4}{5x} - \frac{2}{5x} = \frac{7x^2}{5} - \frac{x^3}{5} - \frac{2}{5x}$$

$$(j) \frac{-4x^2 + 6x^4 - 2x}{-2x} = \frac{-4x^2}{-2x} + \frac{6x^4}{-2x} - \frac{2x}{-2x}$$

$$= \frac{2x^2}{x} - \frac{3x^4}{x} + 1$$

$$= 2x - 3x^3 + 1$$

$$(k) \frac{-x^8 + 9x^4 + 6}{-2x} = \frac{-x^8}{-2x} + \frac{9x^4}{-2x} + \frac{6}{-2x}$$

$$= \frac{x^8}{2x} - \frac{9x^4}{2x} - \frac{3}{x}$$

$$= \frac{x^7}{2} - \frac{9x^3}{2} - \frac{3}{x}$$

$$(l) \frac{-9x^9 - 6x^4 - 2}{-3x} = \frac{-9x^9}{-3x} - \frac{6x^4}{-3x} - \frac{2}{-3x}$$

$$= \frac{3x^9}{x} + \frac{2x^4}{x} + \frac{2}{3x}$$

$$= 3x^8 + 2x^3 + \frac{2}{3x}$$

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Exercise A, Question 2

Question:

Simplify these fractions as far as possible:

$$(a) \frac{(x+3)(x-2)}{(x-2)}$$

$$(b) \frac{(x+4)(3x-1)}{(3x-1)}$$

$$(c) \frac{(x+3)^2}{(x+3)}$$

$$(d) \frac{x^2 + 10x + 21}{(x+3)}$$

$$(e) \frac{x^2 + 9x + 20}{(x+4)}$$

$$(f) \frac{x^2 + x - 12}{(x-3)}$$

$$(g) \frac{x^2 + x - 20}{x^2 + 2x - 15}$$

$$(h) \frac{x^2 + 3x + 2}{x^2 + 5x + 4}$$

$$(i) \frac{x^2 + x - 12}{x^2 - 9x + 18}$$

$$(j) \frac{2x^2 + 7x + 6}{(x-5)(x+2)}$$

$$(k) \frac{2x^2 + 9x - 18}{(x+6)(x+1)}$$

$$(l) \frac{3x^2 - 7x + 2}{(3x-1)(x+2)}$$

$$(m) \frac{2x^2 + 3x + 1}{x^2 - x - 2}$$

$$(n) \frac{x^2 + 6x + 8}{3x^2 + 7x + 2}$$

$$(o) \frac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$$

Solution:

$$(a) \frac{(x+3)(x-2)}{(x-2)}$$

$$= \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= x + 3$$

$$(b) \frac{(x+4)(3x-1)}{(3x-1)}$$

$$= \frac{(x+4)\cancel{(3x-1)}}{\cancel{(3x-1)}}$$

$$= x + 4$$

$$(c) \frac{(x+3)^2}{(x+3)}$$

$$= \frac{(x+3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$= x + 3$$

$$(d) \frac{x^2 + 10x + 21}{x+3}$$

$$= \frac{(x+7)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$= x + 7$$

$$(e) \frac{x^2 + 9x + 20}{x+4}$$

$$= \frac{(x+4)\cancel{(x+5)}}{\cancel{(x+4)}}$$

$$= x + 5$$

$$(f) \frac{x^2 + x - 12}{x-3}$$

$$= \frac{(x-3)(x+4)}{(x-3)}$$

$$= x + 4$$

$$(g) \frac{x^2 + x - 20}{x^2 + 2x - 15}$$

$$= \frac{(x+5)(x-4)}{(x+5)(x-3)}$$

$$= \frac{x-4}{x-3}$$

$$(h) \frac{x^2 + 3x + 2}{x^2 + 5x + 4}$$

$$= \frac{(x+2)(x+1)}{(x+4)(x+1)}$$

$$= \frac{x+2}{x+4}$$

$$(i) \frac{x^2 + x - 12}{x^2 - 9x + 18}$$

$$= \frac{(x+4)(x-3)}{(x-6)(x-3)}$$

$$= \frac{x+4}{x-6}$$

$$(j) \frac{2x^2 + 7x + 6}{(x-5)(x+2)}$$

$$= \frac{(2x+3)(x+2)}{(x-5)(x+2)}$$

$$= \frac{2x+3}{x-5}$$

$$(k) \frac{2x^2 + 9x - 18}{(x+6)(x+1)}$$

$$= \frac{(2x-3)(x+6)}{(x+6)(x+1)}$$

$$= \frac{2x-3}{x+1}$$

$$(l) \frac{3x^2 - 7x + 2}{(3x - 1)(x + 2)}$$

$$= \frac{(3x-1)(x-2)}{(3x-1)(x+2)}$$

$$= \frac{x-2}{x+2}$$

$$(m) \frac{2x^2 + 3x + 1}{x^2 - x - 2}$$

$$= \frac{(2x+1)(x+1)}{(x-2)(x+1)}$$

$$= \frac{2x+1}{x-2}$$

$$(n) \frac{x^2 + 6x + 8}{3x^2 + 7x + 2}$$

$$= \frac{(x+4)(x+2)}{(3x+1)(x+2)}$$

$$= \frac{x+4}{3x+1}$$

$$(o) \frac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$$

$$= \frac{(2x+1)(x-3)}{(2x-3)(x-3)}$$

$$= \frac{2x+1}{2x-3}$$

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Exercise B, Question 1

Question:

Divide:

(a) $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

(b) $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

(c) $x^3 + 7x^2 - 3x - 54$ by $(x + 6)$

(d) $x^3 + 9x^2 + 18x - 10$ by $(x + 5)$

(e) $x^3 - x^2 + x + 14$ by $(x + 2)$

(f) $x^3 + x^2 - 7x - 15$ by $(x - 3)$

(g) $x^3 - 5x^2 + 8x - 4$ by $(x - 2)$

(h) $x^3 - 3x^2 + 8x - 6$ by $(x - 1)$

(i) $x^3 - 8x^2 + 13x + 10$ by $(x - 5)$

(j) $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

Solution:

$$\begin{array}{r}
 \overline{x^2 + 5x + 3} \\
 x + 1 \overline{) x^3 + 6x^2 + 8x + 3} \\
 \underline{x^3 + x^2} \\
 5x^2 + 8x + 3 \\
 \underline{5x^2 + 5x} \\
 3x + 3 \\
 \underline{3x + 3} \\
 0
 \end{array}$$

Answer is $x^2 + 5x + 3$

$$\begin{array}{r}
 x^2 + 6x + 1 \\
 x + 4 \overline{) x^3 + 10x^2 + 25x + 4} \\
 \underline{x^3 + 4x^2} \\
 6x^2 + 25x \\
 \underline{6x^2 + 24x} \\
 x + 4 \\
 \underline{x + 4} \\
 0
 \end{array}$$

(b)

Answer is $x^2 + 6x + 1$

$$\begin{array}{r}
 x^2 + x - 9 \\
 x + 6 \overline{) x^3 + 7x^2 - 3x - 54} \\
 \underline{x^3 + 6x^2} \\
 x^2 - 3x \\
 \underline{x^2 + 6x} \\
 -9x - 54 \\
 \underline{-9x - 54} \\
 0
 \end{array}$$

(c)

Answer is $x^2 + x - 9$

$$\begin{array}{r}
 x^2 + 4x - 2 \\
 x + 5 \overline{) x^3 + 9x^2 + 18x - 10} \\
 \underline{x^3 + 5x^2} \\
 4x^2 + 18x \\
 \underline{4x^2 + 20x} \\
 -2x - 10 \\
 \underline{-2x - 10} \\
 0
 \end{array}$$

(d)

Answer is $x^2 + 4x - 2$

$$\begin{array}{r}
 x^2 - 3x + 7 \\
 x + 2 \overline{) x^3 - x^2 + x + 14} \\
 \underline{x^3 + 2x^2} \\
 -3x^2 + x \\
 \underline{-3x^2 - 6x} \\
 7x + 14 \\
 \underline{7x + 14} \\
 0
 \end{array}$$

(e)

Answer is $x^2 - 3x + 7$

$$\begin{array}{r}
 x^2 + 4x + 5 \\
 x - 3 \overline{) x^3 + x^2 - 7x - 15} \\
 \underline{x^3 - 3x^2} \\
 4x^2 - 7x \\
 \underline{4x^2 - 12x} \\
 5x - 15 \\
 \underline{5x - 15} \\
 0
 \end{array}$$

(f)

Answer is $x^2 + 4x + 5$

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x - 2 \overline{) x^3 - 5x^2 + 8x - 4} \\
 \underline{x^3 - 2x^2} \\
 - 3x^2 + 8x \\
 \underline{- 3x^2 + 6x} \\
 2x - 4 \\
 \underline{2x - 4} \\
 0
 \end{array}$$

(g)

Answer is $x^2 - 3x + 2$

$$\begin{array}{r}
 x^2 - 2x + 6 \\
 x - 1 \overline{) x^3 - 3x^2 + 8x - 6} \\
 \underline{x^3 - x^2} \\
 - 2x^2 + 8x \\
 \underline{- 2x^2 + 2x} \\
 6x - 6 \\
 \underline{6x - 6} \\
 0
 \end{array}$$

(h)

Answer is $x^2 - 2x + 6$

$$\begin{array}{r}
 x^2 - 3x - 2 \\
 x - 5 \overline{) x^3 - 8x^2 + 13x + 10} \\
 \underline{x^3 - 5x^2} \\
 - 3x^2 + 13x \\
 \underline{- 3x^2 + 15x} \\
 - 2x + 10 \\
 \underline{- 2x + 10} \\
 0
 \end{array}$$

(i)

Answer is $x^2 - 3x - 2$

$$\begin{array}{r}
 x^2 + 2x + 8 \\
 x - 7 \overline{) x^3 - 5x^2 - 6x - 56} \\
 \underline{x^3 - 7x^2} \\
 2x^2 - 6x \\
 \underline{2x^2 - 14x} \\
 8x - 56 \\
 \underline{8x - 56} \\
 0
 \end{array}$$

(j)

Answer is $x^2 + 2x + 8$

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Exercise B, Question 2

Question:

Divide:

(a) $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

(b) $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

(c) $3x^3 - 10x^2 - 10x + 8$ by $(x - 4)$

(d) $3x^3 - 5x^2 - 4x - 24$ by $(x - 3)$

(e) $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$

(f) $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

(g) $-3x^3 + 2x^2 - 2x - 7$ by $(x + 1)$

(h) $-2x^3 + 5x^2 + 17x - 20$ by $(x - 4)$

(i) $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

(j) $-4x^3 + 9x^2 - 3x + 2$ by $(x - 2)$

Solution:

$$\begin{array}{r}
 \overline{6x^2 + 3x + 2} \\
 x + 4 \overline{) 6x^3 + 27x^2 + 14x + 8} \\
 \underline{6x^3 + 24x^2} \\
 3x^2 + 14x \\
 \underline{3x^2 + 12x} \\
 2x + 8 \\
 \underline{2x + 8} \\
 0
 \end{array}$$

(a)

Answer is $6x^2 + 3x + 2$

$$\begin{array}{r}
 4x^2 + x - 5 \\
 x + 2 \overline{) 4x^3 + 9x^2 - 3x - 10} \\
 \underline{4x^3 + 8x^2} \\
 x^2 - 3x \\
 \underline{x^2 + 2x} \\
 -5x - 10 \\
 \underline{-5x - 10} \\
 0
 \end{array}$$

(b)

Answer is $4x^2 + x - 5$

$$\begin{array}{r}
 3x^2 + 2x - 2 \\
 x - 4 \overline{) 3x^3 - 10x^2 - 10x + 8} \\
 \underline{3x^3 - 12x^2} \\
 2x^2 - 10x \\
 \underline{2x^2 - 8x} \\
 -2x + 8 \\
 \underline{-2x + 8} \\
 0
 \end{array}$$

(c)

Answer is $3x^2 + 2x - 2$

$$\begin{array}{r}
 3x^2 + 4x + 8 \\
 x - 3 \overline{) 3x^3 - 5x^2 - 4x - 24} \\
 \underline{3x^3 - 9x^2} \\
 4x^2 - 4x \\
 \underline{4x^2 - 12x} \\
 8x - 24 \\
 \underline{8x - 24} \\
 0
 \end{array}$$

(d)

Answer is $3x^2 + 4x + 8$

$$\begin{array}{r}
 2x^2 - 2x - 3 \\
 x + 3 \overline{) 2x^3 + 4x^2 - 9x - 9} \\
 \underline{2x^3 + 6x^2} \\
 -2x^2 - 9x \\
 \underline{-2x^2 - 6x} \\
 -3x - 9 \\
 \underline{-3x - 9} \\
 0
 \end{array}$$

(e)

Answer is $2x^2 - 2x - 3$

$$\begin{array}{r}
 \overline{2x^2 - 3x - 4} \\
 x - 6 \overline{) 2x^3 - 15x^2 + 14x + 24} \\
 \underline{2x^3 - 12x^2} \\
 - 3x^2 + 14x \\
 \underline{- 3x^2 + 18x} \\
 - 4x + 24 \\
 \underline{- 4x + 24} \\
 0
 \end{array}$$

Answer is $2x^2 - 3x - 4$

$$\begin{array}{r}
 \overline{- 3x^2 + 5x - 7} \\
 x + 1 \overline{) - 3x^3 + 2x^2 - 2x - 7} \\
 \underline{- 3x^3 - 3x^2} \\
 5x^2 - 2x \\
 \underline{5x^2 + 5x} \\
 - 7x - 7 \\
 \underline{- 7x - 7} \\
 0
 \end{array}$$

Answer is $- 3x^2 + 5x - 7$

$$\begin{array}{r}
 \overline{- 2x^2 - 3x + 5} \\
 x - 4 \overline{) - 2x^3 + 5x^2 + 17x - 20} \\
 \underline{- 2x^3 + 8x^2} \\
 - 3x^2 + 17x \\
 \underline{- 3x^2 + 12x} \\
 5x - 20 \\
 \underline{5x - 20} \\
 0
 \end{array}$$

Answer is $- 2x^2 - 3x + 5$

$$\begin{array}{r}
 \overline{- 5x^2 + 3x + 5} \\
 x + 6 \overline{) - 5x^3 - 27x^2 + 23x + 30} \\
 \underline{- 5x^3 - 30x^2} \\
 3x^2 + 23x \\
 \underline{3x^2 + 18x} \\
 5x + 30 \\
 \underline{5x + 30} \\
 0
 \end{array}$$

Answer is $- 5x^2 + 3x + 5$

$$\begin{array}{r}
 \phantom{\overline{)} } - 4x^2 + x \\
 x - 2 \overline{) } \\
 \underline{- 4x^3 + 8x^2} \\
 x^2 - 3x \\
 \underline{x^2 - 2x} \\
 - x + 2 \\
 \underline{- x + 2} \\
 0
 \end{array}$$

Answer is $-4x^2 + x - 1$

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Exercise B, Question 3

Question:

Divide:

(a) $x^4 + 5x^3 + 2x^2 - 7x + 2$ by $(x + 2)$

(b) $x^4 + 11x^3 + 25x^2 - 29x - 20$ by $(x + 5)$

(c) $4x^4 + 14x^3 + 3x^2 - 14x - 15$ by $(x + 3)$

(d) $3x^4 - 7x^3 - 23x^2 + 14x - 8$ by $(x - 4)$

(e) $-3x^4 + 9x^3 - 10x^2 + x + 14$ by $(x - 2)$

(f) $3x^5 + 17x^4 + 2x^3 - 38x^2 + 5x - 25$ by $(x + 5)$

(g) $6x^5 - 19x^4 + x^3 + x^2 + 13x + 6$ by $(x - 3)$

(h) $-5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7$ by $(x - 1)$

(i) $2x^6 - 11x^5 + 14x^4 - 16x^3 + 36x^2 - 10x - 24$ by $(x - 4)$

(j) $-x^6 + 4x^5 - 4x^4 + 4x^3 - 5x^2 + 7x - 3$ by $(x - 3)$

Solution:

$$\begin{array}{r}
 x^3 + 3x^2 - 4x + 1 \\
 x + 2 \overline{) x^4 + 5x^3 + 2x^2 - 7x + 2} \\
 \underline{x^4 + 2x^3} \\
 3x^3 + 2x^2 \\
 \underline{3x^3 + 6x^2} \\
 -4x^2 - 7x + 2 \\
 \underline{-4x^2 - 8x} \\
 x + 2 - \\
 \underline{x + 2} \\
 0
 \end{array}$$

Answer is $x^3 + 3x^2 - 4x + 1$

$$\begin{array}{r}
 x^3 + 6x^2 \quad - \quad 5x - 4 \\
 x + 5 \overline{) \quad x^4 + 11x^3 + 25x^2 - 29x - 20} \\
 \underline{x^4 + \quad 5x^3} \\
 6x^3 + 25x^2 \\
 \underline{6x^3 + 30x^2} \\
 - 5x^2 - 29x \\
 \underline{- 5x^2 - 25x} \\
 - 4x - 20 \\
 \underline{- 4x - 20} \\
 0
 \end{array}$$

(b)

Answer is $x^3 + 6x^2 - 5x - 4$

$$\begin{array}{r}
 4x^3 + 2x^2 \quad - \quad 3x - 5 \\
 x + 3 \overline{) \quad 4x^4 + 14x^3 + 3x^2 - 14x - 15} \\
 \underline{4x^4 + 12x^3} \\
 2x^3 + 3x^2 \\
 \underline{2x^3 + 6x^2} \\
 - 3x^2 - 14x \\
 \underline{- 3x^2 - \quad 9x} \\
 - 5x - 15 \\
 \underline{- 5x - 15} \\
 0
 \end{array}$$

(c)

Answer is $4x^3 + 2x^2 - 3x - 5$

$$\begin{array}{r}
 3x^3 + 5x^2 \quad - \quad 3x + 2 \\
 x - 4 \overline{) \quad 3x^4 - \quad 7x^3 - 23x^2 + 14x - 8} \\
 \underline{3x^4 - 12x^3} \\
 5x^3 - 23x^2 \\
 \underline{5x^3 - 20x^2} \\
 - 3x^2 + 14x \\
 \underline{- 3x^2 + 12x} \\
 2x - 8 \\
 \underline{2x - 8} \\
 0
 \end{array}$$

(d)

Answer is $3x^3 + 5x^2 - 3x + 2$

$$\begin{array}{r}
 -3x^3 + 3x^2 \quad - \quad 4x - 7 \\
 x - 2 \overline{) -3x^4 + 9x^3 - 10x^2 + \quad x + 14} \\
 \underline{-3x^4 + 6x^3} \\
 3x^3 - 10x^2 \\
 \underline{3x^3 - \quad 6x^2} \\
 -4x^2 + \quad x \\
 \underline{-4x^2 + 8x} \\
 -7x + 14 \\
 \underline{-7x + 14} \\
 0
 \end{array}$$

(e)

Answer is $-3x^3 + 3x^2 - 4x - 7$

$$\begin{array}{r}
 3x^4 + 2x^3 \quad - \quad 8x^2 + 2x - 5 \\
 x + 5 \overline{) 3x^5 + 17x^4 + \quad 2x^3 - 38x^2 + \quad 5x - 25} \\
 \underline{3x^5 + 15x^4} \\
 2x^4 + \quad 2x^3 \\
 \underline{2x^4 + 10x^3} \\
 -8x^3 - 38x^2 \\
 \underline{-8x^3 - 40x^2} \\
 2x^2 + \quad 5x \\
 \underline{2x^2 + 10x} \\
 -5x - 25 \\
 \underline{-5x - 25} \\
 0
 \end{array}$$

(f)

Answer is $3x^4 + 2x^3 - 8x^2 + 2x - 5$

$$\begin{array}{r}
 6x^4 - x^3 \quad - \quad 2x^2 - 5x - 2 \\
 x - 3 \overline{) 6x^5 - 19x^4 + x^3 + \quad x^2 + 13x + 6} \\
 \underline{6x^5 - 18x^4} \\
 -x^4 + x^3 \\
 \underline{-x^4 + 3x^3} \\
 -2x^3 + \quad x^2 \\
 \underline{-2x^3 + 6x^2} \\
 -5x^2 + 13x \\
 \underline{-5x^2 + 15x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

(g)

Answer is $6x^4 - x^3 - 2x^2 - 5x - 2$

$$\begin{array}{r}
 - 5x^4 + 2x^3 \quad + \quad 4x^2 - 3x + 7 \\
 x-1 \overline{) 5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7} \\
 - 5x^5 + 5x^4 \\
 2x^4 + 2x^3 \\
 2x^4 - 2x^3 \\
 4x^3 - 7x^2 \\
 4x^3 - 4x^2 \\
 - 3x^2 + 10x \\
 - 3x^2 + 3x \\
 7x - 7 \\
 7x - 7 \\
 0
 \end{array}$$

Answer is $-5x^4 + 2x^3 + 4x^2 - 3x + 7$

$$\begin{array}{r}
 2x^5 - 3x^4 \quad + \quad 2x^3 - 8x^2 + 4x + 6 \\
 x-4 \overline{) 2x^6 - 11x^5 + 14x^4 - 16x^3 + 36x^2 - 10x - 24} \\
 2x^6 - 8x^5 \\
 - 3x^5 + 14x^4 \\
 - 3x^5 + 12x^4 \\
 2x^4 - 16x^3 \\
 2x^4 - 8x^3 \\
 - 8x^3 + 36x^2 \\
 - 8x^3 + 32x^2 \\
 4x^2 - 10x \\
 4x^2 - 16x \\
 6x - 24 \\
 6x - 24 \\
 0
 \end{array}$$

Answer is $2x^5 - 3x^4 + 2x^3 - 8x^2 + 4x + 6$

$$\begin{array}{r}
 -x^5 + x^4 \quad - \quad x^3 + x^2 - 2x + 1 \\
 x - 3 \overline{) \quad -x^6 + 4x^5 - 4x^4 + 4x^3 - 5x^2 + 7x - 3} \\
 \underline{-x^6 + 3x^5} \\
 x^5 - 4x^4 \\
 \underline{x^5 - 3x^4} \\
 -x^4 + 4x^3 \\
 \underline{-x^4 + 3x^3} \\
 x^3 - 5x^2 \\
 \underline{x^3 - 3x^2} \\
 -2x^2 + 7x \\
 \underline{-2x^2 + 6x} \\
 x - 3 \\
 \underline{x - 3} \\
 0
 \end{array}$$

Answer is $-x^5 + x^4 - x^3 + x^2 - 2x + 1$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 1

Question:

Divide:

(a) $x^3 + x + 10$ by $(x + 2)$

(b) $2x^3 - 17x + 3$ by $(x + 3)$

(c) $-3x^3 + 50x - 8$ by $(x - 4)$

Solution:

$$\begin{array}{r}
 x^2 - 2x + 5 \\
 x + 2 \overline{) x^3 + 0x^2 + x + 10} \\
 \underline{x^3 + 2x^2} \\
 -2x^2 + x \\
 \underline{-2x^2 - 4x} \\
 5x + 10 \\
 \underline{5x + 10} \\
 0
 \end{array}$$

Answer is $x^2 - 2x + 5$

$$\begin{array}{r}
 2x^2 - 6x + 1 \\
 x + 3 \overline{) 2x^3 + 0x^2 - 17x + 3} \\
 \underline{2x^3 + 6x^2} \\
 -6x^2 - 17x \\
 \underline{-6x^2 - 18x} \\
 x + 3 \\
 \underline{x + 3} \\
 0
 \end{array}$$

Answer is $2x^2 - 6x + 1$

$$\begin{array}{r}
 - 3x^2 - 12x + 2 \\
 x - 4 \overline{) - 3x^3 + 0x^2 + 50x - 8} \\
 - 3x^3 + 12x^2 \\
 - 12x^2 + 50x \\
 - 12x^2 + 48x \\
 2x - 8 \\
 2x - 8 \\
 0
 \end{array}$$

Answer is $-3x^2 - 12x + 2$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 2

Question:

Divide:

(a) $x^3 + x^2 - 36$ by $(x - 3)$

(b) $2x^3 + 9x^2 + 25$ by $(x + 5)$

(c) $-3x^3 + 11x^2 - 20$ by $(x - 2)$

Solution:

$$\begin{array}{r}
 x^2 + 4x + 12 \\
 x - 3 \overline{) x^3 + x^2 + 0x - 36} \\
 \underline{x^3 - 3x^2} \\
 4x^2 + 0x \\
 \underline{4x^2 - 12x} \\
 12x - 36 \\
 \underline{12x - 36} \\
 0
 \end{array}$$

(a)

Answer is $x^2 + 4x + 12$

$$\begin{array}{r}
 2x^2 - x + 5 \\
 x + 5 \overline{) 2x^3 + 9x^2 + 0x + 25} \\
 \underline{2x^3 + 10x^2} \\
 -x^2 + 0x \\
 \underline{-x^2 - 5x} \\
 5x + 25 \\
 \underline{5x + 25} \\
 0
 \end{array}$$

(b)

Answer is $2x^2 - x + 5$

$$\begin{array}{r}
 - 3x^2 + 5x + 10 \\
 x - 2 \overline{) } \\
 - 3x^3 + 11x^2 + 0x - 20 \\
 \\
 5x^2 + 0x \\
 5x^2 - 10x \\
 10x - 20 \\
 10x - 20 \\
 0
 \end{array}$$

Answer is $-3x^2 + 5x + 10$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 3

Question:

Divide:

(a) $x^3 + 2x^2 - 5x - 10$ by $(x + 2)$

(b) $2x^3 - 6x^2 + 7x - 21$ by $(x - 3)$

(c) $-3x^3 + 21x^2 - 4x + 28$ by $(x - 7)$

Solution:

$$\begin{array}{r}
 x^2 - 5 \\
 x + 2 \overline{) x^3 + 2x^2 - 5x - 10} \\
 \underline{x^3 + 2x^2} \\
 0 \\
 - 5x - 10 \\
 \underline{- 5x - 10} \\
 0
 \end{array}$$

(a)

Answer is $x^2 - 5$

$$\begin{array}{r}
 2x^2 + 7 \\
 x - 3 \overline{) 2x^3 - 6x^2 + 7x - 21} \\
 \underline{2x^3 - 6x^2} \\
 0 + 7x - 21 \\
 \underline{7x - 21} \\
 0
 \end{array}$$

(b)

Answer is $2x^2 + 7$

$$\begin{array}{r}
 -3x^2 - 4 \\
 x - 7 \overline{) -3x^3 + 21x^2 - 4x + 28} \\
 \underline{-3x^3 + 21x^2} \\
 0 \\
 - 4x + 28 \\
 \underline{- 4x + 28} \\
 0
 \end{array}$$

(c)

Answer is $-3x^2 - 4$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 4

Question:

Find the remainder when:

(a) $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

(b) $3x^3 - 20x^2 + 10x + 5$ is divided by $(x - 6)$

(c) $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

Solution:

$$\begin{array}{r}
 x^2 - x + 2 \\
 x + 5 \overline{) x^3 + 4x^2 - 3x + 2} \\
 \underline{x^3 + 5x^2} \\
 -x^2 - 3x \\
 \underline{-x^2 - 5x} \\
 2x + 2 \\
 \underline{2x + 10} \\
 -8
 \end{array}$$

(a)

The remainder is -8 .

$$\begin{array}{r}
 3x^2 - 2x - 2 \\
 x - 6 \overline{) 3x^3 - 20x^2 + 10x + 5} \\
 \underline{3x^3 - 18x^2} \\
 -2x^2 + 10x \\
 \underline{-2x^2 + 12x} \\
 -2x + 5 \\
 \underline{-2x + 12} \\
 -7
 \end{array}$$

(b)

The remainder is -7 .

$$\begin{array}{r}
 - 2x^2 - 5x - 8 \\
 x - 4 \overline{) - 2x^3 + 3x^2 + 12x + 20} \\
 - 2x^3 + 8x^2 \\
 - 5x^2 + 12x \\
 - 5x^2 + 20x \\
 - 8x + 20 \\
 - 8x + 32 \\
 - 12
 \end{array}$$

(c)

The remainder is -12 .

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 5

Question:

Show that when $3x^3 - 2x^2 + 4$ is divided by $(x - 1)$ the remainder is 5.

Solution:

$$\begin{array}{r}
 3x^2 + x + 1 \\
 x - 1 \overline{) 3x^3 - 2x^2 + 0x + 4} \\
 \underline{3x^3 - 3x^2} \\
 x^2 + 0x \\
 \underline{x^2 - x} \\
 x + 4 \\
 \underline{x - 1} \\
 5
 \end{array}$$

So the remainder is 5.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 6

Question:

Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x + 1)$ the remainder is -1 .

Solution:

$$\begin{array}{r}
 3x^3 - 11x^2 + 21x - 24 \\
 x + 1 \overline{) 3x^4 - 8x^3 + 10x^2 - 3x - 25} \\
 \underline{3x^4 + 3x^3} \\
 -11x^3 + 10x^2 \\
 \underline{-11x^3 - 11x^2} \\
 21x^2 - 3x \\
 \underline{21x^2 + 21x} \\
 -24x - 25 \\
 \underline{-24x - 24} \\
 -1
 \end{array}$$

So the remainder is -1 .

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 7

Question:

Show that $(x + 4)$ is the factor of $5x^3 - 73x + 28$.

Solution:

$$\begin{array}{r}
 5x^2 - 20x + 7 \\
 x + 4 \overline{) 5x^3 + 0x^2 - 73x + 28} \\
 \underline{5x^3 + 20x^2} \\
 -20x^2 - 73x \\
 \underline{-20x^2 - 80x} \\
 7x + 28 \\
 \underline{7x + 28} \\
 0
 \end{array}$$

The remainder is 0, so $x + 4$ is a factor of $5x^3 - 73x + 28$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 8

Question:

Simplify $\frac{3x^3 - 8x - 8}{x - 2}$.

Solution:

$$\begin{array}{r}
 3x^2 + 6x + 4 \\
 x - 2 \overline{) 3x^3 + 0x^2 - 8x - 8} \\
 \underline{3x^3 - 6x^2} \\
 6x^2 - 8x \\
 \underline{6x^2 - 12x} \\
 4x - 8 \\
 \underline{4x - 8} \\
 0
 \end{array}$$

So $\frac{3x^3 - 8x - 8}{x - 2} = 3x^2 + 6x + 4$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 9

Question:

Divide $x^3 - 1$ by $(x - 1)$.

Solution:

$$\begin{array}{r}
 x^2 + x + 1 \\
 x - 1 \overline{) x^3 + 0x^2 + 0x - 1} \\
 \underline{x^3 - x^2} \\
 x^2 + 0x \\
 \underline{x^2 - x} \\
 x - 1 \\
 \underline{x - 1} \\
 0
 \end{array}$$

Answer is $x^2 + x + 1$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 10

Question:

Divide $x^4 - 16$ by $(x + 2)$.

Solution:

$$\begin{array}{r}
 x^3 - 2x^2 + 4x - 8 \\
 x + 2 \overline{) x^4 + 0x^3 + 0x^2 + 0x - 16} \\
 \underline{x^4 + 2x^3} \\
 - 2x^3 + 0x^2 \\
 \underline{- 2x^3 - 4x^2} \\
 4x^2 + 0x \\
 \underline{4x^2 + 8x} \\
 - 8x - 16 \\
 \underline{- 8x - 16} \\
 0
 \end{array}$$

Answer is $x^3 - 2x^2 + 4x - 8$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 1

Question:

Use the factor theorem to show:

(a) $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

(b) $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

(c) $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$

Solution:

(a) $f(x) = 4x^3 - 3x^2 - 1$

$$f(1) = 4(1)^3 - 3(1)^2 - 1 = 4 - 3 - 1 = 0$$

So $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

(b) $f(x) = 5x^4 - 45x^2 - 6x - 18$

$$f(-3) = 5(-3)^4 - 45(-3)^2 - 6(-3) - 18$$

$$f(-3) = 5(81) - 45(9) + 18 - 18 = 405 - 405 = 0$$

So $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

(c) $f(x) = -3x^3 + 13x^2 - 6x + 8$

$$f(4) = -3(4)^3 + 13(4)^2 - 6(4) + 8$$

$$f(4) = -192 + 208 - 24 + 8 = 0$$

So $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 2

Question:

Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely.

Solution:

$$f(x) = x^3 + 6x^2 + 5x - 12$$

$$f(1) = (1)^3 + 6(1)^2 + 5(1) - 12 = 1 + 6 + 5 - 12 = 0$$

So $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$

$$\begin{array}{r}
 x^2 + 7x + 12 \\
 x - 1 \overline{) x^3 + 6x^2 + 5x - 12} \\
 \underline{x^3 - x^2} \\
 7x^2 + 5x \\
 \underline{7x^2 - 7x} \\
 12x - 12 \\
 \underline{12x - 12} \\
 0
 \end{array}$$

$$\text{Now } x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$\text{So } x^3 + 6x^2 + 5x - 12 = (x - 1)(x + 3)(x + 4)$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 3

Question:

Show that $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$ and hence factorise the expression completely.

Solution:

$$f(x) = x^3 + 3x^2 - 33x - 35$$

$$f(-1) = (-1)^3 + 3(-1)^2 - 33(-1) - 35 = -1 + 3 + 33 - 35 = 0$$

So $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$

$$\begin{array}{r}
 x^2 + 2x - 35 \\
 x + 1 \overline{) x^3 + 3x^2 - 33x - 35} \\
 \underline{x^3 + x^2} \\
 2x^2 - 33x \\
 \underline{2x^2 + x} \\
 -35x - 35 \\
 \underline{-35x - 35} \\
 0
 \end{array}$$

$$\text{Now } x^2 + 2x - 35 = (x + 7)(x - 5)$$

$$\text{So } x^3 + 3x^2 - 33x - 35 = (x + 1)(x + 7)(x - 5)$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 4

Question:

Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely.

Solution:

$$f(x) = x^3 - 7x^2 + 2x + 40$$

$$f(5) = (5)^3 - 7(5)^2 + 2(5) + 40$$

$$f(5) = 125 - 175 + 10 + 40 = 0$$

So $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$

$$\begin{array}{r}
 x^2 - 2x - 8 \\
 x - 5 \overline{) x^3 - 7x^2 + 2x + 40} \\
 \underline{x^3 - 5x^2} \\
 -2x^2 + 2x \\
 \underline{-2x^2 + 10x} \\
 8x + 40 \\
 \underline{-8x + 40} \\
 0
 \end{array}$$

$$\text{Now } x^2 - 2x - 8 = (x - 4)(x + 2)$$

$$\text{So } x^3 - 7x^2 + 2x + 40 = (x - 5)(x - 4)(x + 2).$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 5

Question:

Show that $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$ and hence factorise the expression completely.

Solution:

$$f(x) = 2x^3 + 3x^2 - 18x + 8$$

$$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8 = 16 + 12 - 36 + 8 = 0$$

So $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$

$$\begin{array}{r}
 2x^2 + 7x - 4 \\
 x - 2 \overline{) 2x^3 + 3x^2 - 18x + 8} \\
 \underline{2x^3 - 4x^2} \\
 7x^2 - 18x \\
 \underline{7x^2 - 14x} \\
 -4x + 8 \\
 \underline{-4x + 8} \\
 0
 \end{array}$$

$$\text{Now } 2x^2 + 7x - 4 = (2x - 1)(x + 4)$$

$$\text{So } 2x^3 + 3x^2 - 18x + 8 = (x - 2)(2x - 1)(x + 4)$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 6

Question:

Each of these expressions has a factor $(x \pm p)$. Find a value of p and hence factorise the expression completely.

(a) $x^3 - 10x^2 + 19x + 30$

(b) $x^3 + x^2 - 4x - 4$

(c) $x^3 - 4x^2 - 11x + 30$

Solution:

(a) $f(x) = x^3 - 10x^2 + 19x + 30$

$f(-1) = (-1)^3 - 10(-1)^2 + 19(-1) + 30 = -1 - 10 - 19 + 30 = 0$

So $(x + 1)$ is a factor.

$$\begin{array}{r}
 x^2 - 11x + 30 \\
 x + 1 \overline{) x^3 - 10x^2 + 19x + 30} \\
 \underline{x^3 + x^2} \\
 -11x^2 + 19x \\
 \underline{-11x^2 - 11x} \\
 30x + 30 \\
 \underline{30x + 30} \\
 0
 \end{array}$$

Now $x^2 - 11x + 30 = (x - 5)(x - 6)$

So $x^3 - 10x^2 + 19x + 30 = (x + 1)(x - 5)(x - 6)$.

(b) $f(x) = x^3 + x^2 - 4x - 4$

$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$

So $(x + 1)$ is a factor.

$$\begin{array}{r}
 x^2 - 4 \\
 x + 1 \overline{) x^3 + x^2 - 4x - 4} \\
 \underline{x^3 + x^2} \\
 0 - 4x - 4 \\
 \underline{-4x - 4} \\
 0
 \end{array}$$

Now $x^2 - 4 = (x - 2)(x + 2)$

So $x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$

(c) $f(x) = x^3 - 4x^2 - 11x + 30$

$f(2) = (2)^3 - 4(2)^2 - 11(2) + 30 = 8 - 16 - 22 + 30 = 0$

So $(x - 2)$ is a factor.

$$\begin{array}{r}
 x^2 - 2x - 15 \\
 x - 2 \overline{) x^3 - 4x^2 - 11x + 30} \\
 \underline{x^3 - 2x^2} \\
 - 2x^2 - 11x \\
 \underline{- 2x^2 + 4x} \\
 - 15x + 30 \\
 \underline{- 15x + 30} \\
 0
 \end{array}$$

Now $x^2 - 2x - 15 = (x + 3)(x - 5)$

So $x^3 - 4x^2 - 11x + 30 = (x - 2)(x + 3)(x - 5)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 7

Question:

Factorise:

(a) $2x^3 + 5x^2 - 4x - 3$

(b) $2x^3 - 17x^2 + 38x - 15$

(c) $3x^3 + 8x^2 + 3x - 2$

(d) $6x^3 + 11x^2 - 3x - 2$

(e) $4x^3 - 12x^2 - 7x + 30$

Solution:

(a) $f(x) = 2x^3 + 5x^2 - 4x - 3$

$f(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3 = 2 + 5 - 4 - 3 = 0$

So $(x - 1)$ is a factor.

$$\begin{array}{r}
 2x^2 + 7x + 3 \\
 x - 1 \overline{) 2x^3 + 5x^2 - 4x - 3} \\
 \underline{2x^3 - 2x^2} \\
 7x^2 - 4x \\
 \underline{7x^2 - 7x} \\
 3x - 3 \\
 \underline{3x - 3} \\
 0
 \end{array}$$

Now $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

So $2x^3 + 5x^2 - 4x - 3 = (x - 1)(2x + 1)(x + 3)$.

(b) $f(x) = 2x^3 - 17x^2 + 38x - 15$

$f(3) = 2(3)^3 - 17(3)^2 + 38(3) - 15 = 54 - 153 + 114 - 15 = 0$

So $(x - 3)$ is a factor.

$$\begin{array}{r}
 2x^2 - 11x + 5 \\
 x - 3 \overline{) 2x^3 - 17x^2 + 38x - 15} \\
 \underline{2x^3 - 6x^2} \\
 -11x^2 + 38x \\
 \underline{-11x^2 + 33x} \\
 5x - 15 \\
 \underline{5x - 15} \\
 0
 \end{array}$$

Now $2x^2 - 11x + 5 = (2x - 1)(x - 5)$

So $2x^3 - 17x^2 + 38x - 15 = (x - 3)(2x - 1)(x - 5)$.

$$(c) f(x) = 3x^3 + 8x^2 + 3x - 2$$

$$f(-1) = 3(-1)^3 + 8(-1)^2 + 3(-1) - 2 = -3 + 8 - 3 - 2 = 0$$

So $(x + 1)$ is a factor.

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x + 1 \overline{) 3x^3 + 8x^2 + 3x - 2} \\ \underline{3x^3 + 3x^2} \\ 5x^2 + 3x \\ \underline{5x^2 + 5x} \\ -2x - 2 \\ \underline{-2x - 2} \\ 0 \end{array}$$

$$\text{Now } 3x^2 + 5x - 2 = (3x - 1)(x + 2)$$

$$\text{So } 3x^3 + 8x^2 + 3x - 2 = (x + 1)(3x - 1)(x + 2).$$

$$(d) f(x) = 6x^3 + 11x^2 - 3x - 2$$

$$f(-2) = 6(-2)^3 + 11(-2)^2 - 3(-2) - 2 = -48 + 44 + 6 - 2 = 0$$

So $(x + 2)$ is a factor.

$$\begin{array}{r} 6x^2 - x - 1 \\ x + 2 \overline{) 6x^3 + 11x^2 - 3x - 2} \\ \underline{6x^3 + 12x^2} \\ -x^2 - 3x \\ \underline{-x^2 - 2x} \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$$\text{Now } 6x^2 - x - 1 = (3x + 1)(2x - 1)$$

$$\text{So } 6x^3 + 11x^2 - 3x - 2 = (x + 2)(3x + 1)(2x - 1).$$

$$(e) f(x) = 4x^3 - 12x^2 - 7x + 30$$

$$f(2) = 4(2)^3 - 12(2)^2 - 7(2) + 30 = 32 - 48 - 14 + 30 = 0$$

So $(x - 2)$ is a factor.

$$\begin{array}{r} 4x^2 - 4x - 15 \\ x - 2 \overline{) 4x^3 - 12x^2 - 7x + 30} \\ \underline{4x^3 - 8x^2} \\ -4x^2 - 7x \\ \underline{-4x^2 + 8x} \\ -15x + 30 \\ \underline{-15x + 30} \\ 0 \end{array}$$

$$\text{Now } 4x^2 - 4x - 15 = (2x + 3)(2x - 5)$$

$$\text{So } 4x^3 - 12x^2 - 7x + 30 = (x - 2)(2x + 3)(2x - 5).$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 8

Question:

Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$ find the value of a .

Solution:

$$f(x) = 5x^3 - 9x^2 + 2x + a$$

$$f(1) = 0$$

$$\text{So } 5(1)^3 - 9(1)^2 + 2(1) + a = 0$$

$$5 - 9 + 2 + a = 0$$

$$a = 2$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 9

Question:

Given that $(x + 3)$ is a factor of $6x^3 - bx^2 + 18$ find the value of b .

Solution:

$$f(x) = 6x^3 - bx^2 + 18$$

$$f(-3) = 0$$

$$\text{So } 6(-3)^3 - b(-3)^2 + 18 = 0$$

$$-162 - 9b + 18 = 0$$

$$9b = -144$$

$$b = -16$$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 10

Question:

Given that $(x - 1)$ and $(x + 1)$ are factors of $px^3 + qx^2 - 3x - 7$ find the value of p and q .

Solution:

$$f(x) = px^3 + qx^2 - 3x - 7$$

$$\textcircled{1} \quad f(1) = 0$$

$$p(1)^3 + q(1)^2 - 3(1) - 7 = 0$$

$$p + q - 3 - 7 = 0$$

$$p + q = 10$$

$$\textcircled{2} \quad f(-1) = 0$$

$$p(-1)^3 + q(-1)^2 - 3(-1) - 7 = 0$$

$$-p + q + 3 - 7 = 0$$

$$-p + q = 4$$

Solve simultaneously:

$$p + q = 10$$

$$-p + q = 4$$

$$2q = 14$$

$$q = 7$$

$$p + q = 10, \text{ so } p = 3.$$

Answer is $p = 3, q = 7$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 1

Question:

Find the remainder when:

- (a) $4x^3 - 5x^2 + 7x + 1$ is divided by $(x - 2)$
- (b) $2x^5 - 32x^3 + x - 10$ is divided by $(x - 4)$
- (c) $-2x^3 + 6x^2 + 5x - 3$ is divided by $(x + 1)$
- (d) $7x^3 + 6x^2 - 45x + 1$ is divided by $(x + 3)$
- (e) $4x^4 - 4x^2 + 8x - 1$ is divided by $(2x - 1)$
- (f) $243x^4 - 27x^3 - 3x + 7$ is divided by $(3x - 1)$
- (g) $64x^3 + 32x^2 + 16x + 9$ is divided by $(4x + 1)$
- (h) $81x^3 - 81x^2 + 9x + 6$ is divided by $(3x - 2)$
- (i) $243x^6 - 780x^2 + 6$ is divided by $(3x + 4)$
- (j) $125x^4 + 5x^3 - 9x$ is divided by $(5x + 3)$

Solution:

$$\begin{aligned} \text{(a) } f(x) &= 4x^3 - 5x^2 + 7x + 1 \\ f(2) &= 4(2)^3 - 5(2)^2 + 7(2) + 1 \\ f(2) &= 32 - 20 + 14 + 1 = 27 \\ \text{Remainder is } 27. \end{aligned}$$

$$\begin{aligned} \text{(b) } f(x) &= 2x^5 - 32x^3 + x - 10 \\ f(4) &= 2(4)^5 - 32(4)^3 + (4) - 10 \\ f(4) &= 2048 - 2048 + 4 - 10 = -6 \\ \text{Remainder is } -6. \end{aligned}$$

$$\begin{aligned} \text{(c) } f(x) &= -2x^3 + 6x^2 + 5x - 3 \\ f(-1) &= -2(-1)^3 + 6(-1)^2 + 5(-1) - 3 \\ f(-1) &= 2 + 6 - 5 - 3 = 0 \\ \text{Remainder is } 0. \end{aligned}$$

$$\begin{aligned} \text{(d) } f(x) &= 7x^3 + 6x^2 - 45x + 1 \\ f(-3) &= 7(-3)^3 + 6(-3)^2 - 45(-3) + 1 \\ f(-3) &= -189 + 54 + 135 + 1 = 1 \\ \text{Remainder is } 1. \end{aligned}$$

$$\begin{aligned} \text{(e) } f(x) &= 4x^4 - 4x^2 + 8x - 1 \\ f\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 1 \end{aligned}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} - 1 + 4 - 1 = 2\frac{1}{4}$$

Remainder is $2\frac{1}{4}$.

$$(f) f(x) = 243x^4 - 27x^3 - 3x + 7$$

$$f\left(\frac{1}{3}\right) = 243\left(\frac{1}{3}\right)^4 - 27\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right) + 7$$

$$f\left(\frac{1}{3}\right) = 3 - 1 - 1 + 7 = 8$$

Remainder is 8.

$$(g) f(x) = 64x^3 + 32x^2 - 16x + 9$$

$$f\left(-\frac{1}{4}\right) = 64\left(-\frac{1}{4}\right)^3 + 32\left(-\frac{1}{4}\right)^2 - 16\left(-\frac{1}{4}\right) + 9$$

$$f\left(-\frac{1}{4}\right) = -1 + 2 + 4 + 9 = 14$$

Remainder is 14.

$$(h) f(x) = 81x^3 - 81x^2 + 9x + 6$$

$$f\left(\frac{2}{3}\right) = 81\left(\frac{2}{3}\right)^3 - 81\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right) + 6$$

$$f\left(\frac{2}{3}\right) = 24 - 36 + 6 + 6 = 0$$

Remainder is 0.

$$(i) f(x) = 243x^6 - 780x^2 + 6$$

$$f\left(-\frac{4}{3}\right) = 243\left(-\frac{4}{3}\right)^6 - 780\left(-\frac{4}{3}\right)^2 + 6$$

$$f\left(-\frac{4}{3}\right) = \frac{4096}{3} - \frac{4160}{3} + 6 = -\frac{64}{3} + 6 = -21\frac{1}{3} + 6 = -15\frac{1}{3}$$

Remainder is $-15\frac{1}{3}$.

$$(j) f(x) = 125x^4 + 5x^3 - 9x$$

$$f\left(-\frac{3}{5}\right) = 125\left(-\frac{3}{5}\right)^4 + 5\left(-\frac{3}{5}\right)^3 - 9\left(-\frac{3}{5}\right)$$

$$f\left(-\frac{3}{5}\right) = \frac{405}{25} - \frac{27}{25} + \frac{27}{5} = \frac{378}{25} + \frac{135}{25} = \frac{513}{25} = 20\frac{13}{25}$$

Remainder is $20\frac{13}{25} \left(= 20.52 \right)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 2

Question:

When $2x^3 - 3x^2 - 2x + a$ is divided by $(x - 1)$ the remainder is -4 . Find the value of a .

Solution:

$$f(x) = 2x^3 - 3x^2 - 2x + a$$

$$f(1) = -4$$

$$\text{So } 2(1)^3 - 3(1)^2 - 2(1) + a = -4$$

$$2 - 3 - 2 + a = -4$$

$$a = -1$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 3

Question:

When $-3x^3 + 4x^2 + bx + 6$ is divided by $(x + 2)$ the remainder is 10. Find the value of b .

Solution:

$$f(x) = -3x^3 + 4x^2 + bx + 6$$

$$f(-2) = 10$$

$$\text{So } -3(-2)^3 + 4(-2)^2 + b(-2) + 6 = 10$$

$$24 + 16 - 2b + 6 = 10$$

$$2b = 36$$

$$b = 18$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 4

Question:

When $16x^3 - 32x^2 + cx - 8$ is divided by $(2x - 1)$ the remainder is 1. Find the value of c .

Solution:

$$f(x) = 16x^3 - 32x^2 + cx - 8$$

$$f\left(\frac{1}{2}\right) = 1$$

$$\text{So } 16\left(\frac{1}{2}\right)^3 - 32\left(\frac{1}{2}\right)^2 + c\left(\frac{1}{2}\right) - 8 = 1$$

$$2 - 8 + \frac{1}{2}c - 8 = 1$$

$$\frac{1}{2}c = 15$$

$$c = 30$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 5

Question:

Show that $(x - 3)$ is a factor of $x^6 - 36x^3 + 243$.

Solution:

$$f(x) = x^6 - 36x^3 + 243$$

$$f(3) = (3)^6 - 36(3)^3 + 243$$

$$f(3) = 729 - 972 + 243 = 0$$

Remainder is 0, so $(x - 3)$ is a factor.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 6

Question:

Show that $(2x - 1)$ is a factor of $2x^3 + 17x^2 + 31x - 20$.

Solution:

$$f(x) = 2x^3 + 17x^2 + 31x - 20$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 17\left(\frac{1}{2}\right)^2 + 31\left(\frac{1}{2}\right) - 20$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{17}{4} + \frac{31}{2} - 20 = \frac{1 + 17 + 62 - 80}{4} = 0$$

Remainder is 0, so $(2x - 1)$ is a factor.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 7

Question:

$f(x) = x^2 + 3x + q$. Given $f(2) = 3$, find $f(-2)$.

Solution:

$$f(x) = x^2 + 3x + q$$

$$\text{Given } f(2) = 3.$$

$$\text{So } (2)^2 + 3(2) + q = 3$$

$$4 + 6 + q = 3$$

$$q = -7$$

$$f(x) = x^2 + 3x - 7$$

$$f(-2) = (-2)^2 + 3(-2) - 7$$

$$f(-2) = 4 - 6 - 7 = -9$$

Answer is -9 .

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 8

Question:

$g(x) = x^3 + ax^2 + 3x + 6$. Given $g(-1) = 2$, find the remainder when $g(x)$ is divided by $(3x - 2)$.

Solution:

$$g(x) = x^3 + ax^2 + 3x + 6$$

$$\text{Given } g(-1) = 2.$$

$$\text{So } (-1)^3 + a(-1)^2 + 3(-1) + 6 = 2$$

$$-1 + a - 3 + 6 = 2$$

$$a = 0$$

$$g(x) = x^3 + 3x + 6$$

$$g\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 + 3\left(\frac{2}{3}\right) + 6$$

$$g\left(\frac{2}{3}\right) = \frac{8}{27} + 2 + 6 = 8\frac{8}{27}$$

Answer is $8\frac{8}{27}$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 9

Question:

The expression $2x^3 - x^2 + ax + b$ gives a remainder 14 when divided by $(x - 2)$ and a remainder -86 when divided by $(x + 3)$. Find the value of a and b .

Solution:

$$f(x) = 2x^3 - x^2 + ax + b$$

$$\textcircled{1} \quad f(2) = 14$$

$$\text{So } 2(2)^3 - (2)^2 + a(2) + b = 14$$

$$16 - 4 + 2a + b = 14$$

$$2a + b = 2$$

$$\textcircled{2} \quad f(-3) = -86$$

$$\text{So } 2(-3)^3 - (-3)^2 + a(-3) + b = -86$$

$$-54 - 9 - 3a + b = -86$$

$$-3a + b = -23$$

Solve simultaneously:

$$2a + b = 2$$

$$-3a + b = -23$$

$$5a = 25$$

$$a = 5$$

$$2a + b = 2$$

Substitute $a = 5$:

$$2(5) + b = 2$$

$$10 + b = 2$$

$$b = -8$$

Check $a = 5, b = -8$ by substitution:

$$-3a + b = -3(5) + (-8) = -15 - 8 = -23 \quad \checkmark$$

Answer is $a = 5, b = -8$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 10

Question:

The expression $3x^3 + 2x^2 - px + q$ is divisible by $(x - 1)$ but leaves a remainder of 10 when divided by $(x + 1)$. Find the value of a and b .

Solution:

$$f(x) = 3x^3 + 2x^2 - px + q$$

$$\textcircled{1} \quad f(1) = 0$$

$$\text{So } 3(1)^3 + 2(1)^2 - p(1) + q = 0$$

$$3 + 2 - p + q = 0$$

$$-p + q = -5$$

$$\textcircled{2} \quad f(-1) = 10$$

$$\text{So } 3(-1)^3 + 2(-1)^2 - p(-1) + q = 0$$

$$-3 + 2 + p + q = 10$$

$$p + q = 11$$

Solve simultaneously:

$$-p + q = -5$$

$$p + q = 11$$

$$2q = 6$$

$$q = 3$$

Substitute $q = 3$:

$$p + q = 11$$

$$p + 3 = 11$$

$$p = 8$$

$$\text{Check: } -p + q = -8 + 3 = -5 \checkmark$$

Answer is $p = 8, q = 3$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 1

Question:

Simplify these fractions as far as possible:

$$(a) \frac{3x^4 - 21x}{3x}$$

$$(b) \frac{x^2 - 2x - 24}{x^2 - 7x + 6}$$

$$(c) \frac{2x^2 + 7x - 4}{2x^2 + 9x + 4}$$

Solution:

$$(a) \frac{3x^4 - 21x}{3x} = \frac{3x^4}{3x} - \frac{21x}{3x} = x^3 - 7$$

$$(b) \frac{x^2 - 2x - 24}{x^2 - 7x + 6}$$

$$= \frac{(x-6)(x+4)}{(x-6)(x-1)}$$

$$= \frac{x+4}{x-1}$$

$$(c) \frac{2x^2 + 7x - 4}{2x^2 + 9x + 4}$$

$$= \frac{(2x-1)(x+4)}{(2x+1)(x+4)}$$

$$= \frac{2x-1}{2x+1}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 2

Question:

Divide $3x^3 + 12x^2 + 5x + 20$ by $(x + 4)$.

Solution:

$$\begin{array}{r}
 3x^2 \quad + \quad 5 \\
 x + 4 \overline{) 3x^3 + 12x^2 + 5x + 20} \\
 \underline{3x^3 + 12x^2} \\
 0 5x + 20 \\
 \underline{5x + 20} \\
 0
 \end{array}$$

Answer is $3x^2 + 5$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 3

Question:

Simplify $\frac{2x^3 + 3x + 5}{x + 1}$.

Solution:

$$\begin{array}{r}
 x + 1 \overline{) \begin{array}{r} 2x^3 - 2x \quad + 5 \\ 2x^3 + 0x^2 + 3x + 5 \\ \hline 2x^3 + 2x^2 \\ - 2x^2 + 3x \\ - 2x^2 - 2x \\ 5x + 5 \\ 5x + 5 \\ \hline 0 \end{array} \\
 \end{array}$$

So $\frac{2x^3 + 3x + 5}{x + 1} = 2x^2 - 2x + 5$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 4

Question:

Show that $(x - 3)$ is a factor of $2x^3 - 2x^2 - 17x + 15$. Hence express $2x^3 - 2x^2 - 17x + 15$ in the form $(x - 3) \left(Ax^2 + Bx + C \right)$, where the values A , B and C are to be found.

Solution:

$$f(x) = 2x^3 - 2x^2 - 17x + 15$$

$$f(3) = 2(3)^3 - 2(3)^2 - 17(3) + 15$$

$$f(3) = 54 - 18 - 51 + 15 = 0$$

So $(x - 3)$ is a factor.

$$\begin{array}{r}
 2x^2 + 4x - 5 \\
 x - 3 \overline{) 2x^3 - 2x^2 - 17x + 15} \\
 \underline{2x^3 - 6x^2} \\
 4x^2 - 17x \\
 \underline{4x^2 - 12x} \\
 -5x + 15 \\
 \underline{-5x + 15} \\
 0
 \end{array}$$

$$\text{So } 2x^3 - 2x^2 - 17x + 15 = (x - 3)(2x^2 + 4x - 5).$$

$$\text{So } A = 2, B = 4, C = -5$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 5

Question:

Show that $(x - 2)$ is a factor of $x^3 + 4x^2 - 3x - 18$. Hence express $x^3 + 4x^2 - 3x - 18$ in the form $(x - 2)(px + q)^2$, where the values p and q are to be found.

Solution:

$$f(x) = x^3 + 4x^2 - 3x - 18$$

$$f(2) = (2)^3 + 4(2)^2 - 3(2) - 18$$

$$f(2) = 8 + 16 - 6 - 18 = 0$$

So $(x - 2)$ is a factor.

$$\begin{array}{r}
 x^2 + 6x + 9 \\
 x - 2 \overline{) x^3 + 4x^2 - 3x - 18} \\
 \underline{x^3 - 2x^2} \\
 6x^2 - 3x \\
 \underline{6x^2 - 12x} \\
 9x - 18 \\
 \underline{9x - 18} \\
 0
 \end{array}$$

$$\text{Now } x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$$

$$\text{So } x^3 + 4x^2 - 3x - 18 = (x - 2)(x + 3)^2.$$

$$\text{So } p = 1, q = 3.$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 6

Question:

Factorise completely $2x^3 + 3x^2 - 18x + 8$.

Solution:

$$f(x) = 2x^3 + 3x^2 - 18x + 8$$

$$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8$$

$$f(2) = 16 + 12 - 36 + 8 = 0$$

So $(x - 2)$ is a factor.

$$\begin{array}{r}
 2x^2 + 7x - 4 \\
 x - 2 \overline{) 2x^3 + 3x^2 - 18x + 8} \\
 \underline{2x^3 - 4x^2} \\
 7x^2 - 18x \\
 \underline{7x^2 - 14x} \\
 -4x + 8 \\
 \underline{-4x + 8} \\
 0
 \end{array}$$

$$\text{Now } 2x^2 + 7x - 4 = (2x - 1)(x + 4)$$

$$\text{So } 2x^3 + 3x^2 - 18x + 8 = (x - 2)(2x - 1)(x + 4).$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 7

Question:

Find the value of k if $(x - 2)$ is a factor of $x^3 - 3x^2 + kx - 10$.

Solution:

$$f(x) = x^3 - 3x^2 + kx - 10$$

$$f(2) = 0$$

$$\text{So } (2)^3 - 3(2)^2 + k(2) - 10 = 0$$

$$8 - 12 + 2k - 10 = 0$$

$$2k = 14$$

$$k = 7$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 8

Question:

Find the remainder when $16x^5 - 20x^4 + 8$ is divided by $(2x - 1)$.

Solution:

$$f(x) = 16x^5 - 20x^4 + 8$$

$$f\left(\frac{1}{2}\right) = 16\left(\frac{1}{2}\right)^5 - 20\left(\frac{1}{2}\right)^4 + 8$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{5}{4} + 8 = 7\frac{1}{4}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 9

Question:

$f(x) = 2x^2 + px + q$. Given that $f(-3) = 0$, and $f(4) = 2$:

(a) find the value of p and q

(b) factorise $f(x)$

Solution:

(a) $f(x) = 2x^2 + px + q$

① $f(-3) = 0$

So $2(-3)^2 + p(-3) + q = 0$

$18 - 3p + q = 0$

$3p - q = 18$

② $f(4) = 21$

So $2(4)^2 + p(4) + q = 21$

$4p + q = -11$

Solving simultaneously:

$$3p - q = 18$$

$$4p + q = -11$$

$$7p = 7$$

$$p = 1$$

Substitute $p = 1$ into $4p + q = -11$:

$$4(1) + q = -11$$

$$q = -15$$

Check: $3p - q = 3(1) - (-15) = 3 + 15 = 18$ ✓

So $p = 1, q = -15$

(b) $f(x) = 2x^2 + x - 15 = (2x - 5)(x + 3)$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 10

Question:

$h(x) = x^3 + 4x^2 + rx + s$. Given $h(-1) = 0$, and $h(2) = 30$:

(a) find the value of r and s

(b) find the remainder when $h(x)$ is divided by $(3x - 1)$

Solution:

(a) $h(x) = x^3 + 4x^2 + rx + s$

① $h(-1) = 0$

So $(-1)^3 + 4(-1)^2 + r(-1) + s = 0$

$-1 + 4 - r + s = 0$

$-r + s = -3$

② $h(2) = 30$

So $(2)^3 + 4(2)^2 + r(2) + s = 30$

$8 + 16 + 2r + s = 30$

$2r + s = 6$

Solving simultaneously:

$$2r + s = 6$$

$$-r + s = -3$$

$$3r = 9$$

$r = 3$

Substitute $r = 3$ into $-r + s = -3$:

$-3 + s = -3$

$s = 0$

Check: $2r + s = 2(3) + (0) = 6$ ✓

So $r = 3$, $s = 0$

(b) $h(x) = x^3 + 4x^2 + 3x$

$$h\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)$$

$$h\left(\frac{1}{3}\right) = \frac{1}{27} + \frac{4}{9} + 1 = 1\frac{13}{27}$$

Remainder is $1\frac{13}{27}$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 11

Question:

$$g(x) = 2x^3 + 9x^2 - 6x - 5.$$

(a) Factorise $g(x)$

(b) Solve $g(x) = 0$

Solution:

$$(a) g(x) = 2x^3 + 9x^2 - 6x - 5$$

$$g(1) = 2(1)^3 + 9(1)^2 - 6(1) - 5$$

$$g(1) = 2 + 9 - 6 - 5 = 0$$

So $(x - 1)$ is a factor.

$$\begin{array}{r}
 2x^2 + 11x + 5 \\
 x - 1 \overline{) 2x^3 + 9x^2 - 6x - 5} \\
 \underline{2x^3 - 2x^2} \\
 11x^2 - 6x \\
 \underline{11x^2 - 11x} \\
 5x - 5 \\
 \underline{5x - 5} \\
 0
 \end{array}$$

$$\text{Now } 2x^2 + 11x + 5 = (2x + 1)(x + 5)$$

$$\text{So } g(x) = (x - 1)(2x + 1)(x + 5)$$

$$(b) g(x) = 0$$

$$(x - 1)(2x + 1)(x + 5) = 0$$

$$\text{So } x = 1, x = -\frac{1}{2}, x = -5.$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 12

Question:

The remainder obtained when $x^3 - 5x^2 + px + 6$ is divided by $(x + 2)$ is equal to the remainder obtained when the same expression is divided by $(x - 3)$.
Find the value of p .

Solution:

$$g(x) = x^3 - 5x^2 + px + 6$$

$$\textcircled{1} \quad g(-2) = R$$

$$\text{So } (-2)^3 - 5(-2)^2 + p(-2) + 6 = R$$

$$-8 - 20 - 2p + 6 = R$$

$$-2p - 22 = R$$

$$\textcircled{2} \quad g(3) = R$$

$$\text{So } (3)^3 - 5(3)^2 + p(3) + 6 = R$$

$$27 - 45 + 3p + 6 = R$$

$$3p - 12 = R$$

Solving simultaneously:

$$-2p - 22 = 3p - 12$$

$$-5p = 10$$

$$p = -2$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 13

Question:

The remainder obtained when $x^3 + dx^2 - 5x + 6$ is divided by $(x - 1)$ is twice the remainder obtained when the same expression is divided by $(x + 1)$.
Find the value of d .

Solution:

$$f(x) = x^3 + dx^2 - 5x + 6$$

$$\text{Let } f(-1) = R$$

$$\text{So } (-1)^3 + d(-1)^2 - 5(-1) + 6 = R$$

$$-1 + d + 5 + 6 = R$$

$$d + 10 = R$$

$$\text{Now } f(1) = 2R$$

$$\text{So } (1)^3 + d(1)^2 - 5(1) + 6 = 2R$$

$$1 + d - 5 + 6 = 2R$$

$$d + 2 = 2R$$

Solving simultaneously:

$$d + 2 = 2(d + 10)$$

$$d + 2 = 2d + 20$$

$$2 = d + 20$$

$$d = -18$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 14

Question:

(a) Show that $(x - 2)$ is a factor of $f(x) = x^3 + x^2 - 5x - 2$.

(b) Hence, or otherwise, find the exact solutions of the equation $f(x) = 0$.

[E]

Solution:

$$(a) f(x) = x^3 + x^2 - 5x - 2$$

$$f(2) = (2)^3 + (2)^2 - 5(2) - 2$$

$$f(2) = 8 + 4 - 10 - 2 = 0$$

So $(x - 2)$ is a factor.

$$x - 2 \overline{) \begin{array}{r} x^3 + 3x^2 + 1 \\ x^3 + 2x^2 - 5x - 2 \\ \hline x^2 - 5x - 2 \end{array}}$$

$$(b) \begin{array}{r} 3x^2 - 5x \\ 3x^2 - 6x \\ \hline x - 2 \\ x - 2 \\ \hline 0 \end{array}$$

$$\text{So } f(x) = (x - 2)(x^2 + 3x + 1)$$

$$\text{Now } f(x) = 0 \text{ when } x = 2$$

$$\text{and } x^2 + 3x + 1 = 0$$

$$\text{i.e. } x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} \quad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{5}}{2}$$

$$\text{So } x = 2, x = \frac{-3 + \sqrt{5}}{2}, x = \frac{-3 - \sqrt{5}}{2}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 15

Question:

Given that -1 is a root of the equation $2x^3 - 5x^2 - 4x + 3$, find the two positive roots.

[E]

Solution:

$$\begin{array}{r}
 2x^2 - 7x + 3 \\
 x + 1 \overline{) 2x^3 - 5x^2 - 4x + 3} \\
 \underline{2x^3 + 2x^2} \\
 -7x^2 - 4x \\
 \underline{-7x^2 - 7x} \\
 3x + 3 \\
 \underline{3x + 3} \\
 0
 \end{array}$$

Now $2x^2 - 7x + 3 = (2x - 1)(x - 3)$

So $2x^3 - 5x^2 - 4x + 3 = (x + 1)(2x - 1)(x - 3)$.

The roots are -1 , $\frac{1}{2}$ and 3 .

The positive roots are $x = \frac{1}{2}$ and $x = 3$.