Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise A, Question 1

Question:

Simplify each of the following expressions:

(a) $1 - \cos^2 \frac{1}{2}\theta$

(b) $5 \sin^2 3\theta + 5 \cos^2 3\theta$

(c) $\sin^2 A - 1$

(d) $\frac{\sin \theta}{\tan \theta}$

(e) $\frac{\sqrt{1 - \cos^2 x°}}{\cos x°}$

(f) $\frac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}}$

(g) $(1 + \sin x)^2 + (1 - \sin x)^2 + 2 \cos^2 x$

(h) $\sin^4 \theta + \sin^2 \theta \cos^2 \theta$

(i) $\sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

Solution:

(a) As $\sin^2 \frac{1}{2}\theta + \cos^2 \frac{1}{2}\theta \equiv 1$

So $1 - \cos^2 \frac{1}{2}\theta = \sin^2 \frac{1}{2}\theta$

(b) As $\sin^2 3\theta + \cos^2 3\theta \equiv 1$
So $5 \sin^2 3\theta + 5 \cos^2 3\theta = 5 (\sin^2 3\theta + \cos^2 3\theta) = 5$

(c) As $\sin^2 A + \cos^2 A \equiv 1$
So $\sin^2 A - 1 \equiv - \cos^2 A$

(d) $\frac{\sin \theta}{\tan \theta} = \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}}$

$= \cancel{\sin \theta} \times \frac{\cos \theta}{\cancel{\sin \theta}}$

$= \cos \theta$

(e) $\frac{\sqrt{1 - \cos^2 x°}}{\cos x°} = \frac{\sqrt{\sin^2 x°}}{\cos x°} = \frac{\sin x°}{\cos x°} = \tan x°$

(f) $\frac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}} = \frac{\sqrt{\sin^2 3A}}{\sqrt{\cos^2 3A}} = \frac{\sin 3A}{\cos 3A} = \tan 3A$

(g) $(1 + \sin x°)^2 + (1 - \sin x°)^2 + 2 \cos^2 x°$
$= 1 + 2 \sin x° + \sin^2 x° + 1 - 2 \sin x° + \sin^2 x° + 2 \cos^2 x°$
$= 2 + 2 \sin^2 x° + 2 \cos^2 x°$
$= 2 + 2 (\sin^2 x° + \cos^2 x°)$
$= 2 + 2$
$= 4$

(h) $\sin^4 \theta + \sin^2 \theta \cos^2 \theta = \sin^2 \theta (\sin^2 \theta + \cos^2 \theta) = \sin^2 \theta$

(i) $\sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 = 1^2 = 1$

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Exercise A, Question 2

Question:

Given that $2 \sin \theta = 3 \cos \theta$, find the value of $\tan \theta$.

Solution:

Given $2 \sin \theta = 3 \cos \theta$

So $\frac{\sin \theta}{\cos \theta} = \frac{3}{2}$ (divide both sides by $2 \cos \theta$)

So $\tan \theta = \frac{3}{2}$

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Trigonometrical identities and simple equations

Exercise A, Question 3

Question:

Given that $\sin x \cos y = 3 \cos x \sin y$, express $\tan x$ in terms of $\tan y$.

Solution:

As $\sin x \cos y = 3 \cos x \sin y$

So $\frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} = 3 \frac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y}$

So $\tan x = 3 \tan y$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise A, Question 4

Question:

Express in terms of $\sin\theta$ only:

(a) $\cos^2\theta$

(b) $\tan^2\theta$

(c) $\cos\theta\ \tan\theta$

(d) $\frac{\cos\theta}{\tan\theta}$

(e) $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

Solution:

(a) As $\sin^2\theta + \cos^2\theta \equiv 1$
So $\cos^2\theta \equiv 1 - \sin^2\theta$

(b) $\tan^2\theta \equiv \frac{\sin^2\theta}{\cos^2\theta} \equiv \frac{\sin^2\theta}{1 - \sin^2\theta}$

(c) $\cos\theta \quad \tan\theta$

$$= \cancel{\cos\theta} \times \frac{\sin\theta}{\cancel{\cos\theta}}$$

$= \sin\theta$

(d) $\frac{\cos\theta}{\tan\theta} = \frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}} = \cos\theta \times \frac{\cos\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

So $\frac{\cos\theta}{\tan\theta} = \frac{1 - \sin^2\theta}{\sin\theta}$ or $\frac{1}{\sin\theta} - \sin\theta$

(e) $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \cos^2\theta - \sin^2\theta = (1 - \sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta$

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Trigonometrical identities and simple equations

Exercise A, Question 5

Question:

Using the identities $\sin^2 A + \cos^2 A \equiv 1$ and/or $\tan A \equiv \frac{\sin A}{\cos A}$ $\left(\cos A \neq 0 \right)$, prove that:

(a) $(\sin \theta + \cos \theta)^2 \equiv 1 + 2 \sin \theta \cos \theta$

(b) $\frac{1}{\cos \theta} - \cos \theta \equiv \sin \theta \tan \theta$

(c) $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

(d) $\cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$

(e) $(2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2 \equiv 5$

(f) $2 - (\sin \theta - \cos \theta)^2 \equiv (\sin \theta + \cos \theta)^2$

(g) $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \sin^2 x - \sin^2 y$

Solution:

(a) LHS $= (\sin \theta + \cos \theta)^2$
$= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$
$= (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta$
$= 1 + 2 \sin \theta \cos \theta$
= RHS

(b) LHS $= \frac{1}{\cos \theta} - \cos \theta$

$= \frac{1 - \cos^2 \theta}{\cos \theta}$

$= \frac{\sin^2 \theta}{\cos \theta}$

$= \sin \theta \times \frac{\sin \theta}{\cos \theta}$

$= \sin \theta \tan \theta$
= RHS

(c) LHS $= \tan x^\circ + \frac{1}{\tan x^\circ}$

$= \frac{\sin x^\circ}{\cos x^\circ} + \frac{\cos x^\circ}{\sin x^\circ}$

$= \frac{\sin^2 x^\circ + \cos^2 x^\circ}{\sin x^\circ \cos x^\circ}$

$= \frac{1}{\sin x° \cos x°}$

= RHS

(d) LHS $= \cos^2 A - \sin^2 A$

$\equiv \cos^2 A - (1 - \cos^2 A)$

$\equiv \cos^2 A - 1 + \cos^2 A$

$\equiv 2 \cos^2 A - 1$ ✓

$\equiv 2(1 - \sin^2 A) - 1$

$\equiv 2 - 2 \sin^2 A - 1$

$\equiv 1 - 2 \sin^2 A$ ✓

(e) LHS $= (2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2$

$\equiv 4 \sin^2 \theta - 4 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta + 4 \sin \theta \cos \theta + 4 \cos^2 \theta$

$\equiv 5 \sin^2 \theta + 5 \cos^2 \theta$

$\equiv 5(\sin^2 \theta + \cos^2 \theta)$

$\equiv 5$

$\equiv$ RHS

(f) LHS $\equiv 2 - (\sin \theta - \cos \theta)^2$

$= 2 - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$

$= 2 - (1 - 2 \sin \theta \cos \theta)$

$= 1 + 2 \sin \theta \cos \theta$

$= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$

$= (\sin \theta + \cos \theta)^2$

= RHS

(g) LHS $= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$

$= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x)\sin^2 y$

$= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y$

$= \sin^2 x - \sin^2 y$

= RHS

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Trigonometrical identities and simple equations

Exercise A, Question 6

Question:

Find, without using your calculator, the values of:

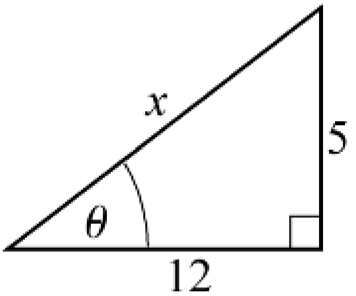
(a) sin θ and cos θ, given that tan $\theta = \frac{5}{12}$ and θ is acute.

(b) sin θ and tan θ, given that cos $\theta = -\frac{3}{5}$ and θ is obtuse.

(c) cos θ and tan θ, given that sin $\theta = -\frac{7}{25}$ and $270° < \theta < 360°$.

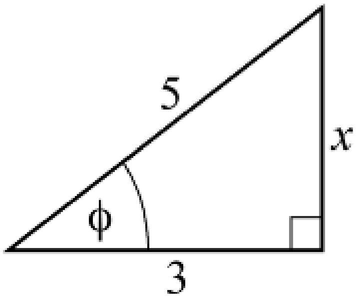
Solution:

(a)

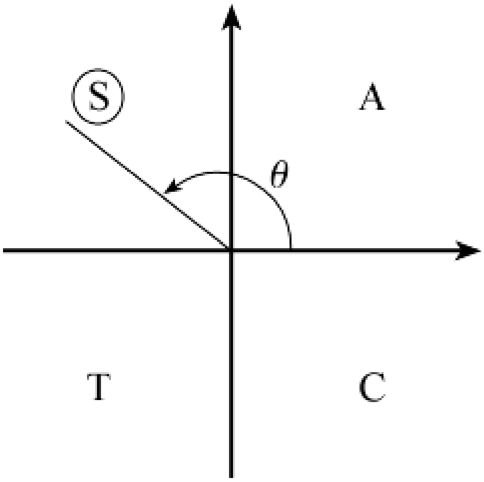


Using Pythagoras' Theorem,
$x^2 = 12^2 + 5^2 = 169$
$x = 13$
So sin $\theta = \frac{5}{13}$ and cos $\theta = \frac{12}{13}$

(b)



Using Pythagoras' Theorem, $x = 4$.
So sin $\phi = \frac{4}{5}$ and tan $\phi = \frac{4}{3}$



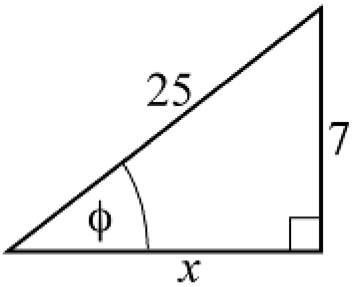
As θ is obtuse,

$\sin\theta = \sin\phi = \frac{4}{5}$

and

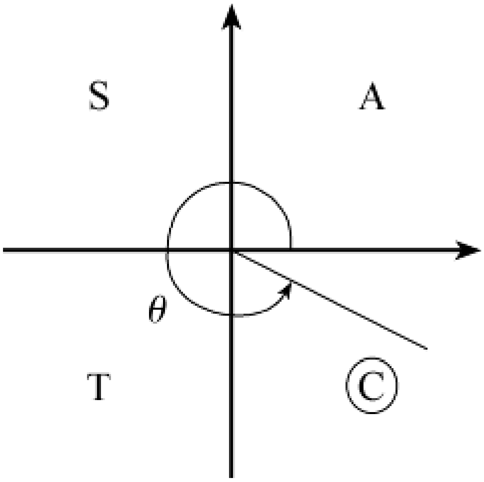
$\tan\theta = -\tan\phi = -\frac{4}{3}$

(c)



Using Pythagoras' Theorem,
$x^2 + 7^2 = 25^2$
$x^2 = 25^2 - 7^2 = 576$
$x = 24$

So $\cos\phi = \frac{24}{25}$ and $\tan\phi = \frac{7}{24}$



As θ is in the 4th quadrant,

$\cos\theta = +\cos\phi = +\frac{24}{25}$

and

$\tan\ \theta = -\tan\ \phi \quad = -\ \frac{7}{24}$

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Trigonometrical identities and simple equations

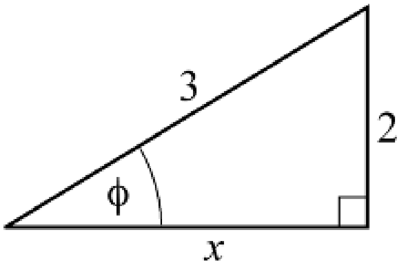
Exercise A, Question 7

Question:

Given that $\sin \theta = \frac{2}{3}$ and that θ is obtuse, find the exact value of: (a) $\cos \theta$, (b) $\tan \theta$.

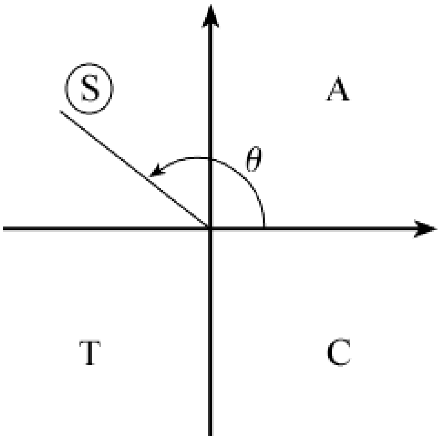
Solution:

Consider the angle ϕ where $\sin \phi = \frac{2}{3}$.



Using Pythagoras' Theorem, $x = \sqrt{5}$

(a) So $\cos \phi = \frac{\sqrt{5}}{3}$



As θ is obtuse, $\cos \theta = - \cos \phi = - \frac{\sqrt{5}}{3}$

(b) From the triangle,

$\tan \phi = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Using the quadrant diagram,

$\tan \theta = - \tan \phi = - \frac{2\sqrt{5}}{5}$

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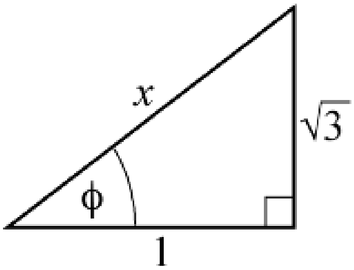
Trigonometrical identities and simple equations
Exercise A, Question 8

Question:

Given that $\tan \theta = -\sqrt{3}$ and that θ is reflex, find the exact value of: (a) $\sin \theta$, (b) $\cos \theta$.

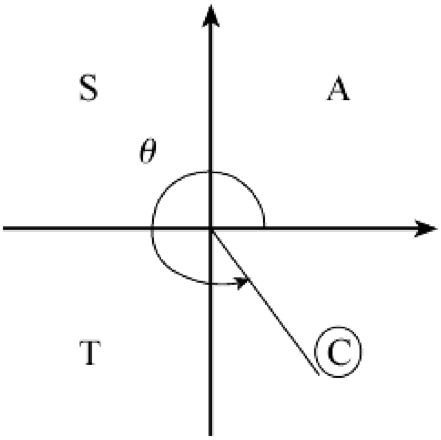
Solution:

Draw a right-angled triangle with $\tan \phi = +\sqrt{3} = \frac{\sqrt{3}}{1}$



Using Pythagoras' Theorem,
$x^2 = (\sqrt{3})^2 + 1^2 = 4$
So $x = 2$

(a) $\sin \phi = \frac{\sqrt{3}}{2}$



As θ is reflex and $\tan \theta$ is − ve, θ is in the 4th quadrant.
So $\sin \theta = -\sin \phi = \frac{-\sqrt{3}}{2}$

(b) $\cos \phi = \frac{1}{2}$

As $\cos \theta = \cos \phi$, $\cos \theta = \frac{1}{2}$

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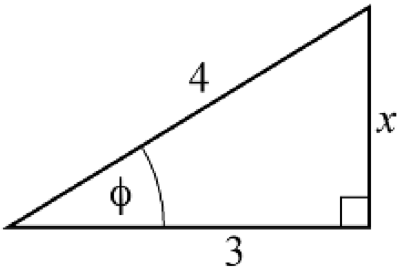
Trigonometrical identities and simple equations
Exercise A, Question 9

Question:

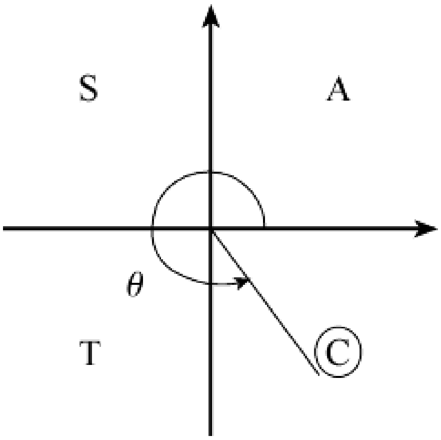
Given that $\cos \theta = \frac{3}{4}$ and that θ is reflex, find the exact value of: (a) $\sin \theta$, (b) $\tan \theta$.

Solution:

Draw a right-angled triangle with $\cos \phi = \frac{3}{4}$



Using Pythagoras' Theorem,
$x^2 + 3^2 = 4^2$
$x^2 = 4^2 - 3^2 = 7$
$x = \sqrt{7}$
So $\sin \phi = \frac{\sqrt{7}}{4}$ and $\tan \phi = \frac{\sqrt{7}}{3}$



As θ is reflex and $\cos \theta$ is +ve, θ is in the 4th quadrant.

(a) $\sin \theta = - \sin \phi = - \frac{\sqrt{7}}{4}$

(b) $\tan \theta = - \tan \phi = - \frac{\sqrt{7}}{3}$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise A, Question 10

Question:

In each of the following, eliminate θ to give an equation relating x and y:

(a) $x = \sin \theta, y = \cos \theta$

(b) $x = \sin \theta, y = 2 \cos \theta$

(c) $x = \sin \theta, y = \cos^2 \theta$

(d) $x = \sin \theta, y = \tan \theta$

(e) $x = \sin \theta + \cos \theta, y = \cos \theta - \sin \theta$

Solution:

(a) As $\sin^2 \theta + \cos^2 \theta \equiv 1$
$x^2 + y^2 = 1$

(b) $\sin \theta = x$ and $\cos \theta = \frac{y}{2}$

So, using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$x^2 + \left(\frac{y}{2} \right)^2 = 1$ or $x^2 + \frac{y^2}{4} = 1$ or $4x^2 + y^2 = 4$

(c) As $\sin \theta = x$, $\sin^2 \theta = x^2$
Using $\sin^2 \theta + \cos^2 \theta \equiv 1$
$x^2 + y = 1$

(d) As $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\cos \theta = \frac{\sin \theta}{\tan \theta}$

So $\cos \theta = \frac{x}{y}$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$x^2 + \frac{x^2}{y^2} = 1$ or $x^2 y^2 + x^2 = y^2$

(e) $\sin \theta + \cos \theta = x$
$-\sin \theta + \cos \theta = y$
Adding up the two equations: $2 \cos \theta = x + y$

So $\cos \theta = \frac{x + y}{2}$

Subtracting the two equations: $2 \sin \theta = x - y$

So $\sin \theta = \frac{x - y}{2}$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$\left(\frac{x-y}{2}\right)^2+\left(\frac{x+y}{2}\right)^2=1$$

$x^2-2xy+y^2+x^2+2xy+y^2=4$

$2x^2+2y^2=4$

$x^2+y^2=2$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise B, Question 1

Question:

Solve the following equations for θ, in the interval $0 < \theta \leq 360°$:

(a) $\sin \theta = -1$

(b) $\tan \theta = \sqrt{3}$

(c) $\cos \theta = \frac{1}{2}$

(d) $\sin \theta = \sin 15°$

(e) $\cos \theta = -\cos 40°$

(f) $\tan \theta = -1$

(g) $\cos \theta = 0$

(h) $\sin \theta = -0.766$

(i) $7 \sin \theta = 5$

(j) $2 \cos \theta = -\sqrt{2}$

(k) $\sqrt{3} \sin \theta = \cos \theta$

(l) $\sin \theta + \cos \theta = 0$

(m) $3 \cos \theta = -2$

(n) $(\sin \theta - 1) (5 \cos \theta + 3) = 0$

(o) $\tan \theta = \tan \theta (2 + 3 \sin \theta)$

Solution:

(a) Using the graph of $y = \sin \theta$
$\sin \theta = -1$ when $\theta = 270°$

(b) $\tan \theta = \sqrt{3}$
The calculator solution is $60°$ ($\tan^{-1} \sqrt{3}$) and, as $\tan \theta$ is +ve, θ lies in the 1st and 3rd quadrants.
$\theta = 60°$ and $(180° + 60°) = 60° , 240°$

(c) $\cos \theta = \frac{1}{2}$

Calculator solution is 60° and as $\cos \theta$ is +ve, θ lies in the 1st and 4th quadrants.
$\theta = 60°$ and $(360° - 60°) = 60° , 300°$

(d) $\sin \theta = \sin 15°$
The acute angle satisfying the equation is $\theta = 15°$.
As $\sin \theta$ is +ve, θ lies in the 1st and 2nd quadrants, so

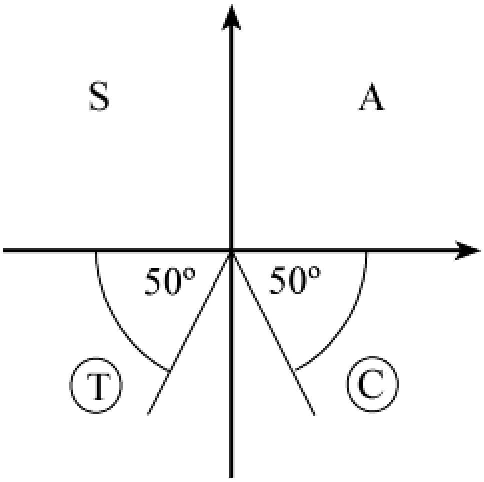
$\theta = 15°$ and $(180° - 15°) = 15°, 165°$

(e) A first solution is $\cos^{-1}(-\cos 40°) = 140°$
A second solution of $\cos\theta = k$ is $360° - $ 1st solution.
So second solution is 220°
(Use the quadrant diagram as a check.)

(f) A first solution is $\tan^{-1}(-1) = -45°$
Use the quadrant diagram, noting that as tan is − ve, solutions are in the 2nd and 4th quadrants.
($-45°$ is not in the given interval)
So solutions are 135° and 315°.

(g) From the graph of $y = \cos\theta$
$\cos\theta = 0$ when $\theta = 90°, 270°$

(h) The calculator solution is $-50.0°$ (3 s.f.)
As $\sin\theta$ is − ve, θ lies in the 3rd and 4th quadrants.



Solutions are 230° and 310°.
[These are $180° + \alpha$ and $360° - \alpha$ where $\alpha = \cos^{-1}(-0.766)$]

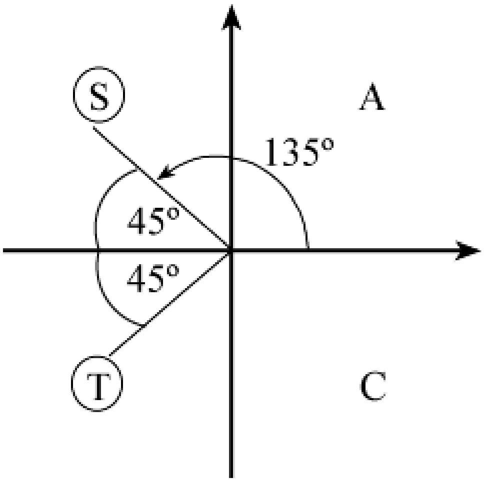
(i) $\sin\theta = \frac{5}{7}$

First solution is $\sin^{-1}\left(\frac{5}{7}\right) = 45.6°$

Second solution is $180° - 45.6° = 134.4°$

(j) $\cos\theta = -\frac{\sqrt{2}}{2}$

Calculator solution is 135°
As $\cos\theta$ is − ve, θ is in the 2nd and 3rd quadrants.



Solutions are 135° and 225° (135° and $360° - 135°$)

(k) $\sqrt{3} \sin \theta = \cos \theta$

So $\tan \theta = \frac{1}{\sqrt{3}}$ dividing both sides by $\sqrt{3} \cos \theta$

Calculator solution is 30°
As $\tan \theta$ is +ve, θ is in the 1st and 3rd quadrants.
Solutions are 30°, 210° (30° and $180° + 30°$)

(l) $\sin \theta + \cos \theta = 0$
So $\sin \theta = - \cos \theta \Rightarrow \tan \theta = -1$
Calculator solution $(-45°)$ is not in given interval
As $\tan \theta$ is $-$ ve, θ is in the 2nd and 4th quadrants.
Solutions are 135° and 315° $[180° + \tan^{-1} (-1) , 360° + \tan^{-1} (-1)]$

(m) Calculator solution is $\cos^{-1} \left(- \frac{2}{3} \right) = 131.8°$ (1 d.p.)

Second solution is $360° - 131.8° = 228.2°$

(n) As $(\sin \theta - 1) (5 \cos \theta + 3) = 0$
either $\sin \theta - 1 = 0$ or $5 \cos \theta + 3 = 0$

So $\sin \theta = 1$ or $\cos \theta = - \frac{3}{5}$

Use the graph of $y = \sin \theta$ to read off solutions of $\sin \theta = 1$
$\sin \theta = 1 \Rightarrow \theta = 90°$

For $\cos \theta = - \frac{3}{5}$,

calculator solution is $\cos^{-1} \left(- \frac{3}{5} \right) = 126.9°$

second solution is $360° - 126.9° = 233.1°$
Solutions are 90°, 126.9°, 233.1°

(o) Rearrange as
$\tan \theta (2 + 3 \sin \theta) - \tan \theta = 0$
$\tan \theta [(2 + 3 \sin \theta) - 1] = 0$ factorising
$\tan \theta (3 \sin \theta + 1) = 0$

So $\tan \theta = 0$ or $\sin \theta = - \frac{1}{3}$

From graph of $y = \tan \theta$, $\tan \theta = 0 \Rightarrow \theta = 180°$, 360° (0° not in given interval)

For $\sin\theta = -\frac{1}{3}$, calculator solution $(-19.5°)$ is not in interval.

Solutions are $180° - \sin^{-1}\left(-\frac{1}{3}\right)$ and $360° + \sin^{-1}\left(-\frac{1}{3}\right)$ or use quadrant diagram.

Complete set of solutions 180°, 199.5°, 340.5°, 360°

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise B, Question 2

Question:

Solve the following equations for *x*, giving your answers to 3 significant figures where appropriate, in the intervals indicated:

(a) $\sin x° = -\frac{\sqrt{3}}{2}, -180 \le x \le 540$

(b) $2 \sin x° = -0.3, -180 \le x \le 180$

(c) $\cos x° = -0.809, -180 \le x \le 180$

(d) $\cos x° = 0.84, -360 < x < 0$

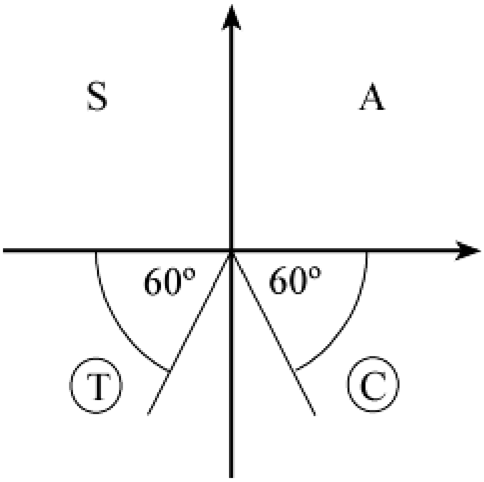
(e) $\tan x° = -\frac{\sqrt{3}}{3}, 0 \le x \le 720$

(f) $\tan x° = 2.90, 80 \le x \le 440$

Solution:

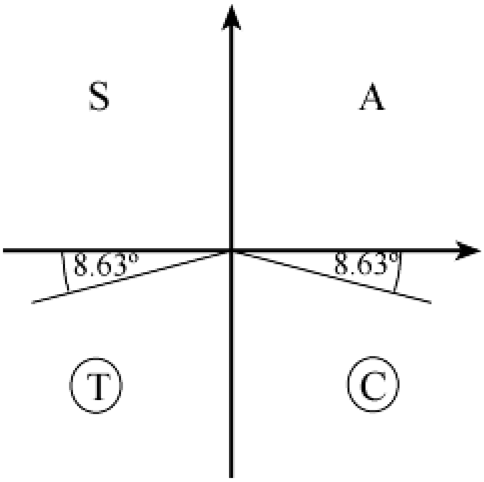
(a) Calculator solution of $\sin x° = -\frac{\sqrt{3}}{2}$ is $x = -60$

As $\sin x°$ is $-$ve, *x* is in the 3rd and 4th quadrants.



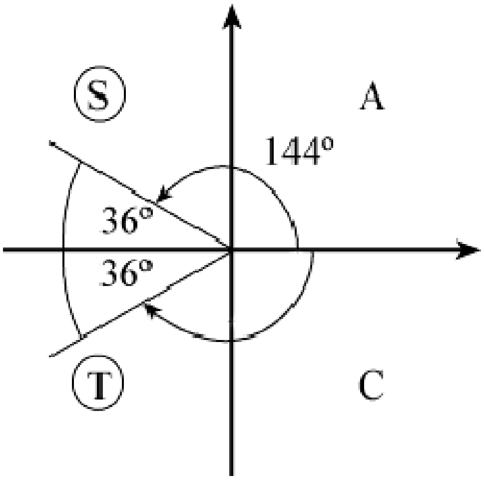
Read off all solutions in the interval $-180 \le x \le 540$
$x = -120, -60, 240, 300$

(b) $2 \sin x° = -0.3$
$\sin x° = -0.15$
First solution is $x = \sin^{-1}(-0.15) = -8.63$ (3 s.f.)
As $\sin x°$ is $-$ve, *x* is in the 3rd and 4th quadrants.



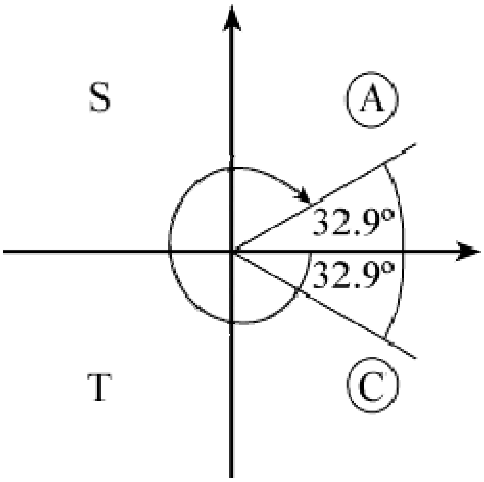
Read off all solutions in the interval $-180 \leq x \leq 180$
$x = -171.37, -8.63 = -171, -8.63$ (3 s.f.)

(c) $\cos x° = -0.809$
Calculator solution is 144 (3 s.f.)
As $\cos x°$ is $-$ ve, x is in the 2nd and 3rd quadrants.



Read off all solutions in the interval $-180 \leq x \leq 180$
$x = -144, +144$
[*Note:* Here solutions are $\cos^{-1}(-0.809)$ and $\{360 - \cos^{-1}(-0.809)\}\{-360\}$]

(d) $\cos x° = 0.84$
Calculator solution is 32.9 (3 s.f.) (not in interval)
As $\cos x°$ is +ve, x is in the 1st and 4th quadrants.

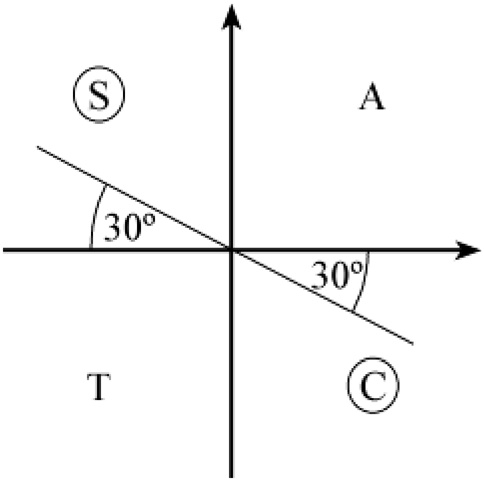


Read off all solutions in the interval $-360 < x < 0$
$x = -327, -32.9$ (3 s.f.)
[*Note:* Here solutions are $\cos^{-1}(0.84) - 360$ and $\{360 - \cos^{-1}(0.84)\} - 360$]

(e) $\tan x° = -\frac{\sqrt{3}}{3}$

Calculator solution is $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -30$ (not in interval)

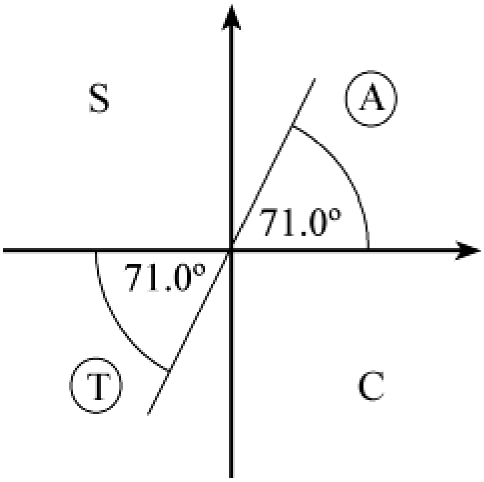
As $\tan x°$ is $-$ve, x is in the 2nd and 4th quadrants.



Read off all solutions in the interval $0 \leq x \leq 720$
$x = 150, 330, 510, 690$

[*Note:* Here solutions are $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 180$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 360$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 540$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 720$]

(f) $\tan x° = 2.90$
Calculator solution is $\tan^{-1}(2.90) = 71.0$ (3 s.f.) (not in interval)
As $\tan x°$ is +ve, x is in the 1st and 3rd quadrants.



Read off all solutions in the interval 80 ≤ x ≤ 440
$x = 251, 431$
[*Note:* Here solutions are $\tan^{-1}(2.90) + 180$, $\tan^{-1}(2.90) + 360$]

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise B, Question 3

Question:

Solve, in the intervals indicated, the following equations for θ, where θ is measured in radians. Give your answer in terms of π or 2 decimal places.

(a) $\sin \theta = 0, \ -2\pi < \theta \leq 2\pi$

(b) $\cos \theta = -\frac{1}{2}, \ -2\pi < \theta \leq \pi$

(c) $\sin \theta = \frac{1}{\sqrt{2}}, \ -2\pi < \theta \leq \pi$

(d) $\sin \theta = \tan \theta, 0 < \theta \leq 2\pi$

(e) $2 (1 + \tan \theta) = 1 - 5 \tan \theta , \ -\pi < \theta \leq 2\pi$

(f) $2 \cos \theta = 3 \sin \theta, 0 < \theta \leq 2\pi$

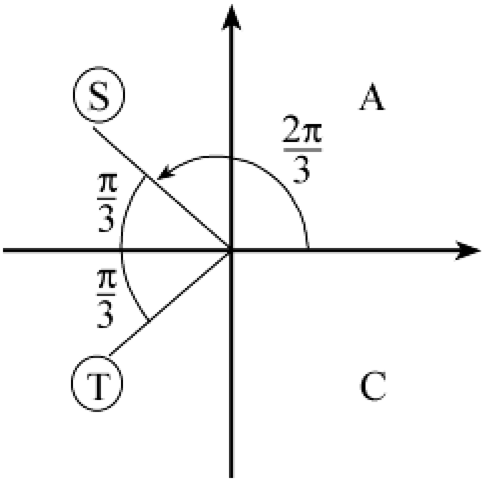
Solution:

(a) Use your graph of $y = \sin \theta$ to read off values of θ for which $\sin \theta = 0$.
In the interval $-2\pi < \theta \leq 2\pi$, solutions are $-\pi, 0, \pi, 2\pi$.

(b) Calculator solution of $\cos \theta = -\frac{1}{2}$ is $\cos^{-1}\left(-\frac{1}{2}\right) = 2.09$ radians

[You should know that $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$]

As $\cos \theta$ is $-$ve, θ is in 2nd and 3rd quadrants.

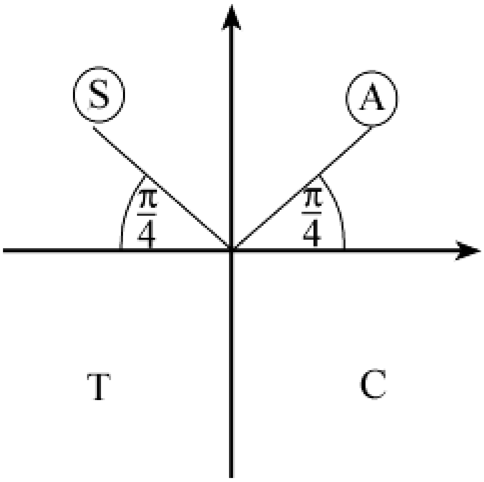


Read off all solutions in the interval $-2\pi < \theta \leq \pi$

$\theta = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$ $(-4.19, -2.09, +2.09)$

(c) Calculator solution of $\sin\ \theta = \frac{1}{\sqrt{2}}$ is $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 0.79$ radians or $\frac{\pi}{4}$

As sin θ is +ve, θ is in the 1st and 2nd quadrants.



Read off all solutions in the interval $-2\pi < \theta \leq \pi$

$\theta = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

(d) $\sin\ \theta = \tan\ \theta$

$\sin\ \theta = \frac{\sin\ \theta}{\cos\ \theta}$

(multiply through by cos θ)
$\sin\ \theta\ \cos\ \theta = \sin\ \theta$
$\sin\ \theta\ \cos\ \theta - \sin\ \theta = 0$
$\sin\ \theta (\cos\ \theta - 1) = 0$
So $\sin\ \theta = 0$ or $\cos\ \theta = 1$ for $0 < \theta \leq 2\pi$
From the graph if $y = \sin\ \theta$, $\sin\ \theta = 0$ where $\theta = \pi, 2\pi$
From the graph of $y = \cos\ \theta$, $\cos\ \theta = 1$ where $\theta = 2\pi$
So solutions are $\pi, 2\pi$

(e) $2 (1 + \tan\ \theta) = 1 - 5\ \tan\ \theta$

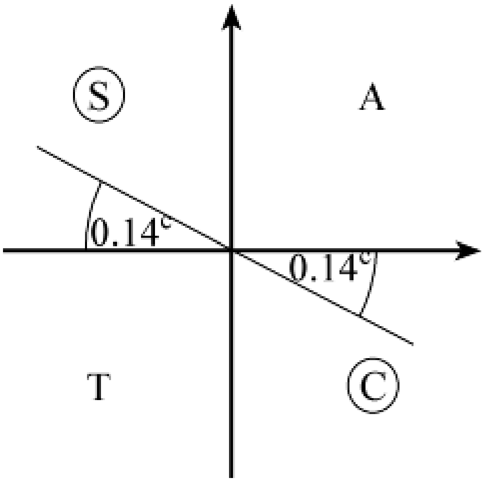
$\Rightarrow\ 2 + 2\ \tan\ \theta = 1 - 5\ \tan\ \theta$

$\Rightarrow\ 7\ \tan\ \theta = -1$

$\Rightarrow\ \tan\ \theta = -\frac{1}{7}$

Calculator solution is $\theta = \tan^{-1}\left(-\frac{1}{7}\right) = -0.14$ radians (2 d.p.)

As tan θ is − ve, θ is in the 2nd and 4th quadrants.



Read off all solutions in the interval $-\pi < \theta \leq 2\pi$

$\theta = -0.14, 3.00, 6.14 \left[\tan^{-1}\left(-\frac{1}{7}\right), \tan^{-1}\left(-\frac{1}{7}\right)+\pi, \tan^{-1}\left(-\frac{1}{7}\right)+2\pi \right]$

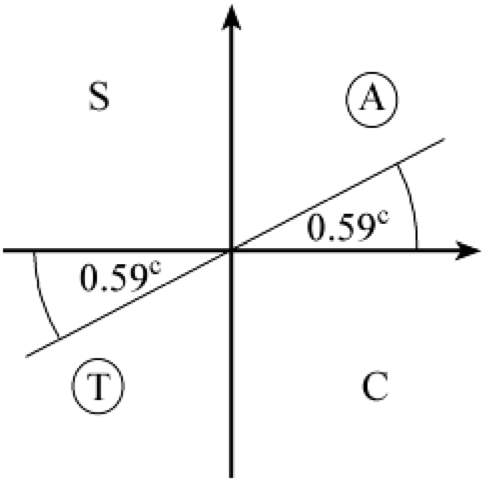
(f) As $2\cos\theta = 3\sin\theta$

$$\frac{2\cancel{\cos\theta}}{3\cancel{\cos\theta}} = \frac{\cancel{3}\sin\theta}{\cancel{3}\cos\theta}$$

So $\tan\theta = \frac{2}{3}$

Calculator solution is $\theta = \tan^{-1}\left(\frac{2}{3}\right) = 0.59$ radians (2 d.p.)

As $\tan\theta$ is +ve, θ is in the 1st and 3rd quadrants.



Read off all solutions in the interval $0 < \theta \leq 2\pi$

$\theta = 0.59, 3.73 \left[\tan^{-1}\left(\frac{2}{3}\right), \tan^{-1}\left(\frac{2}{3}\right)+\pi \right]$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise C, Question 1

Question:

Find the values of θ, in the interval $0 \leq \theta \leq 360°$, for which:

(a) $\sin 4\theta = 0$

(b) $\cos 3\theta = -1$

(c) $\tan 2\theta = 1$

(d) $\cos 2\theta = \frac{1}{2}$

(e) $\tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}}$

(f) $\sin \left(-\theta \right) = \frac{1}{\sqrt{2}}$

(g) $\tan (45° - \theta) = -1$

(h) $2 \sin (\theta - 20°) = 1$

(i) $\tan (\theta + 75°) = \sqrt{3}$

(j) $\cos (50° + 2\theta) = -1$

Solution:

(a) $\sin 4\theta = 0 \quad 0 \leq \theta \leq 360°$
Let $X = 4\theta$ so $0 \leq X \leq 1440°$
Solve $\sin X = 0$ in the interval $0 \leq X \leq 1440°$
From the graph of $y = \sin X$, $\sin X = 0$ where
$X = 0, 180°, 360°, 540°, 720°, 900°, 1080°, 1260°, 1440°$
$\theta = \frac{X}{4} = 0, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°$

(b) $\cos 3\theta = -1 \quad 0 \leq \theta \leq 360°$
Let $X = 3\theta$ so $0 \leq X \leq 1080°$
Solve $\cos X = -1$ in the interval $0 \leq X \leq 1080°$
From the graph of $y = \cos X$, $\cos X = -1$ where
$X = 180°, 540°, 900°$
$\theta = \frac{X}{3} = 60°, 180°, 300°$

(c) $\tan 2\theta = 1 \quad 0 \leq \theta \leq 360°$
Let $X = 2\theta$
Solve $\tan X = 1$ in the interval $0 \leq X \leq 720°$
A solution is $X = \tan^{-1} 1 = 45°$
As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.
So $X = 45°, 225°, 405°, 585°$

$\theta = \frac{X}{2} = 22\frac{1}{2}°, 112\frac{1}{2}°, 202\frac{1}{2}°, 292\frac{1}{2}°$

(d) $\cos 2\theta = \frac{1}{2} \quad 0 \leq \theta \leq 360°$

Let $X = 2\theta$

Solve $\cos X = \frac{1}{2}$ in the interval $0 \leq X \leq 720°$

A solution is $X = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

As cos X is +ve, X is in the 1st and 4th quadrants.
So $X = 60°, 300°, 420°, 660°$

$\theta = \frac{X}{2} = 30°, 150°, 210°, 330°$

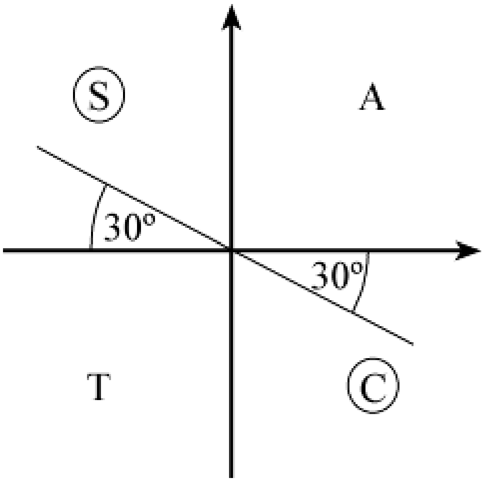
(e) $\tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}} \quad 0 \leq \theta \leq 360°$

Let $X = \frac{1}{2}\theta$

Solve $\tan X = -\frac{1}{\sqrt{3}}$ in the interval $0 \leq X \leq 180°$

A solution is $X = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30°$ (not in interval)

As tan X is − ve, X is in the 2nd and 4th quadrants.



Read off solutions in the interval $0 \leq X \leq 180°$
$X = 150°$
So $\theta = 2X = 300°$

(f) $\sin\left(-\theta\right) = \frac{1}{\sqrt{2}} \quad 0 \leq \theta \leq 360°$

Let $X = -\theta$

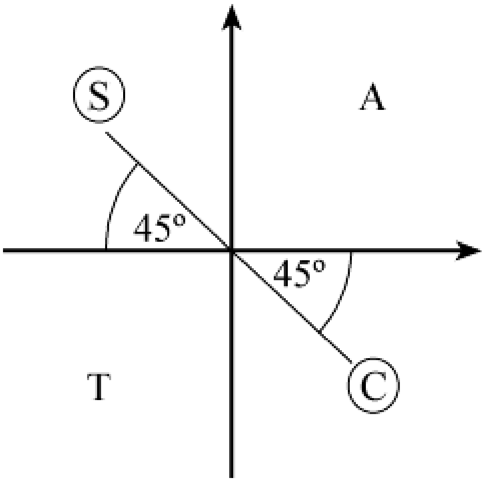
Solve $\sin X = \frac{1}{\sqrt{2}}$ in the interval $0 \geq X \geq -360°$

A solution is $X = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

As sin X is +ve, X is in the 1st and 2nd quadrants.
$X = -315°, -225°$

So $\theta = -X = 225°, 315°$

(g) $\tan(45° - \theta) = -1 \quad 0 \leq \theta \leq 360°$
Let $X = 45° - \theta$ so $0 \geq -\theta \geq -360°$
Solve $\tan X = -1$ in the interval $45° \geq X \geq -315°$
A solution is $X = \tan^{-1}(-1) = -45°$
As $\tan X$ is $-$ve, X is in the 2nd and 4th quadrants.



$X = -225°, -45°$
So $\theta = 45° - X = 90°, 270°$

(h) $2\sin(\theta - 20°) = 1$ so $\sin\left(\theta - 20°\right) = \frac{1}{2} \quad 0 \leq \theta \leq 360°$

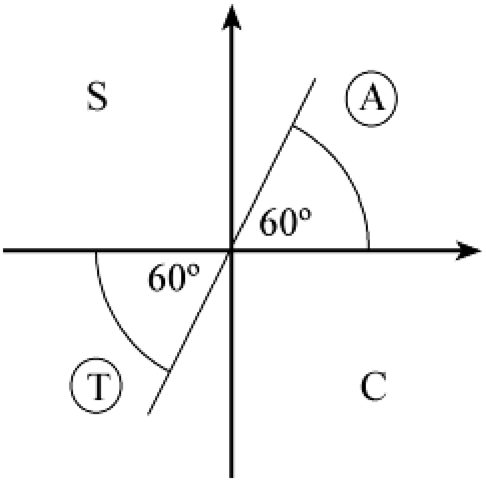
Let $X = \theta - 20°$

Solve $\sin X = \frac{1}{2}$ in the interval $-20° \leq X \leq 340°$

A solution is $X = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

As $\sin X$ is +ve, solutions are in the 1st and 2nd quadrants.
$X = 30°, 150°$
So $\theta = X + 20° = 50°, 170°$

(i) Solve $\tan X = \sqrt{3}$ where $X = (\theta + 75°)$
Interval for X is $75° \leq X \leq 435°$
One solution is $\tan^{-1}(\sqrt{3}) = 60°$ (not in the interval)
As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



$X = 240°, 420°$
So $\theta = X - 75° = 165°, 345°$

(j) Solve $\cos X = -1$ where $X = (50° + 2\theta)$
Interval for X is $50° \leq X \leq 770°$
From the graph of $y = \cos X$, $\cos X = -1$ where
$X = 180°, 540°$
So $2\theta + 50° = 180°, 540°$
$2\theta = 130°, 490°$
$\theta = 65°, 245°$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations
Exercise C, Question 2

Question:

Solve each of the following equations, in the interval given.
Give your answers to 3 significant figures where appropriate.

(a) $\sin\left(\theta - 10°\right) = -\frac{\sqrt{3}}{2}, 0 < \theta \leq 360°$

(b) $\cos(70 - x)° = 0.6, -180 < x \leq 180$

(c) $\tan(3x + 25)° = -0.51, -90 < x \leq 180$

(d) $5\sin 4\theta + 1 = 0, -90° \leq \theta \leq 90°$

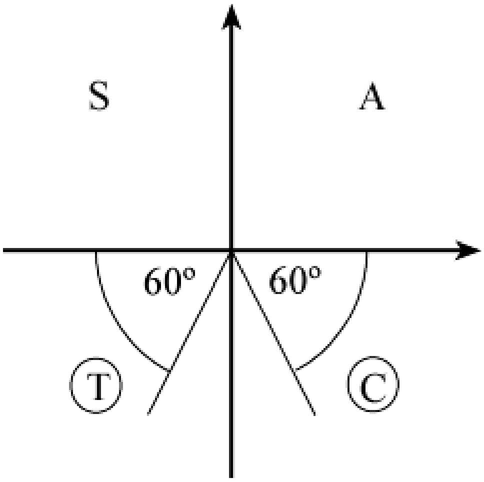
Solution:

(a) Solve $\sin X = -\frac{\sqrt{3}}{2}$ where $X = (\theta - 10°)$

Interval for X is $-10° < X \leq 350°$

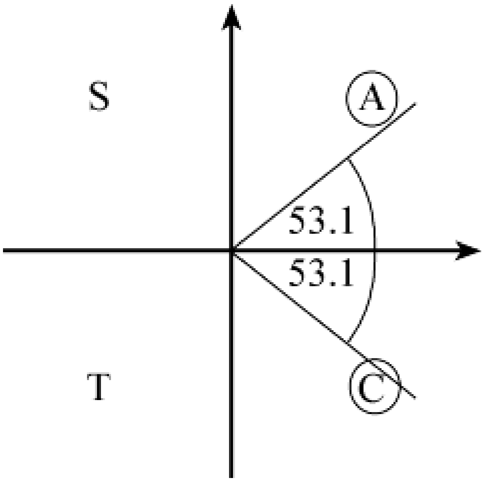
First solution is $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60°$ (not in interval)

As $\sin X$ is $-$ve, X is in the 3rd and 4th quadrants.



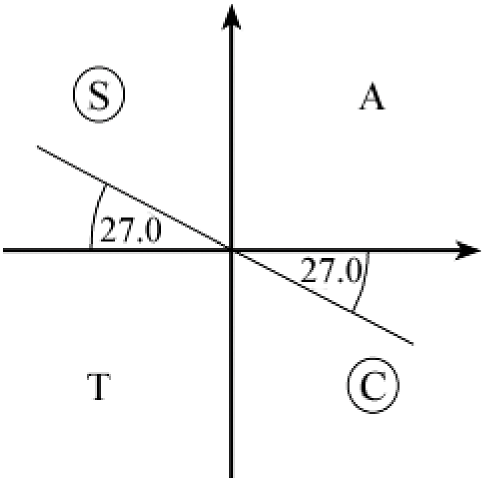
Read off solutions in the interval $-10° < X \leq 350°$
$X = 240°, 300°$
So $\theta = X + 10° = 250°, 310°$

(b) Solve $\cos X° = 0.6$ where $X = (70 - x)$
Interval for X is $180 + 70 > X \geq -180 + 70$ i.e. $-110 \leq X < 250$
First solution is $\cos^{-1}(0.6) = 53.1°$
As $\cos X°$ is +ve, X is in the 1st and 4th quadrants.



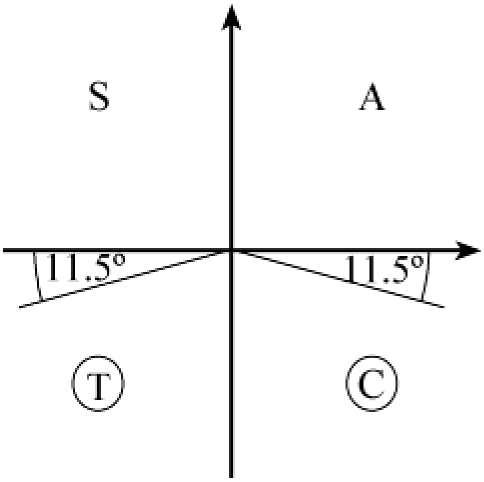
$X = -53.1, +53.1$
So $x = 70 - X =$ 16.9, 123 (3 s.f.)

(c) Solve $\tan X° = -0.51$ where $X = 3x + 25$
Interval for x is $-90 < x \leq 180$
So interval for X is $-245 < X \leq 565$
First solution is $\tan^{-1}(-0.51) = -27.0$
As $\tan X$ is $-$ve, X is in the 2nd and 4th quadrants.



Read off solutions in the interval $-245 < X \leq 565$
$X = -207, -27, 153, 333, 513$
$3x + 25 = -207, -27, 153, 333, 513$
$3x = -232, -52, 128, 308, 488$
So $x = -77.3, -17.3, 42.7, 103, 163$

(d) $5 \sin 4\theta + 1 = 0$
$5 \sin 4\theta = -1$
$\sin 4\theta = -0.2$
Solve $\sin X = -0.2$ where $X = 4\theta$
Interval for X is $-360° \leq X \leq 360°$
First solution is $\sin^{-1}(-0.2) = -11.5°$
As $\sin X$ is $-$ve, X is in the 3rd and 4th quadrants.



Read off solutions in the interval $-360° \leq X \leq 360°$
$X = -168.5°, -11.5°, 191.5°, 348.5°$

So $\theta = \frac{X}{4} = -42.1°, -2.88°, 47.9°, 87.1°$

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Trigonometrical identities and simple equations

Exercise C, Question 3

Question:

Solve the following equations for θ, in the intervals indicated. Give your answers in radians.

(a) $\sin\left(\theta-\frac{\pi}{6}\right)=-\frac{1}{\sqrt{2}},\ -\pi<\theta\leq\pi$

(b) $\cos\left(2\theta+0.2^{c}\right)=-0.2,\ -\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$

(c) $\tan\left(2\theta+\frac{\pi}{4}\right)=1,\ 0\leq\theta\leq2\pi$

(d) $\sin\left(\theta+\frac{\pi}{3}\right)=\tan\frac{\pi}{6},\ 0\leq\theta\leq2\pi$

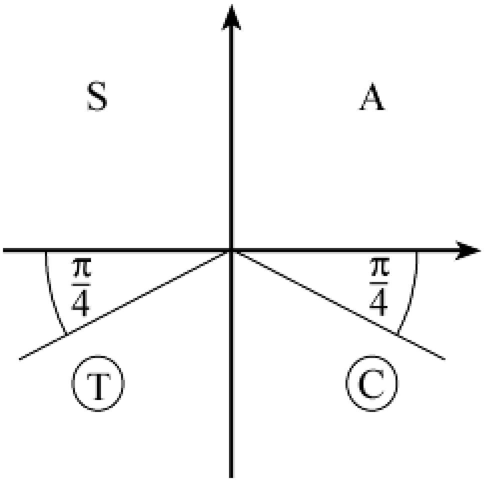
Solution:

(a) Solve $\sin X=-\frac{1}{\sqrt{2}}$ where $X=\theta-\frac{\pi}{6}$

Interval for X is $-\frac{7\pi}{6}\leq X\leq\frac{5\pi}{6}$

First solution is $X=\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)=-\frac{\pi}{4}$

As $\sin X$ is $-$ ve, X is in the 3rd and 4th quadrants.

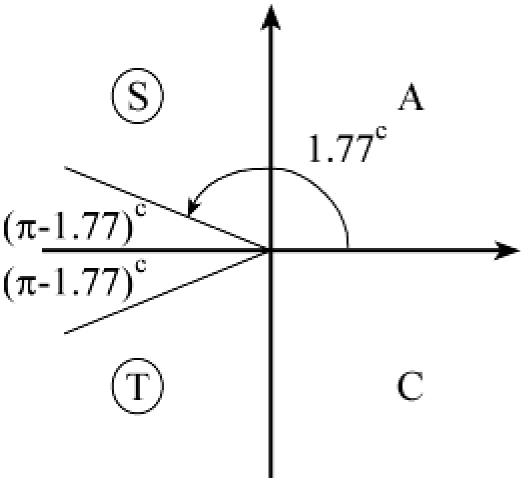


Read off solutions for X in the interval $-\frac{7\pi}{6}\leq X\leq\frac{5\pi}{6}$

$X=-\frac{3\pi}{4},\ -\frac{\pi}{4}$

So $\theta = X + \frac{\pi}{6} = \frac{\pi}{6} - \frac{3\pi}{4}, \frac{\pi}{6} - \frac{\pi}{4} = -\frac{7\pi}{12}, -\frac{\pi}{12}$

(b) Solve $\cos X = -0.2$ where $X = 2\theta + 0.2$ radians
Interval for X is $-\pi + 0.2 \leq X \leq \pi + 0.2$ i.e. $-2.94 \leq X \leq 3.34$
First solution is $X = \cos^{-1}(-0.2) = 1.77\ \ldots$ radians
As $\cos X$ is $-$ve, X is in the 2nd and 3rd quadrants.



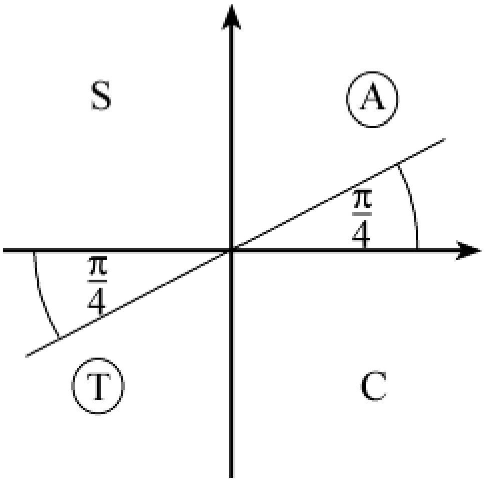
Read off solutions for X in the interval $-2.94 \leq X \leq 3.34$
$X = -1.77, +1.77$ radians
$2\theta + 0.2 = -1.77, +1.77$
$2\theta = -1.97, +1.57$
So $\theta = -0.986, 0.786$

(c) Solve $\tan X = 1$ where $X = 2\theta + \frac{\pi}{4}$

Interval for X is $\frac{\pi}{4} \leq X \leq \frac{17\pi}{4}$

First solution is $X = \tan^{-1} 1 = \frac{\pi}{4}$

As tan is +ve, X is in the 1st and 3rd quadrants.



Read off solutions in the interval $\frac{\pi}{4} \leq X \leq \frac{17\pi}{4}$

$X = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$

$2\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

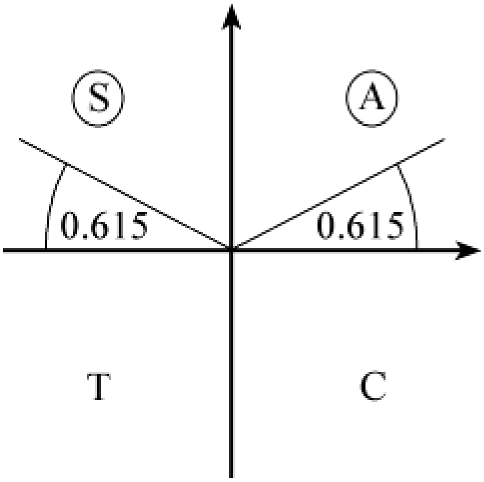
So $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(d) Solve $\sin X = \frac{\sqrt{3}}{3}$ where $X = \theta + \frac{\pi}{3}$

Interval for X is $\frac{\pi}{3} \leq X \leq \frac{7\pi}{3}$ or 1.047 radians $\leq X \leq$ 7.33 radians

First solution is $\sin^{-1}\left(\frac{\sqrt{3}}{3}\right) = 0.615$

As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.



$X = \pi - 0.615, 2\pi + 0.615 = 2.526, 6.899$

So $\theta = X - \frac{\pi}{3} = 1.48, 5.85$

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Trigonometrical identities and simple equations

Exercise D, Question 1

Question:

Solve for θ, in the interval $0 \leq \theta \leq 360°$, the following equations.
Give your answers to 3 significant figures where they are not exact.

(a) $4 \cos^2 \theta = 1$

(b) $2 \sin^2 \theta - 1 = 0$

(c) $3 \sin^2 \theta + \sin \theta = 0$

(d) $\tan^2 \theta - 2 \tan \theta - 10 = 0$

(e) $2 \cos^2 \theta - 5 \cos \theta + 2 = 0$

(f) $\sin^2 \theta - 2 \sin \theta - 1 = 0$

(g) $\tan^2 2\theta = 3$

(h) $4 \sin \theta = \tan \theta$

(i) $\sin \theta + 2 \cos^2 \theta + 1 = 0$

(j) $\tan^2 (\theta - 45°) = 1$

(k) $3 \sin^2 \theta = \sin \theta \cos \theta$

(l) $4 \cos \theta (\cos \theta - 1) = - 5 \cos \theta$

(m) $4 (\sin^2 \theta - \cos \theta) = 3 - 2 \cos \theta$

(n) $2 \sin^2 \theta = 3 (1 - \cos \theta)$

(o) $4 \cos^2 \theta - 5 \sin \theta - 5 = 0$

(p) $\cos^2 \frac{\theta}{2} = 1 + \sin \frac{\theta}{2}$

Solution:

(a) $4 \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = \frac{1}{4}$

So $\cos \theta = \pm \frac{1}{2}$

Solutions are 60°, 120°, 240°, 300°

(b) $2 \sin^2 \theta - 1 = 0 \Rightarrow \sin^2 \theta = \frac{1}{2}$

So $\sin\theta = \pm\frac{1}{\sqrt{2}}$

Solutions are in all four quadrants at 45° to the horizontal.
So $\theta = 45°, 135°, 225°, 315°$

(c) Factorising, $\sin\theta(3\sin\theta + 1) = 0$

So $\sin\theta = 0$ or $\sin\theta = -\frac{1}{3}$

Solutions of $\sin\theta = 0$ are $\theta = 0°, 180°, 360°$ (from graph)

Solutions of $\sin\theta = -\frac{1}{3}$ are $\theta = 199°, 341°$ (3 s.f.) (3rd and 4th quadrants)

(d) $\tan^2\theta - 2\tan\theta - 10 = 0$

So $\tan\theta = \frac{2 \pm \sqrt{4+40}}{2} = \frac{2 \pm \sqrt{44}}{2}\left(= -2.3166\ \ldots \text{ or } 4.3166\ \ldots \right)$

Solutions of $\tan\theta = \frac{2-\sqrt{44}}{2}$ are in the 2nd and 4th quadrants.

So $\theta = 113.35°, 293.3°$

Solutions of $\tan\theta = \frac{2+\sqrt{44}}{2}$ are in the 1st and 3rd quadrants.

So $\theta = 76.95\ \ldots°, 256.95\ \ldots°$
Solution set: 77.0°, 113°, 257°, 293°

(e) Factorise LHS of $2\cos^2\theta - 5\cos\theta + 2 = 0$
$(2\cos\theta - 1)(\cos\theta - 2) = 0$
So $2\cos\theta - 1 = 0$ or $\cos\theta - 2 = 0$
As $\cos\theta \leq 1$, $\cos\theta = 2$ has no solutions.

Solutions of $\cos\theta = \frac{1}{2}$ are $\theta = 60°, 300°$

(f) $\sin^2\theta - 2\sin\theta - 1 = 0$

So $\sin\theta = \frac{2 \pm \sqrt{8}}{2}$

Solve $\sin\theta = \frac{2-\sqrt{8}}{2}$ as $\frac{2+\sqrt{8}}{2} > 1$

$\theta = 204°, 336°$ (solutions are in 3rd and 4th quadrants as $\frac{2-\sqrt{8}}{2} < 0$)

(g) $\tan^2 2\theta = 3 \Rightarrow \tan 2\theta = \pm\sqrt{3}$
Solve $\tan X = +\sqrt{3}$ and $\tan X = -\sqrt{3}$, where $X = 2\theta$
Interval for X is $0 \leq X \leq 720°$
For $\tan X = \sqrt{3}$, $X = 60°, 240°, 420°, 600°$

So $\theta = \frac{X}{2} = 30°, 120°, 210°, 300°$

For $\tan X = -\sqrt{3}$, $X = 120°, 300°, 480°, 660°$
So $\theta = 60°, 150°, 240°, 330°$
Solution set: $\theta = 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$

(h) $4\sin\theta = \tan\theta$

So $4\sin\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow\ 4\sin\theta\cos\theta = \sin\theta$

$\Rightarrow\ 4\sin\theta\cos\theta - \sin\theta = 0$

$\Rightarrow\ \sin\theta(4\cos\theta - 1) = 0$

So $\sin\theta = 0$ or $\cos\theta = \frac{1}{4}$

Solutions of $\sin\ \theta = 0$ are 0°, 180°, 360°

Solutions of $\cos\ \theta = \frac{1}{4}$ are $\cos^{-1}\left(\frac{1}{4}\right)$ and $360° - \cos^{-1}\left(\frac{1}{4}\right)$

Solution set: 0°, 75.5°, 180°, 284°, 360°

(i) $\sin\ \theta + 2\ \cos^2\ \theta + 1 = 0$

So $\sin\ \theta + 2\left(1 - \sin^2\ \theta\right) + 1 = 0$ using $\sin^2\ \theta + \cos^2\ \theta \equiv 1$

⇒ $2\ \sin^2\ \theta - \sin\ \theta - 3 = 0$

⇒ $\left(2\ \sin\ \theta - 3\right)\left(\sin\ \theta + 1\right) = 0$

So $\sin\ \theta = -1$ ($\sin\ \theta = \frac{3}{2}$ has no solution)

⇒ $\theta = 270\ °$

(j) $\tan^2\ \left(\theta - 45\ °\ \right) = 1$

So $\tan\ \left(\theta - 45\ °\ \right) = 1$ or $\tan\ \left(\theta - 45\ °\ \right) = -1$

So $\theta - 45\ ° = 45\ °$, 225° (1st and 3rd quadrants)

or $\theta - 45\ ° = -45\ °$, 135°, 315° (2nd and 4th quadrants)

⇒ $\theta = 0\ °$, 90°, 180°, 270°, 360°

(k) $3\ \sin^2\ \theta = \sin\ \theta\ \cos\ \theta$

⇒ $3\ \sin^2\ \theta - \sin\ \theta\ \cos\ \theta = 0$

⇒ $\sin\ \theta\left(3\ \sin\ \theta - \cos\ \theta\right) = 0$

So $\sin\ \theta = 0$ or $3\ \sin\ \theta - \cos\ \theta = 0$

Solutions of $\sin\ \theta = 0$ are $\theta = 0\ °$, 180°, 360°

For $3\ \sin\ \theta - \cos\ \theta = 0$

$3\ \sin\ \theta = \cos\ \theta$

$$\frac{\cancel{3}\sin\theta}{\cancel{3}\cos\theta} = \frac{\cancel{\cos\theta}1}{3\cancel{\cos\theta}}$$

$\tan\ \theta = \frac{1}{3}$

Solutions are $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ and $180° + \tan^{-1}\left(\frac{1}{3}\right) = 18.4°$, 198°

Solution set: 0°, 18.4°, 180°, 198°, 360°

(l) $4\ \cos\ \theta\left(\cos\ \theta - 1\right) = -5\ \cos\ \theta$

⇒ $\cos\ \theta\left[4\ \left(\cos\ \theta - 1\right) + 5\right] = 0$

⇒ $\cos\ \theta\left(4\ \cos\ \theta + 1\right) = 0$

So $\cos\ \theta = 0$ or $\cos\ \theta = -\frac{1}{4}$

Solutions of $\cos\ \theta = 0$ are 90°, 270°

Solutions of $\cos\ \theta = -\frac{1}{4}$ are 104°, 256° (3 s.f.) (2nd and 3rd quadrants)

Solution set: 90°, 104°, 256°, 270°

(m) $4\ \sin^2\ \theta - 4\ \cos\ \theta = 3 - 2\ \cos\ \theta$

⇒ $4\left(1 - \cos^2\ \theta\right) - 4\ \cos\ \theta = 3 - 2\ \cos\ \theta$

⇒ $4\ \cos^2\ \theta + 2\ \cos\ \theta - 1 = 0$

So $\cos\ \theta = \frac{-2 \pm \sqrt{20}}{8}\left(= \frac{-1 \pm \sqrt{5}}{4}\right)$

Solutions of $\cos\ \theta = \frac{-2+\sqrt{20}}{8}$ are 72°, 288° (1st and 4th quadrants)

Solutions of $\cos\ \theta = \frac{-2-\sqrt{20}}{8}$ are 144°, 216° (2nd and 3rd quadrants)

Solution set: 72.0°, 144°, 216°, 288°

(n) $2\ \sin^2\ \theta = 3(1 - \cos\ \theta)$

$\Rightarrow\ 2(1 - \cos^2\ \theta) = 3(1 - \cos\ \theta)$

$\Rightarrow\ 2(1 - \cos\ \theta)(1 + \cos\ \theta) = 3(1 - \cos\ \theta)$ or write as $a\ \cos^2\ \theta + b\ \cos\ \theta + c \equiv 0$

$\Rightarrow\ (1 - \cos\ \theta)[2(1 + \cos\ \theta) - 3] = 0$

$\Rightarrow\ (1 - \cos\ \theta)(2\ \cos\ \theta - 1) = 0$

So $\cos\ \theta = 1$ or $\cos\ \theta = \frac{1}{2}$

Solutions are 0°, 60°, 300°, 360°

(o) $4\ \cos^2\ \theta - 5\ \sin\ \theta - 5 = 0$

$\Rightarrow\ 4(1 - \sin^2\ \theta) - 5\ \sin\ \theta - 5 = 0$

$\Rightarrow\ 4\ \sin^2\ \theta + 5\ \sin\ \theta + 1 = 0$

$\Rightarrow\ (4\ \sin\ \theta + 1)(\sin\ \theta + 1) = 0$

So $\sin\ \theta = -1$ or $\sin\ \theta = -\frac{1}{4}$

Solution of $\sin\ \theta = -1$ is $\theta = 270°$

Solutions of $\sin\ \theta = -\frac{1}{4}$ are $\theta = 194°$, 346° (3 s.f.) (3rd and 4th quadrants)

Solution set: 194°, 270°, 346°

(p) $\cos^2\ \frac{\theta}{2} = 1 + \sin\ \frac{\theta}{2}$

$\Rightarrow\ 1 - \sin^2\ \frac{\theta}{2} = 1 + \sin\ \frac{\theta}{2}$

$\Rightarrow\ \sin^2\ \frac{\theta}{2} + \sin\ \frac{\theta}{2} = 0$

$\Rightarrow\ \sin\ \frac{\theta}{2}\left(\sin\ \frac{\theta}{2} + 1\right) = 0$

So $\sin\ \frac{\theta}{2} = 0$ or $\sin\ \frac{\theta}{2} = -1$

Solve $\sin\ X = 0$ and $\sin\ X = -1$ where $X = \frac{\theta}{2}$

Interval for X is $0 \le X \le 180°$
$X = 0°$, 180° ($\sin\ X = -1$ has no solutions in the interval)
So $\theta = 2X = 0°$, 360°

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise D, Question 2

Question:

Solve for θ, in the interval $-180° \leq \theta \leq 180°$, the following equations.
Give your answers to 3 significant figures where they are not exact.

(a) $\sin^2 2\theta = 1$

(b) $\tan^2 \theta = 2 \tan \theta$

(c) $\cos \theta (\cos \theta - 2) = 1$

(d) $\sin^2 (\theta + 10°) = 0.8$

(e) $\cos^2 3\theta - \cos 3\theta = 2$

(f) $5 \sin^2 \theta = 4 \cos^2 \theta$

(g) $\tan \theta = \cos \theta$

(h) $2 \sin^2 \theta + 3 \cos \theta = 1$

Solution:

(a) Solve $\sin^2 X = 1$ where $X = 2\theta$
Interval for X is $-360° \leq X \leq 360°$
$\sin X = +1$ gives $X = -270°, 90°$
$\sin X = -1$ gives $X = -90°, +270°$
$X = -270°, -90°, +90°, +270°$
So $\theta = \frac{X}{2} = -135°, -45°, +45°, +135°$

(b) $\tan^2 \theta = 2 \tan \theta$
$\Rightarrow \tan^2 \theta - 2 \tan \theta = 0$
$\Rightarrow \tan \theta (\tan \theta - 2) = 0$
So $\tan \theta = 0$ or $\tan \theta = 2$ (1st and 3rd quadrants)
Solutions are $(-180°, 0°, 180°), (-116.6°, 63.4°)$
Solution set: $-180°, -117°, 0°, 63.4°, 180°$

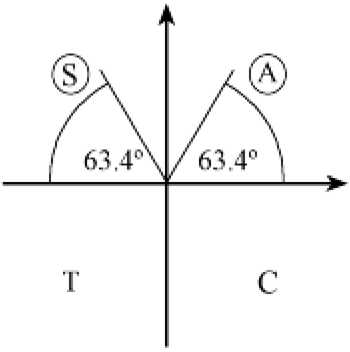
(c) $\cos^2 \theta - 2 \cos \theta = 1$
$\Rightarrow \cos^2 \theta - 2 \cos \theta - 1 = 0$

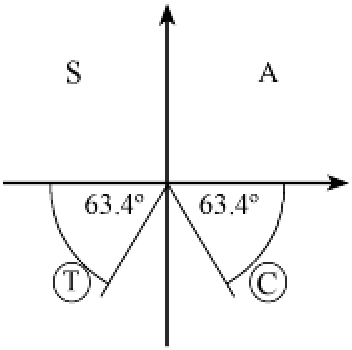
So $\cos \theta = \frac{2 \pm \sqrt{8}}{2}$

$\Rightarrow \cos \theta = \frac{2 - \sqrt{8}}{2}$ (as $\frac{2 + \sqrt{8}}{2} > 1$)

Solutions are $\pm 114°$ (2nd and 3rd quadrants)

(d) $\sin^2 (\theta + 10°) = 0.8$
$\Rightarrow \sin (\theta + 10°) = +\sqrt{0.8}$ or $\sin (\theta + 10°) = -\sqrt{0.8}$
Either $(\theta + 10°) = 63.4°, 116.6°$ or $(\theta + 10°) = -116.6°, -63.4°$





So $\theta = -127°, -73.4°, 53.4°, 107°$ (3 s.f.)

(e) $\cos^2 3\theta - \cos 3\theta - 2 = 0$
$(\cos 3\theta - 2)(\cos 3\theta + 1) = 0$
So $\cos 3\theta = -1$ $(\cos 3\theta \neq 2)$
Solve $\cos X = -1$ where $X = 3\theta$
Interval for X is $-540° \leq X \leq 540°$
From the graph of $y = \cos X$, $\cos X = -1$ where
$X = -540°, -180°, 180°, 540°$

So $\theta = \frac{X}{3} = -180°, -60°, +60°, +180°$

(f) $5\sin^2\theta = 4\cos^2\theta$

$\Rightarrow \tan^2\theta = \frac{4}{5}$ as $\tan\theta = \frac{\sin\theta}{\cos\theta}$

So $\tan\theta = \pm\sqrt{\frac{4}{5}}$

There are solutions from each of the quadrants (angle to horizontal is 41.8°)
$\theta = \pm 138°, \pm 41.8°$

(g) $\tan\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \cos\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin^2\theta + \sin\theta - 1 = 0$

So $\sin\theta = \frac{-1 \pm \sqrt{5}}{2}$

Only solutions from $\sin\theta = \frac{-1 + \sqrt{5}}{2}$ (as $\frac{-1 - \sqrt{5}}{2} < -1$)

Solutions are $\theta =$ 38.2°, 142° (1st and 2nd quadrants)

(h) $2\sin^2\theta + 3\cos\theta = 1$
$\Rightarrow 2(1 - \cos^2\theta) + 3\cos\theta = 1$
$\Rightarrow 2\cos^2\theta - 3\cos\theta - 1 = 0$

So $\cos\theta = \frac{3 \pm \sqrt{17}}{4}$

Only solutions of $\cos\theta = \frac{3 - \sqrt{17}}{4}$ (as $\frac{3 + \sqrt{17}}{4} > 1$)

Solutions are $\theta = \pm 106°$ (2nd and 3rd quadrants)

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise D, Question 3

Question:

Solve for x, in the interval $0 \leq x \leq 2\pi$, the following equations.

Give your answers to 3 significant figures unless they can be written in the form $\frac{a}{b}\pi$, where a and b are integers.

(a) $\tan^2 \frac{1}{2}x = 1$

(b) $2 \sin^2 \left(x + \frac{\pi}{3} \right) = 1$

(c) $3 \tan x = 2 \tan^2 x$

(d) $\sin^2 x + 2 \sin x \cos x = 0$

(e) $6 \sin^2 x + \cos x - 4 = 0$

(f) $\cos^2 x - 6 \sin x = 5$

(g) $2 \sin^2 x = 3 \sin x \cos x + 2 \cos^2 x$

Solution:

(a) $\tan^2 \frac{1}{2}x = 1$

$\Rightarrow \tan \frac{1}{2}x = \pm 1$

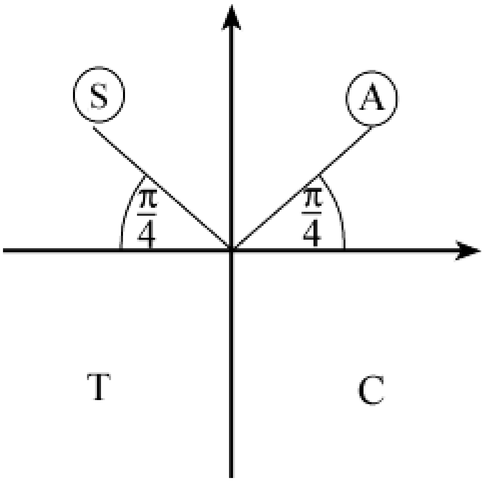
$\Rightarrow \frac{1}{2}x = \frac{\pi}{4}, \frac{3\pi}{4} \quad \left(0 \leq \frac{1}{2}x \leq \pi \right)$

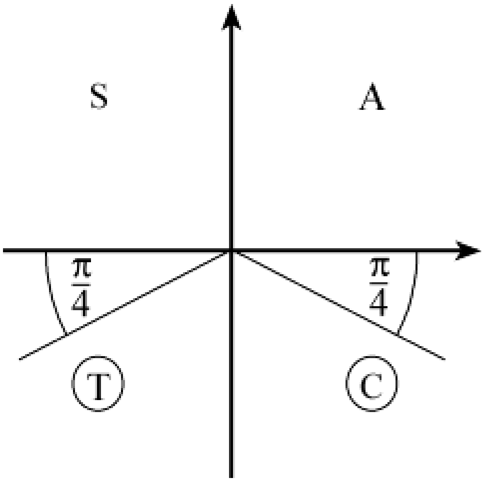
So $x = \frac{\pi}{2}, \frac{3\pi}{2}$

(b) $2 \sin^2 \left(x + \frac{\pi}{3} \right) = 1$ for $\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{7\pi}{3}$

$\Rightarrow \sin^2 \left(x + \frac{\pi}{3} \right) = \frac{1}{2}$

So $\sin \left(x + \frac{\pi}{3} \right) = \frac{1}{\sqrt{2}}$ or $\sin \left(x + \frac{\pi}{3} \right) = - \frac{1}{\sqrt{2}}$





$x + \frac{\pi}{3} = \frac{3\pi}{4}, \frac{9\pi}{4}$ or $x + \frac{\pi}{3} = + \frac{5\pi}{4}, + \frac{7\pi}{4}$

So $x = \frac{3\pi}{4} - \frac{\pi}{3}, \frac{9\pi}{4} - \frac{\pi}{3}$ or $x = \frac{5\pi}{4} - \frac{\pi}{3}, \frac{7\pi}{4} - \frac{\pi}{3}$

Solutions are $x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$

(c) $3 \tan x = 2 \tan^2 x$

$\Rightarrow \quad 2 \tan^2 x - 3 \tan x = 0$

$\Rightarrow \quad \tan x (2 \tan x - 3) = 0$

So $\tan x = 0$ or $\tan x = \frac{3}{2}$

$x = (0 , \pi, 2\pi), (0.983 , \pi + 0.983) = 0 , 0.983, \pi, 4.12, 2\pi$

(d) $\sin^2 x + 2 \sin x \cos x = 0$

$\Rightarrow \quad \sin x (\sin x + 2 \cos x) = 0$

So $\sin x = 0$ or $\sin x + 2 \cos x = 0$
$\sin x = 0$ gives $x = 0, \pi, 2\pi$
$\sin x + 2 \cos x = 0 \quad \Rightarrow \quad \tan x = -2$
Solutions are 2.03, 5.18 radians (2nd and 4th quadrants)
Solution set: 0, 2.03, π, 5.18, 2π

(e) $6 \sin^2 x + \cos x - 4 = 0$

$\Rightarrow \quad 6 (1 - \cos^2 x) + \cos x - 4 = 0$

$\Rightarrow \quad 6 \cos^2 x - \cos x - 2 = 0$

$\Rightarrow \quad (3\cos x - 2)(2\cos x + 1) = 0$

So $\cos x = +\frac{2}{3}$ or $\cos x = -\frac{1}{2}$

Solutions of $\cos x = +\frac{2}{3}$ are $\cos^{-1}\left(\frac{2}{3}\right), 2\pi - \cos^{-1}\left(\frac{2}{3}\right) = 0.841, 5.44$

Solutions of $\cos x = -\frac{1}{2}$ are $\cos^{-1}\left(-\frac{1}{2}\right), 2\pi - \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}$

Solutions are 0.841, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, 5.44

(f) $\cos^2 x - 6\sin x = 5$

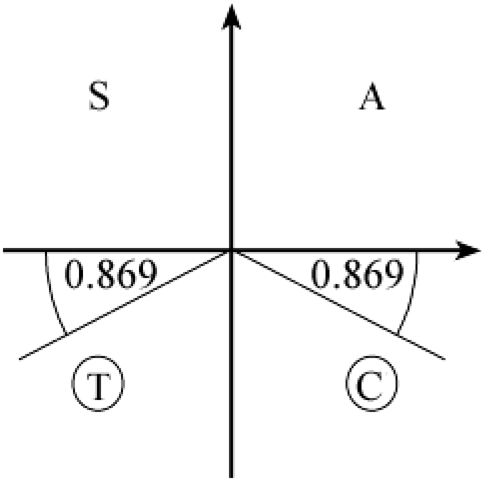
$\Rightarrow \quad (1 - \sin^2 x) - 6\sin x = 5$

$\Rightarrow \quad \sin^2 x + 6\sin x + 4 = 0$

So $\sin x = \frac{-6 \pm \sqrt{20}}{2} \left(= -3 \pm \sqrt{5} \right)$

As $\frac{-6 - \sqrt{20}}{2} < -1$, there are no solutions of $\sin x = \frac{-6 - \sqrt{20}}{2}$

Consider solutions of $\sin x = \frac{-6 + \sqrt{20}}{2}$



$\sin^{-1}\left(\frac{-6 + \sqrt{20}}{2}\right) = -0.869$ (not in given interval)

Solutions are $\pi + 0.869, 2\pi - 0.869 = 4.01, 5.41$

(g) $2\sin^2 x - 3\sin x\cos x - 2\cos^2 x = 0$

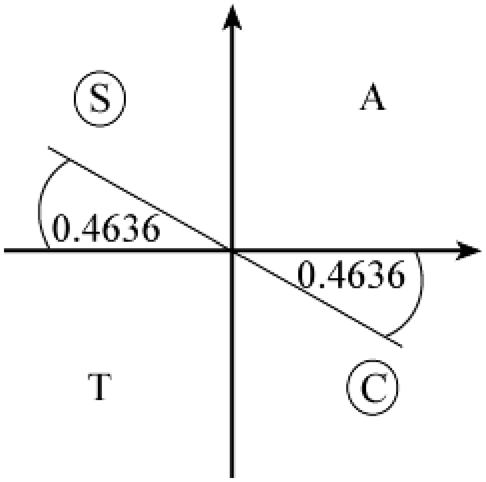
$\Rightarrow \quad (2\sin x + \cos x)(\sin x - 2\cos x) = 0$

$\Rightarrow \quad 2\sin x + \cos x = 0$ or $\sin x - 2\cos x = 0$

So $\tan x = -\frac{1}{2}$ or $\tan x = 2$

Consider solutions of $\tan x = -\frac{1}{2}$

First solution is $\tan^{-1}\left(-\frac{1}{2}\right) = -0.4636 \quad \ldots \quad$ (not in interval)



Solutions are $\pi - 0.4636$, $2\pi - 0.4636 = 2.68, 5.82$
Solutions of $\tan x = 2$ are $\tan^{-1} 2, \pi + \tan^{-1} 2 = 1.11, 4.25$
Solution set: $x = 1.11, 2.68, 4.25, 5.82$ (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise E, Question 1

Question:

Given that angle A is obtuse and $\cos A = -\sqrt{\frac{7}{11}}$, show that $\tan A = \frac{-2\sqrt{7}}{7}$.

Solution:

Using $\sin^2 A + \cos^2 A \equiv 1$

$$\sin^2 A + \left(-\sqrt{\frac{7}{11}} \right)^2 = 1$$

$$\sin^2 A = 1 - \frac{7}{11} = \frac{4}{11}$$

$$\sin A = \pm \frac{2}{\sqrt{11}}$$

But A is in the second quadrant (obtuse), so $\sin A$ is + ve.

So $\sin A = + \frac{2}{\sqrt{11}}$

Using $\tan A = \frac{\sin A}{\cos A}$

$$\tan A = \frac{\left(\frac{2}{\sqrt{11}} \right)}{-\sqrt{\frac{7}{11}}} = -\frac{2}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{7}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7}$$ (rationalising the denominator)

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Edexcel Modular Mathematics for AS and A-Level

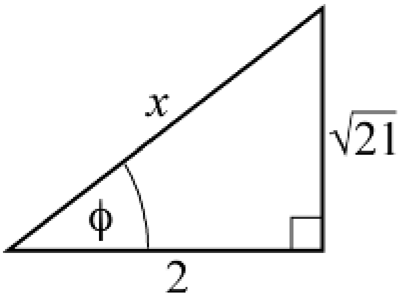
Trigonometrical identities and simple equations
Exercise E, Question 2

Question:

Given that angle B is reflex and $\tan B = + \frac{\sqrt{21}}{2}$, find the exact value of: (a) $\sin B$, (b) $\cos B$.

Solution:

Draw a right-angled triangle with an angle ϕ where $\tan \phi = + \frac{\sqrt{21}}{2}$.



Using Pythagoras' Theorem to find the hypotenuse:
$x^2 = 2^2 + (\sqrt{21})^2 = 4 + 21 = 25$
So $x = 5$

(a) $\sin \phi = \frac{\sqrt{21}}{5}$

As B is reflex and $\tan B$ is + ve, B is in the third quadrant.

So $\sin B = - \sin \phi = - \frac{\sqrt{21}}{5}$

(b) From the diagram $\cos \phi = \frac{2}{5}$

B is in the third quadrant, so $\cos B = - \cos \phi = - \frac{2}{5}$

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Trigonometrical identities and simple equations

Exercise E, Question 3

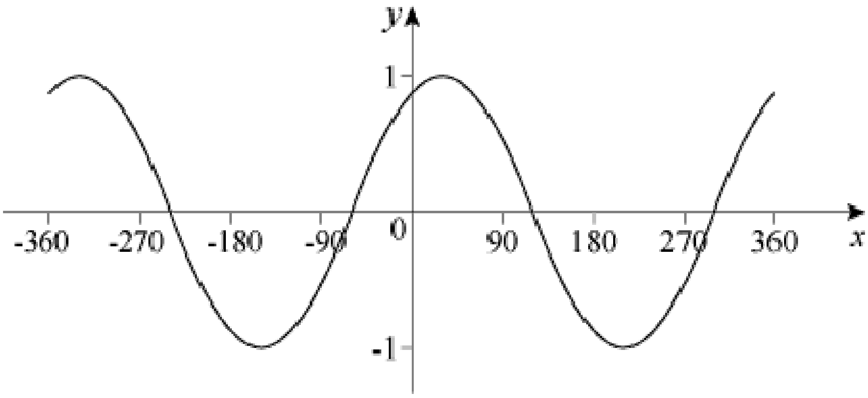
Question:

(a) Sketch the graph of $y = \sin \left(x + 60 \right) °$, in the interval $- 360 \leq x \leq 360$, giving the coordinates of points of intersection with the axes.

(b) Calculate the values of the x-coordinates of the points in which the line $y = \frac{1}{2}$ intersects the curve.

Solution:

(a) The graph of $y = \sin \left(x + 60 \right) °$ is the graph of $y = \sin x °$ translated by 60 to the left.



The curve meets the x-axis at
$\left(- 240 , 0 \right)$, $\left(- 60 , 0 \right)$, $\left(120 , 0 \right)$ and $\left(300 , 0 \right)$
The curve meets the y-axis, where $x = 0$.

So $y = \sin 60 ° = \frac{\sqrt{3}}{2}$

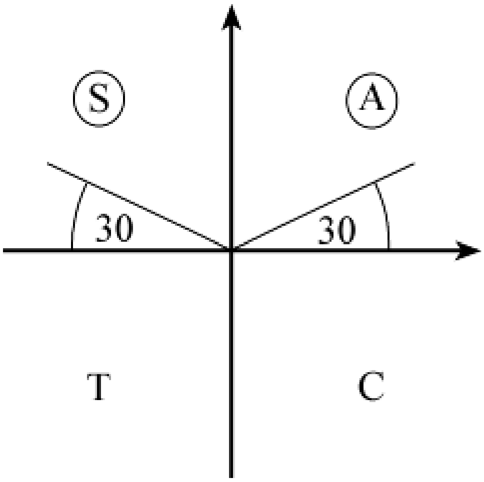
Coordinates are $\left(0 , \frac{\sqrt{3}}{2} \right)$

(b) The line meets the curve where $\sin \left(x + 60 \right) ° = \frac{1}{2}$

Let $\left(x + 60 \right) = X$ and solve $\sin X ° = \frac{1}{2}$ where $- 300 \leq X \leq 420$

$\sin X ° = \frac{1}{2}$

First solution is $X = 30$ (your calculator solution)
As $\sin X$ is + ve, X is in the 1st and 2nd quadrants.



Read off all solutions in the interval $-300 \leq X \leq 420$

$X = -210, 30, 150, 390$

$x + 60 = -210, 30, 150, 390$

So $x = -270, -30, 90, 330$

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Trigonometrical identities and simple equations

Exercise E, Question 4

Question:

Simplify the following expressions:

(a) $\cos^4 \theta - \sin^4 \theta$

(b) $\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$

(c) $\cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta$

Solution:

(a) Factorise $\cos^4 \theta - \sin^4 \theta$ (difference of two squares)
$\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta + \sin^2 \theta) (\cos^2 \theta - \sin^2 \theta) = (1) (\cos^2 \theta - \sin^2 \theta)$ (as $\sin^2 \theta + \cos^2 \theta \equiv 1$)
So $\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$

(b) Factorise $\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$
$\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$
$= \sin^2 3\theta (1 - \cos^2 3\theta)$ use $\sin^2 3\theta + \cos^2 3\theta \equiv 1$
$= \sin^2 3\theta (\sin^2 3\theta)$
$= \sin^4 3\theta$

(c) $\cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta = (\cos^2 \theta + \sin^2 \theta)^2 = 1$
since $\sin^2 \theta + \cos^2 \theta \equiv 1$

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Trigonometrical identities and simple equations
Exercise E, Question 5

Question:

(a) Given that $2 (\sin x + 2 \cos x) = \sin x + 5 \cos x$, find the exact value of $\tan x$.

(b) Given that $\sin x \cos y + 3 \cos x \sin y = 2 \sin x \sin y - 4 \cos x \cos y$, express $\tan y$ in terms of $\tan x$.

Solution:

(a) $2 (\sin x + 2 \cos x) = \sin x + 5 \cos x$

$\Rightarrow \quad 2 \sin x + 4 \cos x = \sin x + 5 \cos x$

$\Rightarrow \quad 2 \sin x - \sin x = 5 \cos x - 4 \cos x$

$\Rightarrow \quad \sin x = \cos x$ divide both sides by $\cos x$

So $\tan x = 1$

(b) $\sin x \cos y + 3 \cos x \sin y = 2 \sin x \sin y - 4 \cos x \cos y$

$$\Rightarrow \quad \frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} + \frac{3 \cancel{\cos x} \sin y}{\cancel{\cos x} \cos y} = \frac{2 \sin x \sin y}{\cos x \cos y} - \frac{4 \cancel{\cos x} \cancel{\cos y}}{\cancel{\cos x} \cancel{\cos y}}$$

$\Rightarrow \quad \tan x + 3 \tan y = 2 \tan x \tan y - 4$

$\Rightarrow \quad 2 \tan x \tan y - 3 \tan y = 4 + \tan x$

$\Rightarrow \quad \tan y (2 \tan x - 3) = 4 + \tan x$

So $\tan y = \frac{4 + \tan x}{2 \tan x - 3}$

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Trigonometrical identities and simple equations
Exercise E, Question 6

Question:

Show that, for all values of θ:

(a) $(1 + \sin \theta)^2 + \cos^2 \theta = 2 (1 + \sin \theta)$

(b) $\cos^4 \theta + \sin^2 \theta = \sin^4 \theta + \cos^2 \theta$

Solution:

(a) LHS $= (1 + 2 \sin \theta + \sin^2 \theta) + \cos^2 \theta$
$= 1 + 2 \sin \theta + 1$ since $\sin^2 \theta + \cos^2 \theta \equiv 1$
$= 2 + 2 \sin \theta$
$= 2 (1 + \sin \theta)$
= RHS

(b) LHS $= \cos^4 \theta + \sin^2 \theta$
$= (\cos^2 \theta)^2 + \sin^2 \theta$
$= (1 - \sin^2 \theta)^2 + \sin^2 \theta$ since $\sin^2 \theta + \cos^2 \theta \equiv 1$
$= 1 - 2 \sin^2 \theta + \sin^4 \theta + \sin^2 \theta$
$= (1 - \sin^2 \theta) + \sin^4 \theta$
$= \cos^2 \theta + \sin^4 \theta$ using $\sin^2 \theta + \cos^2 \theta \equiv 1$
= RHS

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Trigonometrical identities and simple equations

Exercise E, Question 7

Question:

Without attempting to solve them, state how many solutions the following equations have in the interval $0 \leq \theta \leq 360°$. Give a brief reason for your answer.

(a) $2 \sin \theta = 3$

(b) $\sin \theta = - \cos \theta$

(c) $2 \sin \theta + 3 \cos \theta + 6 = 0$

(d) $\tan \theta + \frac{1}{\tan \theta} = 0$

Solution:

(a) $\sin \theta = \frac{3}{2}$ has no solutions as $- 1 \leq \sin \theta \leq 1$

(b) $\sin \theta = - \cos \theta$
$\Rightarrow \tan \theta = - 1$
Look at graph of $y = \tan \theta$ in the interval $0 \leq \theta \leq 360°$.
There are 2 solutions

(c) The minimum value of $2 \sin \theta$ is $- 2$
The minimum value of $3 \cos \theta$ is $- 3$
Each minimum value is for a different θ.
So the minimum value of $2 \sin \theta + 3 \cos \theta > - 5$.
There are no solutions of $2 \sin \theta + 3 \cos \theta + 6 = 0$ as the LHS can never be zero.

(d) Solving $\tan \theta + \frac{1}{\tan \theta} = 0$ is equivalent to solving $\tan^2 \theta = - 1$, which has no real solutions, so there are no solutions.

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Trigonometrical identities and simple equations

Exercise E, Question 8

Question:

(a) Factorise $4xy - y^2 + 4x - y$.

(b) Solve the equation $4 \sin \theta \cos \theta - \cos^2 \theta + 4 \sin \theta - \cos \theta = 0$, in the interval $0 \le \theta \le 360°$.

Solution:

(a) $4xy - y^2 + 4x - y \equiv y(4x - y) + (4x - y) = (4x - y)(y + 1)$

(b) Using (a) with $x = \sin \theta$, $y = \cos \theta$

$4 \sin \theta \cos \theta - \cos^2 \theta + 4 \sin \theta - \cos \theta = 0$

$\Rightarrow \quad (4 \sin \theta - \cos \theta)(\cos \theta + 1) = 0$

So $4 \sin \theta - \cos \theta = 0$ or $\cos \theta + 1 = 0$

$4 \sin \theta - \cos \theta = 0 \quad \Rightarrow \quad \tan \theta = \frac{1}{4}$

Calculator solution is $\theta = 14.0°$

$\tan \theta$ is +ve so θ is in the 1st and 3rd quadrants

So $\theta = 14.0°, 194°$

$\cos \theta + 1 = 0 \quad \Rightarrow \quad \cos \theta = -1$

So $\theta = +180°$ (from graph)

Solutions are $\theta = 14.0°, 180°, 194°$

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Trigonometrical identities and simple equations

Exercise E, Question 9

Question:

(a) Express $4\cos 3\theta° - \sin (90 - 3\theta)°$ as a single trigonometric function.

(b) Hence solve $4\cos 3\theta° - \sin (90 - 3\theta)° = 2$ in the interval $0 \le \theta \le 360$. Give your answers to 3 significant figures.

Solution:

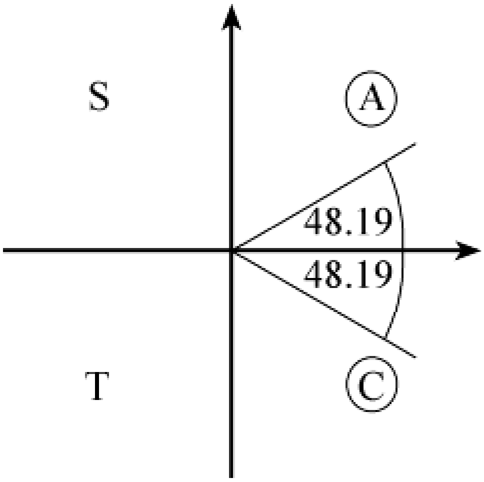
(a) As $\sin (90 - \theta)° \equiv \cos \theta°$, $\sin (90 - 3\theta)° \equiv \cos 3\theta°$
So $4\cos 3\theta° - \sin (90 - 3\theta)° = 4\cos 3\theta° - \cos 3\theta° = 3\cos 3\theta°$

(b) Using (a) $4\cos 3\theta° - \sin (90 - 3\theta)° = 2$
is equivalent to $3\cos 3\theta° = 2$

so $\cos 3\theta° = \frac{2}{3}$

Let $X = 3\theta$ and solve $\cos X° = \frac{2}{3}$ in the interval $0 \le X \le 1080$

The calculator solution is $X = 48.19$
As $\cos X°$ is +ve, X is in the 1st and 4th quadrant.



Read off all solutions in the interval $0 \le X \le 1080$
$X = 48.19, 311.81, 408.19, 671.81, 768.19, 1031.81$

So $\theta = \frac{1}{3}X = 16.1, 104, 136, 224, 256, 344$ (3 s.f.)

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Trigonometrical identities and simple equations

Exercise E, Question 10

Question:

Find, in radians to two decimal places, the value of x in the interval $0 \le x \le 2\pi$, for which $3 \sin^2 x + \sin x - 2 = 0$. **[E]**

Solution:

$3 \sin^2 x + \sin x - 2 = 0$
$(3 \sin x - 2) (\sin x + 1) = 0$ factorising

So $\sin x = \frac{2}{3}$ or $\sin x = -1$

For $\sin x = \frac{2}{3}$ your calculator answer is 0.73 (2 d.p.)

As $\sin x$ is +ve, x is in the 1st and 2nd quadrants.
So second solution is $(\pi - 0.73) = 2.41$ (2 d.p.)

For $\sin x = -1$, $x = \frac{3\pi}{2} = 4.71$ (2 d.p.)

So $x = 0.73, 2.41, 4.71$

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Trigonometrical identities and simple equations
Exercise E, Question 11

Question:

Given that $2\sin 2\theta = \cos 2\theta$:

(a) Show that $\tan 2\theta = 0.5$.

(b) Hence find the value of θ, to one decimal place, in the interval $0 \leq \theta < 360°$ for which $2\sin 2\theta° = \cos 2\theta°$. **[E]**

Solution:

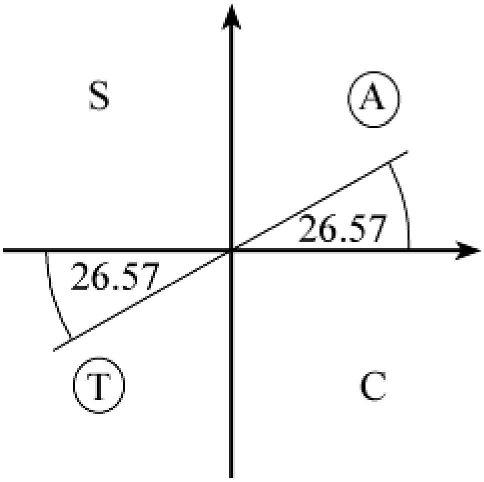
(a) $2\sin 2\theta = \cos 2\theta$

$\Rightarrow \frac{2\sin 2\theta}{\cos 2\theta} = 1$

$\Rightarrow 2\tan 2\theta = 1 \quad \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

So $\tan 2\theta = 0.5$

(b) Solve $\tan 2\theta° = 0.5$ in the interval $0 \leq \theta < 360$
or $\tan X° = 0.5$ where $X = 2\theta$, $0 \leq X < 720$
The calculator solution for $\tan^{-1} 0.5 = 26.57$
As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



Read off solutions for X in the interval $0 \leq X < 720$
$X = 26.57, 206.57, 386.57, 566.57$
$X = 2\theta$

So $\theta = \frac{1}{2}X = 13.3, 103.3, 193.3, 283.3$ (1 d.p.)

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Trigonometrical identities and simple equations

Exercise E, Question 12

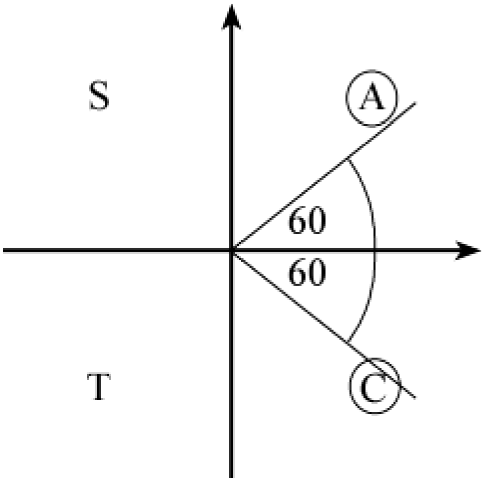
Question:

Find all the values of θ in the interval $0 \leq \theta < 360$ for which:
(a) $\cos(\theta + 75)° = 0.5$.

(b) $\sin 2\theta° = 0.7$, giving your answers to one decimal place. **[E]**

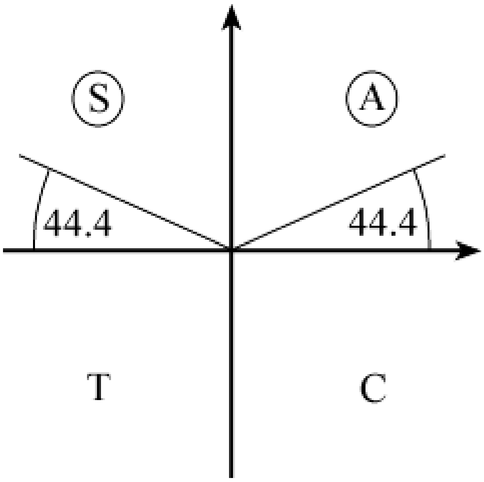
Solution:

(a) $\cos(\theta + 75)° = 0.5$
Solve $\cos X° = 0.5$ where $X = \theta + 75$, $75 \leq X < 435$
Your calculator solution for X is 60
As $\cos X$ is +ve, X is in the 1st and 4th quadrants.



Read off all solutions in the interval $75 \leq X < 435$
$X = 300, 420$
$\theta + 75 = 300, 420$
So $\theta = 225, 345$

(b) $\sin 2\theta° = 0.7$ in the interval $0 \leq \theta < 360$
Solve $\sin X° = 0.7$ where $X = 2\theta$, $0 \leq X < 720$
Your calculator solution is 44.4
As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.



Read off solutions in the interval $0 \leq X < 720$
$X = 44.4, 135.6, 404.4, 495.6$
$X = 2\theta$

So $\theta = \frac{1}{2}X = 22.2, 67.8, 202.2, 247.8$ (1 d.p.)

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Trigonometrical identities and simple equations

Exercise E, Question 13

Question:

(a) Find the coordinates of the point where the graph of $y = 2 \sin \left(2x + \frac{5}{6}\pi \right)$ crosses the y-axis.

(b) Find the values of x, where $0 \le x \le 2\pi$, for which $y = \sqrt{2}$. **[E]**

Solution:

(a) $y = 2 \sin \left(2x + \frac{5}{6}\pi \right)$ crosses the y-axis where $x = 0$

So $y = 2 \sin \frac{5}{6}\pi = 2 \times \frac{1}{2} = 1$

Coordinates are (0, 1)

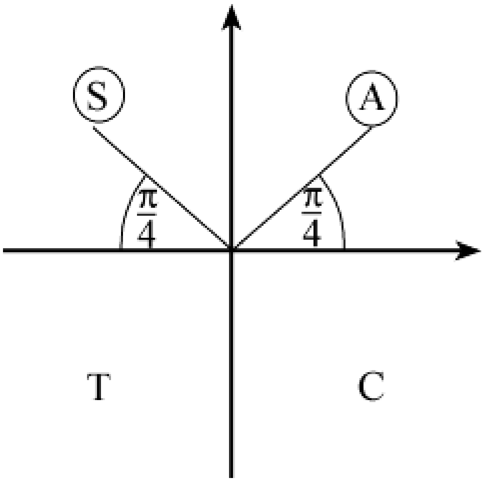
(b) Solve $2 \sin \left(2x + \frac{5}{6}\pi \right) = \sqrt{2}$ in the interval $0 \le x \le 2\pi$

So $\sin \left(2x + \frac{5}{6}\pi \right) = \frac{\sqrt{2}}{2}$

or $\sin X = \frac{\sqrt{2}}{2}$ where $\frac{5}{6}\pi \le X \le 4\frac{5}{6}\pi$

Your calculator solution is $\frac{\pi}{4}$

As $\sin X$ is +ve, X lies in the 1st and 2nd quadrants.



Read off solutions for X in the interval $\frac{5}{6}\pi \le X \le 4\frac{5}{6}\pi$

(**Note**: first value of X in interval is on second revolution.)

$X = \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$

$2x + \frac{5}{6}\pi = \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$

$2x = \frac{9\pi}{4} - \frac{5\pi}{6}, \frac{11\pi}{4} - \frac{5\pi}{6}, \frac{17\pi}{4} - \frac{5\pi}{6}, \frac{19\pi}{4} - \frac{5\pi}{6}$

$2x = \frac{17\pi}{12}, \frac{23\pi}{12}, \frac{41\pi}{12}, \frac{47\pi}{12}$

So $x = \frac{17\pi}{24}, \frac{23\pi}{24}, \frac{41\pi}{24}, \frac{47\pi}{24}$

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Trigonometrical identities and simple equations

Exercise E, Question 14

Question:

Find, giving your answers in terms of π, all values of θ in the interval $0 < \theta < 2\pi$, for which:

(a) $\tan \left(\theta + \frac{\pi}{3} \right) = 1$

(b) $\sin 2\theta = - \frac{\sqrt{3}}{2}$ **[E]**

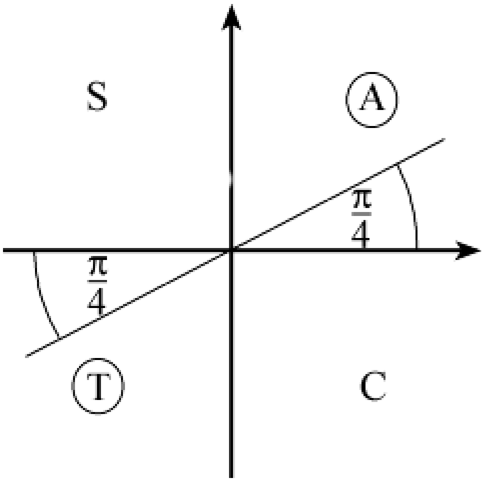
Solution:

(a) $\tan \left(\theta + \frac{\pi}{3} \right) = 1$ in the interval $0 < \theta < 2\pi$

Solve $\tan X = 1$ where $\frac{\pi}{3} < X < \frac{7\pi}{3}$

Calculator solution is $\frac{\pi}{4}$

As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



Read off solutions for X in the interval $\frac{\pi}{3} < X < \frac{7\pi}{3}$

$X = \frac{5\pi}{4}, \frac{9\pi}{4}$

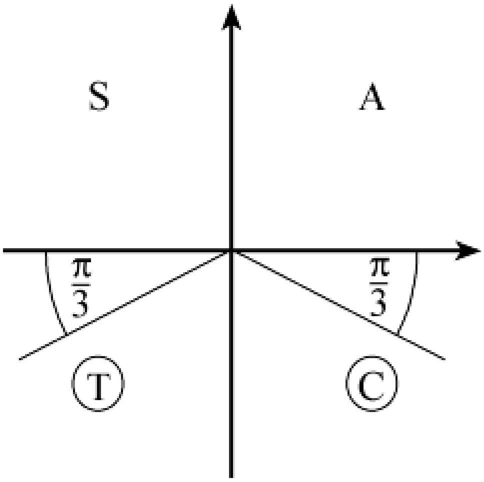
$\theta + \frac{\pi}{3} = \frac{5\pi}{4}, \frac{9\pi}{4}$

So $\theta = \frac{5\pi}{4} - \frac{\pi}{3}, \frac{9\pi}{4} - \frac{\pi}{3} = \frac{11\pi}{12}, \frac{23\pi}{12}$

(b) Solve $\sin X = \frac{- \sqrt{3}}{2}$ where $X = 2\theta, 0 < \theta < 4\pi$

Calculator answer is $-\frac{\pi}{3}$

As $\sin X$ is $-$ve, X is in the 3rd and 4th quadrants.



Read off solutions for X in the interval $0 < \theta < 4\pi$

$X = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

So $\theta = \frac{1}{2}X = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

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Trigonometrical identities and simple equations

Exercise E, Question 15

Question:

Find the values of x in the interval $0 < x < 270°$ which satisfy the equation

$$\frac{\cos 2x + 0.5}{1 - \cos 2x} = 2$$

Solution:

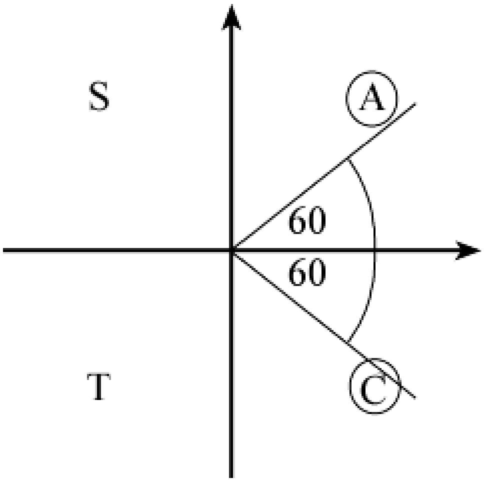
Multiply both sides of equation by $(1 - \cos 2x)$ (providing $\cos 2x \neq 1$)
(**Note**: In the interval given $\cos 2x$ is never equal to 1.)
So $\cos 2x + 0.5 = 2 - 2 \cos 2x$

$\Rightarrow 3 \cos 2x = \frac{3}{2}$

So $\cos 2x = \frac{1}{2}$

Solve $\cos X = \frac{1}{2}$ where $X = 2x$, $0 < X < 540$

Calculator solution is 60°
As $\cos X$ is +ve, X is in 1st and 4th quadrants.



Read off solutions for X in the interval $0 < X < 540$
$X = 60°, 300°, 420°$

So $x = \frac{1}{2} X = 30°, 150°, 210°$

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Trigonometrical identities and simple equations
Exercise E, Question 16

Question:

Find, to the nearest integer, the values of x in the interval $0 \leq x < 180°$ for which $3 \sin^2 3x - 7 \cos 3x - 5 = 0$.

[E]

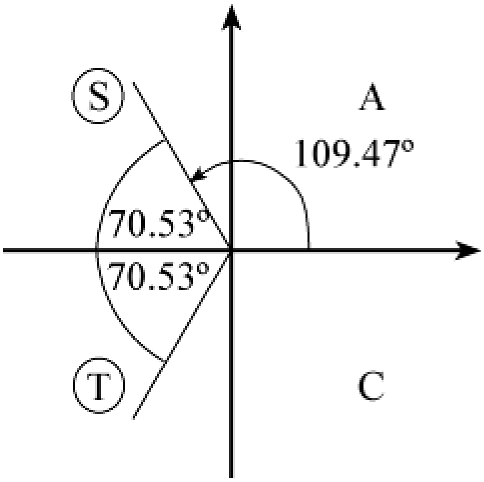
Solution:

Using $\sin^2 3x + \cos^2 3x \equiv 1$
$3 (1 - \cos^2 3x) - 7 \cos 3x - 5 = 0$
$\Rightarrow \quad 3 \cos^2 3x + 7 \cos 3x + 2 = 0$
$\Rightarrow \quad (3 \cos 3x + 1) (\cos 3x + 2) = 0$ factorising
So $3 \cos 3x + 1 = 0$ or $\cos 3x + 2 = 0$
As $\cos 3x = -2$ has no solutions, the only solutions are from
$3 \cos 3x + 1 = 0$ or $\cos 3x = -\frac{1}{3}$

Let $X = 3x$

Solve $\cos X = -\frac{1}{3}$ in the interval $0 \leq X < 540°$

The calculator solution is $X = 109.47°$
As $\cos X$ is $-$ ve, X is in the 2nd and 3rd quadrants.



Read off values of X in the interval $0 \leq X < 540°$
$X = 109.47°, 250.53°, 469.47°$

So $x = \frac{1}{3}X = 36.49°, 83.51°, 156.49° = 36°, 84°, 156°$ (to the nearest integer)

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Trigonometrical identities and simple equations

Exercise E, Question 17

Question:

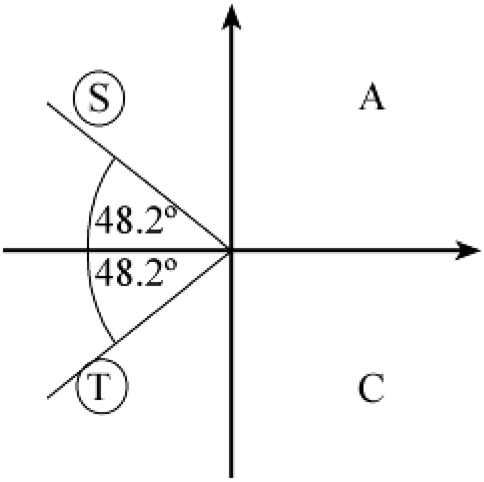
Find, in degrees, the values of θ in the interval $0 \leq \theta < 360°$ for which $2\cos^2\theta - \cos\theta - 1 = \sin^2\theta$
Give your answers to 1 decimal place, where appropriate.

[E]

Solution:

Using $\sin^2\theta + \cos^2\theta \equiv 1$
$2\cos^2\theta - \cos\theta - 1 = 1 - \cos^2\theta$
$\Rightarrow \quad 3\cos^2\theta - \cos\theta - 2 = 0$
$\Rightarrow \quad (3\cos\theta + 2)(\cos\theta - 1) = 0$
So $3\cos\theta + 2 = 0$ or $\cos\theta - 1 = 0$
For $3\cos\theta + 2 = 0$, $\cos\theta = -\frac{2}{3}$

Calculator solution is 131.8°
As $\cos\theta$ is $-$ ve, θ is in the 2nd and 3rd quadrants.



$\theta =$ 131.8°, 228.2°
For $\cos\theta = 1$, $\theta = 0°$ (see graph and note that 360° is not in given interval)
So solutions are $\theta =$ 0°, 131.8°, 228.2°

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Trigonometrical identities and simple equations
Exercise E, Question 18

Question:

Consider the function f(x) defined by
$f(x) \equiv 3 + 2 \sin (2x + k)°$, $0 < x < 360$
where k is a constant and $0 < k < 360$. The curve with equation $y = f(x)$ passes through the point with coordinates $(15, 3 + \sqrt{3})$.

(a) Show that $k = 30$ is a possible value for k and find the other possible value of k.

(b) Given that $k = 30$, solve the equation $f(x) = 1$.

[E]

Solution:

(a) $(15, 3 + \sqrt{3})$ lies on the curve $y = 3 + 2 \sin (2x + k)°$
So $3 + \sqrt{3} = 3 + 2 \sin (30 + k)°$
$2 \sin (30 + k)° = \sqrt{3}$
$$\sin \left(30 + k \right)° = \frac{\sqrt{3}}{2}$$
A solution, from your calculator, is 60°
So $30 + k = 60$ is a possible result
$\Rightarrow \quad k = 30$
As $\sin (30 + k)$ is +ve, answers lie in the 1st and 2nd quadrant.
The other angle is 120°, so $30 + k = 120$
$\Rightarrow \quad k = 90$

(b) For $k = 30$, $f(x) = 1$ is
$3 + 2 \sin (2x + 30)° = 1$
$2 \sin (2x + 30)° = -2$
$\sin (2x + 30)° = -1$
Let $X = 2x + 30$
Solve $\sin X° = -1$ in the interval $30 < X < 750$
From the graph of $y = \sin X°$
$X = +270, 630$
$2x + 30 = 270, 630$
$2x = 240, 600$
So $x = 120, 300$

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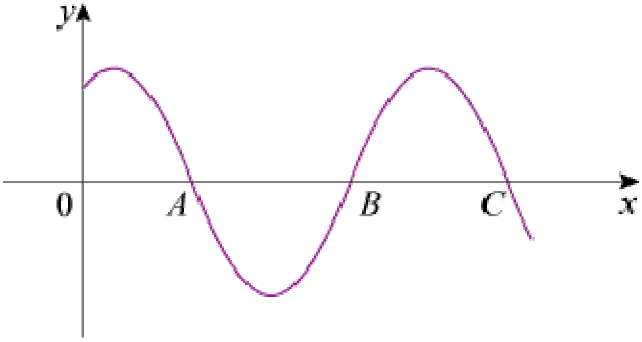
Trigonometrical identities and simple equations

Exercise E, Question 19

Question:

(a) Determine the solutions of the equation
$\cos \left(2x - 30 \right)^{\circ} = 0$ for which $0 \le x \le 360$.



(b) The diagram shows part of the curve with equation $y = \cos \left(px - q \right)^{\circ}$, where p and q are positive constants and $q < 180$. The curve cuts the x-axis at points A, B and C, as shown.
Given that the coordinates of A and B are (100, 0) and (220, 0) respectively:
(i) Write down the coordinates of C.
(ii) Find the value of p and the value of q.

[E]

Solution:

(a) The graph of $y = \cos x^{\circ}$ crosses x-axis $\left(y = 0 \right)$ where $x = 90, 270, \ldots$
Let $X = 2x - 30$
Solve $\cos X^{\circ} = 0$ in the interval $-30 \le X \le 690$
$X = 90, 270, 450, 630$
$2x - 30 = 90, 270, 450, 630$
$2x = 120, 300, 480, 660$
So $x = 60, 150, 240, 330$

(b) (i) As $AB = BC$, C has coordinates (340, 0)
(ii) When $x = 100$, $\cos \left(100p - q \right)^{\circ} = 0$, so $100p - q = 90$①
When $x = 220$, $220p - q = 270$②
When $x = 340$, $340p - q = 450$③
Solving the simultaneous equations ② – ①: $120p = 180 \Rightarrow p = \frac{3}{2}$
Substitute in ①: $150 - q = 90 \Rightarrow q = 60$

Solutionbank C2

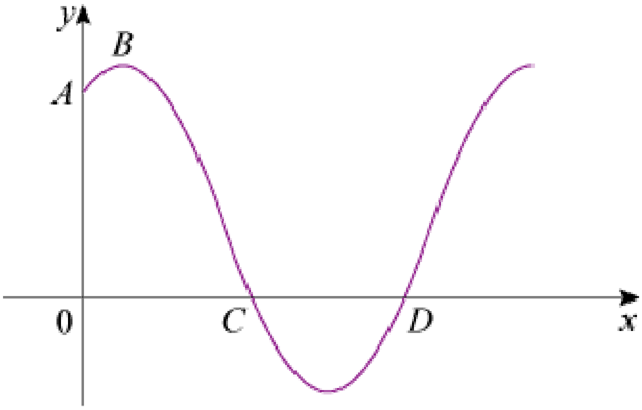
Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise E, Question 20

Question:

The diagram shows part of the curve with equation $y = f(x)$, where $f(x) = 1 + 2\sin(px° + q°)$, p and q being positive constants and $q \leq 90$. The curve cuts the y-axis at the point A and the x-axis at the points C and D. The point B is a maximum point on the curve.



Given that the coordinates of A and C are (0, 2) and (45, 0) respectively:

(a) Calculate the value of q.

(b) Show that $p = 4$.

(c) Find the coordinates of B and D.

[E]

Solution:

(a) Substitute (0, 2) is $y = f(x)$:
$2 = 1 + 2\sin q°$
$2\sin q° = +1$
$\sin q° = +\frac{1}{2}$
As $q \leq 90$, $q = 30$

(b) C is where $1 + 2\sin(px° + q°) = 0$ for the first time.

Solve $\sin\left(px° + 30°\right) = -\frac{1}{2}$(use only first solution)

$45p° + 30° = 210°$ ($x = 45$ at C)
$45p = 180$
$p = 4$

(c) At B $f(x)$ is a maximum.
$1 + 2\sin(4x° + 30°)$ is a maximum when $\sin(4x° + 30°) = 1$
So y value at $B = 1 + 2 = 3$
For x value, solve $4x° + 30° = 90°$ (as B is first maximum)
$\Rightarrow x = 15$
Coordinates of B are (15, 3).

D is the second x value for which $1 + 2 \sin (4x ° + 30 °) = 0$

Solve $\sin \left(4x ° + 30 ° \right) = - \frac{1}{2}$(use second solution)

$4x ° + 30 ° = 330 °$
$4x ° = 300 °$
$x = 75$
Coordinates of D are (75, 0).