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Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise A, Question 1

Question:

Evaluate the following definite integrals:

(a) $\int_1^2 \left(\frac{2}{x^3} + 3x \right) dx$

(b) $\int_0^2 (2x^3 - 4x + 5) dx$

(c) $\int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx$

(d) $\int_1^2 \left(6x - \frac{12}{x^4} + 3 \right) dx$

(e) $\int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1 \right) dx$

Solution:

(a) $\int_1^2 \left(\frac{2}{x^3} + 3x \right) dx$

$= \int_1^2 (2x^{-3} + 3x) dx$

$= \left[\frac{2x^{-2}}{-2} + \frac{3x^2}{2} \right]_1^2$

$= \left[-x^{-2} + \frac{3}{2}x^2 \right]_1^2$

$= \left(-\frac{1}{4} + \frac{3}{2} \times 4 \right) - \left(-1 + \frac{3}{2} \right)$

$= \left(-\frac{1}{4} + 6 \right) - \frac{1}{2}$

$= 5\frac{1}{4}$

(b) $\int_0^2 (2x^3 - 4x + 5) dx$

$= \left[\frac{2x^4}{4} - \frac{4x^2}{2} + 5x \right]_0^2$

$= \left[\frac{x^4}{2} - 2x^2 + 5x \right]_0^2$

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$= \left(\frac{16}{2} - 2 \times 4 + 10 \right) - \left(0 \right)$

$= 8 - 8 + 10$

$= 10$

(c) $\int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx$

$= \int_4^9 \left(x^{\frac{1}{2}} - 6x^{-2} \right) dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^{-1}}{-1} \right]_4^9$

$= \left[\frac{2}{3} x^{\frac{3}{2}} + 6x^{-1} \right]_4^9$

$= \left(\frac{2}{3} \times 9^{\frac{3}{2}} + \frac{6}{9} \right) - \left(\frac{2}{3} \times 4^{\frac{3}{2}} + \frac{6}{4} \right)$

$= \left(\frac{2}{3} \times 3^3 + \frac{2}{3} \right) - \left(\frac{2}{3} \times 2^3 + \frac{3}{2} \right)$

$= 18 + \frac{2}{3} - \frac{16}{3} - \frac{3}{2}$

$= 16 \frac{1}{2} - \frac{14}{3}$

$= 11 \frac{5}{6}$

(d) $\int_1^2 \left(6x - \frac{12}{x^4} + 3 \right) dx$

$= \int_1^2 (6x - 12x^{-4} + 3) dx$

$= \left[\frac{6x^2}{2} - \frac{12x^{-3}}{-3} + 3x \right]_1^2$

$= [3x^2 + 4x^{-3} + 3x]_1^2$

$= \left(3 \times 4 + \frac{4}{8} + 6 \right) - \left(3 + 4 + 3 \right)$

$= 12 + \frac{1}{2} + 6 - 10$

$= 8 \frac{1}{2}$

(e) $\int_1^8 \left(x^{-\frac{1}{3}} + 2x - 1 \right) dx$

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$$= \left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{2x^2}{2} - x \right]_1^8$$

$$= \left[\frac{3}{2}x^{\frac{2}{3}} + x^2 - x \right]_1^8$$

$$= \left(\frac{3}{2} \times 2^2 + 64 - 8 \right) - \left(\frac{3}{2} + 1 - 1 \right)$$

$$= 62 - \frac{3}{2}$$

$$= 60\frac{1}{2}$$

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Solutionbank C2

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Integration

Exercise A, Question 2

Question:

Evaluate the following definite integrals:

(a) $\int_1^3 \left(\frac{x^3 + 2x^2}{x} \right) dx$

(b) $\int_1^4 (\sqrt{x} - 3)^2 dx$

(c) $\int_3^6 \left(x - \frac{3}{x} \right)^2 dx$

(d) $\int_0^1 x^2 \left(\sqrt{x} + \frac{1}{x} \right) dx$

(e) $\int_1^4 \frac{2 + \sqrt{x}}{x^2} dx$

Solution:

(a) $\int_1^3 \left(\frac{x^3 + 2x^2}{x} \right) dx$

$= \int_1^3 (x^2 + 2x) dx$

$= \left[\frac{x^3}{3} + x^2 \right]_1^3$

$= \left(\frac{27}{3} + 9 \right) - \left(\frac{1}{3} + 1 \right)$

$= 18 - \frac{4}{3}$

$= 16 \frac{2}{3}$

(b) $\int_1^4 (\sqrt{x} - 3)^2 dx$

$= \int_1^4 (x - 6 \sqrt{x} + 9) dx$

$= \int_1^4 \left(x - 6x^{\frac{1}{2}} + 9 \right) dx$

$= \left[\frac{x^2}{2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + 9x \right]_1^4$

$= \left[\frac{x^2}{2} - 4x^{\frac{3}{2}} + 9x \right]_1^4$

$= \left(\frac{16}{2} - 4 \times 2^3 + 36 \right) - \left(\frac{1}{2} - 4 + 9 \right)$

$= 8 - 32 + 36 - 5\frac{1}{2}$

$= 12 - 5\frac{1}{2}$

$= 6\frac{1}{2}$

(c) $\int_3^6 \left(x - \frac{3}{x} \right)^2 dx$

$= \int_3^6 \left(x^2 - 6 + \frac{9}{x^2} \right) dx$

$= \int_3^6 (x^2 - 6 + 9x^{-2}) dx$

$= \left[\frac{x^3}{3} - 6x + \frac{9x^{-1}}{-1} \right]_3^6$

$= \left[\frac{x^3}{3} - 6x - 9x^{-1} \right]_3^6$

$= \left(\frac{216}{3} - 36 - \frac{9}{6} \right) - \left(\frac{27}{3} - 18 - \frac{9}{3} \right)$

$= 72 - 36 - \frac{3}{2} - 9 + 18 + 3$

$= 48 - \frac{3}{2}$

$= 46\frac{1}{2}$

(d) $\int_0^1 x^2 \left(\sqrt{x} + \frac{1}{x} \right) dx$

$= \int_0^1 \left(x^{\frac{5}{2}} + x \right) dx$

$= \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{x^2}{2} \right]_0^1$

$= \left[\frac{2}{7} x^{\frac{7}{2}} + \frac{x^2}{2} \right]_0^1$

$= \left(\frac{2}{7} + \frac{1}{2} \right) - \left(0 \right)$

$= \frac{4}{14} + \frac{7}{14}$

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$= \frac{11}{14}$

(e) $\int_1^4 \left(\frac{2 + \sqrt{x}}{x^2} \right) dx$

$= \int_1^4 \left(\frac{2}{x^2} + \frac{1}{x^{\frac{3}{2}}} \right) dx$

$= \int_1^4 \left(2x^{-2} + x^{-\frac{3}{2}} \right) dx$

$= \left[\frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_1^4$

$= \left[-2x^{-1} - 2x^{-\frac{1}{2}} \right]_1^4$

$= \left(-\frac{2}{4} - \frac{2}{2} \right) - \left(-2 - 2 \right)$

$= -1\frac{1}{2} + 4$

$= 2\frac{1}{2}$

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Integration
Exercise B, Question 1

Question:

Find the area between the curve with equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ in each of the following cases:

(a) $f(x) = 3x^2 - 2x + 2$; $a = 0$, $b = 2$

(b) $f(x) = x^3 + 4x$; $a = 1$, $b = 2$

(c) $f(x) = \sqrt{x} + 2x$; $a = 1$, $b = 4$

(d) $f(x) = 7 + 2x - x^2$; $a = -1$, $b = 2$

(e) $f\left(x\right) = \frac{8}{x^3} + \sqrt{x}$; $a = 1$, $b = 4$

Solution:

(a) $A = \int_0^2 (3x^2 - 2x + 2)\, dx$

$= \left[\frac{3x^3}{3} - \frac{2x^2}{2} + 2x \right]_0^2$

$= [x^3 - x^2 + 2x]_0^2$

$= (8 - 4 + 4) - (0)$

$= 8$

(b) $A = \int_1^2 (x^3 + 4x)\, dx$

$= \left[\frac{x^4}{4} + \frac{4x^2}{2} \right]_1^2$

$= \left(\frac{16}{4} + 2 \times 4 \right) - \left(\frac{1}{4} + 2 \right)$

$= 4 + 8 - 2\frac{1}{4}$

$= 9\frac{3}{4}$

(c) $A = \int_1^4 (\sqrt{x} + 2x)\, dx$

$= \int_1^4 \left(x^{\frac{1}{2}} + 2x \right) dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x^2 \right]_1^4$

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$= \left[\frac{2}{3}x^{\frac{3}{2}} + x^2 \right]_1^4$

$= \left(\frac{2}{3} \times 2^3 + 16 \right) - \left(\frac{2}{3} + 1 \right)$

$= \frac{16}{3} + 16 - \frac{2}{3} - 1$

$= 15 + \frac{14}{3}$

$= 19\frac{2}{3}$

(d) $A = \int_{-1}^{2} (7 + 2x - x^2) \, dx$

$= \left[7x + x^2 - \frac{x^3}{3} \right]_{-1}^{2}$

$= \left(14 + 4 - \frac{8}{3} \right) - \left(-7 + 1 + \frac{1}{3} \right)$

$= 18 - \frac{8}{3} + 6 - \frac{1}{3}$

$= 24 - \frac{9}{3}$

$= 21$

(e) $A = \int_1^4 \left(\frac{8}{x^3} + \sqrt{x} \right) dx$

$= \int_1^4 \left(8x^{-3} + x^{\frac{1}{2}} \right) dx$

$= \left[\frac{8x^{-2}}{-2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$

$= \left[-4x^{-2} + \frac{2}{3}x^{\frac{3}{2}} \right]_1^4$

$= \left(-\frac{4}{16} + \frac{2}{3} \times 2^3 \right) - \left(-4 + \frac{2}{3} \right)$

$= -\frac{1}{4} + \frac{16}{3} + 4 - \frac{2}{3}$

$= 3\frac{3}{4} + 4\frac{2}{3}$

$= 8\frac{5}{12}$

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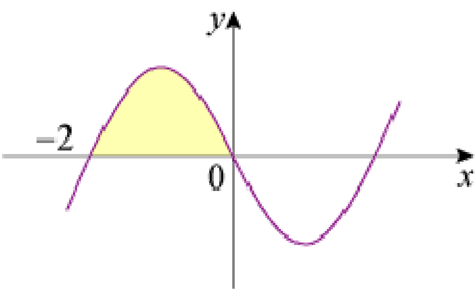
Integration
Exercise B, Question 2

Question:

The sketch shows part of the curve with equation $y = x(x^2 - 4)$.
Find the area of the shaded region.



Solution:

$A = \int_{-2}^{0} x(x^2 - 4)\,dx$

$= \int_{-2}^{0} (x^3 - 4x)\,dx$

$= \left[\frac{x^4}{4} - \frac{4x^2}{2} \right]_{-2}^{0}$

$= \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^{0}$

$= \left(0 \right) - \left(\frac{16}{4} - 2 \times 4 \right)$

$= -4 + 8$

$= 4$

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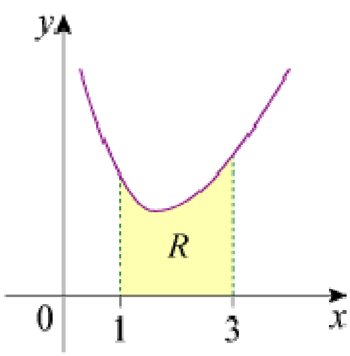
Integration
Exercise B, Question 3

Question:

The diagram shows a sketch of the curve with equation $y = 3x + \frac{6}{x^2} - 5$, $x > 0$.

The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$.
Find the area of R.



Solution:

$$A = \int_1^3 \left(3x + \frac{6}{x^2} - 5 \right) dx$$
$$= \int_1^3 (3x + 6x^{-2} - 5) dx$$
$$= \left[\frac{3x^2}{2} + \frac{6x^{-1}}{-1} - 5x \right]_1^3$$
$$= \left[\frac{3}{2}x^2 - 6x^{-1} - 5x \right]_1^3$$
$$= \left(\frac{3}{2} \times 9 - \frac{6}{3} - 15 \right) - \left(\frac{3}{2} - 6 - 5 \right)$$
$$= \frac{27}{2} - 17 - \frac{3}{2} + 11$$
$$= \frac{24}{2} - 6$$
$$= 6$$

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Integration
Exercise B, Question 4

Question:

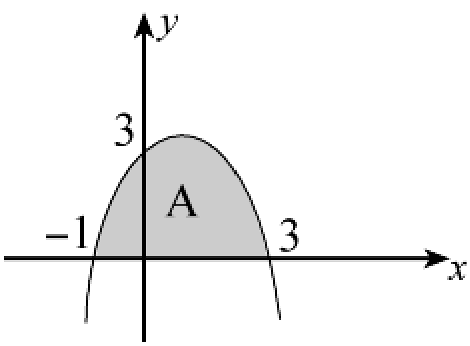
Find the area of the finite region between the curve with equation $y = (3 - x)(1 + x)$ and the x-axis.

Solution:

$y = (3 - x)(1 + x)$ is ∩ shaped

$y = 0 \Rightarrow x = 3, -1$

$x = 0 \Rightarrow y = 3$



$$A = \int_{-1}^{3} (3 - x)(1 + x)\,dx$$

$$= \int_{-1}^{3} (3 + 2x - x^2)\,dx$$

$$= \left[3x + x^2 - \frac{x^3}{3} \right]_{-1}^{3}$$

$$= \left(9 + 9 - \frac{27}{3} \right) - \left(-3 + 1 + \frac{1}{3} \right)$$

$$= 9 + 1\frac{2}{3}$$

$$= 10\frac{2}{3}$$

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Integration

Exercise B, Question 5

Question:

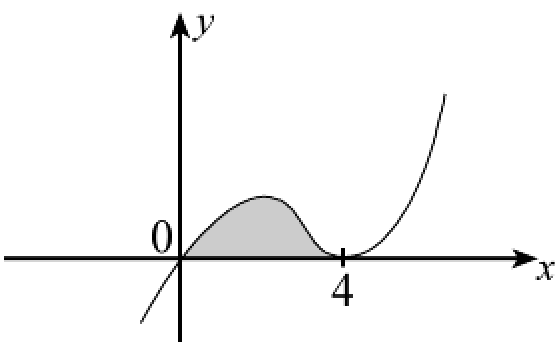
Find the area of the finite region between the curve with equation $y = x(x-4)^2$ and the x-axis.

Solution:

$y = x(x-4)^2$

$y = 0 \Rightarrow x = 0, 4$ (twice)

Turning point at (4, 0)



$$\text{Area} = \int_0^4 x(x-4)^2 dx$$

$$= \int_0^4 x(x^2 - 8x + 16)\, dx$$

$$= \int_0^4 (x^3 - 8x^2 + 16x)\, dx$$

$$= \left[\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2 \right]_0^4$$

$$= \left(64 - \frac{8}{3} \times 64 + 128 \right) - \left(0 \right)$$

$$= \frac{64}{3} \text{ or } 21\frac{1}{3}$$

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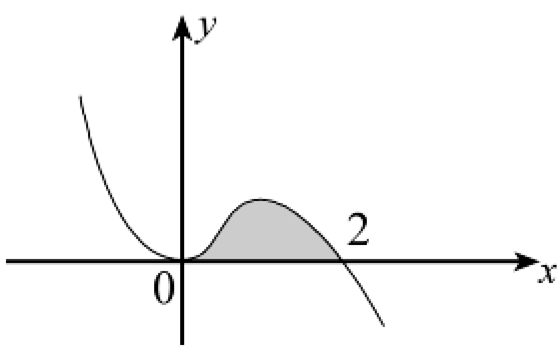
Integration
Exercise B, Question 6

Question:

Find the area of the finite region between the curve with equation $y = x^2 (2 - x)$ and the x-axis.

Solution:

$y = x^2 (2 - x)$
$y = 0 \Rightarrow x = 0$ (twice), 2
Turning point at (0, 0)
$x \to -\infty , y \to \infty$
$x \to \infty , y \to -\infty$



Area $= \int_0^2 x^2 (2 - x) \, dx$

$= \int_0^2 (2x^2 - x^3) \, dx$

$= \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$

$= \left(\frac{16}{3} - \frac{16}{4} \right) - \left(0 \right)$

$= \frac{4}{3}$ or $1 \frac{1}{3}$

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Solutionbank C2
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Integration
Exercise C, Question 1

Question:

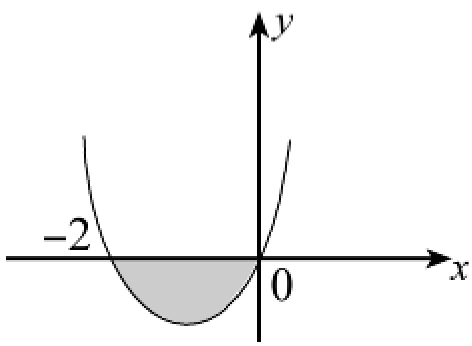
Sketch the following and find the area of the finite region or regions bounded by the curve and the x-axis:

$y = x(x + 2)$

Solution:

$y = x(x + 2)$ is $\cup$ shaped

$y = 0 \Rightarrow x = 0, -2$



Area $= -\int_{-2}^{0} x(x + 2)\,dx$

$= -\int_{-2}^{0} (x^2 + 2x)\,dx$

$= -\left[\frac{x^3}{3} + x^2\right]_{-2}^{0}$

$= -\left\{ \left(0\right) - \left(-\frac{8}{3} + 4\right) \right\}$

$= -\left(-\frac{4}{3}\right)$

$= \frac{4}{3}$ or $1\frac{1}{3}$

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Integration
Exercise C, Question 2

Question:

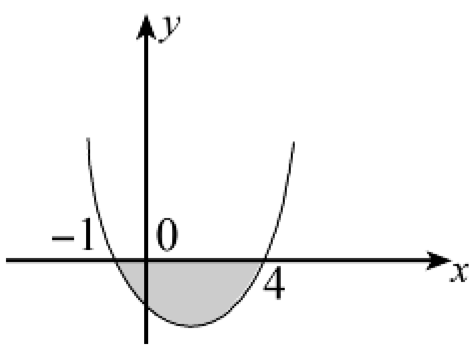
Sketch the following and find the area of the finite region or regions bounded by the curve and the x-axis:

$y = (x + 1)(x - 4)$

Solution:

$y = (x + 1)(x - 4)$ is $\cup$ shaped

$y = 0 \Rightarrow x = -1, 4$



$$\int_{-1}^{4} (x + 1)(x - 4)\,dx$$

$$= \int_{-1}^{4} (x^2 - 3x - 4)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} - 4x \right]_{-1}^{4}$$

$$= \left(\frac{64}{3} - \frac{3}{2} \times 16 - 16 \right) - \left(-\frac{1}{3} - \frac{3}{2} + 4 \right)$$

$$= \frac{64}{3} - 40 + \frac{11}{6} - 4$$

$$= -20\frac{5}{6}$$

So area $= 20\frac{5}{6}$

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Integration
Exercise C, Question 3

Question:

Sketch the following and find the area of the finite region or regions bounded by the curve and the x-axis:

$y = (x+3)x(x-3)$

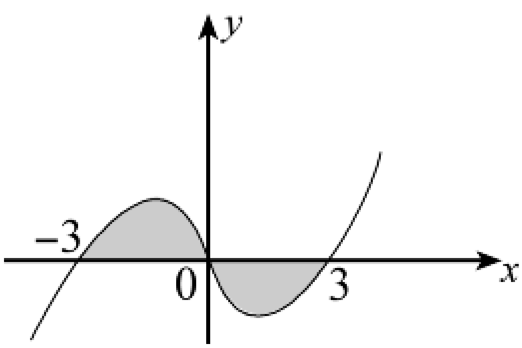
Solution:

$y = (x+3)x(x-3)$

$y = 0 \quad \Rightarrow \quad x = -3, 0, 3$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



$$\int y\,dx = \int \left(x^3 - 9x \right) dx = \left[\frac{x^4}{4} - \frac{9}{2}x^2 \right]$$

$$\int_{-3}^{0} y\,dx = \left(0 \right) - \left(\frac{81}{4} - \frac{9}{2} \times 9 \right) = +\frac{81}{4}$$

$$\int_{0}^{3} y\,dx = \left(\frac{81}{4} - \frac{9}{2} \times 9 \right) - \left(0 \right) = -\frac{81}{4}$$

So area $= \frac{81}{4} + \frac{81}{4} = \frac{81}{2}$ or $40\frac{1}{2}$

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Integration
Exercise C, Question 4

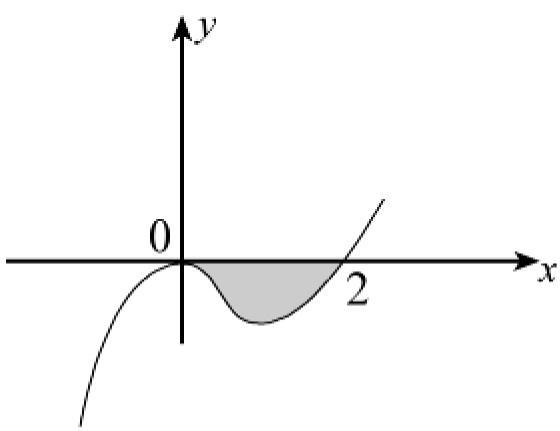
Question:

Sketch the following and find the area of the finite region or regions bounded by the curves and the x-axis:

$y = x^2 (x - 2)$

Solution:

$y = x^2 (x - 2)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at (0, 0)
$x \to \infty , y \to \infty$
$x \to -\infty , y \to -\infty$



$$\text{Area} = -\int_0^2 x^2 (x - 2)\, dx$$
$$= -\int_0^2 (x^3 - 2x^2)\, dx$$
$$= -\left[\frac{x^4}{4} - \frac{2}{3}x^3 \right]_0^2$$
$$= -\left\{ \left(\frac{16}{4} - \frac{2}{3} \times 8 \right) - \left(0 \right) \right\}$$
$$= -\left(4 - \frac{16}{3} \right)$$
$$= \frac{4}{3} \text{ or } 1\frac{1}{3}$$

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Integration
Exercise C, Question 5

Question:

Sketch the following and find the area of the finite region or regions bounded by the curve and the x-axis:

$y = x(x-2)(x-5)$

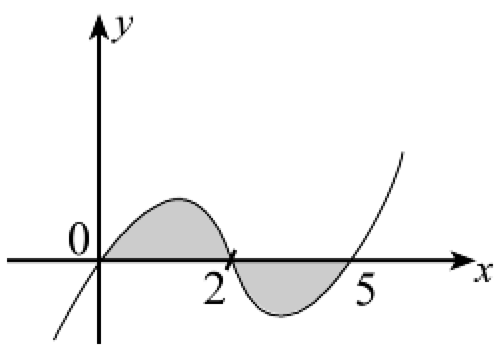
Solution:

$y = x(x-2)(x-5)$

$y = 0 \Rightarrow x = 0, 2, 5$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



$\int y\,dx = \int x(x^2 - 7x + 10)\,dx = \int (x^3 - 7x^2 + 10x)\,dx$

$\int y\,dx = \left[\frac{x^4}{4} - \frac{7}{3}x^3 + 5x^2 \right]$

$\int_0^2 y\,dx = \left(\frac{16}{4} - \frac{7}{3} \times 8 + 20 \right) - \left(0 \right) = 24 - \frac{56}{3} = 5\frac{1}{3}$

$\int_2^5 y\,dx = \left(\frac{625}{4} - \frac{7}{3} \times 125 + 125 \right) - \left(5\frac{1}{3} \right) = -15\frac{3}{4}$

So area $= 5\frac{1}{3} + 15\frac{3}{4} = 21\frac{1}{12}$

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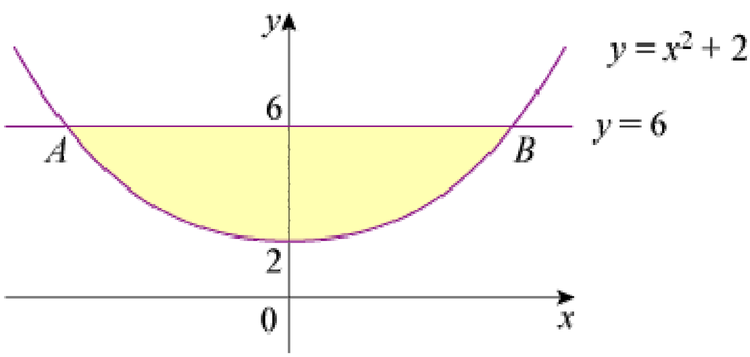
Integration
Exercise D, Question 1

Question:

The diagram shows part of the curve with equation $y = x^2 + 2$ and the line with equation $y = 6$. The line cuts the curve at the points A and B.



(a) Find the coordinates of the points A and B.

(b) Find the area of the finite region bounded by AB and the curve.

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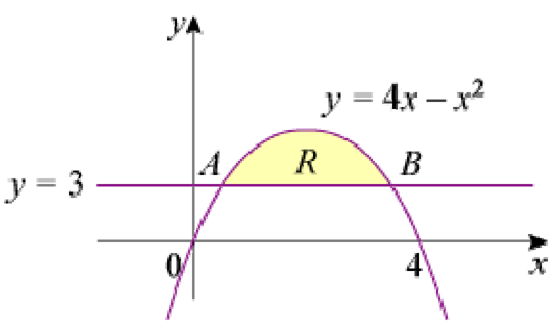
Exercise D, Question 2

Question:

The diagram shows the finite region, R, bounded by the curve with equation $y = 4x - x^2$ and the line $y = 3$. The line cuts the curve at the points A and B.



(a) Find the coordinates of the points A and B.

(b) Find the area of R.

Solution:

(a) A, B are given by

$3 = 4x - x^2$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 1, 3$

So A is (1, 3) and B is (3, 3)

(b) Area $= \int_1^3 [(4x - x^2) - 3] \, dx$

$= \int_1^3 (4x - x^2 - 3) \, dx$

$= \left[2x^2 - \frac{x^3}{3} - 3x \right]_1^3$

$= \left(18 - 9 - 9 \right) - \left(2 - \frac{1}{3} - 3 \right)$

$= 1 \frac{1}{3}$

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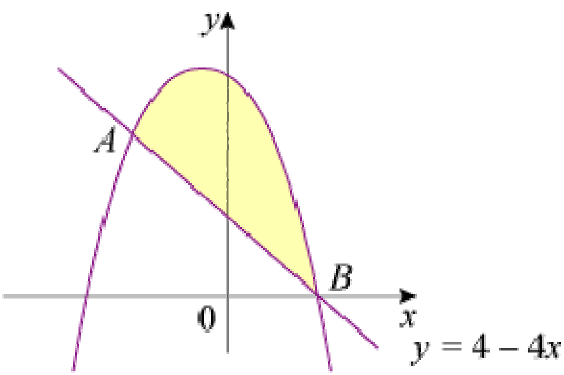
Exercise D, Question 3

Question:

The diagram shows a sketch of part of the curve with equation $y = 9 - 3x - 5x^2 - x^3$ and the line with equation $y = 4 - 4x$. The line cuts the curve at the points $A(-1, 8)$ and $B(1, 0)$.



Find the area of the shaded region between AB and the curve.

Solution:

Area $= \int_{-1}^{1} (\text{curve} - \text{line})\, dx$

$= \int_{-1}^{1} [\, 9 - 3x - 5x^2 - x^3 - (4 - 4x)\,]\, dx$

$= \int_{-1}^{1} (5 + x - 5x^2 - x^3)\, dx$

$$= \left[5x + \frac{x^2}{2} - \frac{5}{3}x^3 - \frac{x^4}{4} \right]_{-1}^{1}$$

$$= \left(5 + \frac{1}{2} - \frac{5}{3} - \frac{1}{4} \right) - \left(-5 + \frac{1}{2} + \frac{5}{3} - \frac{1}{4} \right)$$

$$= 10 - \frac{10}{3}$$

$$= \frac{20}{3} \text{ or } 6\frac{2}{3}$$

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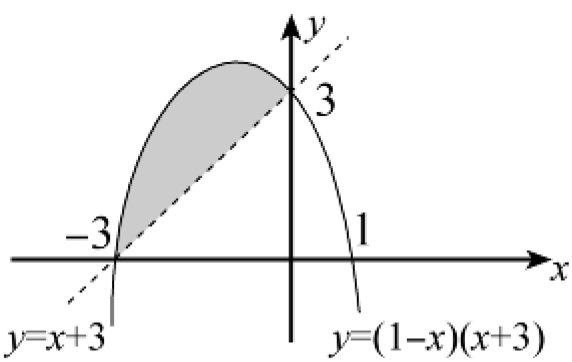
Exercise D, Question 4

Question:

Find the area of the finite region bounded by the curve with equation $y = (1 - x)(x + 3)$ and the line $y = x + 3$.

Solution:

$y = (1 - x)(x + 3)$ is $\cap$ shaped and crosses the x-axis at (1, 0) and $(-3, 0)$
$y = x + 3$ is a straight line passing through $(-3, 0)$ and (0, 3)



Intersections when
$x + 3 = (1 - x)(x + 3)$
$0 = (x + 3)(1 - x - 1)$
$0 = -x(x + 3)$
$x = -3$ or 0

Area $= \int_{-3}^{0} [(1 - x)(x + 3) - (x + 3)] dx$

$= \int_{-3}^{0} (-x^2 - 3x) dx$

$= \left[-\frac{x^3}{3} - \frac{3}{2}x^2 \right]_{-3}^{0}$

$= \left(0 \right) - \left(\frac{27}{3} - \frac{27}{2} \right)$

$= \frac{27}{6}$ or $\frac{9}{2}$ or 4.5

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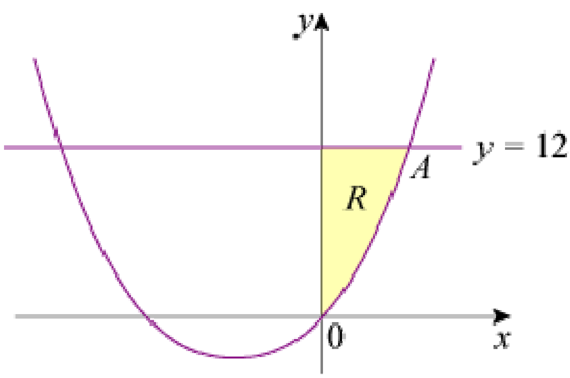
Exercise D, Question 5

Question:

The diagram shows the finite region, R, bounded by the curve with equation $y = x(4 + x)$, the line with equation $y = 12$ and the y-axis.



(a) Find the coordinate of the point A where the line meets the curve.

(b) Find the area of R.

Solution:

(a) A is given by

$x(4 + x) = 12$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x = 2$ or -6

So A is (2, 12)

(b) R is given by taking $\int_0^2 x(4 + x)\,dx$ away from a rectangle of area $12 \times 2 = 24$.

So area of R

$= 24 - \int_0^2 (x^2 + 4x)\,dx$

$= 24 - \left[\frac{x^3}{3} + 2x^2 \right]_0^2$

$= 24 - \left\{ \left(\frac{8}{3} + 8 \right) - \left(0 \right) \right\}$

$= 24 - \frac{32}{3}$

$= \frac{40}{3}$ or $13\frac{1}{3}$

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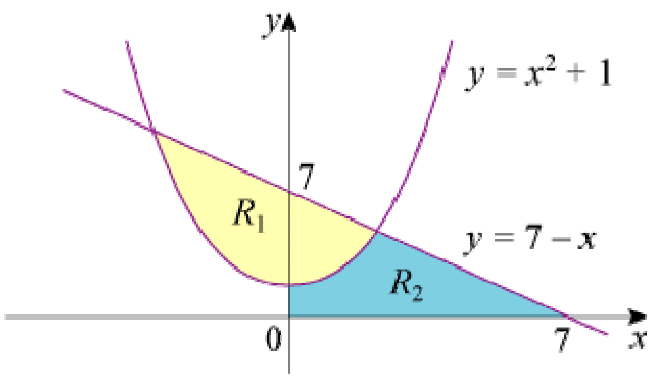
Exercise D, Question 6

Question:

The diagram shows a sketch of part of the curve with equation $y = x^2 + 1$ and the line with equation $y = 7 - x$. The finite region R_1 is bounded by the line and the curve. The finite region R_2 is below the curve and the line and is bounded by the positive x- and y-axes as shown in the diagram.



(a) Find the area of R_1.

(b) Find the area of R_2.

Solution:

(a) Intersections when
$7 - x = x^2 + 1$
$0 = x^2 + x - 6$
$0 = (x + 3) (x - 2)$
$x = 2$ or $- 3$

(a) Area of R_1 is given by $\int_{-3}^{2} [7 - x - (x^2 + 1)] dx$

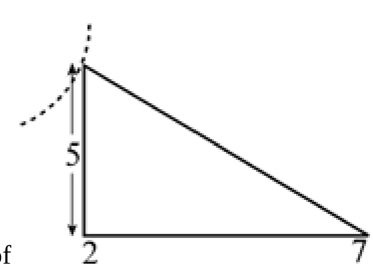
$= \int_{-3}^{2} (6 - x - x^2) dx$

$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-3}^{2}$

$= \left(12 - \frac{4}{2} - \frac{8}{3} \right) - \left(- 18 - \frac{9}{2} + \frac{27}{3} \right)$

$= 20 \frac{5}{6}$

(b) Area of R_2 is given by $\int_0^2 (x^2 + 1) dx +$ area of



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$= \left[\frac{x^3}{3} + x \right]_0^2 + \frac{1}{2} \times 5 \times 5$

$= \left(\frac{8}{3} + 2 \right) - \left(0 \right) + \frac{25}{2}$

$= 17 \frac{1}{6}$

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Exercise D, Question 7

Question:

The curve C has equation $y = x^{\frac{2}{3}} - \frac{2}{x^{\frac{1}{3}}} + 1$.

(a) Verify that C crosses the x-axis at the point (1, 0).

(b) Show that the point A (8, 4) also lies on C.

(c) The point B is (4, 0). Find the equation of the line through AB.
The finite region R is bounded by C, AB and the positive x-axis.

(d) Find the area of R.

Solution:

(a) $x = 1$, $y = 1 - \frac{2}{1} + 1 = 0$

So (1, 0) lies on C

(b) $x = 8$, $y = 8^{\frac{2}{3}} - \frac{2}{8^{\frac{1}{3}}} + 1 = 2^2 - \frac{2}{2} + 1 = 4$

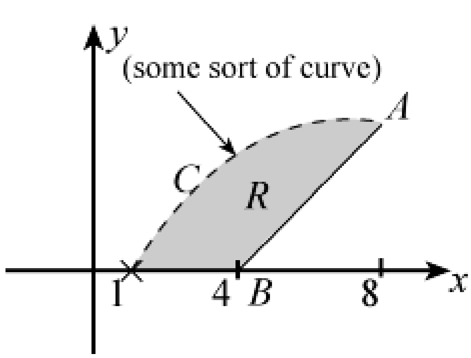
So (8, 4) lies on C

(c) A is (8, 4) and B is (4, 0)

Gradient of line through AB is $\frac{4-0}{8-4} = 1$.

So equation is $y - 0 = x - 4$, i.e. $y = x - 4$

(d)



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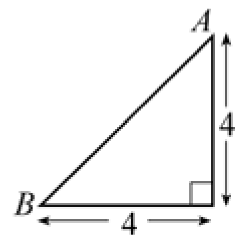
The area of R is given by $\int_1^8$ (curve) dx − area of 

$$= \int_1^8 \left(x^{\frac{2}{3}} - \frac{2}{x^{\frac{1}{3}}} + 1 \right) dx - \frac{1}{2} \times 4 \times 4$$

$$= \left[\frac{3}{5} x^{\frac{5}{3}} - \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + x \right]_1^8 - 8$$

$$= \left(\frac{3}{5} \times 32 - 3 \times 4 + 8 \right) - \left(\frac{3}{5} - 3 + 1 \right) - 8$$

$$= \frac{93}{5} - 4 + 2 - 8$$

$$= 8 \frac{3}{5}$$

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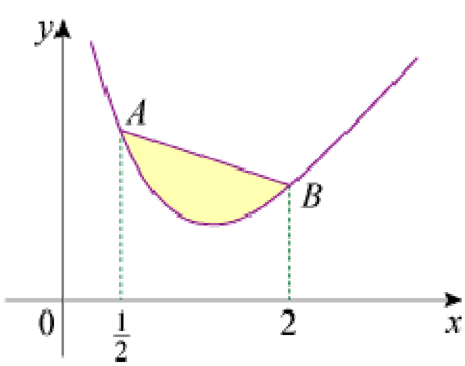
Exercise D, Question 8

Question:

The diagram shows part of a sketch of the curve with equation $y = \frac{2}{x^2} + x$.

The points A and B have x-coordinates $\frac{1}{2}$ and 2 respectively.



Find the area of the finite region between AB and the curve.

Solution:

$$\text{Area} = \int_{\frac{1}{2}}^{2} \left[\text{line } AB - \left(\frac{2}{x^2} + x \right) \right] dx$$

A is $\left(\frac{1}{2}, 8\frac{1}{2} \right)$ and B is $\left(2, 2\frac{1}{2} \right)$

$$\text{Gradient} = -\frac{6}{1\frac{1}{2}} = -4$$

So equation is $y - 2\frac{1}{2} = -4\left(x - 2 \right)$, i.e. $y = 10\frac{1}{2} - 4x$

$$\text{Area} = \int_{\frac{1}{2}}^{2} \left(10\frac{1}{2} - 5x - 2x^{-2} \right) dx$$

$$= \left[\frac{21}{2}x - \frac{5}{2}x^2 - \frac{2x^{-1}}{-1} \right]_{\frac{1}{2}}^{2}$$

$$= \left[\frac{21}{2}x - \frac{5}{2}x^2 + \frac{2}{x} \right]_{\frac{1}{2}}^{2}$$

$$= \left(21 - 10 + 1 \right) - \left(\frac{21}{4} - \frac{5}{8} + 4 \right)$$

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$= 12 - 8\frac{5}{8}$

$= 3\frac{3}{8}$ or 3.375 or 3.38 (3 s.f.)

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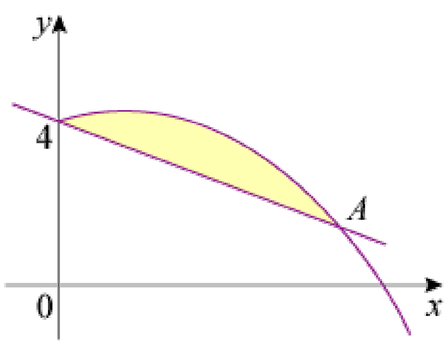
Exercise D, Question 9

Question:

The diagram shows part of the curve with equation $y = 3\sqrt{x} - \sqrt{x^3} + 4$ and the line with equation $y = 4 - \frac{1}{2}x$.



(a) Verify that the line and the curve cross at the point $A(4, 2)$.

(b) Find the area of the finite region bounded by the curve and the line.

Solution:

(a) $x = 4$ in line gives $y = 4 - \frac{1}{2} \times 4 = 2$

$x = 4$ in curve gives $y = 3 \times \sqrt{4} - \sqrt{64} + 4 = 6 - 8 + 4 = 2$

So (4, 2) lies on line and curve.

(b) Area $= \int_0^4 \left[3x^{\frac{1}{2}} - x^{\frac{3}{2}} + 4 - \left(4 - \frac{1}{2}x \right) \right] dx$

$= \int_0^4 \left(3x^{\frac{1}{2}} - x^{\frac{3}{2}} + \frac{1}{2}x \right) dx$

$= \left[\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{4} \right]_0^4$

$= \left[2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + \frac{x^2}{4} \right]_0^4$

$= \left(2 \times 8 - \frac{2}{5} \times 32 + 4 \right) - \left(0 \right)$

$= 20 - \frac{64}{5}$

$= \frac{36}{5}$ or 7.2

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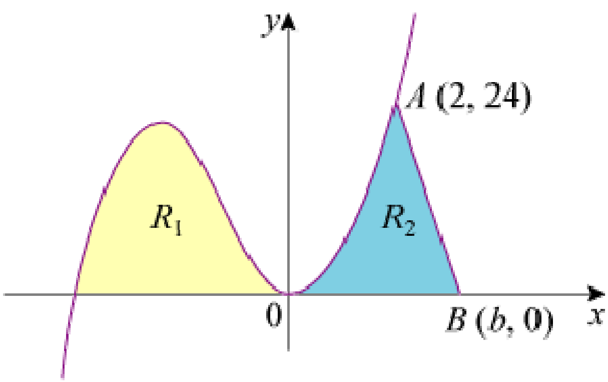
Integration
Exercise D, Question 10

Question:



The sketch shows part of the curve with equation $y = x^2 (x + 4)$. The finite region R_1 is bounded by the curve and the negative x-axis. The finite region R_2 is bounded by the curve, the positive x-axis and AB, where $A (2 , 24)$ and $B (b , 0)$.

The area of R_1 = the area of R_2.

(a) Find the area of R_1.

(b) Find the value of b.

Solution:

(a) $y = x^2 (x + 4)$

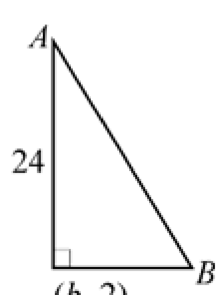
$y = 0 \quad \Rightarrow \quad x = 0 \quad \text{(twice)}, \; - 4$

Area of R_1 is $\int_{-4}^{0} (x^3 + 4x^2) \, dx$

$$= \left[\frac{x^4}{4} + \frac{4}{3}x^3 \right]_{-4}^{0}$$

$$= \left(0 \right) - \left(\frac{4^4}{4} - \frac{4^4}{3} \right)$$

$$= \frac{4^4}{12} = \frac{4^3}{3} = \frac{64}{3} \text{ or } 21 \frac{1}{3}$$

(b) Area of R_2 is $\int_0^2 (x^3 + 4x^2) \, dx +$ area of 

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$= \left[\frac{x^4}{4} + \frac{4}{3}x^3 \right]_0^2 + 12 \left(b - 2 \right)$

$= \left(\frac{16}{4} + \frac{32}{3} \right) - \left(0 \right) + 12 \left(b - 2 \right)$

$= 14\frac{2}{3} + 12b - 24$

$= -9\frac{1}{3} + 12b$

Area of R_2 = area of R_1 $\Rightarrow$ $-9\frac{1}{3} + 12b = 21\frac{1}{3}$

So $12b = 30\frac{2}{3}$ $\Rightarrow$ $b = 2\frac{5}{9}$ or 2.56 (3 s.f.)

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Exercise E, Question 1

Question:

Copy and complete the table below and use the trapezium rule to estimate $\int_1^3 \frac{1}{x^2+1} dx$:

x	1	1.5	2	2.5	3
$y = \frac{1}{x^2+1}$	0.5	0.308		0.138	

Solution:

$x = 2, y = 0.2; x = 3, y = 0.1$
$h = 0.5$

So $A \approx \frac{1}{2} \times 0.5 \left[0.5 + 2 \left(0.308 + 0.2 + 0.138 \right) + 0.1 \right]$

$= \frac{1}{4} \left[1.892 \right]$

$= 0.473$

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Integration
Exercise E, Question 2

Question:

Use the table below to estimate $\int_1^{2.5} \sqrt{(2x-1)}\,dx$ with the trapezium rule:

x	1	1.25	1.5	1.75	2	2.25	2.5
$y = \sqrt{(2x-1)}$	1	1.225	1.414	1.581	1.732	1.871	2

Solution:

$$A \approx \frac{1}{2} \times 0.25 \left[1 + 2 \left(1.225 + 1.414 + 1.581 + 1.732 + 1.871 \right) + 2 \right]$$

$$= \frac{1}{8} \left[18.646 \right]$$

$= 2.33075$
$= 2.33$ (3 s.f.)

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Exercise E, Question 3

Question:

Copy and complete the table below and use it, together with the trapezium rule, to estimate $\int_0^2 \sqrt{(x^3+1)}\, dx$:

x	0	0.5	1	1.5	2
$y = \sqrt{(x^3+1)}$	1	1.061	1.414		

Solution:

$x = 1.5$, $y = \sqrt{(1.5^3+1)} = 2.09165 \quad \ldots \quad$ or 2.092 (4 s.f.)
$x = 2$, $y = \sqrt{(2^3+1)} = 3$
$\int_0^2 \sqrt{(x^3+1)}\, dx$

$$\approx \frac{1}{2} \times 0.5 \left[1 + 2 \left(1.061 + 1.414 + 2.092 \right) + 3 \right]$$

$$= \frac{1}{4} \left[13.134 \right]$$

$= 3.2835$
$= 3.28$ (3 s.f.)

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Exercise E, Question 4

Question:

(a) Use the trapezium rule with 8 strips to estimate $\int_0^2 2^x dx$.

(b) With reference to a sketch of $y = 2^x$ explain whether your answer in part (a) is an underestimate or an overestimate of $\int_0^2 2^x dx$.

Solution:

$h = 0.25$

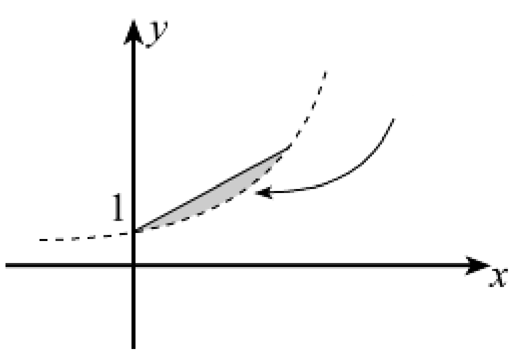
x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
y	1	1.189	1.414	1.682	2	2.378	2.828	3.364	4

$\int_0^2 2^x dx$

$$\approx \frac{1}{2} \times 0.25 \left[1 + 2 \left(1.189 + 1.414 + 1.682 + 2 + 2.378 + 2.828 + 3.364 \right) + 4 \right]$$

$$= \frac{1}{8} \left[34.71 \right]$$

$= 4.33875$
$= 4.34$ (3 s.f.)



(b)

Curve bends beneath straight line of trapezium so trapezium rule will **overestimate**.

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Exercise E, Question 5

Question:

Use the trapezium rule with 6 strips to estimate $\int_0^3 \frac{1}{\sqrt{(x^2+1)}} dx$.

Solution:

$h = 0.5$

x	0	0.5	1	1.5	2	2.5	3
y	1	0.894	0.707	0.555	0.447	0.371	0.316

$A \approx \frac{1}{2} \times 0.5 \left[1 + 2 \left(0.894 + 0.707 + 0.555 + 0.447 + 0.371 \right) + 0.316 \right]$

$= \frac{1}{4} \left[7.264 \right]$

$= 1.816$ or 1.82 (3 s.f.)

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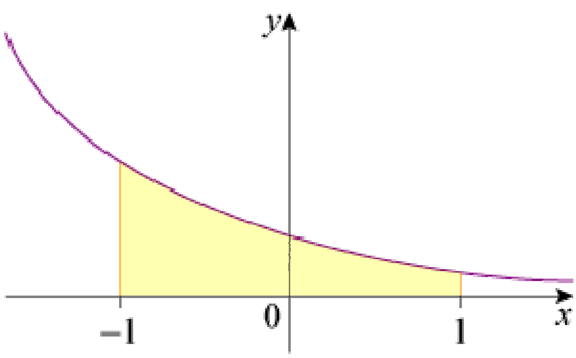
Exercise E, Question 6

Question:

The diagram shows a sketch of part of the curve with equation $y = \frac{1}{x+2}$, $x > -2$.



(a) Copy and complete the table below and use the trapezium rule to estimate the area bounded by the curve, the *x*-axis and the lines $x = -1$ and $x = 1$.

x	-1	-0.6	-0.2	0.2	0.6	1
$y = \frac{1}{x+2}$	1	0.714			0.385	0.333

(b) State, with a reason, whether your answer in part (a) is an overestimate or an underestimate.

Solution:

(a) $h = 0.4$

$x = -0.2$, $y = \frac{1}{1.8} = 0.555 \ldots = 0.556$ (3 d.p.)

$x = 0.2$, $y = \frac{1}{2.2} = 0.4545 \ldots = 0.455$ (3 d.p.)

$$\text{area} \approx \frac{1}{2} \times 0.4 \left[1 + 2 \left(0.714 + 0.556 + 0.455 + 0.385 \right) + 0.333 \right]$$

$= 0.2 [5.553]$
$= 1.1106$
$= 1.11$ (3 s.f.)

(b) Curve bends down below the straight lines of the trapezia so trapezium rule will give an **overestimate**.

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Exercise E, Question 7

Question:

(a) Sketch the curve with equation $y = x^3 + 1$, for $-2 < x < 2$.

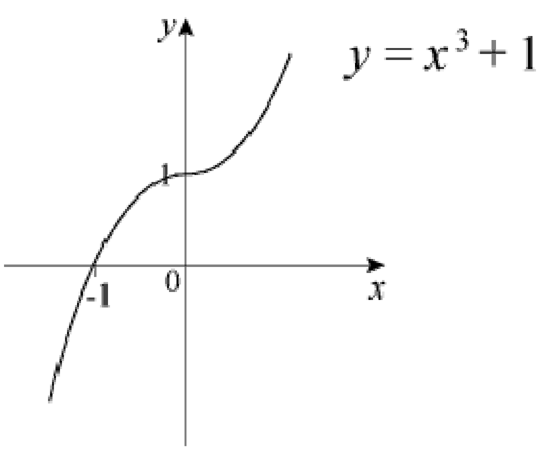
(b) Use the trapezium rule with 4 strips to estimate the value of $\int_{-1}^{1} (x^3 + 1) dx$.

(c) Use integration to find the exact value of $\int_{-1}^{1} (x^3 + 1) dx$.

(d) Comment on your answers to parts (b) and (c).

Solution:

(a) $y = x^3 + 1$ is a vertical translation $(+ 1)$ of $y = x^3$



(b) $h = 0.5$

x	-1	-0.5	0	0.5	1
y	0	0.875	1	1.125	2

$$\int_{-1}^{1} \left(x^3 + 1 \right) dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.875 + 1 + 1.125 \right) + 2 \right] = \frac{1}{4} \left[8 \right] = 2$$

(c) $$\int_{-1}^{1} \left(x^3 + 1 \right) dx = \left[\frac{x^4}{4} + x \right]_{-1}^{1} = \left(\frac{1}{4} + 1 \right) - \left(\frac{1}{4} - 1 \right) = 2$$

(d) Same. Curve has rotational symmetry of order 2 about (0, 1) and trapezia cut curve above and below symmetrically.

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Integration
Exercise E, Question 8

Question:

Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{(3^x - 1)}\, dx$.

Solution:

$h = 0.5$

x	0	0.5	1	1.5	2
y	0	0.856	1.414	2.048	2.828

$$\int_0^2 \sqrt{\left(3^x - 1\right)}\, dx \approx \frac{1}{2} \times 0.5 \left[0 + 2\left(0.856 + 1.414 + 2.048 \right) + 2.828 \right]$$

$$= \frac{1}{4}\left[11.464 \right]$$

$= 2.866$
$= 2.87$ (3 s.f.)

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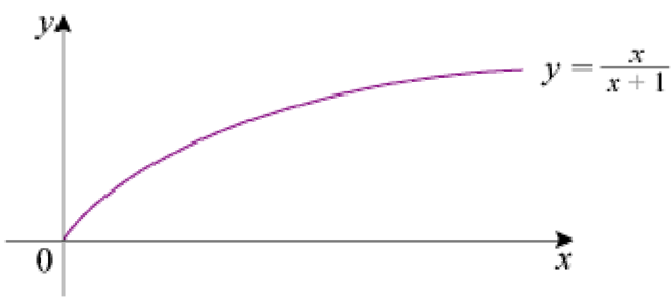
Integration

Exercise E, Question 9

Question:

The sketch shows part of the curve with equation $y = \frac{x}{x+1}$, $x \geq 0$.



(a) Use the trapezium rule with 6 strips to estimate $\int_0^3 \frac{x}{x+1} dx$.

(b) With reference to the sketch state, with a reason, whether the answer in part (a) is an overestimate or an underestimate.

Solution:

(a) $h = 0.5$

x	0	0.5	1	1.5	2	2.5	3
y	0	0.333	0.5	0.6	0.667	0.714	0.75

$$\int_0^3 \frac{x}{x+1} dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.333 + 0.5 + 0.6 + 0.667 + 0.714 \right) + 0.75 \right]$$

$$= \frac{1}{4} \left[6.378 \right]$$

$= 1.5945$

$= 1.59$ (3 s.f.)

(b) Curve bends outwards above straight lines of trapezia so trapezium rule is an **underestimate**.

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Integration
Exercise E, Question 10

Question:

(a) Use the trapezium rule with n strips to estimate $\int_0^2 \sqrt{x}\ dx$ in the cases (i) $n = 4$ (ii) $n = 6$.

(b) Compare your answers from part (a) with the exact value of the integral and calculate the percentage error in each case.

Solution:

(a) (i) $h = 0.5$

x	0	0.5	1	1.5	2
y	0	0.707	1	1.225	1.414

$$\int_0^2 \sqrt{x}\ dx \approx \frac{1}{2} \times 0.5 \left[0 + 2 \left(0.707 + 1 + 1.225 \right) + 1.414 \right] = \frac{1}{4} \left[7.278 \right] = 1.8195$$

(ii) $h = \frac{1}{3}$

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
y	0	0.577	0.816	1	1.155	1.291	1.414

$$\int_0^2 \sqrt{x}\ dx \approx \frac{1}{2} \times \frac{1}{3} \left[0 + 2 \left(0.577 + 0.816 + 1 + 1.155 + 1.291 \right) + 1.414 \right] = \frac{1}{6} \left[11.092 \right]$$
$$= 1.8486$$

(b) $$\int_0^2 \sqrt{x}\ dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^2 = \left(\frac{2}{3} \times 2 \sqrt{2} \right) - \left(0 \right) = \frac{4}{3} \sqrt{2} = 1.8856 \ \ldots$$

(i) % error $$= \frac{100 \left(\frac{4}{3} \sqrt{2} - 1.8195 \right)}{\frac{4}{3} \sqrt{2}} = 3.51\ \%$$

(ii) % error $$= \frac{100 \left(\frac{4}{3} \sqrt{2} - 1.8486 \right)}{\frac{4}{3} \sqrt{2}} = 1.96\ \%$$

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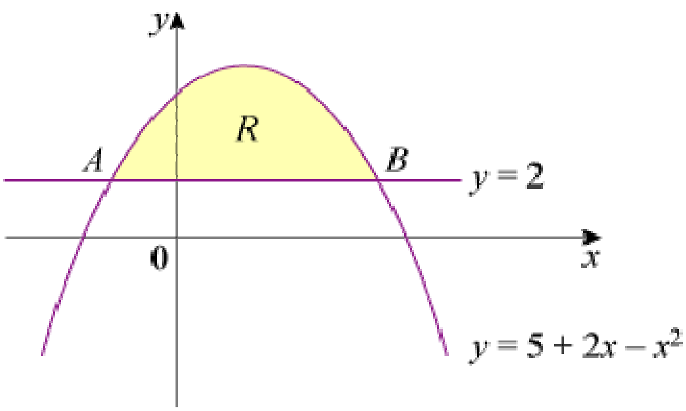
Integration
Exercise F, Question 1

Question:

The diagram shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B.



(a) Find the x-coordinates of A and B.

(b) The shaded region R is bounded by the curve and the line. Find the area of R.

[E]

Solution:

(a) $2 = 5 + 2x - x^2$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad (x - 3) (x + 1) = 0$

$\Rightarrow \quad x = -1 \, (A), 3 \, (B)$

(b) Area of $R = \int_{-1}^{3} (5 + 2x - x^2 - 2) \, dx$

$= \int_{-1}^{3} (3 + 2x - x^2) \, dx$

$= \left[3x + x^2 - \frac{1}{3}x^3 \right]_{-1}^{3}$

$= \left(9 + 9 - \frac{27}{3} \right) - \left(-3 + 1 + \frac{1}{3} \right)$

$= 9 + 2 - \frac{1}{3}$

$= 10 \frac{2}{3}$

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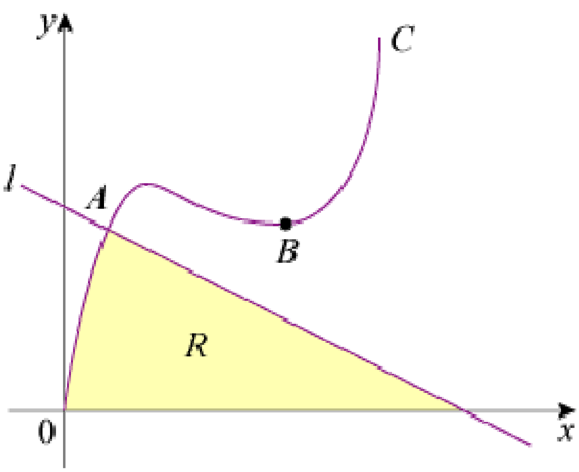
Exercise F, Question 2

Question:

The diagram shows part of the curve C with equation $y = x^3 - 9x^2 + px$, where p is a constant. The line l has equation $y + 2x = q$, where q is a constant. The point A is the intersection of C and l, and C has a minimum at the point B. The x-coordinates of A and B are 1 and 4 respectively.



(a) Show that $p = 24$ and calculate the value of q.

(b) The shaded region R is bounded by C, l and the x-axis. Using calculus, showing all the steps in your working and using the values of p and q found in part (a), find the area of R.

[E]

Solution:

(a) When $x = 1$: $q - 2x = x^3 - 9x^2 + px$

$\Rightarrow \quad q - 2 = 1 - 9 + p$

$\Rightarrow \quad q + 6 = p$ ①

When $x = 4$: $\frac{dy}{dx} = 3x^2 - 18x + p = 0$

$\Rightarrow \quad 48 - 72 + p = 0$

$\Rightarrow \quad p = 24$

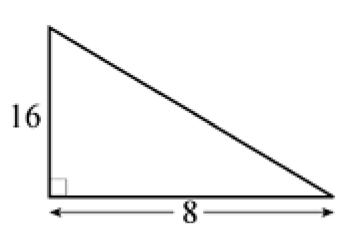
Substitute into ①: $q = p - 6 = 18$

(b) Line is $y = 18 - 2x$

So A is (1, 16) and the line cuts the x-axis at (9, 0)

Area of R is given by

$\int_0^1 (x^3 - 9x^2 + 24x) \, dx +$ area of



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$$= \left[\frac{x^4}{4} - \frac{9}{3}x^3 + \frac{24}{2}x^2 \right]_0^1 + \frac{1}{2} \times 8 \times 16$$

$$= \left[\frac{x^4}{4} - 3x^3 + 12x^2 \right]_0^1 + 64$$

$$= \left(\frac{1}{4} - 3 + 12 \right) - \left(0 \right) + 64$$

$$= 73\frac{1}{4}$$

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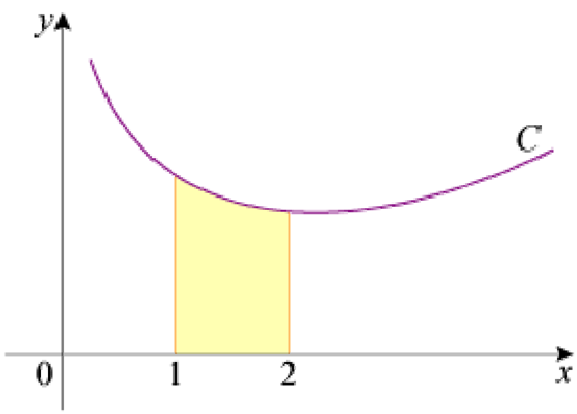
Integration

Exercise F, Question 3

Question:

The diagram shows part of the curve C with equation $y = f(x)$, where $f(x) = 16x^{-\frac{1}{2}} + x^{\frac{3}{2}}$, $x > 0$.



(a) Use calculus to find the x-coordinate of the minimum point of C, giving your answer in the form $k\sqrt{3}$, where k is an exact fraction.
The shaded region shown in the diagram is bounded by C, the x-axis and the lines with equations $x = 1$ and $x = 2$.

(b) Using integration and showing all your working, find the area of the shaded region, giving your answer in the form $a + b\sqrt{2}$, where a and b are exact fractions.

[E]

Solution:

(a) $f'(x) = -8x^{-\frac{3}{2}} + \frac{3}{2}x^{\frac{1}{2}}$

$f'(x) = 0 \Rightarrow \frac{8}{x^{\frac{3}{2}}} = \frac{3}{2}x^{\frac{1}{2}}$ or $x^2 = \frac{16}{3}$

(x must be positive) So $x = \frac{4}{\sqrt{3}}$ or $\frac{4}{3}\sqrt{3}$

(b) Area $= \int_1^2 \left(16x^{-\frac{1}{2}} + x^{\frac{3}{2}}\right) dx$

$= \left[\frac{16x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_1^2$

$= \left[32x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}}\right]_1^2$

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$$= \left(32\sqrt{2} + \frac{2}{5} \times 2^2 \sqrt{2} \right) - \left(32 + \frac{2}{5} \right)$$

$$= \frac{168}{5}\sqrt{2} - \frac{162}{5}$$

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Integration
Exercise F, Question 4

Question:

(a) Find $\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx$.

(b) Use your answer to part (a) to evaluate

$\int_1^4 \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx$.

giving your answer as an exact fraction.

[E]

Solution:

(a) $\left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) = 1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 = 5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}}$

$\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx = 5x - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$

(b) $\int_1^4 \left(x^{\frac{1}{2}} - 4 \right) \left(x^{\frac{-1}{2}} - 1 \right) dx$

$= \left[5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} \right]_1^4$

$= \left(20 - 8 \times 2 - \frac{2}{3} \times 2^3 \right) - \left(5 - 8 - \frac{2}{3} \right)$

$= 4 - \frac{16}{3} + 3 + \frac{2}{3}$

$= 7 - \frac{14}{3}$

$= \frac{7}{3}$ or $2\frac{1}{3}$

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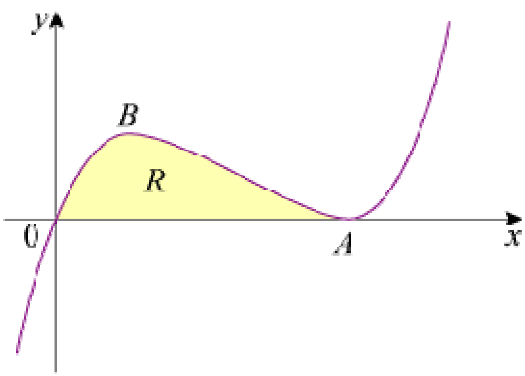
Integration
Exercise F, Question 5

Question:

The diagram shows part of the curve with equation $y = x^3 - 6x^2 + 9x$. The curve touches the x-axis at A and has a maximum turning point at B.



(a) Show that the equation of the curve may be written as $y = x(x-3)^2$, and hence write down the coordinates of A.

(b) Find the coordinates of B.

(c) The shaded region R is bounded by the curve and the x-axis. Find the area of R.

[E]

Solution:

(a) $(x-3)^2 = x^2 - 6x + 9$
So $x(x-3)^2 = x^3 - 6x^2 + 9x$
$y = 0 \Rightarrow x = 0$ [i.e. (0, 0)] or 3 (twice)
So A is (3, 0)

(b) $\frac{dy}{dx} = 0 \Rightarrow 0 = 3x^2 - 12x + 9$

$\Rightarrow 0 = 3(x^2 - 4x + 3)$

$\Rightarrow 0 = 3(x-3)(x-1)$

$\Rightarrow x = 1$ or 3

$x = 3$ at A, the minimum, so B is (1, 4)

(c) Area of $R = \int_0^3 (x^3 - 6x^2 + 9x)\,dx$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right]_0^3$$

$$= \left(\frac{81}{4} - 2 \times 27 + \frac{9}{2} \times 9\right) - \left(0\right)$$

$$= 6\frac{3}{4}$$

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Exercise F, Question 6

Question:

Given that $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$:

(a) Show that $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$, where A and B are constants to be found.

(b) Hence find $\int y \ dx$.

(c)Using your answer from part (b) determine the exact value of $\int_1^8 y dx$.

[E]

Solution:

(a) $y = \left(x^{\frac{1}{3}} + 3 \right)^2 = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \quad (A = 6, B = 9)$

(b) $\int y \ dx = \left[\frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c \right]$

$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$

(c) $\int_1^8 y \ dx = \left[\frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x \right]_1^8$

$= \left(\frac{3}{5} \times 32 + \frac{9}{2} \times 16 + 72 \right) - \left(\frac{3}{5} + \frac{9}{2} + 9 \right)$

$= \frac{93}{5} + 135 - \frac{9}{2}$

$= 149 \frac{1}{10}$ or 149.1

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Exercise F, Question 7

Question:

Considering the function $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$, $x > 0$:

(a) Find $\frac{dy}{dx}$.

(b) Find $\int y \ dx$.

(c) Hence show that $\int_1^3 y \ dx = A + B\sqrt{3}$, where A and B are integers to be found.

[E]

Solution:

(a) $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2} \times 4x^{-\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

(b) $\int y dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \right) dx$

$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$

(c) $\int_1^3 y dx = \left[2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} \right]_1^3$

$= \left(2 \times 3\sqrt{3} - 8\sqrt{3} \right) - \left(2 - 8 \right)$
$= -2\sqrt{3} + 6$
$= 6 - 2\sqrt{3}$

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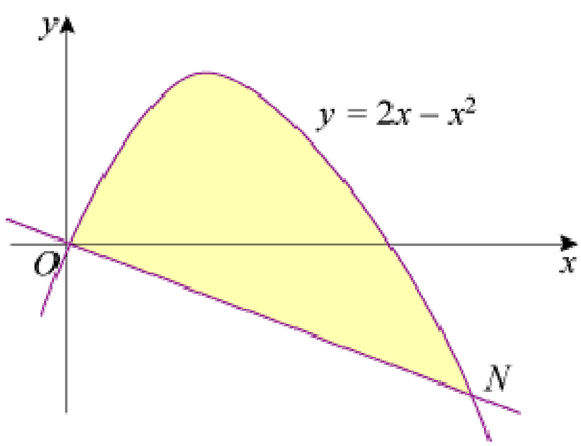
Exercise F, Question 8

Question:

The diagram shows a sketch of the curve with equation $y = 2x - x^2$ and the line *ON* which is the normal to the curve at the origin *O*.



(a) Find an equation of *ON*.

(b) Show that the *x*-coordinate of the point *N* is $2\frac{1}{2}$ and determine its *y*-coordinate.

(c) The shaded region shown is bounded by the curve and the line *ON*. Without using a calculator, determine the area of the shaded region.

Solution:

(a) $y = 2x - x^2$

$\frac{dy}{dx} = 2 - 2x$

Gradient of tangent at $(0 , \ 0)$ is 2.

Gradient of *ON* $= - \frac{1}{2}$

So equation of *ON* is $y = - \frac{1}{2}x$ or $2y + x = 0$

(b) *N* is point of intersection of *ON* and the curve, so

$- \frac{1}{2}x = 2x - x^2$

$2x^2 - 5x = 0$

$x (2x - 5) = 0$

$x = 0, \frac{5}{2}$

So *N* is $\left(2\frac{1}{2} , \ - 1\frac{1}{4} \right)$

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(c) Area $= \int_0^{2\frac{1}{2}} (\text{curve} - \text{line})\, dx$

$$= \int_0^{2\frac{1}{2}} \left[2x - x^2 - \left(-\frac{1}{2}x \right) \right] dx$$

$$= \int_0^{2\frac{1}{2}} \left(\frac{5}{2}x - x^2 \right) dx$$

$$= \left[\frac{5}{4}x^2 - \frac{x^3}{3} \right]_0^{2\frac{1}{2}}$$

$$= \left(\frac{31.25}{4} - \frac{15.625}{3} \right) - \left(0 \right)$$

$$= \frac{125}{48}$$

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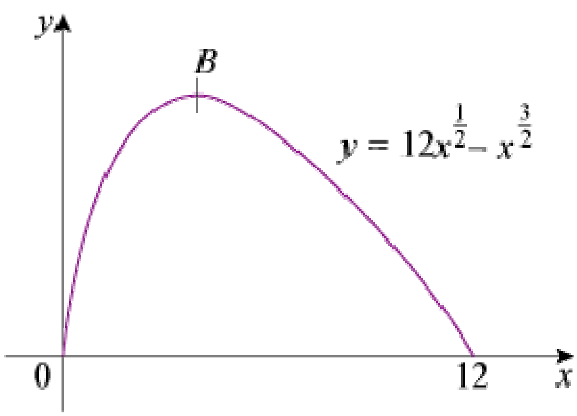
Integration
Exercise F, Question 9

Question:



The diagram shows a sketch of the curve with equation
$y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$ for $0 \le x \le 12$.

(a) Show that $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}\left(4 - x\right)$.

(b) At the point B on the curve the tangent to the curve is parallel to the x-axis. Find the coordinates of the point B.

(c) Find, to 3 significant figures, the area of the finite region bounded by the curve and the x-axis.

[E]

Solution:

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}\left(4 - x\right)$

(b) $\frac{dy}{dx} = 0 \Rightarrow x = 4, y = 12 \times 2 - 2^3 = 16$

So B is (4, 16)

(c) Area $= \int_0^{12}\left(12x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$

$= \left[\frac{12x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^{12}$

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$$= \left[8x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} \right]_0^{12}$$

$$= \left(8 \times \sqrt{12^3} - \frac{2}{5}\sqrt{12^5} \right) - \left(0 \right)$$

$= 133.0215 \quad \ldots$

$= 133$ (3 s.f.)

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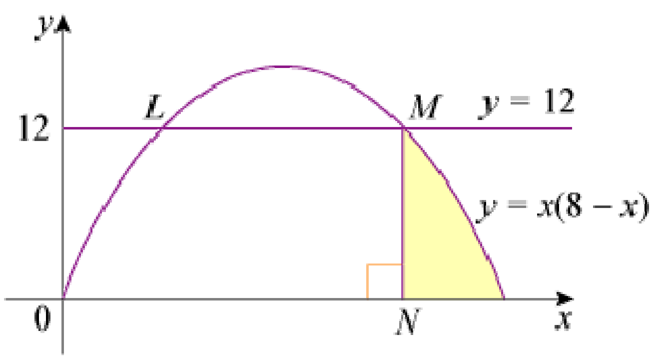
Integration
Exercise F, Question 10

Question:

The diagram shows the curve C with equation $y = x(8 - x)$ and the line with equation $y = 12$ which meet at the points L and M.



(a) Determine the coordinates of the point M.

(b) Given that N is the foot of the perpendicular from M on to the x-axis, calculate the area of the shaded region which is bounded by NM, the curve C and the x-axis.

[E]

Solution:

(a) $x(8 - x) = 12$

$\Rightarrow \quad 8x - x^2 = 12$

$\Rightarrow \quad 0 = x^2 - 8x + 12$

$\Rightarrow \quad 0 = (x - 6)(x - 2)$

$\Rightarrow \quad x = 2$ or 6

M is on the right of L, so M is (6, 12)

(b) Area $= \int_6^8 (8x - x^2)\,dx$

$$= \left[4x^2 - \frac{x^3}{3} \right]_6^8$$

$$= \left(4 \times 64 - \frac{512}{3} \right) - \left(4 \times 36 - \frac{216}{3} \right)$$

$$= 256 - 170\frac{2}{3} - 144 + 72$$

$$= 13\frac{1}{3}$$

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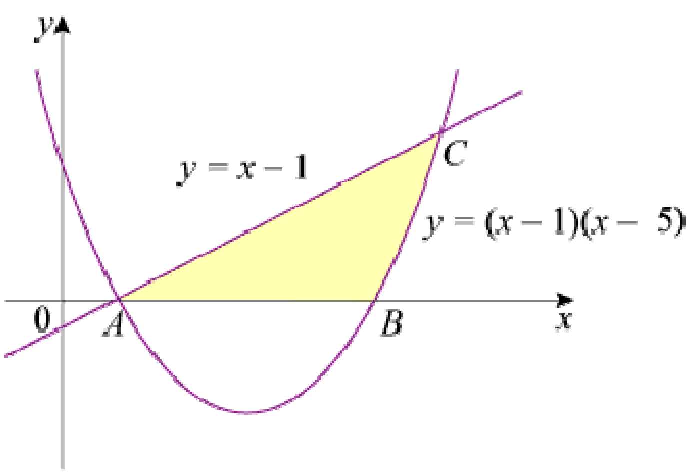
Integration

Exercise F, Question 11

Question:

The diagram shows the line $y = x - 1$ meeting the curve with equation $y = (x - 1)(x - 5)$ at A and C. The curve meets the x-axis at A and B.



(a) Write down the coordinates of A and B and find the coordinates of C.

(b) Find the area of the shaded region bounded by the line, the curve and the x-axis.

Solution:

(a) A is $(1, 0)$, B is $(5, 0)$

$x - 1 = (x - 1)(x - 5)$

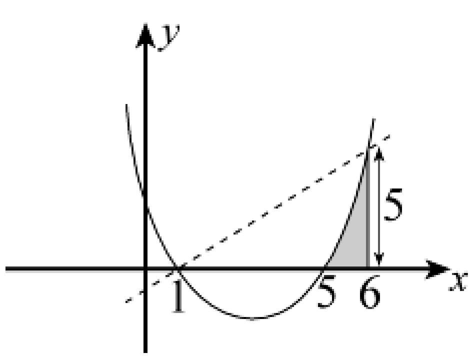
$\Rightarrow \quad 0 = (x - 1)(x - 5 - 1)$

$\Rightarrow \quad 0 = (x - 1)(x - 6)$

$\Rightarrow \quad x = 1, 6$

So C is $(6, 5)$

(b)



Shaded region is $\int_5^6 (x - 1)(x - 5)\,dx = \int_5^6 (x^2 - 6x + 5)\,dx$

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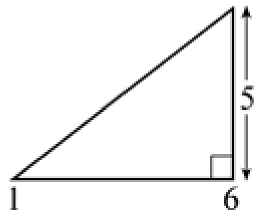
Required area = area of 

$- \int_5^6 (x^2 - 6x + 5) \, dx$

$= \frac{1}{2} \times 5 \times 5 - \left[\frac{x^3}{3} - 3x^2 + 5x \right]_5^6$

$= 12 \frac{1}{2} - \left[\left(\frac{216}{3} - 3 \times 36 + 30 \right) - \left(\frac{125}{3} - 75 + 25 \right) \right]$

$= 12 \frac{1}{2} + 6 - 50 + 41 \frac{2}{3}$

$= 10 \frac{1}{6}$

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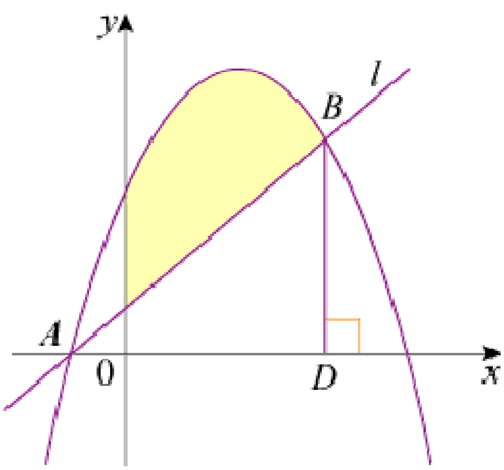
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Integration
Exercise F, Question 12

Question:



A and B are two points which lie on the curve C, with equation $y = -x^2 + 5x + 6$. The diagram shows C and the line l passing through A and B.

(a) Calculate the gradient of C at the point where $x = 2$.
The line l passes through the point with coordinates (2, 3) and is parallel to the tangent to C at the point where $x = 2$.

(b) Find an equation of l.

(c) Find the coordinates of A and B.
The point D is the foot of the perpendicular from B on to the x-axis.

(d) Find the area of the region bounded by C, the x-axis, the y-axis and BD.

(e) Hence find the area of the shaded region.

[E]

Solution:

(a) $\frac{dy}{dx} = -2x + 5$

When $x = 2$ gradient of C is $-4 + 5 = 1$

(b) Equation of l is $y - 3 = 1 (x - 2)$ i.e. $y = x + 1$

(c) A is $(-1, 0)$
B is given by
$x + 1 = -x^2 + 5x + 6$
$x^2 - 4x - 5 = 0$
$(x - 5) (x + 1) = 0$
$x = -1$ or 5
So B is (5, 6)

(d) Area $= \int_0^5 (-x^2 + 5x + 6) dx$

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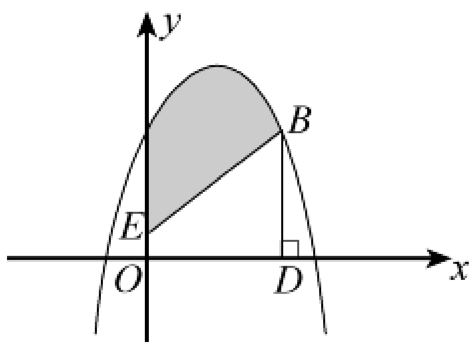
$= \left[-\frac{x^3}{3} + \frac{5x^2}{2} + 6x \right]_0^5$

$= \left(-\frac{125}{3} + \frac{125}{2} + 30 \right) - \left(0 \right)$

$= \frac{125}{6} + 30$

$= 50\frac{5}{6}$

(e)



Required area is (d) − trapezium *OEBD*

Area of trapezium $= \frac{1}{2} \times 5 \times \left(1 + 6 \right) = \frac{35}{2} = 17\frac{1}{2}$

Shaded region $= 50\frac{5}{6} - 17\frac{1}{2} = 33\frac{1}{3}$

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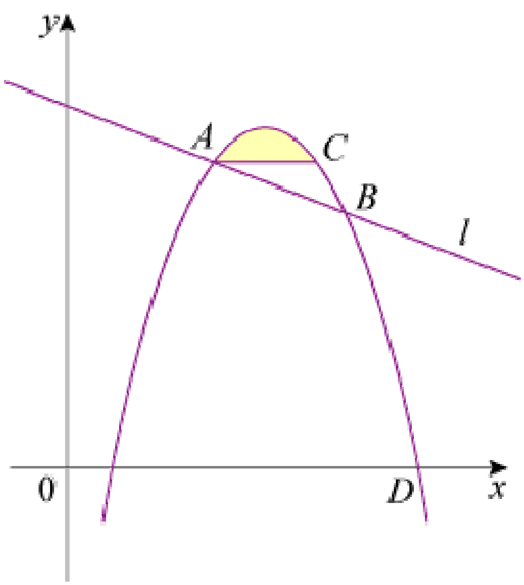
Integration
Exercise F, Question 13

Question:



The diagram shows part of the curve with equation $y = p + 10x - x^2$, where p is a constant, and part of the line l with equation $y = qx + 25$, where q is a constant. The line l cuts the curve at the points A and B. The x-coordinates of A and B are 4 and 8 respectively. The line through A parallel to the x-axis intersects the curve again at the point C.

(a) Show that $p = -7$ and calculate the value of q.

(b) Calculate the coordinates of C.

(c) The shaded region in the diagram is bounded by the curve and the line AC. Using algebraic integration and showing all your working, calculate the area of the shaded region.

[E]

Solution:

(a) Using A which lies on line and curve: $4q + 25 = p + 40 - 16$
i.e. $4q = p - 1$ ①
Using B which lies on line and curve: $8q + 25 = p + 80 - 64$
i.e. $8q = p - 9$ ②
Solving ② – ① $\Rightarrow$ $4q = -8$ $\Rightarrow$ $q = -2$
Substitute into ① $\Rightarrow$ $p = 1 + 4q = -7$

(b) At A, $y = 4q + 25 = 17$
So C is given by
$17 = -7 + 10x - x^2$
$x^2 - 10x + 24 = 0$
$(x - 6) (x - 4) = 0$
$x = 4, 6$
So C is (6, 17)

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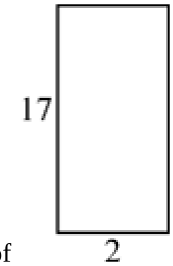
(c) Area $= \int_4^6 \left(-7 + 10x - x^2 \right) dx -$ area of 

$$= \left[-7x + 5x^2 - \frac{1}{3}x^3 \right]_4^6 - 34$$

$$= \left(-42 + 180 - 72 \right) - \left(-28 + 80 - \frac{64}{3} \right) - 34$$

$$= \frac{4}{3} \text{ or } 1\frac{1}{3}$$

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