Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise A, Question 1

Question:

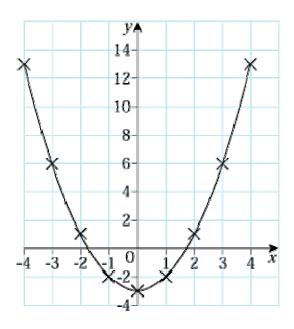
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = x^2 - 3$$

Solution:

$$y = x^2 - 3$$
.

$$x$$
 -4 -33 -2 -1 0 1 2 3 4 $x^2 - 316 - 39 - 34 - 31 - 30 - 31 - 34 - 39 - 316 - 3 y 13 6 1 -2 -3 -2 1 6 $13$$



Equation of line of symmetry is x = 0.

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Quadratic Equations Exercise A, Question 2

Question:

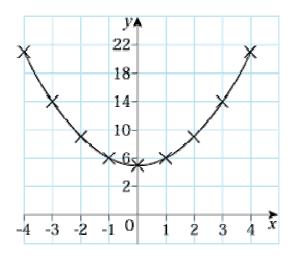
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = x^2 + 5$$

Solution:

$$y = x^2 + 5$$
.

$$x$$
 -4 -33 -2 -1 0 1 2 3 4 $x^2 + 516 + 59 + 54 + 51 + 50 + 51 + 54 + 59 + 516 + 5$ y 21 14 9 6 5 6 9 14 21



Equation of line of symmetry is x = 0.

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Quadratic Equations Exercise A, Question 3

Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

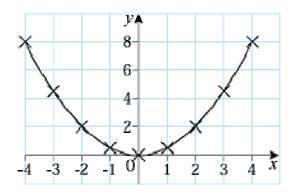
$$y = \frac{1}{2}x^2$$

Solution:

$$y = \frac{1}{2}x^2$$

$$\frac{1}{2}x^2 \, 8$$
 $4\frac{1}{2}$ 2 $\frac{1}{2}$ $0\frac{1}{2}24\frac{1}{2}8$

$$y = 8 + 4\frac{1}{2} + 2 + \frac{1}{2} + 0\frac{1}{2} + 24\frac{1}{2} + 8$$



Equation of line of symmetry is x = 0.

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Quadratic Equations Exercise A, Question 4

Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

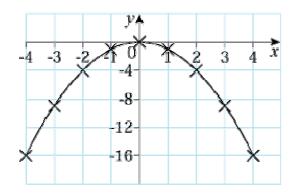
$$y = -x^2$$

Solution:

$$y = -x^2$$

$$x - 4 - 33 - 2 - 101 2 3 4$$

 $-x^2 - 16 - 9 - 4 - 10 - 1 - 4 - 9 - 16$
 $y - 16 - 9 - 4 - 10 - 1 - 4 - 9 - 16$



Equation of line of symmetry is x = 0.

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise A, Question 5

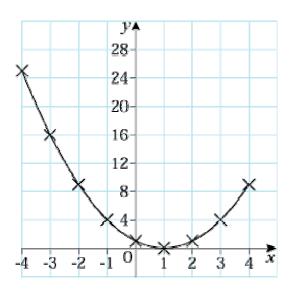
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = (x - 1)^2$$

Solution:

$$y = (x - 1)^2$$



Equation of line of symmetry is x = 1.

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise A, Question 6

Question:

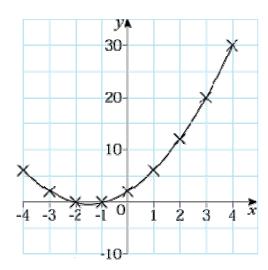
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = x^2 + 3x + 2$$

Solution:

$$y = x^2 + 3x + 2$$

$$x$$
 -4 -33 -2 -1 0 1 2 3 4 $x^2 + 3x + 216 - 12 + 29 - 9 + 24 - 6 + 21 - 3 + 20 + 0 + 21 + 3 + 24 + 6 + 29 + 9 + 216 + 12 + 2$ y 6 2 0 0 2 6 12 20 30



Equation of line of symmetry is $x = -1 \frac{1}{2}$.

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Quadratic Equations Exercise A, Question 7

Question:

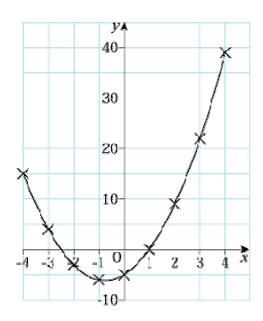
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = 2x^2 + 3x - 5$$

Solution:

$$y = 2x^2 + 3x - 5$$

$$x$$
 -4 -33 -2 -1 0 1 2 3 4 $2x^2 + 3x - 532 - 12 - 518 - 9 - 58 - 6 - 52 - 3 - 50 + 0 - 52 + 3 - 58 + 6 - 518 + 9 - 532 + 12 - 5$ y 15 4 -3 -6 -5 0 9 22 39



Equation of line of symmetry is $x = -\frac{3}{4}$.

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise A, Question 8

Question:

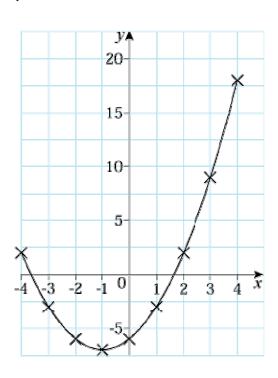
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = x^2 + 2x - 6$$

Solution:

$$y = x^2 + 2x - 6$$

$$x$$
 -4 -33 -2 -1 0 1 2 3 4 $x^2 + 2x - 616 - 8 - 69 - 6 - 64 - 4 - 61 - 2 - 60 + 0 - 61 + 2 - 64 + 4 - 69 + 6 - 616 + 8 - 6$ y 2 -3 -6 -7 -6 -3 2 9 18



Equation of line of symmetry is x = -1.

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Quadratic Equations Exercise A, Question 9

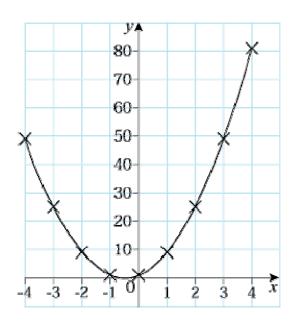
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

$$y = (2x + 1)^{2}$$

Solution:

$$y = (2x + 1)^2$$



Equation of line of symmetry is $x = -\frac{1}{2}$.

Quadratic Equations Exercise B, Question 1

Question:

Solve the following equation:

$$x^2 = 4x$$

Solution:

$$x^{2} - 4x = 0$$

 $x (x - 4) = 0$
 $x = 0$ or $x - 4 = 0$
So $x = 0$ or $x = 4$

Quadratic Equations Exercise B, Question 2

Question:

Solve the following equation:

$$x^2 = 25x$$

Solution:

$$x^{2} - 25x = 0$$

 $x (x - 25) = 0$
 $x = 0 \text{ or } x - 25 = 0$
So $x = 0 \text{ or } x = 25$

Quadratic Equations Exercise B, Question 3

Question:

Solve the following equation:

$$3x^2 = 6x$$

Solution:

$$3x^2 - 6x = 0$$

 $3x (x - 2) = 0$
 $x = 0$ or $x - 2 = 0$
So $x = 0$ or $x = 2$

Quadratic Equations Exercise B, Question 4

Question:

Solve the following equation:

$$5x^2 = 30x$$

Solution:

$$5x^2 - 30x = 0$$

 $5x (x - 6) = 0$
 $x = 0$ or $x - 6 = 0$
So $x = 0$ or $x = 6$

Quadratic Equations Exercise B, Question 5

Question:

Solve the following equation:

$$x^2 + 3x + 2 = 0$$

Solution:

$$(x+1)(x+2) = 0$$

 $x+1=0$ or $x+2=0$
So $x=-1$ or $x=-2$

Quadratic Equations Exercise B, Question 6

Question:

Solve the following equation:

$$x^2 + 5x + 4 = 0$$

Solution:

$$(x+1) (x+4) = 0$$

 $x+1=0 \text{ or } x+4=0$
So $x=-1 \text{ or } x=-4$

Quadratic Equations Exercise B, Question 7

Question:

Solve the following equation:

$$x^2 + 7x + 10 = 0$$

Solution:

$$(x+2)(x+5) = 0$$

 $x+2=0$ or $x+5=0$
 $x=-2$ or $x=-5$

Quadratic Equations Exercise B, Question 8

Question:

Solve the following equation:

$$x^2 - x - 6 = 0$$

Solution:

$$(x-3)(x+2) = 0$$

 $x-3 = 0$ or $x+2 = 0$
So $x = 3$ or $x = -2$

Quadratic Equations Exercise B, Question 9

Question:

Solve the following equation:

$$x^2 - 8x + 15 = 0$$

Solution:

$$(x-3)(x-5) = 0$$

 $x-3 = 0$ or $x-5 = 0$
So $x = 3$ or $x = 5$

Quadratic Equations Exercise B, Question 10

Question:

Solve the following equation:

$$x^2 - 9x + 20 = 0$$

Solution:

$$(x-4)(x-5) = 0$$

 $x-4=0$ or $x-5=0$
So $x=4$ or $x=5$

Quadratic Equations Exercise B, Question 11

Question:

Solve the following equation:

$$x^2 - 5x - 6 = 0$$

Solution:

$$(x-6)(x+1) = 0$$

 $x-6=0$ or $x+1=0$
So $x=6$ or $x=-1$

Quadratic Equations Exercise B, Question 12

Question:

Solve the following equation:

$$x^2 - 4x - 12 = 0$$

Solution:

$$(x-6)(x+2) = 0$$

 $x-6=0$ or $x+2=0$
So $x=6$ or $x=-2$

Quadratic Equations Exercise B, Question 13

Question:

Solve the following equation:

$$2x^2 + 7x + 3 = 0$$

Solution:

$$(2x + 1) (x + 3) = 0$$

 $2x + 1 = 0$ or $x + 3 = 0$
 $2x = -1$ or $x = -3$
So $x = -\frac{1}{2}$ or $x = -3$

Quadratic Equations Exercise B, Question 14

Question:

Solve the following equation:

$$6x^2 - 7x - 3 = 0$$

Solution:

$$(3x + 1) (2x - 3) = 0$$

 $3x + 1 = 0 \text{ or } 2x - 3 = 0$
So $x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$

Quadratic Equations Exercise B, Question 15

Question:

Solve the following equation:

$$6x^2 - 5x - 6 = 0$$

Solution:

$$(3x + 2) (2x - 3) = 0$$

 $3x + 2 = 0 \text{ or } 2x - 3 = 0$
So $x = -\frac{2}{3} \text{ or } x = \frac{3}{2}$

Quadratic Equations Exercise B, Question 16

Question:

Solve the following equation:

$$4x^2 - 16x + 15 = 0$$

Solution:

$$(2x-3)(2x-5) = 0$$

 $2x-3=0 \text{ or } 2x-5=0$
So $x = \frac{3}{2} \text{ or } x = \frac{5}{2}$

Quadratic Equations Exercise B, Question 17

Question:

Solve the following equation:

$$3x^2 + 5x = 2$$

Solution:

$$3x^{2} + 5x - 2 = 0$$

$$(3x - 1) (x + 2) = 0$$

$$3x - 1 = 0 \text{ or } x + 2 = 0$$
So $x = \frac{1}{3} \text{ or } x = -2$

Quadratic Equations Exercise B, Question 18

Question:

Solve the following equation:

$$(2x-3)^2=9$$

Solution:

$$2x - 3 = \pm 3$$

$$2x = \pm 3 + 3$$

$$x = \frac{\pm 3 + 3}{2}$$
So $x = 3$ or $x = 0$

30x - 301x - 0

Quadratic Equations Exercise B, Question 19

Question:

Solve the following equation:

$$(x-7)^2=36$$

Solution:

$$x - 7 = \pm 6$$

 $x = \pm 6 + 7$
So $x = 1$ or $x = 13$

Quadratic Equations Exercise B, Question 20

Question:

Solve the following equation:

$$2x^2 = 8$$

Solution:

$$x^{2} = 4$$

$$x = \pm 2$$
So $x = 2$ or $x = -2$

Quadratic Equations Exercise B, Question 21

Question:

Solve the following equation:

$$3x^2 = 5$$

Solution:

$$x^{2} = \frac{5}{3}$$

$$x = \pm \sqrt{\frac{5}{3}}$$

$$\operatorname{So} x = \sqrt{\frac{5}{3}} \operatorname{or} x = -\sqrt{\frac{5}{3}}$$

Quadratic Equations Exercise B, Question 22

Question:

Solve the following equation:

$$(x-3)^2=13$$

Solution:

$$x - 3 = \pm \sqrt{13}$$

 $x = 3 \pm \sqrt{13}$
So $x = 3 + \sqrt{13}$ or $x = 3 - \sqrt{13}$

Quadratic Equations Exercise B, Question 23

Question:

Solve the following equation:

$$(3x-1)^2=11$$

Solution:

$$3x - 1 = \pm \sqrt{11}$$
$$3x = 1 \pm \sqrt{11}$$
$$x = \frac{1 \pm \sqrt{11}}{3}$$

Quadratic Equations Exercise B, Question 24

Question:

Solve the following equation:

$$5x^2 - 10x^2 = -7 + x + x^2$$

Solution:

$$-6x^{2} - x + 7 = 0$$

$$(1 - x) (7 + 6x) = 0$$

$$x = 1 \text{ or } 6x = -7$$
So $x = 1 \text{ or } x = -\frac{7}{6}$

Quadratic Equations Exercise B, Question 25

Question:

Solve the following equation:

$$6x^2 - 7 = 11x$$

Solution:

$$6x^{2} - 11x - 7 = 0$$

$$(3x - 7) (2x + 1) = 0$$

$$3x - 7 = 0 \text{ or } 2x + 1 = 0$$

$$\text{So } x = \frac{7}{3} \text{ or } x = -\frac{1}{2}$$

Quadratic Equations Exercise B, Question 26

Question:

Solve the following equation:

$$4x^2 + 17x = 6x - 2x^2$$

Solution:

$$6x^{2} + 11x = 0$$

$$x (6x + 11) = 0$$

$$x = 0 \text{ or } 6x + 11 = 0$$
So $x = 0 \text{ or } x = -\frac{11}{6}$

Quadratic Equations Exercise C, Question 1

Question:

Complete the square for the expression:

$$x^2 + 4x$$

Solution:

$$= (x+2)^2-4$$

Quadratic Equations Exercise C, Question 2

Question:

Complete the square for the expression:

$$x^{2} - 6x$$

Solution:

$$= (x-3)^2-9$$

Quadratic Equations Exercise C, Question 3

Question:

Complete the square for the expression:

$$x^2 - 16x$$

Solution:

$$= (x-8)^2-64$$

Quadratic Equations Exercise C, Question 4

Question:

Complete the square for the expression:

$$x^2 + x$$

Solution:

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$

Quadratic Equations Exercise C, Question 5

Question:

Complete the square for the expression:

$$x^2 - 14x$$

Solution:

$$= (x-7)^2 - 49$$

Quadratic Equations Exercise C, Question 6

Question:

Complete the square for the expression:

$$2x^2 + 16x$$

Solution:

$$= 2 (x^{2} + 8x)$$

$$= 2 [(x + 4)^{2} - 16]$$

$$= 2 (x + 4)^{2} - 32$$

Quadratic Equations Exercise C, Question 7

Question:

Complete the square for the expression:

$$3x^2 - 24x$$

Solution:

= 3 (
$$x^2 - 8x$$
)
= 3 [($x - 4$) $^2 - 16$]
= 3 ($x - 4$) $^2 - 48$

Quadratic Equations Exercise C, Question 8

Question:

Complete the square for the expression:

$$2x^2 - 4x$$

Solution:

$$= 2 (x^{2} - 2x)$$

$$= 2 [(x - 1)^{2} - 1]$$

$$= 2 (x - 1)^{2} - 2$$

Quadratic Equations Exercise C, Question 9

Question:

Complete the square for the expression:

$$5x^2 + 20x$$

Solution:

$$= 5 (x^{2} + 4x)$$

$$= 5 [(x + 2)^{2} - 4]$$

$$= 5 (x + 2)^{2} - 20$$

Quadratic Equations Exercise C, Question 10

Question:

Complete the square for the expression:

$$2x^2 - 5x$$

Solution:

$$= 2 \left(x^2 - \frac{5}{2}x \right)$$

$$= 2 \left[\left(x - \frac{5}{4} \right)^2 - \frac{25}{16} \right]$$

$$= 2 \left(x - \frac{5}{4} \right)^2 - \frac{25}{8}$$

Quadratic Equations Exercise C, Question 11

Question:

Complete the square for the expression:

$$3x^2 + 9x$$

Solution:

$$= 3 (x^{2} + 3x)$$

$$= 3 \left[\left(x + \frac{3}{2} \right)^{2} - \frac{9}{4} \right]$$

$$= 3 \left(x + \frac{3}{2} \right)^{2} - \frac{27}{4}$$

Quadratic Equations Exercise C, Question 12

Question:

Complete the square for the expression:

$$3x^{2} - x$$

Solution:

$$= 3 \left(x^{2} - \frac{1}{3}x \right)$$

$$= 3 \left[\left(x - \frac{1}{6} \right)^{2} - \frac{1}{36} \right]$$

$$= 3 \left(x - \frac{1}{6} \right)^{2} - \frac{3}{36}$$

$$= 3 \left(x - \frac{1}{6} \right)^{2} - \frac{1}{12}$$

Quadratic Equations Exercise D, Question 1

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 6x + 1 = 0$$

Solution:

$$x^{2} + 6x = -1$$

$$(x+3)^{2} - 9 = -1$$

$$(x+3)^{2} = -1 + 9$$

$$(x+3)^{2} = 8$$

$$x+3 = \pm \sqrt{8}$$

$$x = -3 \pm \sqrt{8}$$

$$x = -3 \pm \sqrt{2} \sqrt{4}$$

$$x = -3 \pm 2\sqrt{2}$$
So $x = -3 + 2\sqrt{2}$ or $x = -3 - 2\sqrt{2}$

Quadratic Equations Exercise D, Question 2

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 12x + 3 = 0$$

Solution:

$$x^{2} + 12x = -3$$

 $(x+6)^{2} - 36 = -3$
 $(x+6)^{2} = 33$
 $x+6 = \pm \sqrt{33}$
 $x = -6 \pm \sqrt{33}$
So $x = -6 + \sqrt{33}$ or $x = -6 - \sqrt{33}$

Quadratic Equations Exercise D, Question 3

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 - 10x = 5$$

Solution:

$$(x-5)^2 - 25 = 5$$

 $(x-5)^2 = 5 + 25$
 $(x-5)^2 = 30$
 $x-5 = \pm \sqrt{30}$
 $x = 5 \pm \sqrt{30}$
So $x = 5 + \sqrt{30}$ or $x = 5 - \sqrt{30}$

Quadratic Equations Exercise D, Question 4

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 + 4x - 2 = 0$$

Solution:

$$x^{2} + 4x = 2$$

 $(x + 2)^{2} - 4 = 2$
 $(x + 2)^{2} = 6$
 $x + 2 = \pm \sqrt{6}$
So $x = -2 + \sqrt{6}$ or $x = -2 - \sqrt{6}$

Quadratic Equations Exercise D, Question 5

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$x^2 - 3x - 5 = 0$$

Solution:

$$x^{2} - 3x = 5$$

$$\left(x - \frac{3}{2}\right)^{2} - \frac{9}{4} = 5$$

$$\left(x - \frac{3}{2}\right)^{2} = 5 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^{2} = \frac{29}{4}$$

$$x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$
So $x = \frac{3 + \sqrt{29}}{2}$ or $x = \frac{3 - \sqrt{29}}{2}$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise D, Question 6

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$2x^2 - 7 = 4x$$

Solution:

$$2x^2 - 4x = 7$$
$$x^2 - 2x = \frac{7}{2}$$

$$(x-1)^2-1=\frac{7}{2}$$

$$(x-1)^2 = \frac{9}{2}$$

$$x - 1 = \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3\sqrt{2}}{2}$$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise D, Question 7

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$4x^2 - x = 8$$

Solution:

$$x^{2} - \frac{1}{4}x = 2$$

$$\left(x - \frac{1}{8}\right)^{2} - \frac{1}{64} = 2$$

$$\left(x - \frac{1}{8}\right)^{2} = 2 + \frac{1}{64}$$

$$\left(x - \frac{1}{8}\right)^{2} = \frac{129}{64}$$

$$x - \frac{1}{8} = \pm \frac{\sqrt{129}}{8}$$

$$x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$$
So $x = \frac{1 + \sqrt{129}}{8}$ or $x = \frac{1 - \sqrt{129}}{8}$

Quadratic Equations Exercise D, Question 8

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$10 = 3x - x^2$$

Solution:

$$x^{2} - 3x = -10$$

$$\left(x - \frac{3}{2}\right)^{2} - \frac{9}{4} = -10$$

$$\left(x - \frac{3}{2}\right)^{2} = -\frac{31}{4}$$

No real roots as RHS is negative.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise D, Question 9

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$15 - 6x - 2x^2 = 0$$

Solution:

$$2x^{2} + 6x = 15$$

$$x^{2} + 3x = \frac{15}{2}$$

$$\left(x + \frac{3}{2}\right)^{2} - \frac{9}{4} = \frac{15}{2}$$

$$\left(x + \frac{3}{2}\right)^{2} = \frac{39}{4}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{39}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$$
So $x = -\frac{3}{2} + \frac{\sqrt{39}}{2}$ or $x = -\frac{3}{2} - \frac{\sqrt{39}}{2}$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise D, Question 10

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$$5x^2 + 8x - 2 = 0$$

Solution:

$$x^{2} + \frac{8}{5}x = \frac{2}{5}$$

$$\left(x + \frac{4}{5}\right)^{2} - \frac{16}{25} = \frac{2}{5}$$

$$\left(x + \frac{4}{5}\right)^{2} = \frac{26}{25}$$

$$x + \frac{4}{5} = \pm \frac{\sqrt{26}}{5}$$

$$x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$$
So $x = \frac{-4 + \sqrt{26}}{5}$ or $x = \frac{-4 - \sqrt{26}}{5}$

Quadratic Equations Exercise E, Question 1

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 + 3x + 1 = 0$$

Solution:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$
Then $x = \frac{-3 + \sqrt{5}}{2}$ or $x = \frac{-3 - \sqrt{5}}{2}$

Quadratic Equations Exercise E, Question 2

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 - 3x - 2 = 0$$

Solution:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$
Then $x = \frac{3 + \sqrt{17}}{2}$ or $x = \frac{3 - \sqrt{17}}{2}$

Quadratic Equations Exercise E, Question 3

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 + 6x + 6 = 0$$

Solution:

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2 \times 1}$$

$$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = -3 \pm \sqrt{3}$$
Then $x = -3 + \sqrt{3}$ or $x = -3 - \sqrt{3}$

Quadratic Equations Exercise E, Question 4

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$x^2 - 5x - 2 = 0$$

Solution:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{5 \pm \sqrt{33}}{2}$$
Then $x = \frac{5 + \sqrt{33}}{2}$ or $x = \frac{5 - \sqrt{33}}{2}$

Quadratic Equations Exercise E, Question 5

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$3x^2 + 10x - 2 = 0$$

Solution:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-2)}}{2 \times 3}$$

$$x = \frac{-10 \pm \sqrt{100 + 24}}{6}$$

$$x = \frac{-10 \pm \sqrt{124}}{6}$$

$$x = \frac{-10 \pm 2\sqrt{31}}{6}$$
Then $x = \frac{-5 + \sqrt{31}}{3}$ or $x = \frac{-5 - \sqrt{31}}{3}$

Quadratic Equations Exercise E, Question 6

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$4x^2 - 4x - 1 = 0$$

Solution:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$

$$x = \frac{+4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4\sqrt{2}}{8}$$
Then $x = \frac{1 + \sqrt{2}}{2}$ or $x = \frac{1 - \sqrt{2}}{2}$

Quadratic Equations Exercise E, Question 7

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$7x^2 + 9x + 1 = 0$$

Solution:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(7)(1)}}{2 \times 7}$$

$$x = \frac{-9 \pm \sqrt{81 - 28}}{14}$$

$$x = \frac{-9 \pm \sqrt{53}}{14}$$
Then $x = \frac{-9 + \sqrt{53}}{14}$ or $x = \frac{-9 - \sqrt{53}}{14}$

Quadratic Equations Exercise E, Question 8

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$5x^2 + 4x - 3 = 0$$

Solution:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-3)}}{2 \times 5}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{10}$$

$$x = \frac{-4 \pm \sqrt{76}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{19}}{10}$$
Then $x = \frac{-2 + \sqrt{19}}{5}$ or $x = \frac{-2 - \sqrt{19}}{5}$

Quadratic Equations Exercise E, Question 9

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$4x^2 - 7x = 2$$

Solution:

$$4x^{2} - 7x - 2 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^{2} - 4(4)(-2)}}{2 \times 4}$$

$$x = \frac{+7 \pm \sqrt{49 + 32}}{8}$$

$$x = \frac{7 \pm \sqrt{81}}{8}$$

$$x = \frac{7 \pm 9}{8}$$

Then
$$x = 2$$
 or $x = -\frac{1}{4}$

Quadratic Equations Exercise E, Question 10

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$$11x^2 + 2x - 7 = 0$$

Solution:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-7)}}{2 \times 11}$$

$$x = \frac{-2 \pm \sqrt{4 + 308}}{22}$$

$$x = \frac{-2 \pm \sqrt{312}}{22}$$

$$x = \frac{-2 \pm 2\sqrt{78}}{22}$$

$$x = \frac{-1 \pm \sqrt{78}}{11}$$
Then $x = \frac{-1 + \sqrt{78}}{11}$ or $x = \frac{-1 - \sqrt{78}}{11}$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise F, Question 1

Question:

Sketch the graphs of the following equations:

(a)
$$y = x^2 + 3x + 2$$

(b)
$$y = x^2 - 3x + 10$$

(c)
$$y = x^2 + 2x - 15$$

(d)
$$y = 2x^2 + 7x + 3$$

(e)
$$y = 2x^2 + x - 3$$

(f)
$$y = 6x^2 - 19x + 10$$

(g)
$$y = 3x^2 - 2x - 5$$

(h)
$$y = 3x^2 - 13x$$

(i)
$$y = -x^2 + 6x + 7$$

(j)
$$y = 4 - 7x - 2x^2$$

Solution:

(a)
$$a > 0$$
 so graph is a \cup shape.

$$b^2 = 9, 4ac = 8$$

 $b^2 > 4ac$, so there are two different roots of the equation y = 0.

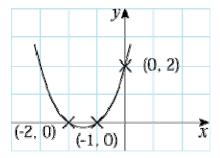
When y = 0,

$$(x+2)(x+1)=0$$

$$x = -2 \text{ or } x = -1$$

So crossing points are (-2,0) and (-1,0).

When x = 0, y = 2, so (0, 2) is a crossing point.



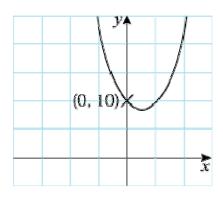
(b)
$$a > 0$$
 so graph is a \cup shape.

$$b^2 = 9, 4ac = 40$$

 $b^2 < 4ac$, so there are no real roots of the equation y = 0.

So there are no crossing points at y = 0.

When x = 0, y = 10, so crossing point is (0, 10).



(c) a > 0 so graph is a \cup shape.

$$b^2 = 4, 4ac = -60$$

 $b^2 > 4ac$, so two different roots of y = 0.

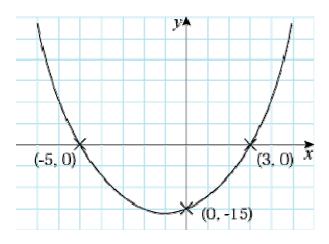
When
$$y = 0$$
,

$$0 = (x+5) (x-3)$$

 $x = -5 \text{ or } x = 3$

$$x = -5 \text{ or } x = 3$$

So crossing points are (-5,0) and (3,0). When x = 0, y = -15, so crossing point is (0,-15).



(d) a > 0 so graph is a \cup shape.

$$b^2 = 49, 4ac = 24$$

 $b^2 > 4ac$, so two different roots of y = 0.

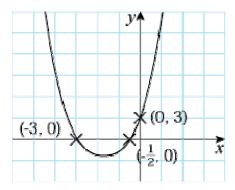
When
$$y = 0$$
,

$$0 = (2x + 1) (x + 3)$$

$$x = -\frac{1}{2}$$
 or $x = -3$

 $\left(\begin{array}{cc} -\frac{1}{2}, 0 \end{array}\right)$ and $\left(\begin{array}{cc} -3, 0 \end{array}\right)$. So crossing points are

When x = 0, y = 3, so crossing point is (0, 3).



(e) a > 0 so graph is a \cup shape.

$$b^2 = 1, 4ac = -24$$

 $b^2 > 4ac$, so two different roots of y = 0.

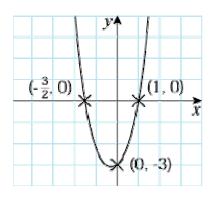
When y = 0,

$$0 = (2x + 3) (x - 1)$$

$$x = -\frac{3}{2} \text{ or } x = 1$$

So crossing points are $\left(-\frac{3}{2},0\right)$ and (1,0).

When x = 0, y = -3, so crossing point is (0, -3).



(f) a > 0 so graph is a \cup shape.

$$b^2 = 361, 4ac = 240$$

 $b^2 > 4ac$, so two different roots of y = 0.

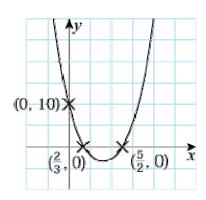
When y = 0,

$$0 = (3x - 2) (2x - 5)$$

$$x = \frac{2}{3} \text{ or } x = \frac{5}{2}$$

So crossing points are $\left(\begin{array}{c} \frac{2}{3} \end{array}, 0 \right)$ and $\left(\begin{array}{c} \frac{5}{2} \end{array}, 0 \right)$.

When x = 0, y = 10, so crossing point is (0, 10).



(g) $a > \text{so graph is a} \cup \text{shape.}$

$$b^2 = 4, 4ac = -60$$

 $b^2 > 4ac$, so two different roots of y = 0.

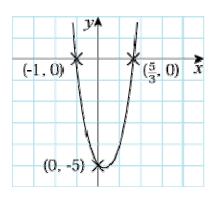
When
$$y = 0$$
,

$$0 = (3x - 5) (x + 1)$$

$$x = \frac{5}{3} \text{ or } x = -1$$

So crossing points are $\left(\begin{array}{c} \frac{5}{3} \\ \end{array}, 0\right)$ and (-1, 0).

When x = 0, y = -5, so crossing point is (0, -5).



(h) a > 0 so graph is a \cup shape.

$$b^2 = 169, 4ac = 0$$

 $b^2 > 4ac$, so two different roots of y = 0.

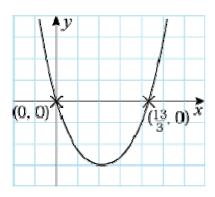
When y = 0,

$$0 = x (3x - 13)$$

$$x = 0 \text{ or } x = \frac{13}{3}$$

So crossing points are (0,0) and $\left(\frac{13}{3},0\right)$.

When x = 0, y = 0, so crossing point is (0, 0).



(i) a < 0 so graph is a \cap

$$b^2 = 36, 4ac = -28$$

 $b^2 > 4ac$, so two different roots of y = 0.

When y = 0,

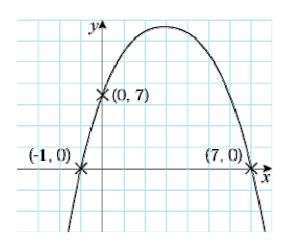
$$0 = (7 - x) (1 + x)$$

 $x = 7 \text{ or } x = -1$

$$x = 7 \text{ or } x = -1$$

So crossing points are (7,0) and (-1,0).

When x = 0, y = 7, so crossing point is (0, 7).



(j)
$$a < 0$$
 so graph is a \cap shape.
 $b^2 = 49, 4ac = -32$

$$b^2 = 49, 4ac = -32$$

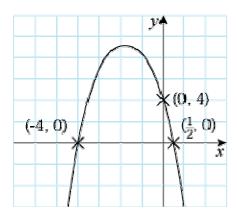
 $b^2 > 4ac$, so two different roots of y = 0.

When
$$y = 0$$
, $0 = (1 - 2x) (4 + x)$

$$x = \frac{1}{2} \text{ or } x = -4$$

So crossing points are $\left(\begin{array}{c} \frac{1}{2} \\ \end{array}, 0\right)$ and $\left(\begin{array}{c} -4 \\ \end{array}, 0\right)$.

When x = 0, y = 4, so crossing point is (0, 4).



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Quadratic Equations Exercise F, Question 2

Question:

Find the values of k for which $x^2 + kx + 4 = 0$ has equal roots.

Solution:

$$x^2 + kx + 4 = 0$$
 has equal roots if $b^2 = 4ac$ i.e. $k^2 = 4 \times 1 \times 4 = 16 \implies k = \pm 4$

Quadratic Equations Exercise F, Question 3

Question:

Find the values of k for which $kx^2 + 8x + k = 0$ has equal roots.

Solution:

$$kx^2 + 8x + k = 0$$
 has equal roots if $b^2 = 4ac$
i.e. $8^2 = 4 \times k \times k = 4k^2$
So $k^2 = \frac{64}{4} = 16 \implies k = \pm 4$

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Quadratic Equations Exercise G, Question 1

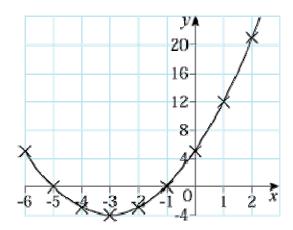
Question:

Draw the graphs with the following equations, choosing appropriate values for x. For each graph write down the equation of the line of symmetry.

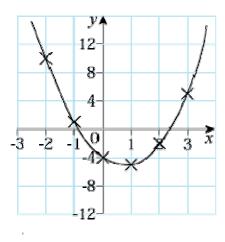
(a)
$$y = x^2 + 6x + 5$$

(b)
$$y = 2x^2 - 3x - 4$$

Solution:



x = -3 is line of symmetry.



 $x = \frac{3}{4}$ is line of symmetry.

Quadratic Equations Exercise G, Question 2

Question:

Solve the following equations:

(a)
$$y^2 + 3y + 2 = 0$$

(b)
$$3x^2 + 13x - 10 = 0$$

(c)
$$5x^2 - 10x = 4x + 3$$

(d)
$$(2x-5)^2=7$$

Solution:

(a)
$$(y+1)(y+2) = 0$$

 $y = -1$ or $y = -2$

(b)
$$(3x-2)(x+5) = 0$$

 $x = \frac{2}{3}$ or $x = -5$

(c)
$$5x^2 - 14x - 3 = 0$$

(5x + 1) (x - 3) = 0
 $x = -\frac{1}{5}$ or $x = 3$

(d)
$$2x - 5 = \pm \sqrt{7}$$

 $2x = \pm \sqrt{7} + 5$
 $x = \frac{5 \pm \sqrt{7}}{2}$

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Quadratic Equations Exercise G, Question 3

Question:

Solve the following equations by:

- (i) Completing the square.
- (ii) Using the formula.

(a)
$$x^2 + 5x + 2 = 0$$

(b)
$$x^2 - 4x - 3 = 0$$

(c)
$$5x^2 + 3x - 1 = 0$$

(d)
$$3x^2 - 5x = 4$$

Solution:

(a) (i)
$$x^2 + 5x = -2$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} = -2$$

$$\left(\begin{array}{c} x + \frac{5}{2} \end{array}\right)^2 = \frac{17}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(ii)
$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2}$$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(b)(i)
$$x^2 - 4x = 3$$

$$(x-2)^2-4=3$$

$$(x-2)^2 = 7$$
$$x-2 = \pm \sqrt{7}$$

$$x - 2 = \pm \sqrt{2}$$

$$x = 2 \pm \sqrt{7}$$

(ii)
$$x = \frac{-(-4) \pm \sqrt{16-4(1)(-3)}}{2}$$

$$x = \frac{+4 \pm \sqrt{16 + 12}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 7}}{2}$$

$$x = \frac{4 \pm 2\sqrt{7}}{2}$$
$$x = 2 \pm \sqrt{7}$$

(c) (i)
$$5x^2 + 3x = 1$$

 $5\left(x^2 + \frac{3}{5}x\right) = 1$
 $5\left[\left(x + \frac{3}{10}\right)^2 - \frac{9}{100}\right] = 1$
 $\left(x + \frac{3}{10}\right)^2 - \frac{9}{100} = \frac{1}{5}$
 $\left(x + \frac{3}{10}\right)^2 = \frac{29}{100}$
 $x + \frac{3}{10} = \pm \frac{\sqrt{29}}{10}$
 $x = \frac{-3 \pm \sqrt{29}}{10}$

(ii)
$$x = \frac{-3 \pm \sqrt{9 - 4(5)(-1)}}{10}$$

$$x = \frac{-3 \pm \sqrt{29}}{10}$$

(d)(i)
$$3\left(x^2 - \frac{5}{3}x\right) = 4$$

$$3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36}\right] = 4$$

$$\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} = \frac{4}{3}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{73}{36}$$

$$x - \frac{5}{6} = \pm \frac{\sqrt{73}}{6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$

(ii)
$$x = \frac{-(-5) \pm \sqrt{25 - 4(3)(-4)}}{6}$$

$$x = \frac{+5 \pm \sqrt{25 + 48}}{6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$

Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations Exercise G, Question 4

Question:

Sketch graphs of the following equations:

(a)
$$y = x^2 + 5x + 4$$

(b)
$$y = 2x^2 + x - 3$$

(c)
$$y = 6 - 10x - 4x^2$$

(d)
$$y = 15x - 2x^2$$

Solution:

(a) a > 0 so \cup shape

$$b^2 = 25, 4ac = 16$$

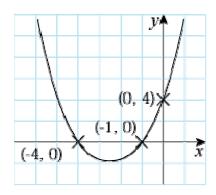
 $b^2 > 4ac$, so two different roots of y = 0.

$$y = 0 \implies 0 = (x + 1) (x + 4)$$

$$x = -1 \text{ or } x = -4$$

So x-axis crossing points are (-1,0) and (-4,0).

 $x = 0 \implies y = 4$ So y-axis crossing point is (0, 4).



(b)
$$a > 0$$
 So \cup shape

$$b^2 = 1,4ac = -24$$

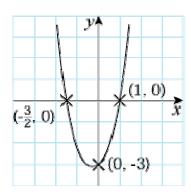
 $b^2 > 4ac$, so two different roots of y = 0.

$$y = 0 \implies 0 = (2x + 3) (x - 1)$$

$$x = -\frac{3}{2} \text{ or } x = 1$$

So x-axis crossing points are $\left(-\frac{3}{2},0\right)$ and (1,0).

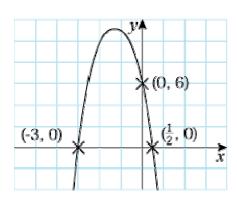
 $x = 0 \implies y = -3$ so y-axis crossing point in (0, -3).



(c)
$$a < 0$$
 So \cap shape $b^2 = 100$, $4ac = -96$ $b^2 > 4ac$, so two different roots of $y = 0$. $y = 0 \Rightarrow 0 = (1 - 2x) (6 + 2x)$ $x = \frac{1}{2}$ or $x = -3$

So x-axis crossing points are $\left(\begin{array}{c} \frac{1}{2} \\ \end{array}, 0 \right)$ and (-3, 0).

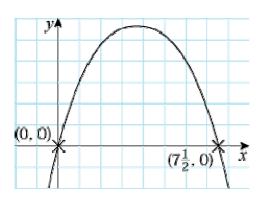
 $x = 0 \implies y = 6$ so y-axis crossing point is (0, 6).



(d)
$$a < 0$$
 so \cap shape $b^2 = 225$, $4ac = 0$ $b^2 > 4ac$, so two different roots of $y = 0$. $y = 0 \Rightarrow 0 = x (15 - 2x)$ $x = 0$ or $x = 7 \frac{1}{2}$

So x-axis crossing points are (0,0) and $\left(7\frac{1}{2},0\right)$.

 $x = 0 \implies y = 0$ So y-axis crossing point is (0, 0).



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Quadratic Equations Exercise G, Question 5

Question:

Given that for all values of x:

$$3x^2 + 12x + 5 = p(x + q)^2 + r$$

- (a) Find the values of p, q and r.
- (b) Solve the equation $3x^2 + 12x + 5 = 0$. **[E]**

Solution:

(a)
$$3x^2 + 12x + 5 = p(x^2 + 2qx + q^2) + r$$

 $3x^2 + 12x + 5 = px^2 + 2pqx + pq^2 + r$
Comparing $x^2 : p = 3$

Comparing
$$x: 2pq = 12$$

Comparing constants :
$$pq^2 + r = 5\Im$$

$$2 \times 3q = 12$$

$$q=2$$

Substitute p = 3 and q = 2 into ③:

$$3\times 2^2+r=5$$

$$12 + r = 5$$

$$r = -7$$

So
$$p = 3$$
, $q = 2$, $r = -7$

(b)
$$3x^{2} + 12x + 5 = 0$$

 $\Rightarrow 3(x+2)^{2} - 7 = 0$
 $\Rightarrow 3(x+2)^{2} = 7$
 $\Rightarrow (x+2)^{2} = \frac{7}{3}$
 $\Rightarrow x+2 = \pm \sqrt{\frac{7}{3}}$
So $x = -2 \pm \sqrt{\frac{7}{3}}$

Quadratic Equations Exercise G, Question 6

Question:

Find, as surds, the roots of the equation

$$2(x+1)(x-4) - (x-2)^2 = 0$$

Solution:

$$2(x^{2} - 3x - 4) - (x^{2} - 4x + 4) = 0$$

$$2x^{2} - 6x - 8 - x^{2} + 4x - 4 = 0$$

$$x^{2} - 2x - 12 = 0$$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(-12)}}{2}$$

$$x = \frac{+2 \pm \sqrt{52}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 13}}{2}$$

$$x = \frac{2 \pm 2\sqrt{13}}{2}$$

$$x = 1 \pm \sqrt{13}$$

Quadratic Equations Exercise G, Question 7

Question:

Use algebra to solve (x-1)(x+2) = 18. **[E]**

Solution:

$$x^{2} + x - 2 = 18$$

$$x^{2} + x - 20 = 0$$

$$(x + 5) (x - 4) = 0$$

$$x = -5 \text{ or } x = 4$$