

# Solutionbank C2

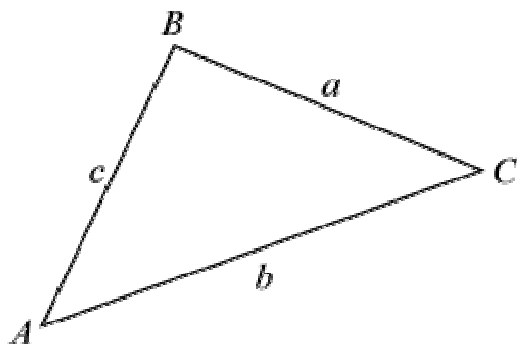
## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise A, Question 1

#### Question:

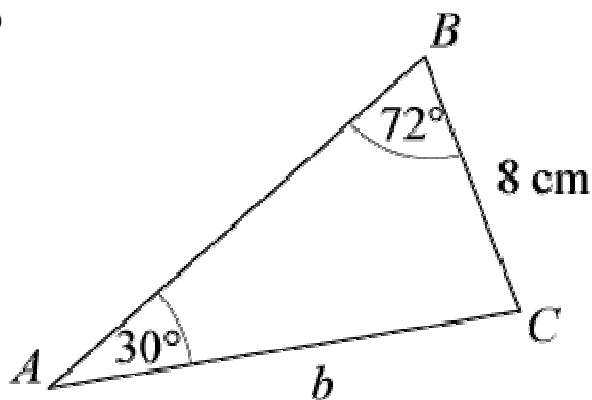
In each of parts (a) to (d), given values refer to the general triangle:



- (a) Given that  $a = 8$  cm,  $A = 30^\circ$ ,  $B = 72^\circ$ , find  $b$ .
- (b) Given that  $a = 24$  cm,  $A = 110^\circ$ ,  $C = 22^\circ$ , find  $c$ .
- (c) Given that  $b = 14.7$  cm,  $A = 30^\circ$ ,  $C = 95^\circ$ , find  $a$ .
- (d) Given that  $c = 9.8$  cm,  $B = 68.4^\circ$ ,  $C = 83.7^\circ$ , find  $a$ .

#### Solution:

(a)

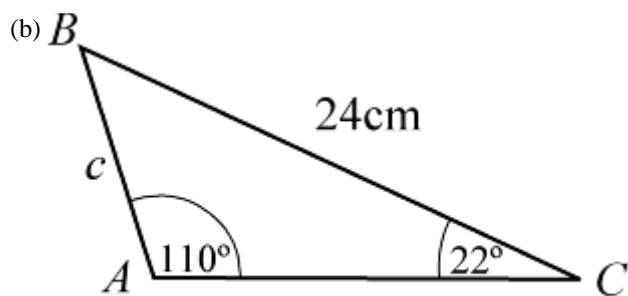


Using  $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{b}{\sin 72^\circ} = \frac{8}{\sin 30^\circ}$$

$$\Rightarrow b = \frac{8 \sin 72^\circ}{\sin 30^\circ} = 15.2 \text{ cm (3 s.f.)}$$

(Check: as  $72^\circ > 30^\circ$ ,  $b > 8$  cm.)

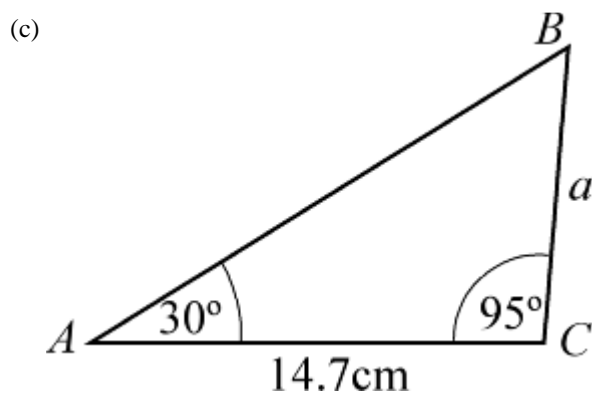


Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{c}{\sin 22^\circ} = \frac{24}{\sin 110^\circ}$$

$$\Rightarrow c = \frac{24 \sin 22^\circ}{\sin 110^\circ} = 9.57 \text{ cm (3 s.f.)}$$

(As  $110^\circ > 22^\circ$ ,  $24\text{cm} > c$ .)

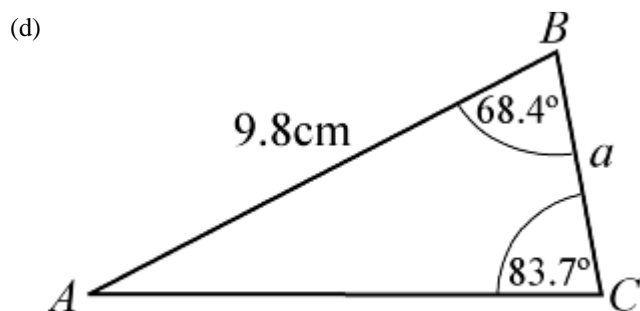


$$\angle ABC = 180^\circ - (30 + 95)^\circ = 55^\circ$$

Using  $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{a}{\sin 30^\circ} = \frac{14.7}{\sin 55^\circ}$$

$$\Rightarrow a = \frac{14.7 \sin 30^\circ}{\sin 55^\circ} = 8.97 \text{ cm (3 s.f.)}$$



$$\angle BAC = 180^\circ - (68.4 + 83.7)^\circ = 27.9^\circ$$

Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{a}{\sin 27.9^\circ} = \frac{9.8}{\sin 83.7^\circ}$$

$$\Rightarrow a = \frac{9.8 \sin 27.9^\circ}{\sin 83.7^\circ} = 4.61 \text{ cm (3 s.f.)}$$

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## Edexcel Modular Mathematics for AS and A-Level

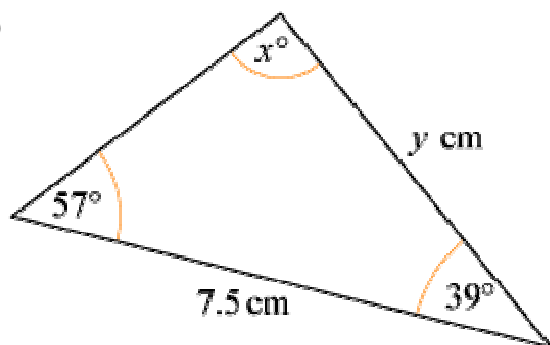
### The sine and cosine rule

#### Exercise A, Question 2

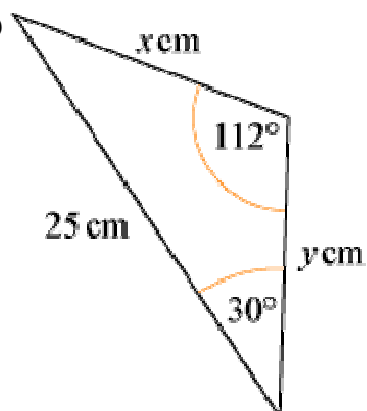
#### Question:

In each of the following triangles calculate the values of  $x$  and  $y$ .

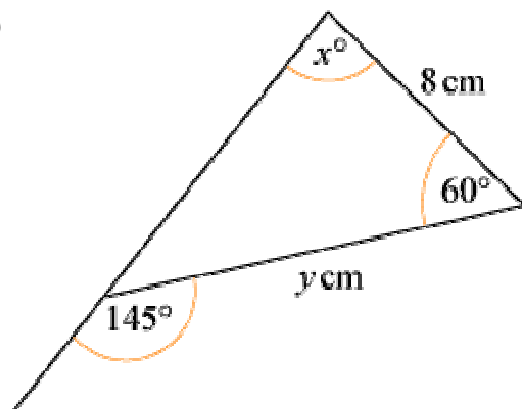
(a)

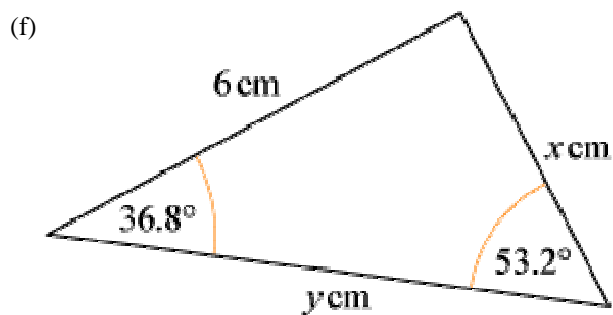
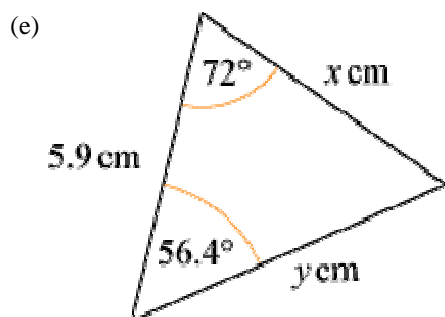
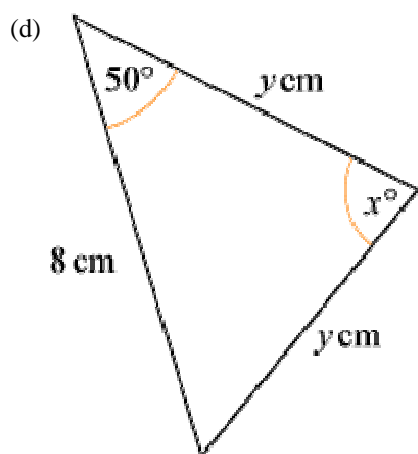


(b)

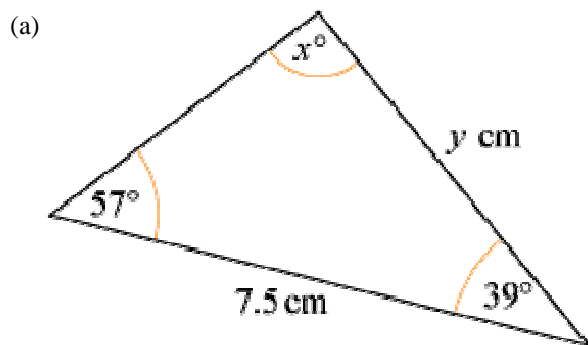


(c)





**Solution:**

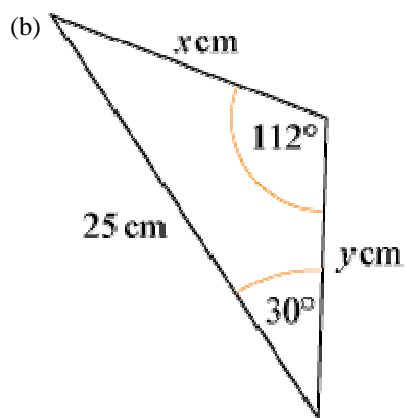


$$x = 180 - (57 + 39) = 84$$

Using  $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\Rightarrow \frac{y}{\sin 57^\circ} = \frac{7.5}{\sin 84^\circ}$$

$$\Rightarrow y = \frac{7.5 \sin 57^\circ}{\sin 84^\circ} = 6.32 \text{ (3 s.f.)}$$



Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{x}{\sin 30^\circ} = \frac{25}{\sin 112^\circ}$$

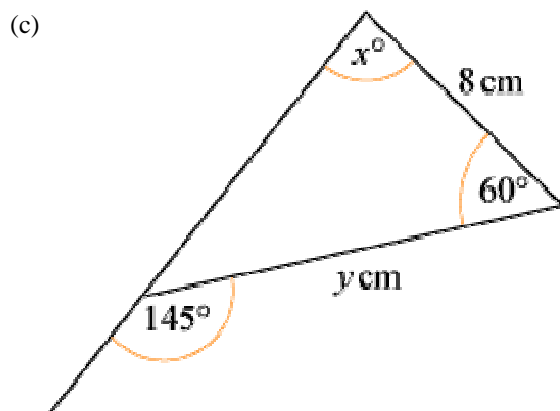
$$\Rightarrow x = \frac{25 \sin 30^\circ}{\sin 112^\circ} = 13.5 \text{ (3 s.f.)}$$

$$\angle B = 180^\circ - (112 + 30)^\circ = 38^\circ$$

Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{y}{\sin 38^\circ} = \frac{25}{\sin 112^\circ}$$

$$\Rightarrow y = \frac{25 \sin 38^\circ}{\sin 112^\circ} = 16.6 \text{ (3 s.f.)}$$

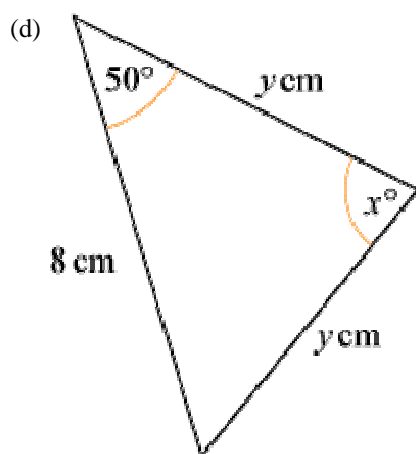


$$x = 180 - (60 + 35) = 85$$

Using  $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 85^\circ} = \frac{8}{\sin 35^\circ}$$

$$\Rightarrow y = \frac{8 \sin 85^\circ}{\sin 35^\circ} = 13.9 \text{ (3 s.f.)}$$



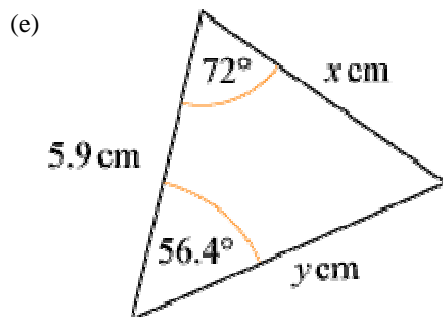
$$x = 180 - (50 + 50) = 80$$

Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{y}{\sin 50^\circ} = \frac{8}{\sin 80^\circ}$$

$$\Rightarrow y = \frac{8 \sin 50^\circ}{\sin 80^\circ} = 6.22 \text{ (3 s.f.)}$$

(Note: You could use the line of symmetry to split the triangle into two right-angled triangles and use  $\cos 50^\circ = \frac{4}{y}$ .)



$$\angle C = 180^\circ - (56.4 + 72)^\circ = 51.6^\circ$$

Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{x}{\sin 56.4^\circ} = \frac{5.9}{\sin 51.6^\circ}$$

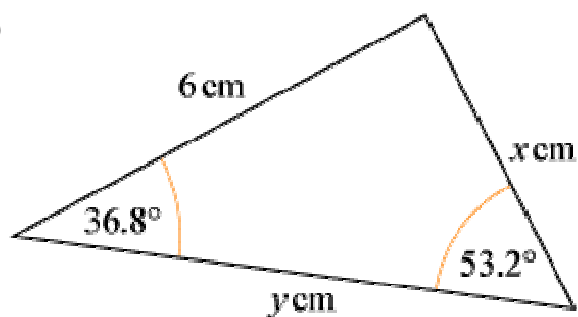
$$\Rightarrow x = \frac{5.9 \sin 56.4^\circ}{\sin 51.6^\circ} = 6.27 \text{ (3 s.f.)}$$

Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{y}{\sin 72^\circ} = \frac{5.9}{\sin 51.6^\circ}$$

$$\Rightarrow y = \frac{5.9 \sin 72^\circ}{\sin 51.6^\circ} = 7.16 \text{ (3 s.f.)}$$

(f)



Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{x}{\sin 36.8^\circ} = \frac{6}{\sin 53.2^\circ}$$

$$\Rightarrow x = \frac{6 \sin 36.8^\circ}{\sin 53.2^\circ} = 4.49 \text{ (3 s.f.)}$$

$$\angle B = 180^\circ - (36.8 + 53.2)^\circ = 90^\circ$$

Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{6}{\sin 53.2^\circ} = \frac{y}{\sin 90^\circ}$$

$$\Rightarrow y = \frac{6 \sin 90^\circ}{\sin 53.2^\circ} = 7.49 \text{ (3 s.f.)}$$

(Note: The third angle is  $90^\circ$  so you could solve the problem using sine and cosine; the sine rule is not necessary.)



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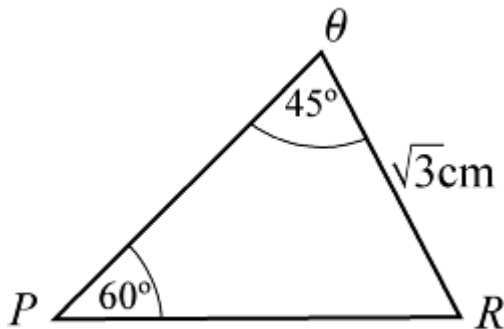
### The sine and cosine rule

#### Exercise A, Question 3

#### Question:

In  $\triangle PQR$ ,  $QR = \sqrt{3}$  cm,  $\angle PQR = 45^\circ$  and  $\angle QPR = 60^\circ$ . Find (a)  $PR$  and (b)  $PQ$ .

#### Solution:



(a) Using  $\frac{q}{\sin Q} = \frac{p}{\sin P}$

$$\Rightarrow \frac{PR}{\sin 45^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow PR = \frac{\sqrt{3} \sin 45^\circ}{\sin 60^\circ} = 1.41 \text{ cm (3 s.f.)}$$

(The exact answer is  $\sqrt{2}$  cm.)

(b) Using  $\frac{r}{\sin R} = \frac{p}{\sin P}$

$$\Rightarrow \frac{PQ}{\sin 75^\circ} = \frac{\sqrt{3}}{\sin 60^\circ} [\text{Angle } R = 180^\circ - (60 + 45)^\circ = 75^\circ]$$

$$\Rightarrow PQ = \frac{\sqrt{3} \sin 75^\circ}{\sin 60^\circ} = 1.93 \text{ cm (3 s.f.)}$$

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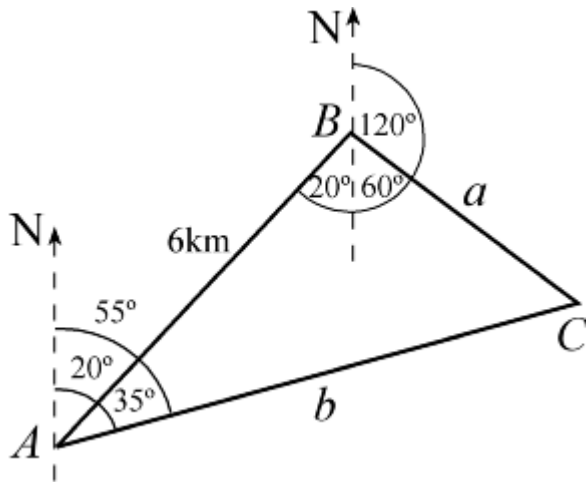
### The sine and cosine rule

#### Exercise A, Question 4

#### Question:

Town  $B$  is 6 km, on a bearing of  $020^\circ$ , from town  $A$ . Town  $C$  is located on a bearing of  $055^\circ$  from town  $A$  and on a bearing of  $120^\circ$  from town  $B$ . Work out the distance of town  $C$  from (a) town  $A$  and (b) town  $B$ .

#### Solution:



$$\angle BAC = 55^\circ - 20^\circ = 35^\circ$$

$$\angle ABC = 20^\circ \text{ ('Z' angles)} + 60^\circ \text{ (angles on a straight line)} = 80^\circ$$

$$\angle ACB = 180^\circ - (80 + 35)^\circ = 65^\circ$$

(a) Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{AC}{\sin 80^\circ} = \frac{6}{\sin 65^\circ}$$

$$\Rightarrow AC = \frac{6 \sin 80^\circ}{\sin 65^\circ} = 6.52 \text{ km (3 s.f.)}$$

(b) Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{BC}{\sin 35^\circ} = \frac{6}{\sin 65^\circ} \Rightarrow BC = \frac{6 \sin 35^\circ}{\sin 65^\circ} = 3.80 \text{ km (3 s.f.)}$$

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## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

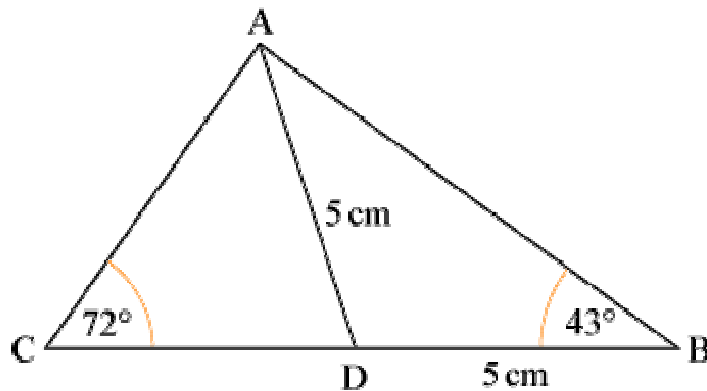
#### Exercise A, Question 5

#### Question:

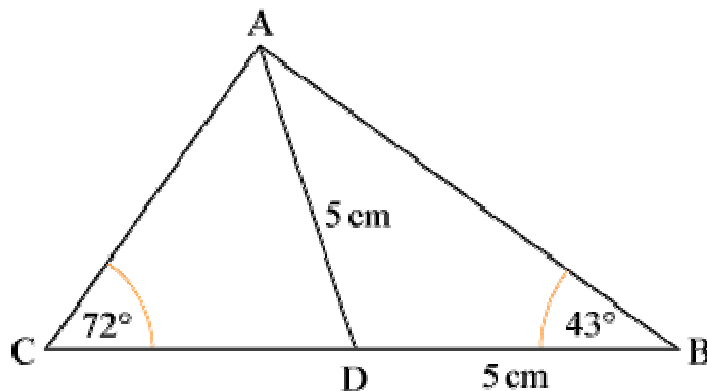
In the diagram  $AD = DB = 5$  cm,  $\angle ABC = 43^\circ$

and  $\angle ACB = 72^\circ$ .

Calculate (a)  $AB$  and (b)  $CD$ .



#### Solution:



(a) In  $\triangle ABD$ ,  $\angle DAB = 43^\circ$  (isosceles  $\triangle$ ).

So  $\angle ADB = 180^\circ - (2 \times 43^\circ) = 94^\circ$

As triangle is isosceles you could work with right-angled triangle, but using sine rule  $\frac{d}{\sin D} = \frac{a}{\sin A}$

$$\Rightarrow \frac{AB}{\sin 94^\circ} = \frac{5}{\sin 43^\circ}$$

$$\Rightarrow AB = \frac{5 \sin 94^\circ}{\sin 43^\circ} = 7.31 \text{ cm (3 s.f.)}$$

(b) In  $\triangle ADC$ ,  $\angle ADC = 180^\circ - 94^\circ = 86^\circ$  (angles on a straight line).

So  $\angle CAD = 180^\circ - (72^\circ + 86^\circ) = 22^\circ$

Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{CD}{\sin 22^\circ} = \frac{5}{\sin 72^\circ}$$

$$\Rightarrow CD = \frac{5 \sin 22^\circ}{\sin 72^\circ} = 1.97 \text{ cm (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise B, Question 1

#### Question:

(Note: Give answers to 3 significant figures, unless they are exact.)

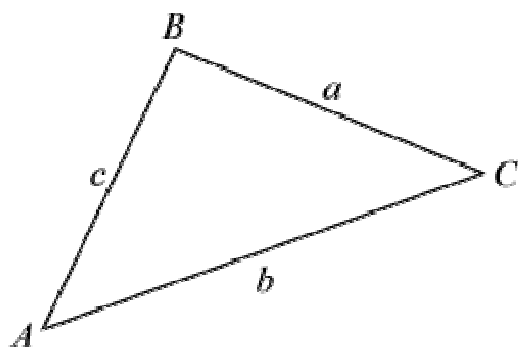
In each of the following sets of data for a triangle  $ABC$ , find the value of  $x$ :

(a)  $AB = 6$  cm,  $BC = 9$  cm,  $\angle BAC = 117^\circ$ ,  $\angle ACB = x^\circ$ .

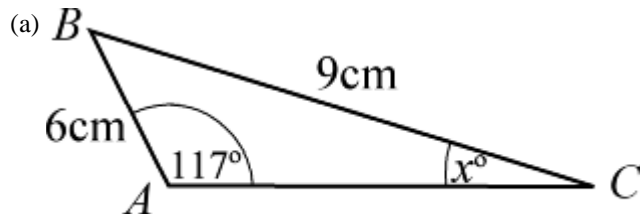
(b)  $AC = 11$  cm,  $BC = 10$  cm,  $\angle ABC = 40^\circ$ ,  $\angle CAB = x^\circ$ .

(c)  $AB = 6$  cm,  $BC = 8$  cm,  $\angle BAC = 60^\circ$ ,  $\angle ACB = x^\circ$ .

(d)  $AB = 8.7$  cm,  $AC = 10.8$  cm,  $\angle ABC = 28^\circ$ ,  $\angle BAC = x^\circ$ .



#### Solution:



Using  $\frac{\sin C}{c} = \frac{\sin A}{a}$

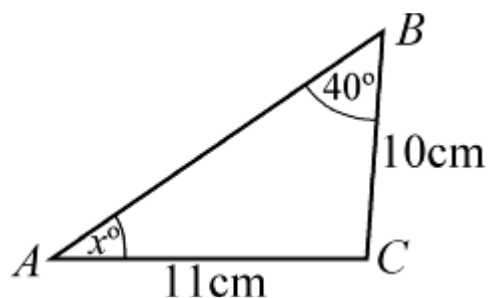
$$\Rightarrow \frac{\sin x^\circ}{6} = \frac{\sin 117^\circ}{9}$$

$$\Rightarrow \sin x^\circ = \frac{6 \sin 117^\circ}{9} (= 0.5940\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{6 \sin 117^\circ}{9} \right) = 36.4^\circ \text{ (3 s.f.)}$$

$$\Rightarrow x = 36.4$$

(b)



Using  $\frac{\sin A}{a} = \frac{\sin B}{b}$

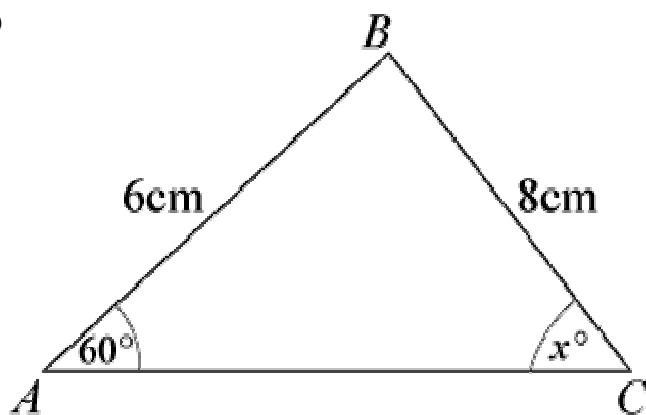
$$\Rightarrow \frac{\sin x^\circ}{10} = \frac{\sin 40^\circ}{11}$$

$$\Rightarrow \sin x^\circ = \frac{10 \sin 40^\circ}{11} (= 0.5843\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{10 \sin 40^\circ}{11} \right) = 35.8^\circ \text{ (3 s.f.)}$$

$$\Rightarrow x = 35.8$$

(c)



Using  $\frac{\sin C}{c} = \frac{\sin A}{a}$

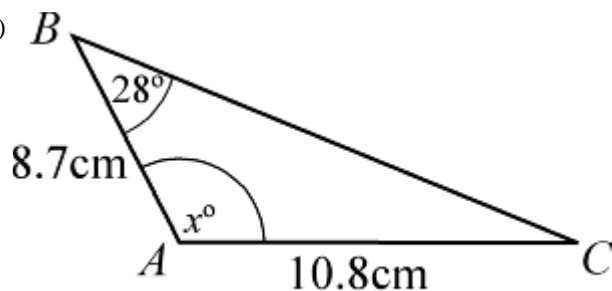
$$\Rightarrow \frac{\sin x^\circ}{6} = \frac{\sin 60^\circ}{8}$$

$$\Rightarrow \sin x^\circ = \frac{6 \sin 60^\circ}{8} (= 0.6495\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{6 \sin 60^\circ}{8} \right) = 40.5^\circ \text{ (3 s.f.)}$$

$$\Rightarrow x = 40.5$$

(d)



Using  $\frac{\sin C}{c} = \frac{\sin B}{b}$

$$\Rightarrow \frac{\sin C^\circ}{8.7} = \frac{\sin 28^\circ}{10.8}$$

$$\Rightarrow \sin C^\circ = \frac{8.7 \sin 28^\circ}{10.8} (= 0.3781\dots)$$

$$\Rightarrow C^\circ = \sin^{-1} \left( \frac{8.7 \sin 28^\circ}{10.8} \right)$$

$$\Rightarrow C = 22.2^\circ \text{ (3 s.f.)}$$

$$\Rightarrow x^\circ = 180^\circ - (28 + 22.2)^\circ = 129.8^\circ = 130^\circ \text{ (3 s.f.)}$$

$$\Rightarrow x = 130$$

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## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

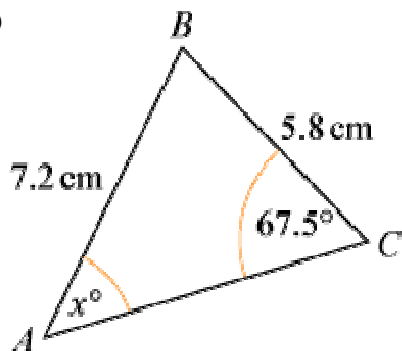
#### Exercise B, Question 2

#### Question:

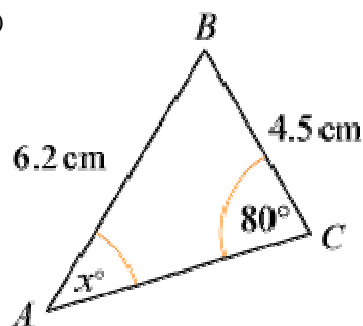
(Note: Give answers to 3 significant figures, unless they are exact.)

In each of the diagrams shown below, work out the value of  $x$ :

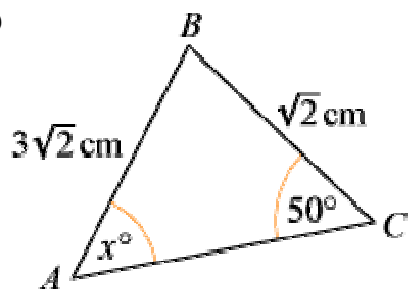
(a)



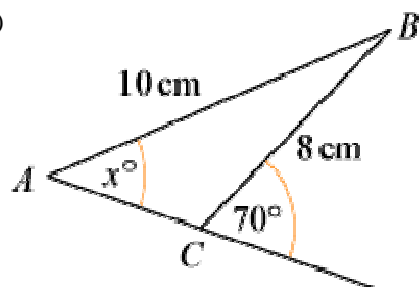
(b)



(c)

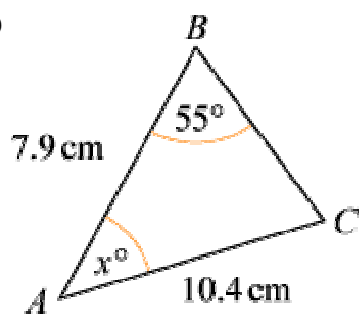


(d)

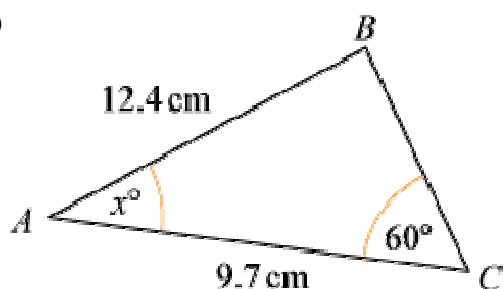




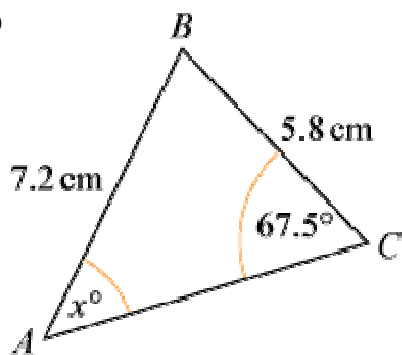
(e)



(f)

**Solution:**

(a)

Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$ 

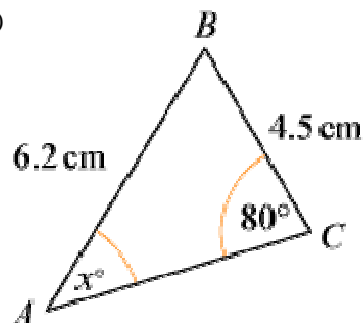
$$\Rightarrow \frac{\sin x^\circ}{5.8} = \frac{\sin 67.5^\circ}{7.2}$$

$$\Rightarrow \sin x^\circ = \frac{5.8 \sin 67.5^\circ}{7.2} (= 0.7442\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{5.8 \sin 67.5^\circ}{7.2} \right) = 48.09^\circ$$

$$\Rightarrow x = 48.1 \text{ (3 s.f.)}$$

(b)



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

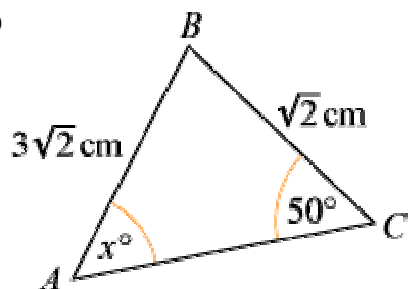
$$\Rightarrow \frac{\sin x^\circ}{4.5} = \frac{\sin 80^\circ}{6.2}$$

$$\Rightarrow \sin x^\circ = \frac{4.5 \sin 80^\circ}{6.2} (= 0.7147\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{4.5 \sin 80^\circ}{6.2} \right) = 45.63^\circ$$

$$\Rightarrow x = 45.6 \text{ (3 s.f.)}$$

(c)



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

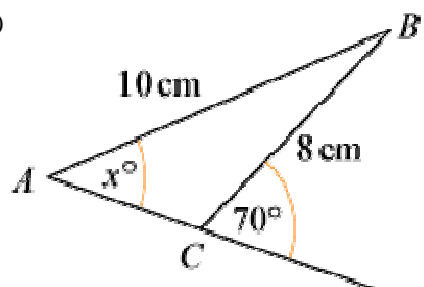
$$\Rightarrow \frac{\sin x^\circ}{\sqrt{2}} = \frac{\sin 50^\circ}{3\sqrt{2}}$$

$$\Rightarrow \sin x^\circ = \frac{\sqrt{2} \sin 50^\circ}{3\sqrt{2}} (= 0.2553 \dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{\sin 50^\circ}{3} \right) = 14.79^\circ$$

$$\Rightarrow x = 14.8 \text{ (3 s.f.)}$$

(d)



$$\text{Angle ACB} = 180^\circ - 70^\circ = 110^\circ$$

Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

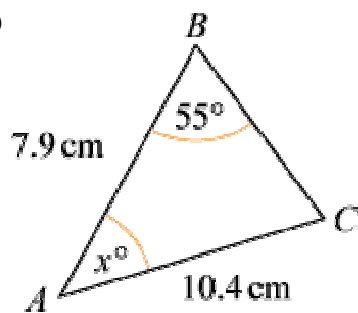
$$\Rightarrow \frac{\sin x^\circ}{8} = \frac{\sin 110^\circ}{10}$$

$$\Rightarrow \sin x^\circ = \frac{8 \sin 110^\circ}{10} (= 0.7517\dots)$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{8 \sin 110^\circ}{10} \right) = 48.74^\circ$$

$$\Rightarrow x = 48.7 \text{ (3 s.f.)}$$

(e)



Using  $\frac{\sin C}{c} = \frac{\sin B}{b}$

$$\Rightarrow \frac{\sin C}{7.9} = \frac{\sin 55^\circ}{10.4}$$

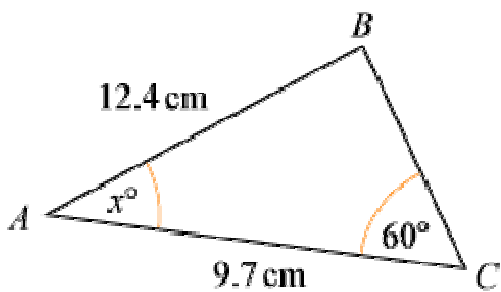
$$\Rightarrow \sin C = \frac{7.9 \sin 55^\circ}{10.4} (= 0.6222\dots)$$

$$\Rightarrow C = \sin^{-1} \left( \frac{7.9 \sin 55^\circ}{10.4} \right) = 38.48^\circ$$

$$x^\circ = 180^\circ - (55 + C)^\circ$$

$$\Rightarrow x = 86.52 = 86.5 \text{ (3 s.f.)}$$

(f)



Using  $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin B}{9.7} = \frac{\sin 60^\circ}{12.4}$$

$$\Rightarrow \sin B = \frac{9.7 \sin 60^\circ}{12.4} (= 0.6774\dots)$$

$$\Rightarrow B = 42.65^\circ$$

$$x^\circ = 180^\circ - (60 + B)^\circ = 77.35^\circ$$

$$\Rightarrow x = 77.4 \text{ (3 s.f.)}$$

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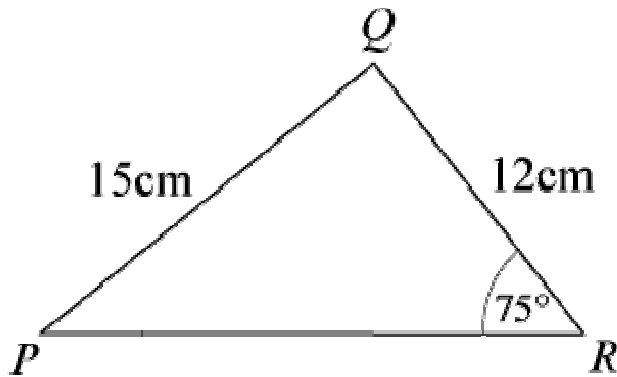
#### Exercise B, Question 3

#### Question:

(Note: Give answers to 3 significant figures, unless they are exact.)

In  $\triangle PQR$ ,  $PQ = 15$  cm,  $QR = 12$  cm and  $\angle PRQ = 75^\circ$ . Find the two remaining angles.

#### Solution:



Using  $\frac{\sin P}{p} = \frac{\sin R}{r}$

$$\Rightarrow \frac{\sin P}{12} = \frac{\sin 75^\circ}{15}$$

$$\Rightarrow \sin P = \frac{12 \sin 75^\circ}{15} (= 0.7727\dots)$$

$$\Rightarrow P = \sin^{-1} \left( \frac{12 \sin 75^\circ}{15} \right) = 50.60^\circ$$

Angle QPR =  $50.6^\circ$  (3 s.f.)

Angle PQR =  $180^\circ - (75 + 50.6)^\circ = 54.4^\circ$  (3 s.f.)

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

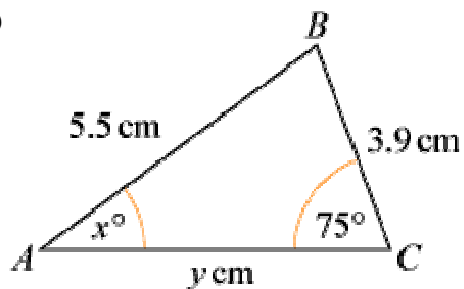
#### Exercise B, Question 4

#### Question:

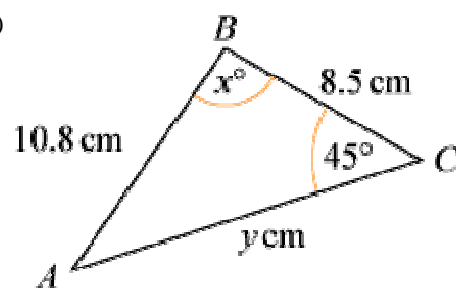
(Note: Give answers to 3 significant figures, unless they are exact.)

In each of the following diagrams work out the values of  $x$  and  $y$ :

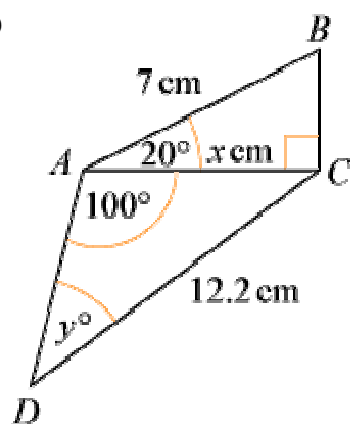
(a)



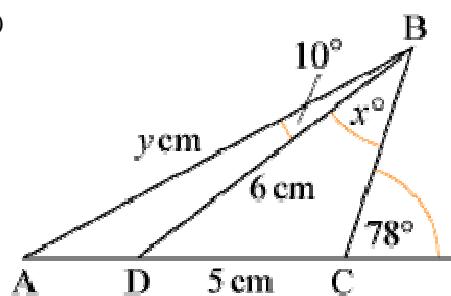
(b)

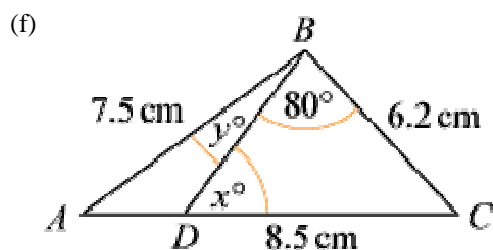
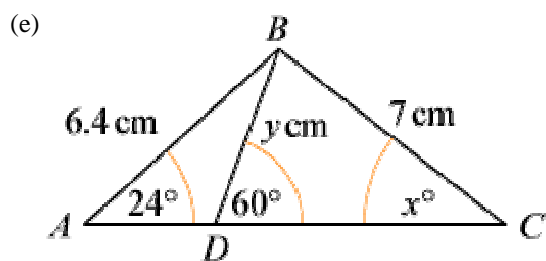


(c)

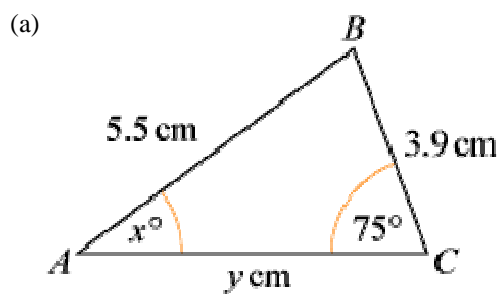


(d)





**Solution:**



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin x^\circ}{3.9} = \frac{\sin 75^\circ}{5.5}$$

$$\Rightarrow \sin x^\circ = \frac{3.9 \sin 75^\circ}{5.5}$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{3.9 \sin 75^\circ}{5.5} \right) = 43.23^\circ$$

$$\Rightarrow x = 43.2 \text{ (3 s.f.)}$$

So  $\angle ABC = 180^\circ - (75 + 43.2)^\circ = 61.8^\circ$

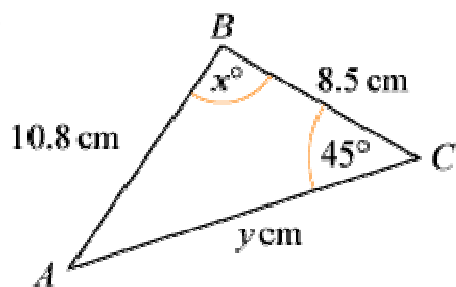
Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{y}{\sin 61.8^\circ} = \frac{5.5}{\sin 75^\circ}$$

$$\Rightarrow y = \frac{5.5 \sin 61.8^\circ}{\sin 75^\circ} = 5.018$$

$$\Rightarrow y = 5.02 \text{ (3 s.f.)}$$

(b)



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin A}{8.5} = \frac{\sin 45^\circ}{10.8}$$

$$\Rightarrow \sin A = \frac{8.5 \sin 45^\circ}{10.8}$$

$$\Rightarrow A = \sin^{-1} \left( \frac{8.5 \sin 45^\circ}{10.8} \right) = 33.815^\circ$$

$$x^\circ = 180^\circ - (45^\circ + A)^\circ = 101.2^\circ$$

$$\Rightarrow x = 101 \text{ (3 s.f.)}$$

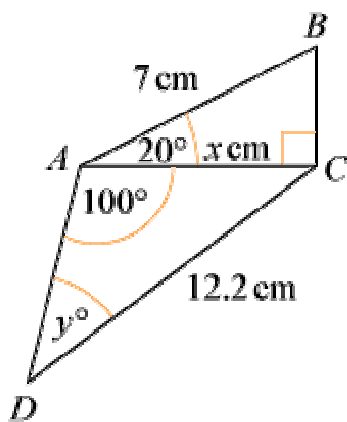
Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{y}{\sin x^\circ} = \frac{10.8}{\sin 45^\circ}$$

$$\Rightarrow y = \frac{10.8 \sin x^\circ}{\sin 45^\circ} = 14.98$$

$$\Rightarrow y = 15.0 \text{ (3 s.f.)}$$

(c)



In  $\triangle ABC$ ,  $\frac{x}{7} = \cos 20^\circ \Rightarrow x = 7 \cos 20^\circ = 6.578 = 6.58 \text{ (3 s.f.)}$

In  $\triangle ADC$ , using  $\frac{\sin D}{d} = \frac{\sin A}{a}$

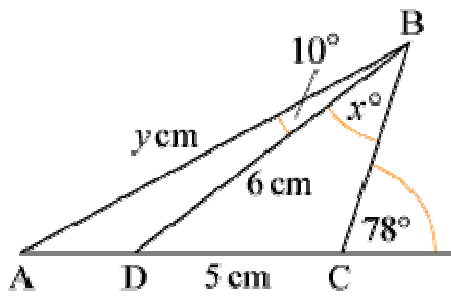
$$\Rightarrow \frac{\sin y^\circ}{x} = \frac{\sin 100^\circ}{12.2}$$

$$\Rightarrow \sin y^\circ = \frac{x \sin 100^\circ}{12.2}$$

$$\Rightarrow y^\circ = \sin^{-1} \left( \frac{x \sin 100^\circ}{12.2} \right) = 32.07^\circ$$

$$\Rightarrow y = 32.1 \text{ (3 s.f.)}$$

(d)



In  $\triangle BDC$ ,  $\angle C = 180^\circ - 78^\circ = 102^\circ$

Using  $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin x^\circ}{5} = \frac{\sin 102^\circ}{6}$$

$$\Rightarrow \sin x^\circ = \frac{5 \sin 102^\circ}{6}$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{5 \sin 102^\circ}{6} \right) = 54.599^\circ$$

$$\Rightarrow x = 54.6 \text{ (3 s.f.)}$$

In  $\triangle ABC$ ,  $\angle BAC = 180^\circ - 102^\circ - (10 + x)^\circ = 13.4^\circ$

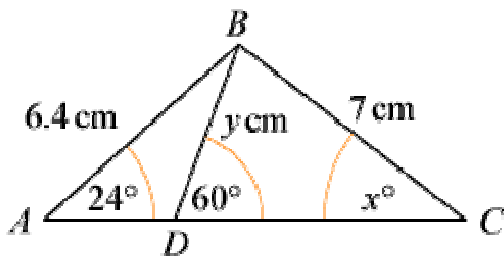
So  $\angle ADB = 180^\circ - 10^\circ - 13.4^\circ = 156.6^\circ$

Using  $\frac{d}{\sin D} = \frac{a}{\sin A}$  in  $\triangle ABD$

$$\Rightarrow \frac{y}{\sin 156.6^\circ} = \frac{6}{\sin 13.4^\circ}$$

$$\Rightarrow y = \frac{6 \sin 156.6^\circ}{\sin 13.4^\circ} = 10.28 = 10.3 \text{ (3 s.f.)}$$

(e)



In  $\triangle ABC$ , using  $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\Rightarrow \frac{\sin x^\circ}{6.4} = \frac{\sin 24^\circ}{7}$$

$$\Rightarrow \sin x^\circ = \frac{6.4 \sin 24^\circ}{7}$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{6.4 \sin 24^\circ}{7} \right) = 21.83^\circ$$

$$\Rightarrow x = 21.8 \text{ (3 s.f.)}$$

In  $\triangle ABD$ , using  $\frac{a}{\sin A} = \frac{d}{\sin D}$

$$\Rightarrow \frac{y}{\sin 24^\circ} = \frac{6.4}{\sin 120^\circ}$$

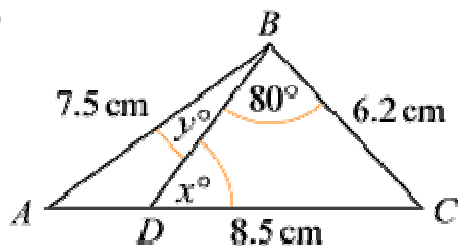


$$\Rightarrow y = \frac{6.4 \sin 24^\circ}{\sin 120^\circ} = 3.0058$$

$$\Rightarrow y = 3.01 \text{ (3 s.f.)}$$

(The above approach finds the two values independently. You could find  $y$  first and then use it to find  $x$ , but then if  $y$  is wrong then so will  $x$  be.)

(f)



Using  $\frac{\sin D}{d} = \frac{\sin B}{b}$  in  $\triangle BDC$

$$\Rightarrow \frac{\sin x^\circ}{6.2} = \frac{\sin 80^\circ}{8.5}$$

$$\Rightarrow x^\circ = \frac{6.2 \sin 80^\circ}{8.5}$$

$$\Rightarrow x^\circ = \sin^{-1} \left( \frac{6.2 \sin 80^\circ}{8.5} \right) = 45.92^\circ$$

$$\Rightarrow x = 45.9 \text{ (3 s.f.)}$$

In  $\triangle ABC$ ,  $\angle ACB = 180^\circ - (80 + x)^\circ = 54.08^\circ$

Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin A}{6.2} = \frac{\sin 54.08^\circ}{7.5}$$

$$\Rightarrow \sin A = \frac{6.2 \sin 54.08^\circ}{7.5}$$

$$\Rightarrow A = \sin^{-1} \left( \frac{6.2 \sin 54.08^\circ}{7.5} \right) = 42.03^\circ$$

So  $y^\circ = 180^\circ - (42.03 + 134.1)^\circ = 3.87 \text{ (3 s.f.)}$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

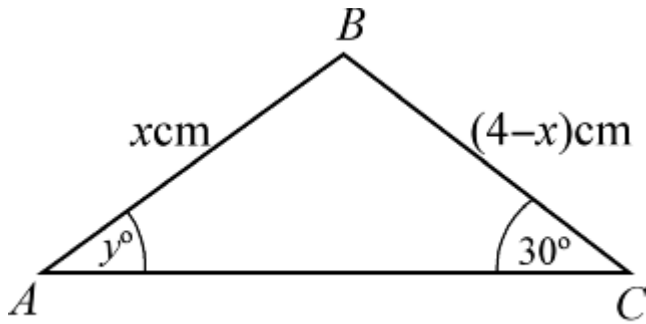
#### Exercise B, Question 5

#### Question:

In  $\triangle ABC$ ,  $AB = x$  cm,  $BC = (4 - x)$  cm,  $\angle BAC = y^\circ$  and  $\angle BCA = 30^\circ$ .

Given that  $\sin y^\circ = \frac{1}{\sqrt{2}}$ , show that  $x = 4(\sqrt{2} - 1)$ .

#### Solution:



Using  $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \frac{4-x}{\sin y^\circ} = \frac{x}{\sin 30^\circ}$$

$$\Rightarrow (4-x) \sin 30^\circ = x \sin y^\circ$$

$$\Rightarrow (4-x) \times \frac{1}{2} = x \times \frac{1}{\sqrt{2}}$$

Multiply throughout by 2:

$$4-x = x\sqrt{2}$$

$$x + \sqrt{2}x = 4$$

$$x(1 + \sqrt{2}) = 4$$

$$x = \frac{4}{1 + \sqrt{2}}$$

Multiply 'top and bottom' by  $\sqrt{2} - 1$ :

$$x = \frac{4(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{4(\sqrt{2}-1)}{2-1} = 4(\sqrt{2}-1)$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise C, Question 1

#### Question:

(Give answers to 3 significant figures.)

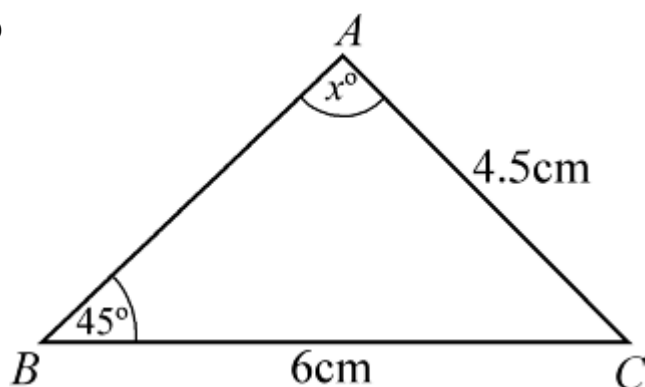
In  $\triangle ABC$ ,  $BC = 6$  cm,  $AC = 4.5$  cm and  $\angle ABC = 45^\circ$ :

(a) Calculate the two possible values of  $\angle BAC$ .

(b) Draw a diagram to illustrate your answers.

#### Solution:

(a)



$$x > 45^\circ$$

So there are two possible results.

$$\text{Using } \frac{\sin A}{a} = \frac{\sin B}{b}$$

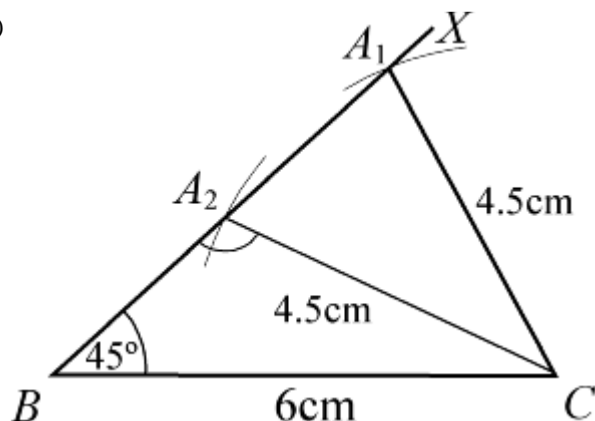
$$\frac{\sin x^\circ}{6} = \frac{\sin 45^\circ}{4.5}$$

$$\sin x^\circ = \frac{6 \sin 45^\circ}{4.5}$$

$$x^\circ = \sin^{-1} \left( \frac{6 \sin 45^\circ}{4.5} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{6 \sin 45^\circ}{4.5} \right)$$

$$x^\circ = 70.5^\circ \text{ (3 s.f.) or } 109.5^\circ$$

(b)



Draw  $BC = 6$  cm.

Measure angle of  $45^\circ$  at  $B$  ( $BX$ ).

Put compass point at  $C$  and open out to 4.5 cm. Where arc meets  $BX$  are the two possible positions of  $A$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

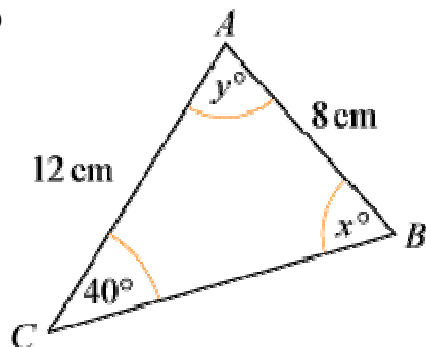
#### Exercise C, Question 2

#### Question:

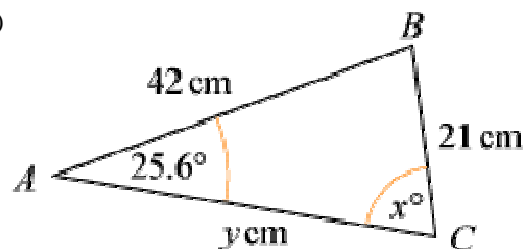
(Give answers to 3 significant figures.)

In each of the diagrams shown below, calculate the possible values of  $x$  and the corresponding values of  $y$ :

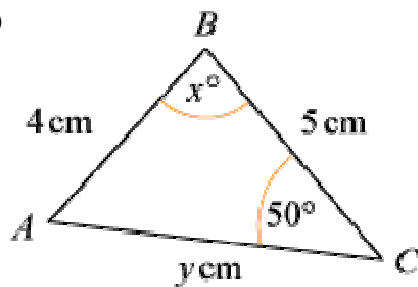
(a)



(b)

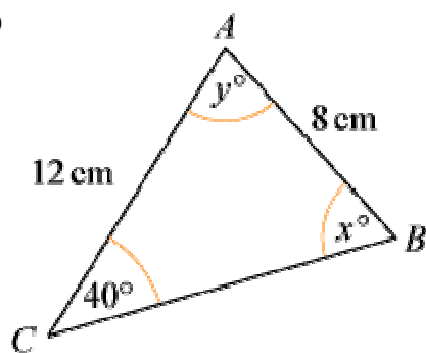


(c)



#### Solution:

(a)



Using  $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin x^\circ}{12} = \frac{\sin 40^\circ}{8}$$

$$\sin x^\circ = \frac{12 \sin 40^\circ}{8}$$

$$x^\circ = \sin^{-1} \left( \frac{12 \sin 40^\circ}{8} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{12 \sin 40^\circ}{8} \right)$$

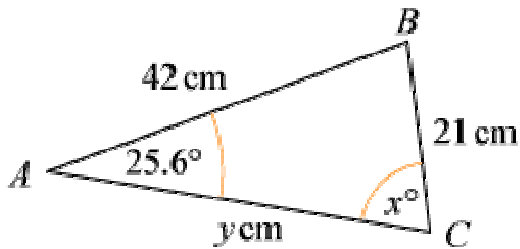
$$x^\circ = 74.6^\circ \text{ or } 105.4^\circ$$

$$x = 74.6 \text{ or } 105 \text{ (3 s.f.)}$$

When  $x = 74.6$ ,  $y = 180 - (74.6 + 40) = 180 - 114.6 = 65.4$  (3 s.f.)

When  $x = 105.4$ ,  $y = 180 - (105.4 + 40) = 180 - 145.4 = 34.6$  (3 s.f.)

(b)



Using  $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin x^\circ}{42} = \frac{\sin 25.6^\circ}{21}$$

$$\sin x^\circ = \frac{42 \sin 25.6^\circ}{21}$$

$$x^\circ = \sin^{-1} (2 \sin 25.6^\circ) \text{ or } 180^\circ - \sin^{-1} (2 \sin 25.6^\circ)$$

$$x^\circ = 59.79^\circ \text{ or } 120.2^\circ$$

$$x = 59.8 \text{ or } 120 \text{ (3 s.f.)}$$

When  $x = 59.8$ ,

$$\text{angle } B = 180^\circ - (59.8^\circ + 25.6^\circ) = 94.6^\circ$$

Using  $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 94.6^\circ} = \frac{21}{\sin 25.6^\circ} \Rightarrow y = \frac{21 \sin 94.6^\circ}{\sin 25.6^\circ} = 48.4 \text{ (3 s.f.)}$$

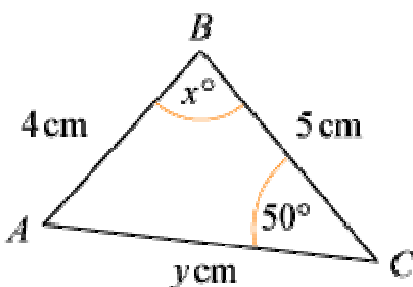
When  $x = 120.2$ ,

$$\text{angle } B = 180^\circ - (120.2^\circ + 25.6^\circ) = 34.2^\circ$$

Using  $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 34.2^\circ} = \frac{21}{\sin 25.6^\circ} \Rightarrow y = \frac{21 \sin 34.2^\circ}{\sin 25.6^\circ} = 27.3 \text{ (3 s.f.)}$$

(c)



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{5} = \frac{\sin 50^\circ}{4}$$

$$\sin A = \frac{5 \sin 50^\circ}{4}$$

$$A = \sin^{-1} \left( \frac{5 \sin 50^\circ}{4} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{5 \sin 50^\circ}{4} \right)$$

$$A = 73.25 \text{ or } 106.75$$

When  $A = 73.247$ ,

$$x = 180 - (50 + 73.247) = 56.753 \dots = 56.8 \text{ (3 s.f.)}$$

Using  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{y}{\sin x^\circ} = \frac{4}{\sin 50^\circ} \Rightarrow y = \frac{4 \sin x^\circ}{\sin 50^\circ} = 4.37 \text{ (3 s.f.)}$$

When  $A = 106.75$ ,

$$x = 180 - (50 + 106.75) = 23.247 = 23.2 \text{ (3 s.f.)}$$

$$\text{As above: } y = \frac{4 \sin x^\circ}{\sin 50^\circ} = 2.06 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise C, Question 3

#### Question:

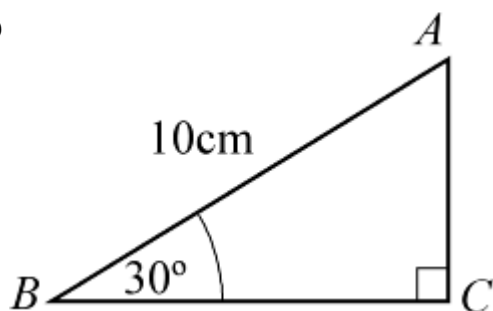
(Give answers to 3 significant figures.)

In each of the following cases  $\triangle ABC$  has  $\angle ABC = 30^\circ$  and  $AB = 10$  cm:

- (a) Calculate the least possible length that  $AC$  could be.
- (b) Given that  $AC = 12$  cm, calculate  $\angle ACB$ .
- (c) Given instead that  $AC = 7$  cm, calculate the two possible values of  $\angle ACB$ .

#### Solution:

(a)



$AC$  is least when it is at right angles to  $BC$ .

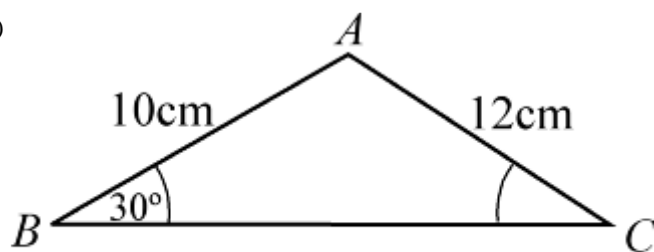
$$\text{Using } \sin B = \frac{AC}{AB}$$

$$\sin 30^\circ = \frac{AC}{10}$$

$$AC = 10 \sin 30^\circ = 5$$

$$AC = 5 \text{ cm}$$

(b)



$$\text{Using } \frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{10} = \frac{\sin 30^\circ}{12}$$

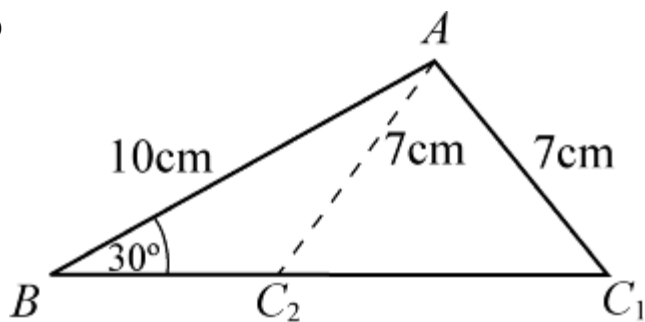
$$\sin C = \frac{10 \sin 30^\circ}{12}$$



$$C = \sin^{-1} \left( \frac{10 \sin 30^\circ}{12} \right) = 24.62^\circ$$

$$\angle ACB = 24.6^\circ \text{ (3 s.f.)}$$

(c)



As 7 cm < 10 cm,  $\angle ACB > 30^\circ$  and there are two possible results.

Using 7 cm instead of 12 cm in (b):

$$\sin C = \frac{10 \sin 30^\circ}{7}$$

$$C = \sin^{-1} \left( \frac{10 \sin 30^\circ}{7} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{10 \sin 30^\circ}{7} \right)$$

$$C = 45.58^\circ \text{ or } 134.4^\circ$$

$$\angle ACB = 45.6^\circ \text{ (3 s.f.) or } 134^\circ \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

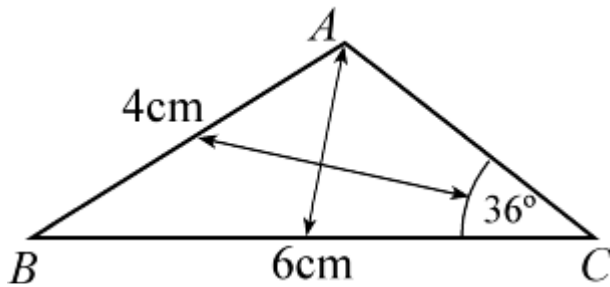
#### Exercise C, Question 4

#### Question:

(Give answers to 3 significant figures.)

Triangle  $ABC$  is such that  $AB = 4$  cm,  $BC = 6$  cm and  $\angle ACB = 36^\circ$ . Show that one of the possible values of  $\angle ABC$  is  $25.8^\circ$  (to 3 s.f.). Using this value, calculate the length of  $AC$ .

#### Solution:



As  $4 < 6$ ,  $36^\circ < \angle BAC$ , so there are two possible values for angle  $A$ .

Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{6} = \frac{\sin 36^\circ}{4}$$

$$\sin A = \frac{6 \sin 36^\circ}{4}$$

$$A = \sin^{-1} \left( \frac{6 \sin 36^\circ}{4} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{6 \sin 36^\circ}{4} \right)$$

$$A = 61.845 \dots^\circ \text{ or } 118.154 \dots^\circ$$

$$\text{When } A = 118.154 \dots^\circ$$

$$\angle ABC = 180^\circ - (36^\circ + 118.154 \dots^\circ) = 25.846 \dots^\circ = 25.8^\circ \text{ (3 s.f.)}$$

Using this value for  $\angle ABC$  and  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{AC}{\sin 25.8^\circ} = \frac{4}{\sin 36^\circ}$$

$$\Rightarrow AC = \frac{4 \sin 25.8^\circ}{\sin 36^\circ} = 2.96 \text{ cm (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

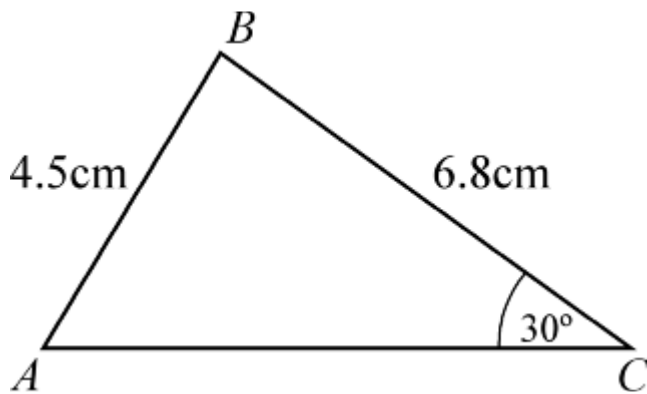
#### Exercise C, Question 5

#### Question:

(Give answers to 3 significant figures.)

Two triangles  $ABC$  are such that  $AB = 4.5$  cm,  $BC = 6.8$  cm and  $\angle ACB = 30^\circ$ . Work out the value of the largest angle in each of the triangles.

#### Solution:



As  $6.8 > 4.5$ , angle  $A > 30^\circ$  and so there are two possible values for  $A$ .

Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{6.8} = \frac{\sin 30^\circ}{4.5}$$

$$A = \sin^{-1} \left( \frac{6.8 \sin 30^\circ}{4.5} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{6.8 \sin 30^\circ}{4.5} \right)$$

$$A = 49.07 \dots^\circ \text{ or } 130.926 \dots^\circ$$

When  $A = 49.07 \dots^\circ$ , angle  $B$  is the largest angle

$$\angle ABC = 180^\circ - (30^\circ + 49.07 \dots^\circ) = 100.9 \dots^\circ = 101^\circ \text{ (3 s.f.)}$$

When  $A = 130.926 \dots^\circ$ , this will be the largest angle

$$\angle BAC = 131^\circ \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

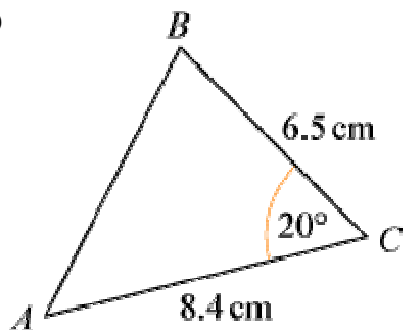
#### Exercise D, Question 1

#### Question:

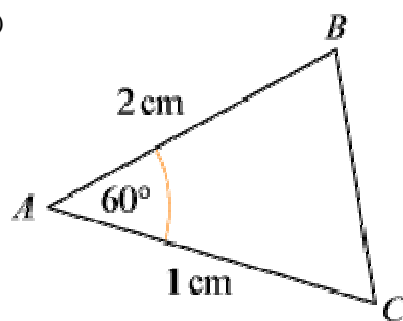
(Note: Give answers to 3 significant figures, where appropriate.)

In each of the following triangles calculate the length of the third side:

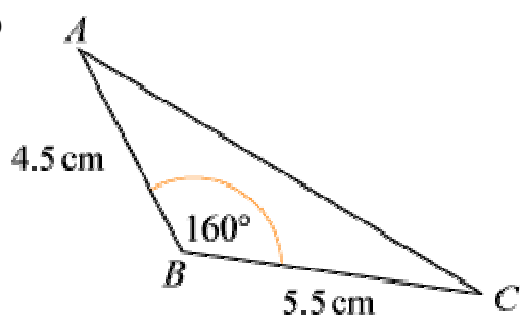
(a)



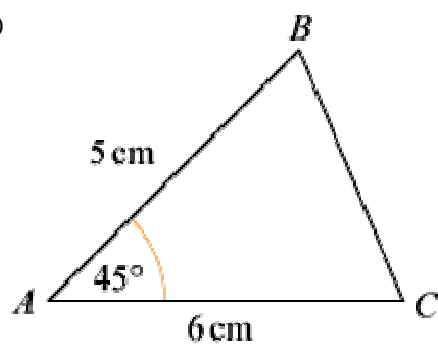
(b)



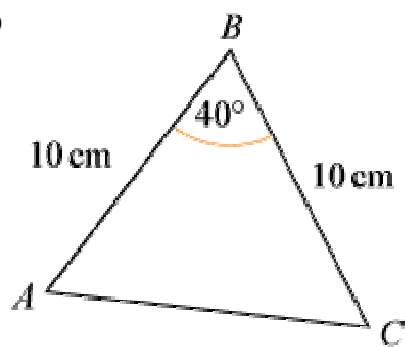
(c)



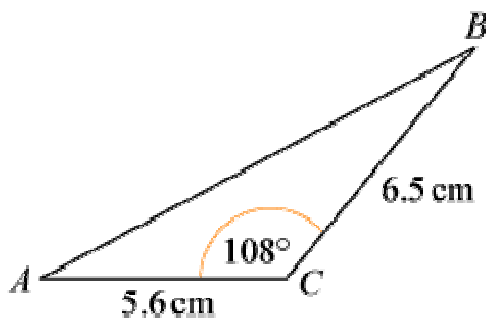
(d)



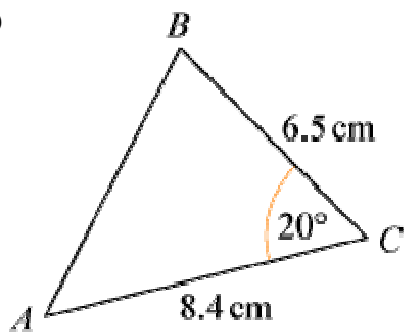
(e)



(f)

**Solution:**

(a)

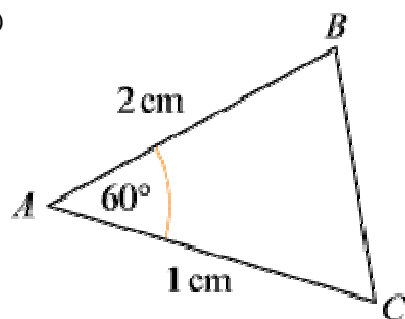
Using  $c^2 = a^2 + b^2 - 2ab \cos C$ 

$$AB^2 = 6.5^2 + 8.4^2 - 2 \times 6.5 \times 8.4 \times \cos 20^\circ$$

$$AB^2 = \frac{10.1955 \dots}{10.1955 \dots}$$

$$AB = \sqrt{10.1955 \dots} = 3.19 \text{ cm (3 s.f.)}$$

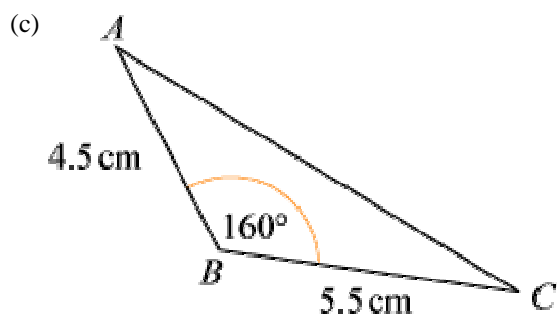
(b)

Using  $a^2 = b^2 + c^2 - 2bc \cos A$ 

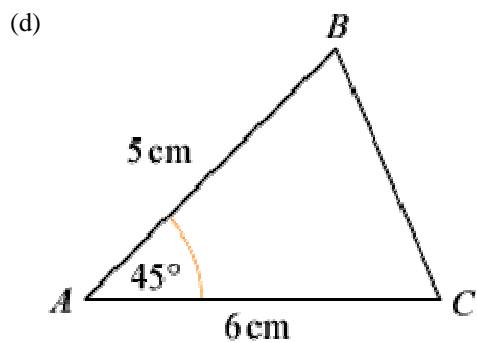
$$BC^2 = 1^2 + 2^2 - 2 \times 1 \times 2 \times \cos 60^\circ$$

$$BC^2 = 3$$

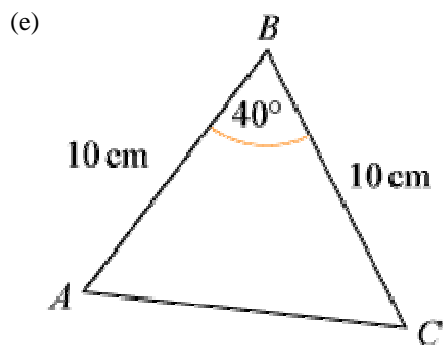
$$BC = \sqrt{3} = 1.73 \text{ cm (3 s.f.)}$$



Using  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $AC^2 = 5.5^2 + 4.5^2 - 2 \times 5.5 \times 4.5 \times \cos 160^\circ$   
 $AC^2 = 97.014 \dots$   
 $AC = \sqrt{97.014 \dots} = 9.85 \text{ cm (3 s.f.)}$

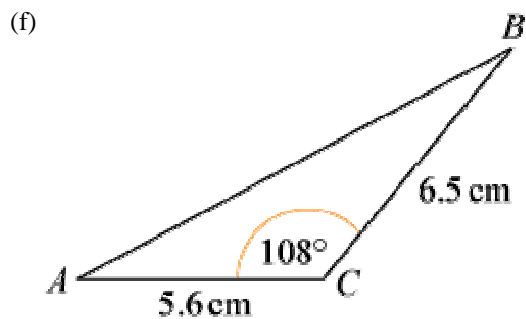


Using  $a^2 = b^2 + c^2 - 2bc \cos A$   
 $BC^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 45^\circ = 18.573 \dots$   
 $BC = \sqrt{18.573 \dots} = 4.31 \text{ cm (3 s.f.)}$



(This is an isosceles triangle and so you could use right-angled triangle work.)

Using  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $AC^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 40^\circ = 46.791 \dots$   
 $AC = \sqrt{46.791 \dots} = 6.84 \text{ cm (3 s.f.)}$



Using  $c^2 = a^2 + b^2 - 2ab \cos C$

$$AB^2 = 6.5^2 + 5.6^2 - 2 \times 6.5 \times 5.6 \times \cos 108^\circ = 96.106 \dots$$

$$AB = \sqrt{96.106 \dots} = 9.80 \text{ cm (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

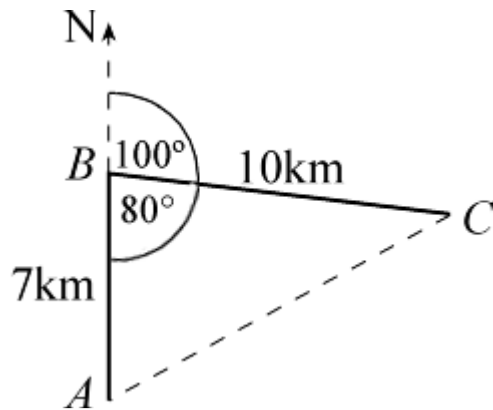
#### Exercise D, Question 2

#### Question:

(Note: Give answers to 3 significant figures, where appropriate.)

From a point  $A$  a boat sails due north for 7 km to  $B$ . The boat leaves  $B$  and moves on a bearing of  $100^\circ$  for 10 km until it reaches  $C$ . Calculate the distance of  $C$  from  $A$ .

#### Solution:



Using the cosine rule:  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $AC^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 80^\circ = 124.689 \dots$   
 $AC = \sqrt{124.689} \dots = 11.2 \text{ km (3 s.f.)}$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

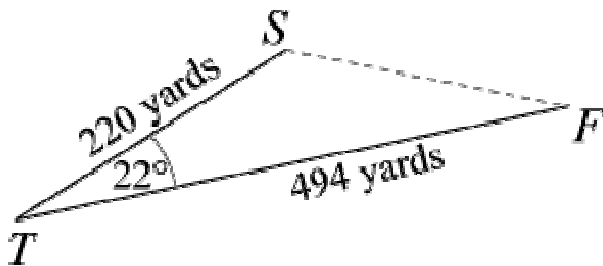
#### Exercise D, Question 3

#### Question:

(Note: Give answers to 3 significant figures, where appropriate.)

The distance from the tee,  $T$ , to the flag,  $F$ , on a particular hole on a golf course is 494 yards. A golfer's tee shot travels 220 yards and lands at the point  $S$ , where  $\angle STF = 22^\circ$ . Calculate how far the ball is from the flag.

#### Solution:



Using the cosine rule:

$$t^2 = f^2 + s^2 - 2fs \cos T$$

$$SF^2 = 220^2 + 494^2 - 2 \times 220 \times 494 \cos 22^\circ = 90903.317 \dots$$

$$SF = \sqrt{90903.317 \dots} = 301.5 \dots \text{ yards} = 302 \text{ yards (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

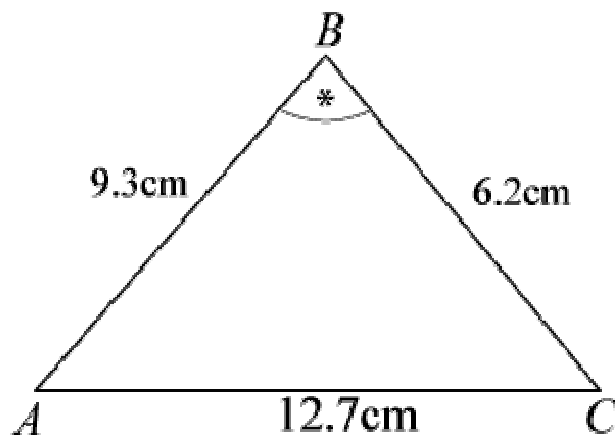
#### Exercise D, Question 4

#### Question:

(Note: Give answers to 3 significant figures, where appropriate.)

In  $\triangle ABC$ ,  $AB = (x - 3)$  cm,  $BC = (x + 3)$  cm,  $AC = 8$  cm and  $\angle BAC = 60^\circ$ . Use the cosine rule to find the value of  $x$ .

#### Solution:



Using  $a^2 = b^2 + c^2 - 2bc \cos A$

$$(x + 3)^2 = (x - 3)^2 + 8^2 - 2 \times 8 \times (x - 3) \cos 60^\circ$$

$$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8(x - 3)$$

$$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8x + 24$$

$$6x + 6x + 8x = 64 + 24$$

$$20x = 88$$

$$x = \frac{88}{20} = 4.4$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

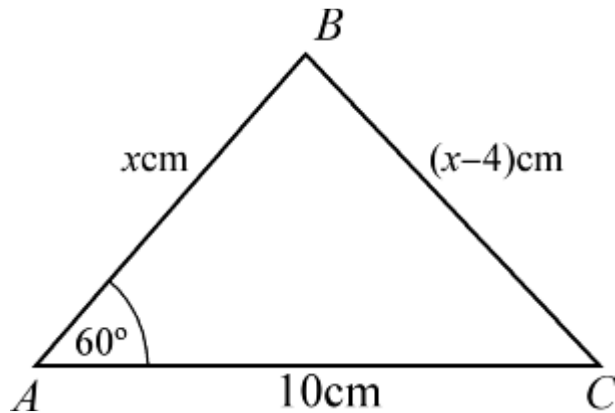
#### Exercise D, Question 5

#### Question:

(Note: Give answers to 3 significant figures, where appropriate.)

In  $\triangle ABC$ ,  $AB = x$  cm,  $BC = (x - 4)$  cm,  $AC = 10$  cm and  $\angle BAC = 60^\circ$ . Calculate the value of  $x$ .

#### Solution:



Using  $a^2 = b^2 + c^2 - 2bc \cos A$

$$(x - 4)^2 = 10^2 + x^2 - 2 \times 10 \times x \cos 60^\circ$$

$$x^2 - 8x + 16 = 100 + x^2 - 10x$$

$$10x - 8x = 100 - 16$$

$$2x = 84$$

$$x = 42$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise D, Question 6

#### Question:

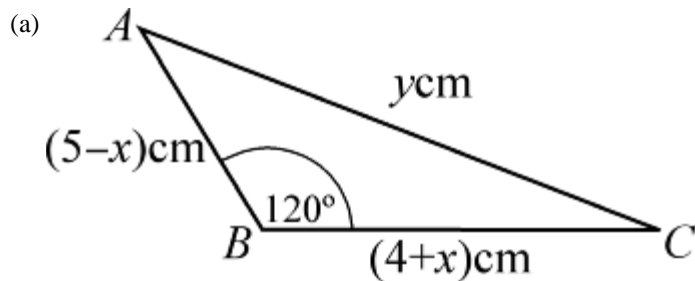
(Note: Give answers to 3 significant figures, where appropriate.)

In  $\triangle ABC$ ,  $AB = (5 - x)$  cm,  $BC = (4 + x)$  cm,  $\angle ABC = 120^\circ$  and  $AC = y$  cm.

(a) Show that  $y^2 = x^2 - x + 61$ .

(b) Use the method of completing the square to find the minimum value of  $y^2$ , and give the value of  $x$  for which this occurs.

#### Solution:



Using  $b^2 = a^2 + c^2 - 2ac \cos B$

$$y^2 = (4 + x)^2 + (5 - x)^2 - 2(4 + x)(5 - x) \cos 120^\circ$$

$$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + (4 + x)(5 - x) \quad (\text{Note: } 2 \cos 120^\circ = -1)$$

$$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + 20 + x - x^2 = x^2 - x + 61$$

(b) Completing the square:  $y^2 = \left(x - \frac{1}{2}\right)^2 + 61 - \frac{1}{4}$

$$\Rightarrow y^2 = \left(x - \frac{1}{2}\right)^2 + 60\frac{3}{4}$$

Minimum value of  $y^2$  occurs when  $\left(x - \frac{1}{2}\right)^2 = 0$ , i.e. when  $x = \frac{1}{2}$ .

So minimum value of  $y^2 = 60.75$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

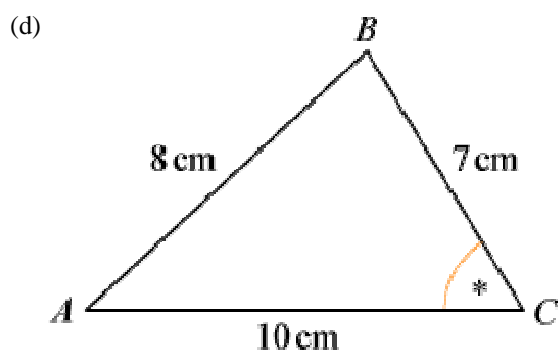
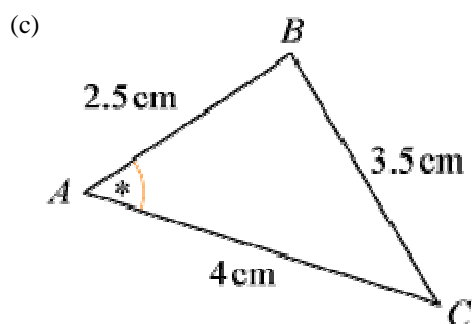
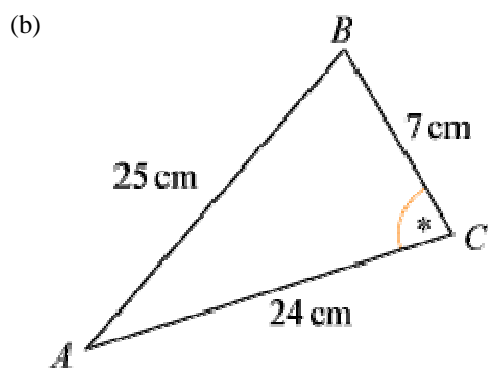
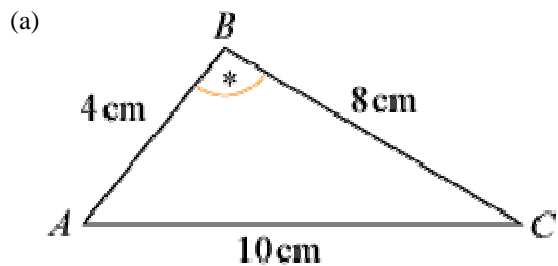
### The sine and cosine rule

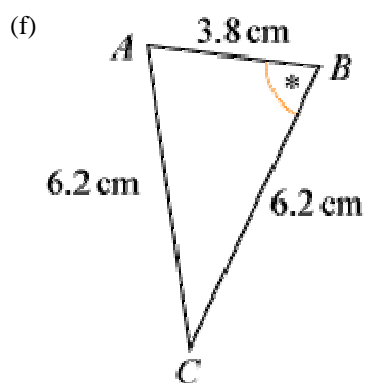
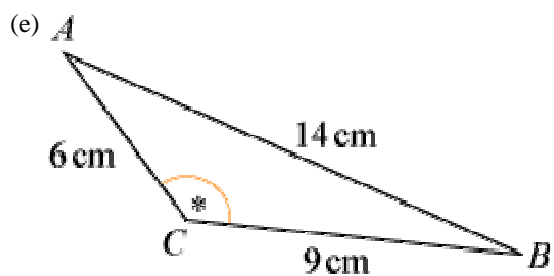
#### Exercise E, Question 1

#### Question:

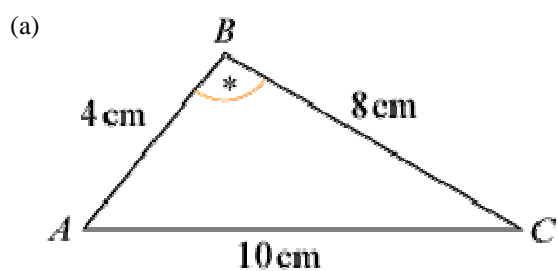
(Give answers to 3 significant figures.)

In the following triangles calculate the size of the angle marked \*:





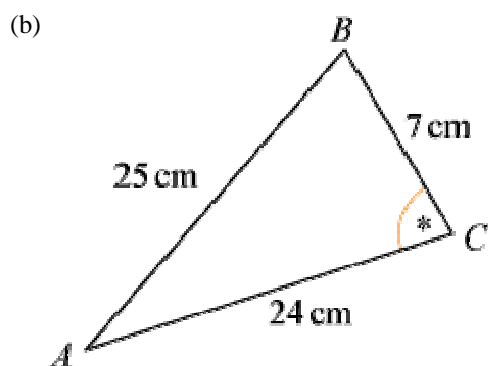
**Solution:**



Using  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\cos B = \frac{8^2 + 4^2 - 10^2}{2 \times 8 \times 4} = -\frac{20}{64} = -\frac{5}{16}$$

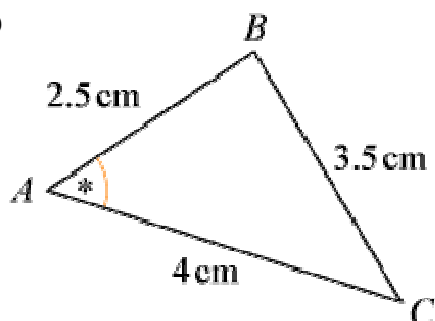
$$B = \cos^{-1} \left( -\frac{5}{16} \right) = 108.2 \dots^\circ = 108^\circ \text{ (3 s.f.)}$$



Using  $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$\cos C = \frac{7^2 + 24^2 - 25^2}{2 \times 7 \times 24} = 0 \Rightarrow C = 90^\circ$$

(c)

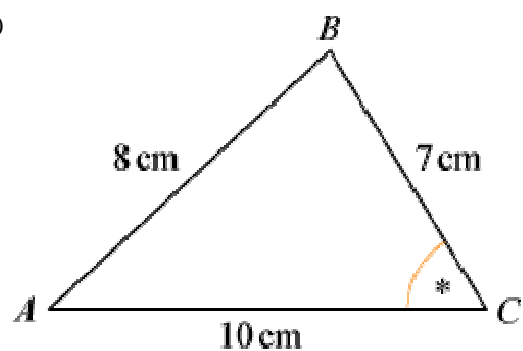


$$\text{Using } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{4^2 + 2.5^2 - 3.5^2}{2 \times 4 \times 2.5} = \frac{1}{2}$$

$$A = \cos^{-1} \left( \frac{1}{2} \right) = 60^\circ$$

(d)

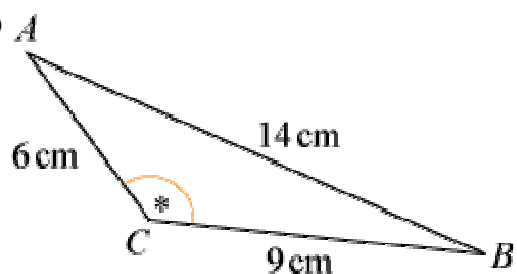


$$\text{Using } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{7^2 + 10^2 - 8^2}{2 \times 7 \times 10} = 0.6071 \dots$$

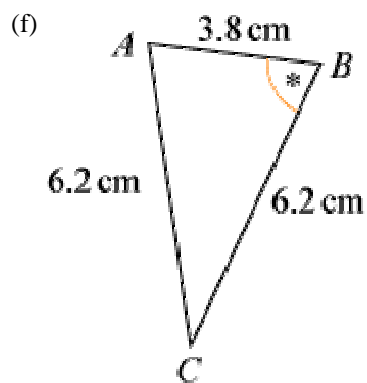
$$C = \cos^{-1}(0.6071\dots) = 52.6^\circ \text{ (3 s.f.)}$$

(e)



$$\text{Using } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{9^2 + 6^2 - 14^2}{2 \times 9 \times 6} = -0.7314 \dots \Rightarrow C = 137^\circ \text{ (3 s.f.)}$$



Using  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\cos B = \frac{6.2^2 + 3.8^2 - 6.2^2}{2 \times 6.2 \times 3.8} = \frac{3.8}{2 \times 6.2} = 0.3064 \dots \Rightarrow B = 72.2^\circ \text{ (3 s.f.)}$$

(This is an isosceles triangle so you could use right-angled triangle trigonometry.)



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

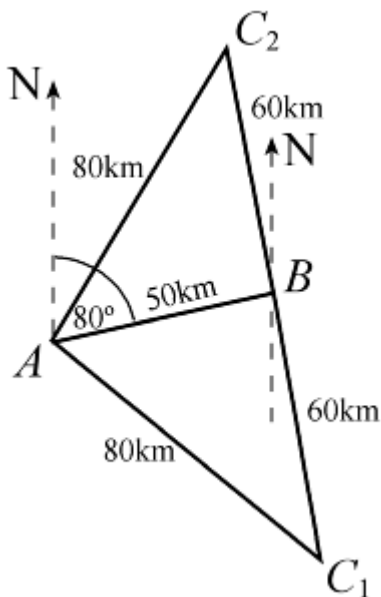
#### Exercise E, Question 2

#### Question:

(Give answers to 3 significant figures.)

A helicopter flies on a bearing of  $080^\circ$  from  $A$  to  $B$ , where  $AB = 50$  km.  
It then flies for 60 km to a point  $C$ .  
Given that  $C$  is 80 km from  $A$ , calculate the bearing of  $C$  from  $A$ .

#### Solution:



The bearing of  $C$  from  $B$  is not given so there are two possibilities for  $C$  using the data.  
The angle  $A$  will be the same in each  $\triangle ABC$ .

$$\text{Using } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{80^2 + 50^2 - 60^2}{2 \times 80 \times 50} = 0.6625$$

$$A = 48.5^\circ$$

$$\text{Bearing of } C \text{ from } A \text{ is } 80^\circ \pm 48.5^\circ = 128.5^\circ \text{ or } 31.5^\circ$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

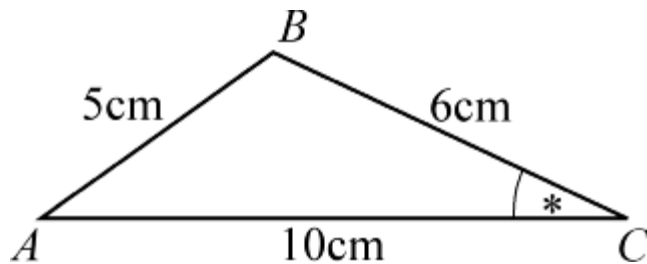
#### Exercise E, Question 3

#### Question:

(Give answers to 3 significant figures.)

In  $\triangle ABC$ ,  $AB = 5$  cm,  $BC = 6$  cm and  $AC = 10$  cm.  
Calculate the value of the smallest angle.

#### Solution:



The smallest angle is  $C$  as this is opposite  $AB$ .

$$\text{Using } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{6^2 + 10^2 - 5^2}{2 \times 6 \times 10} = 0.925$$

$$C = 22.3^\circ \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

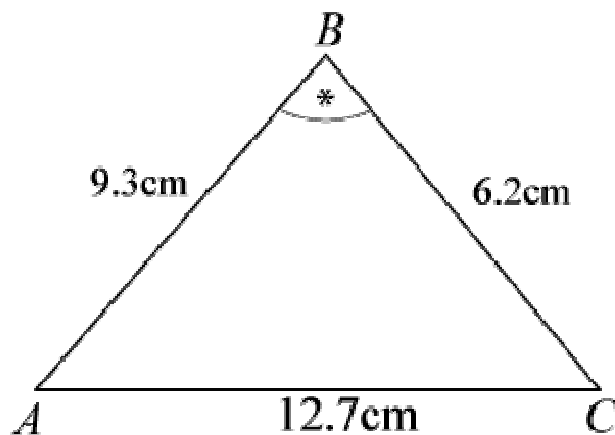
#### Exercise E, Question 4

#### Question:

(Give answers to 3 significant figures.)

In  $\triangle ABC$ ,  $AB = 9.3$  cm,  $BC = 6.2$  cm and  $AC = 12.7$  cm.  
Calculate the value of the largest angle.

#### Solution:



The largest angle is  $B$  as it is opposite  $AC$ .

$$\text{Using } \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{6.2^2 + 9.3^2 - 12.7^2}{2 \times 6.2 \times 9.3} = -0.3152 \dots$$

$$B = 108.37 \dots = 108^\circ \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

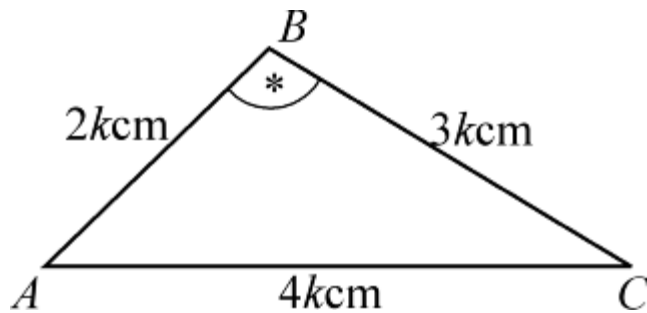
#### Exercise E, Question 5

#### Question:

(Give answers to 3 significant figures.)

The lengths of the sides of a triangle are in the ratio 2:3:4.  
Calculate the value of the largest angle.

#### Solution:



The largest angle will be opposite the side 4k cm.

$$\text{Using } \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{9k^2 + 4k^2 - 16k^2}{2 \times 3k \times 2k} = -0.25$$

$$B = 104.477 \dots^\circ = 104^\circ \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise E, Question 6

#### Question:

(Give answers to 3 significant figures.)

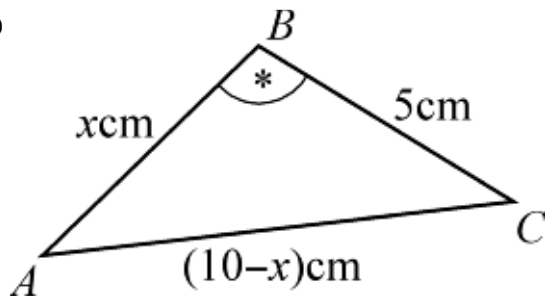
In  $\triangle ABC$ ,  $AB = x$  cm,  $BC = 5$  cm and  $AC = (10 - x)$  cm:

(a) Show that  $\cos \angle ABC = \frac{4x - 15}{2x}$ .

(b) Given that  $\cos \angle ABC = -\frac{1}{7}$ , work out the value of  $x$ .

#### Solution:

(a)



$$\begin{aligned} \cos B &= \frac{5^2 + x^2 - (10 - x)^2}{2 \times 5 \times x} \\ &= \frac{25 + x^2 - (100 - 20x + x^2)}{10x} \\ &= \frac{25 + x^2 - 100 + 20x - x^2}{10x} \\ &= \frac{20x - 75}{10x} \\ &= \frac{5(4x - 15)}{10x} \\ &= \frac{4x - 15}{2x} \end{aligned}$$

(b) As  $\cos B = -\frac{1}{7}$

$$\frac{4x - 15}{2x} = -\frac{1}{7}$$

$$7(4x - 15) = -2x$$

$$28x - 105 = -2x$$

$$30x = 105$$

$$x = \frac{105}{30} = 3\frac{1}{2}$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

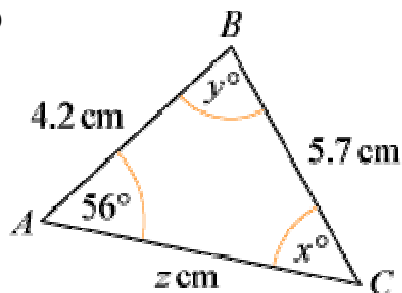
#### Exercise F, Question 1

#### Question:

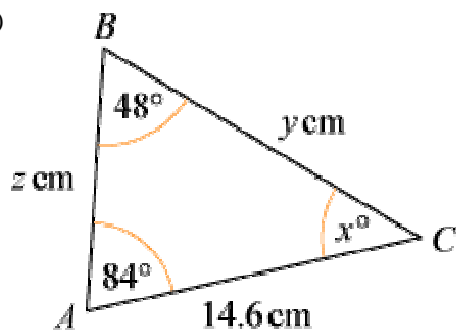
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

In each triangle below find the values of  $x$ ,  $y$  and  $z$ .

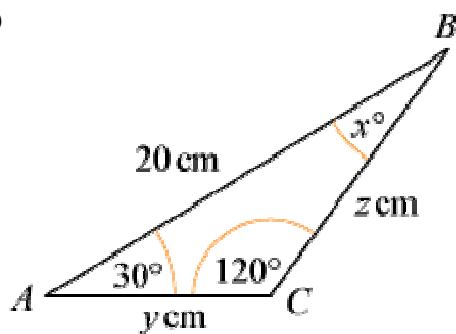
(a)



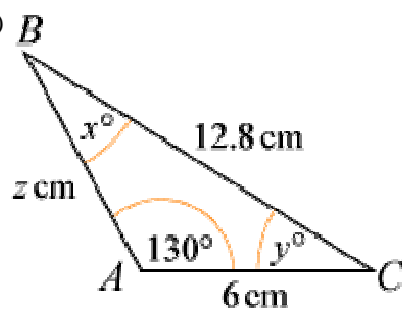
(b)

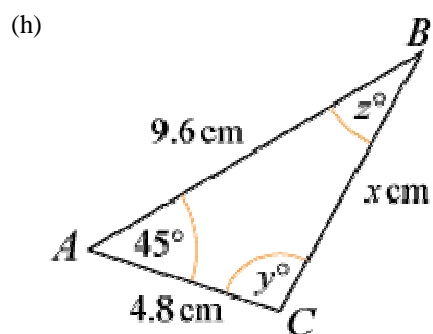
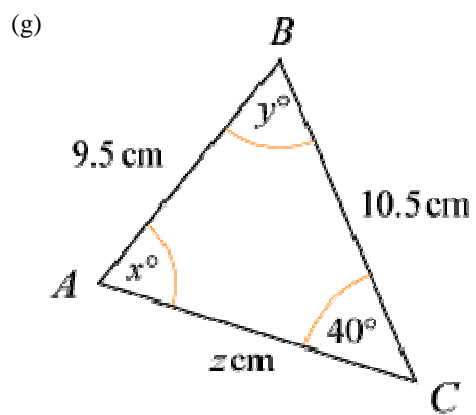
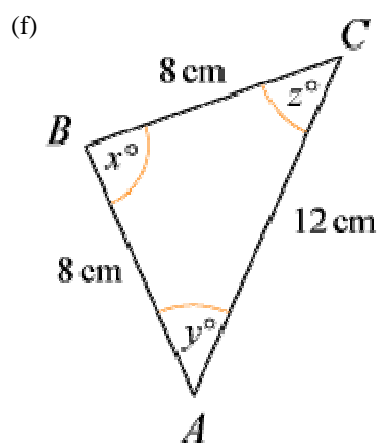
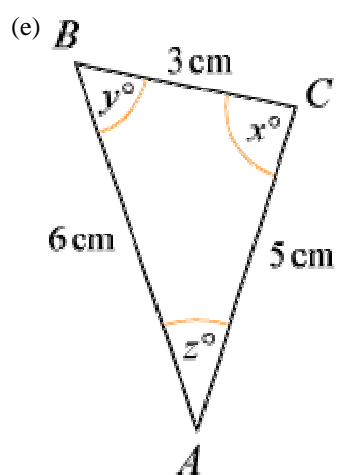


(c)



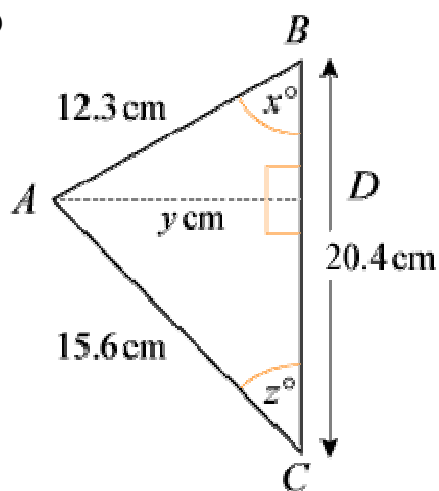
(d)



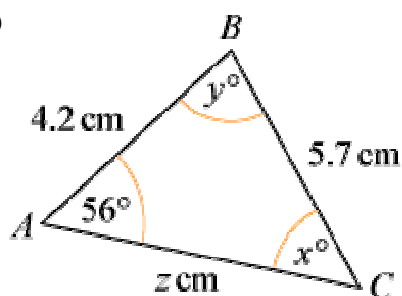




(i)

**Solution:**

(a)



Using  $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin x^\circ}{4.2} = \frac{\sin 56^\circ}{5.7}$$

$$\sin x^\circ = \frac{4.2 \sin 56^\circ}{5.7}$$

$$x^\circ = \sin^{-1} \left( \frac{4.2 \sin 56^\circ}{5.7} \right) = 37.65 \dots^\circ$$

$$x = 37.7 \text{ (3 s.f.)}$$

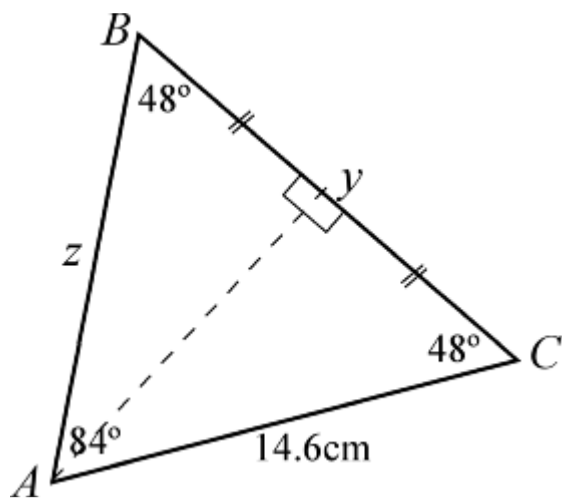
$$\text{So } y^\circ = 180^\circ - (56^\circ + 37.7^\circ) = 86.3^\circ$$

$$y = 86.3 \text{ (3 s.f.)}$$

Using  $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{z}{\sin y^\circ} = \frac{5.7}{\sin 56^\circ} \Rightarrow z = \frac{5.7 \sin y^\circ}{\sin 56^\circ} = 6.86 \text{ (3 s.f.)}$$

$$(b) x^\circ = 180^\circ - (48^\circ + 84^\circ) = 48^\circ \Rightarrow x = 48$$



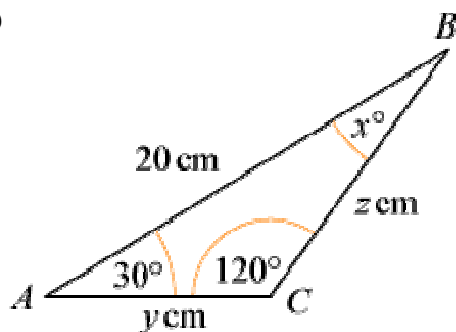
As  $\angle B = \angle c$ ,  $z = 14.6$

Using the line of symmetry through A

$$\cos 48^\circ = \frac{\frac{y}{2}}{14.6}$$

$$\Rightarrow y = 29.2 \cos 48^\circ = 19.5 \text{ (3 s.f.)}$$

(c)



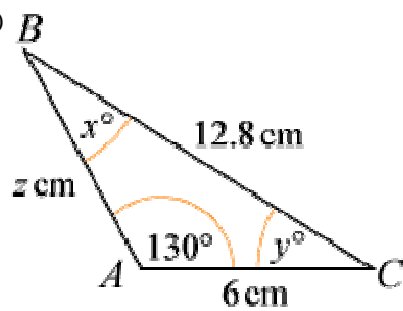
$$x^\circ = 180^\circ - (120^\circ + 30^\circ) = 30^\circ$$

Using the line of symmetry through C

$$\cos 30^\circ = \frac{10}{y} \Rightarrow y = \frac{10}{\cos 30^\circ} = 11.5 \text{ (3 s.f.)}$$

As  $\triangle ABC$  is isosceles with  $AC = CB$ ,  $z = 11.5$  (3 s.f.)

(d)



$$\text{Using } \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 130^\circ}{12.8} = \frac{\sin x^\circ}{6} \Rightarrow \sin x^\circ = \frac{6 \sin 130^\circ}{12.8} = 0.35908 \dots$$

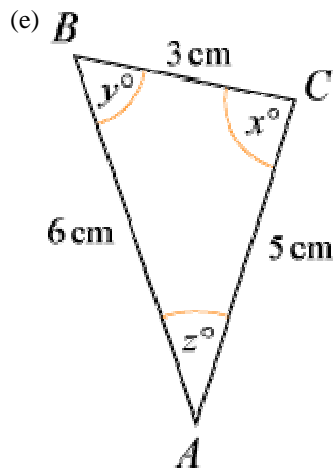
$$\Rightarrow x = 21.0 \text{ (3 s.f.)}$$

$$\text{So } y^\circ = 180^\circ - (130^\circ + x^\circ) = 28.956 \dots^\circ \Rightarrow y = 29.0 \text{ (3 s.f.)}$$

$$\text{Using } \frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{z}{\sin y^\circ} = \frac{12.8}{\sin 130^\circ}$$

$$\Rightarrow z = \frac{12.8 \sin y^\circ}{\sin 130^\circ} = 8.09 \text{ (3 s.f.)}$$



$$\text{Using } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos x^\circ = \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5} = -0.06$$

$$x = 93.8 \text{ (3 s.f.)}$$

$$\text{Using } \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin y^\circ}{5} = \frac{\sin x^\circ}{6}$$

$$\sin y^\circ = \frac{5 \sin x^\circ}{6}$$

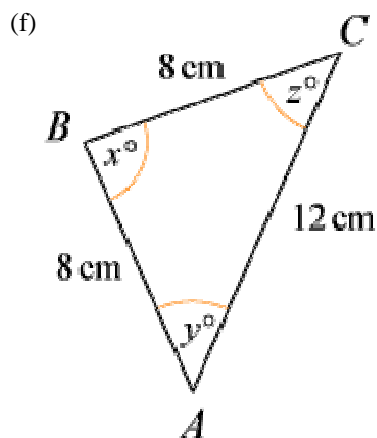
$$y^\circ = \sin^{-1} \left( \frac{5 \sin x^\circ}{6} \right) = 56.25 \dots^\circ$$

$$y = 56.3 \text{ (3 s.f.)}$$

Using angle sum for a triangle

$$z^\circ = 180^\circ - (x + y)^\circ = 29.926 \dots^\circ$$

$$z = 29.9 \text{ (3 s.f.)}$$



Using the line of symmetry through  $B$

$$\cos y^\circ = \frac{6}{8}$$

$$y^\circ = \cos^{-1} \left( \frac{3}{4} \right) = 41.40 \dots$$

$$y = 41.4 \text{ (3 s.f.)}$$

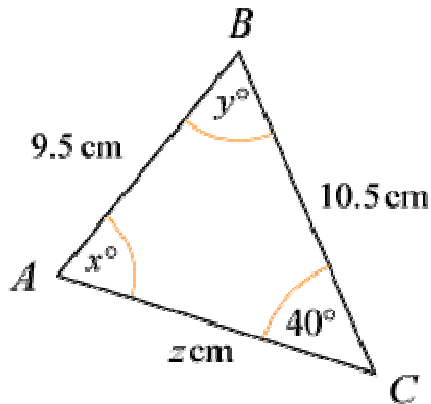
As triangle is isosceles

$$z = y = 41.4 \text{ (3 s.f.)}$$

$$\text{So } x^\circ = 180^\circ - (y + z)^\circ = 97.2^\circ$$

$$x = 97.2 \text{ (3 s.f.)}$$

(g)



Using  $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin x^\circ}{10.5} = \frac{\sin 40^\circ}{9.5}$$

$$\sin x^\circ = \frac{10.5 \sin 40^\circ}{9.5}$$

$$x^\circ = \sin^{-1} \left( \frac{10.5 \sin 40^\circ}{9.5} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{10.5 \sin 40^\circ}{9.5} \right)$$

$$x^\circ = 45.27^\circ \text{ or } 134.728 \dots^\circ$$

$$x = 45.3 \text{ (3 s.f.) or } 135 \text{ (3 s.f.)}$$

Using sine rule:  $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{z}{\sin y^\circ} = \frac{9.5}{\sin 40^\circ}$$

$$z = \frac{9.5 \sin y^\circ}{\sin 40^\circ}$$

When  $x = 45.3$

$$y^\circ = 180^\circ - (40 + 45.3)^\circ = 94.7^\circ \text{ so } y = 94.7 \text{ (3 s.f.)}$$

$$z = \frac{9.5 \sin y^\circ}{\sin 40^\circ} = 14.7 \text{ (3 s.f.)}$$

When  $x = 134.72 \dots^\circ$

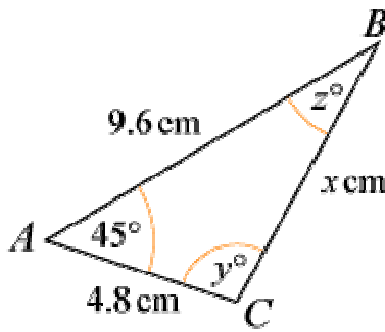
$$y^\circ = 180^\circ - (40 + 134.72 \dots)^\circ = 5.27^\circ \Rightarrow y = 5.27 \text{ (3 s.f.)}$$

$$z = \frac{9.5 \sin y^\circ}{\sin 40^\circ} = 1.36 \text{ (3 s.f.)}$$

$$\text{So } x = 45.3, y = 94.7, z = 14.7$$

$$\text{or } x = 135, y = 5.27, z = 1.36$$

(h)

Using  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$x^2 = 4.8^2 + 9.6^2 - 2 \times 4.8 \times 9.6 \times \cos 45^\circ = 50.03 \quad \dots$$

$$x = 7.07 \text{ (3 s.f.)}$$

Using  $\frac{\sin C}{c} = \frac{\sin A}{a}$  (As  $9.6 > x$ ,  $y > 45$  and there are two possible values for  $y$ .)

$$\frac{\sin y^\circ}{9.6} = \frac{\sin 45^\circ}{x}$$

$$\sin y^\circ = \frac{9.6 \sin 45^\circ}{x}$$

$$y^\circ = \sin^{-1} \left( \frac{9.6 \sin 45^\circ}{x} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{9.6 \sin 45^\circ}{x} \right)$$

$$y^\circ = 73.67 \quad \dots \quad \text{or } 106.32 \quad \dots \quad \text{or } 106 \text{ (3 s.f.)}$$

$$y = 73.7 \text{ (3 s.f.) or } 106 \text{ (3 s.f.)}$$

When  $y = 73.67 \quad \dots$ 

$$z^\circ = 180^\circ - (45 + 73.67 \quad \dots)^\circ = 61.32 \quad \dots \quad \text{or } 61.3 \text{ (3 s.f.)}$$

$$z = 61.3 \text{ (3 s.f.)}$$

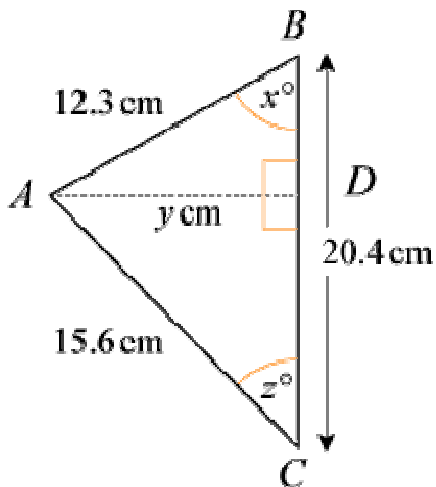
When  $y = 106.32 \quad \dots$ 

$$z^\circ = 180^\circ - (45 + 106.32 \quad \dots)^\circ = 28.67 \quad \dots \quad \text{or } 28.7 \text{ (3 s.f.)}$$

$$z = 28.7 \text{ (3 s.f.)}$$

So  $x = 7.07$ ,  $y = 73.7$ ,  $z = 61.3$ or  $x = 7.07$ ,  $y = 106$ ,  $z = 28.7$ 

(i)

Using  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ 

$$\cos x^\circ = \frac{20.4^2 + 12.3^2 - 15.6^2}{2 \times 20.4 \times 12.3} = 0.6458 \quad \dots$$

$$x^\circ = 49.77 \quad \dots \quad \text{or } 49.8 \text{ (3 s.f.)}$$

In the right-angled  $\triangle ABD$

$$\sin x^\circ = \frac{y}{12.3} \Rightarrow y = 12.3 \sin x^\circ = 9.39 \text{ (3 s.f.)}$$

In right-angled  $\triangle ACD$

$$\sin z^\circ = \frac{y}{15.6} = 0.60199 \dots$$

$$z^\circ = 37.01 \dots^\circ$$

$$z = 37.0 \text{ (3 s.f.)}$$

$$\text{So } x = 49.8, y = 9.39, z = 37.0$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

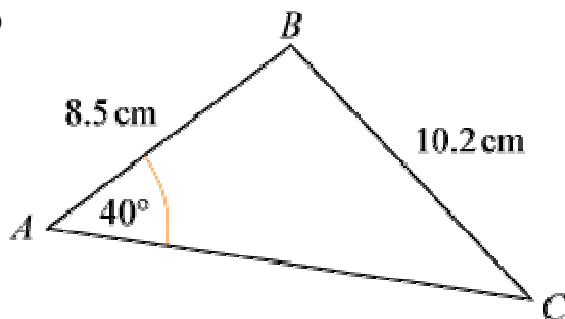
#### Exercise F, Question 2

#### Question:

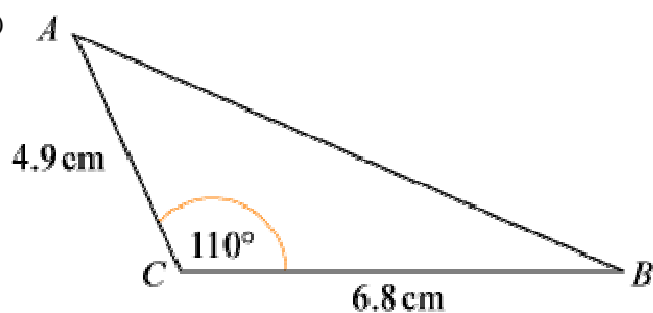
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

Calculate the size of the remaining angles and the length of the third side in the following triangles:

(a)

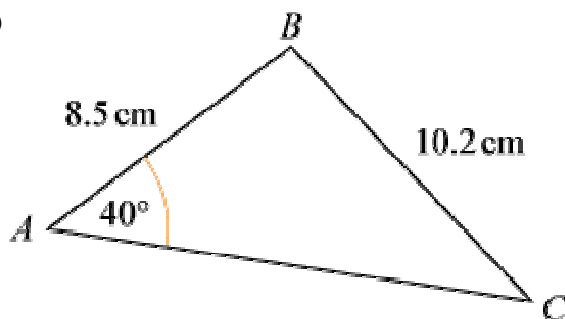


(b)



#### Solution:

(a)



$$\text{Using } \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{8.5} = \frac{\sin 40^\circ}{10.2}$$

$$\sin C = \frac{8.5 \sin 40^\circ}{10.2}$$

$$C = \sin^{-1} \left( \frac{8.5 \sin 40^\circ}{10.2} \right) = 32.388 \dots^\circ$$

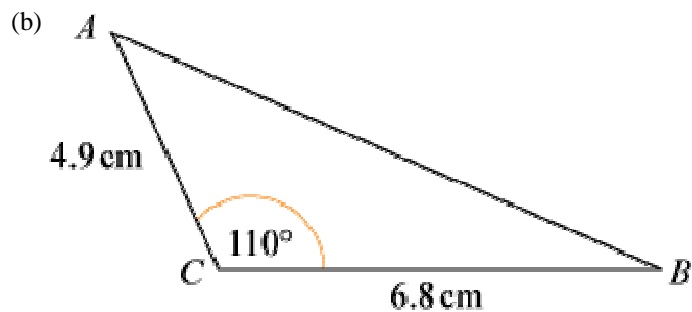
$$C = 32.4^\circ \text{ (3 s.f.)}$$

$$\text{Angle } B = 180^\circ - (40 + C)^\circ = 107.6 \dots^\circ$$

$$B = 108^\circ \text{ (3 s.f.)}$$

$$\text{Using } \frac{b}{\sin B} = \frac{a}{\sin A}$$

$$b = \frac{10.2 \sin B}{\sin 40^\circ} = 15.1 \text{ cm (3 s.f.)}$$



$$\text{Using } c^2 = a^2 + b^2 - 2ab \cos C$$

$$AB^2 = 6.8^2 + 4.9^2 - 2 \times 6.8 \times 4.9 \times \cos 110^\circ = 93.04 \dots$$

$$AB = 9.6458 \dots = 9.65 \text{ cm (3 s.f.)}$$

$$\text{Using } \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\sin A = \frac{6.8 \sin 110^\circ}{AB} = 0.66245 \dots$$

$$A = 41.49^\circ = 41.5^\circ \text{ (3 s.f.)}$$

$$\text{So } B = 180^\circ - (110 + A)^\circ = 28.5^\circ \text{ (3 s.f.)}$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

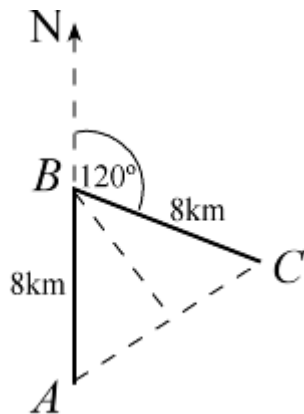
#### Exercise F, Question 3

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

A hiker walks due north from  $A$  and after 8 km reaches  $B$ . She then walks a further 8 km on a bearing of  $120^\circ$  to  $C$ . Work out (a) the distance from  $A$  to  $C$  and (b) the bearing of  $C$  from  $A$ .

#### Solution:



(a)  $\angle ABC = 180^\circ - 120^\circ = 60^\circ$

As  $\angle A = \angle C$ , all angles are  $60^\circ$ ; it is an equilateral triangle.

So  $AC = 8$  km.

(b) As  $\angle BAC = 60^\circ$ ,

the bearing of  $C$  from  $A$  is  $060^\circ$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

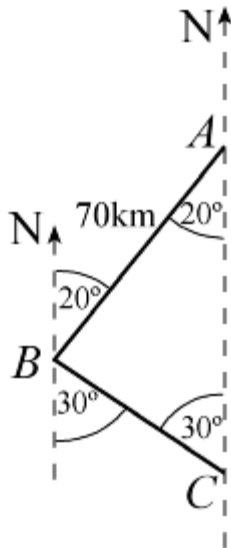
#### Exercise F, Question 4

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

A helicopter flies on a bearing of  $200^\circ$  from  $A$  to  $B$ , where  $AB = 70$  km. It then flies on a bearing of  $150^\circ$  from  $B$  to  $C$ , where  $C$  is due south of  $A$ . Work out the distance of  $C$  from  $A$ .

#### Solution:



From the diagram  $\angle ABC = 180^\circ - (20 + 30)^\circ = 130^\circ$

$$\text{Using } \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{AC}{\sin 130^\circ} = \frac{70}{\sin 30^\circ}$$

$$AC = \frac{70 \sin 130^\circ}{\sin 30^\circ} = 107.246 \dots$$

$$AC = 107 \text{ km (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

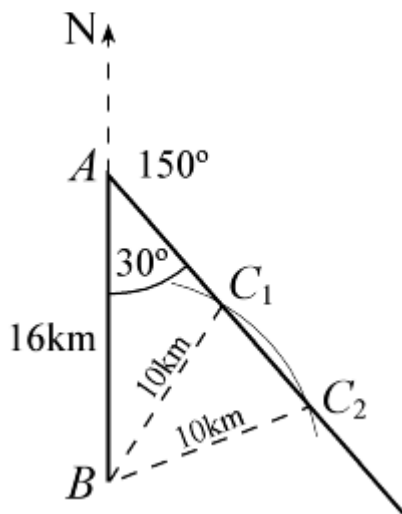
#### Exercise F, Question 5

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

Two radar stations  $A$  and  $B$  are 16 km apart and  $A$  is due north of  $B$ . A ship is known to be on a bearing of  $150^\circ$  from  $A$  and 10 km from  $B$ . Show that this information gives two positions for the ship, and calculate the distance between these two positions.

#### Solution:



Using the sine rule:  $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin C}{16} = \frac{\sin 30^\circ}{10}$$

$$\sin C = \frac{16 \sin 30^\circ}{10} = 0.8$$

$$C = \sin^{-1}(0.8) \text{ or } 180^\circ - \sin^{-1}(0.8)$$

$$C = 53.1^\circ \text{ or } 126.9^\circ$$

$$\angle AC_2B = 53.1^\circ, \angle AC_1B = 127^\circ \text{ (3 s.f.)}$$

(Store the correct values; these are not required answers.)

Triangle  $BC_1C_2$  is isosceles, so  $C_1C_2$  can be found using this triangle, without finding  $AC_1$  and  $AC_2$ .

Use the line of symmetry through  $B$ :

$$\cos \angle C_1C_2B = \frac{\frac{1}{2}C_1C_2}{10}$$

$$\Rightarrow C_1C_2 = 20 \cos \angle C_1C_2B = 20 \cos \angle AC_2B = 20 \cos 53.1^\circ$$

$$\Rightarrow C_1C_2 = 12 \text{ km}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

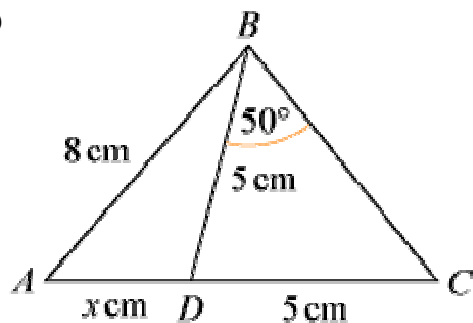
#### Exercise F, Question 6

#### Question:

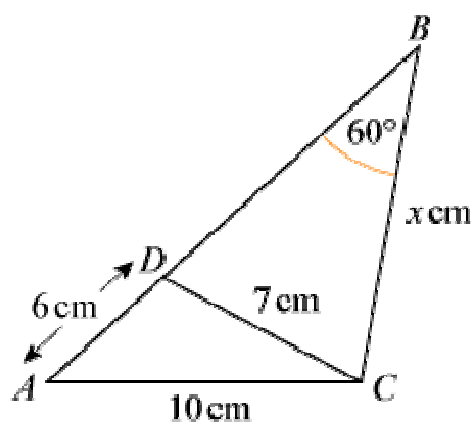
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

Find  $x$  in each of the following diagrams:

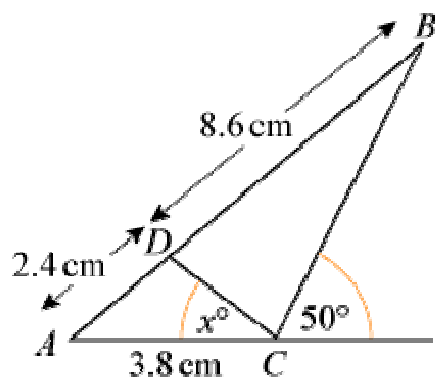
(a)



(b)

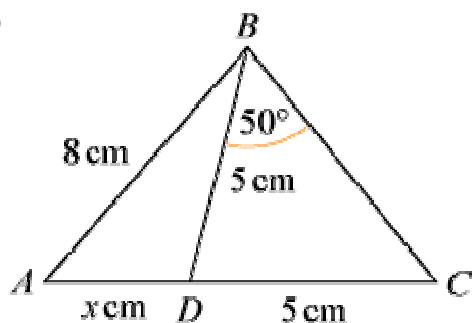


(c)



#### Solution:

(a)



In the isosceles  $\triangle BDC$ :  $\angle BDC = 180^\circ - (50 + 50)^\circ = 80^\circ$

So  $\angle BDA = 180^\circ - 80^\circ = 100^\circ$

Using the sine rule in  $\triangle ABD$ :  $\frac{\sin A}{a} = \frac{\sin D}{d}$

$$\Rightarrow \frac{\sin A}{5} = \frac{\sin 100^\circ}{8}$$

$$\Rightarrow \sin A = \frac{5 \sin 100^\circ}{8}$$

$$\text{So } A = \sin^{-1} \left( \frac{5 \sin 100^\circ}{8} \right) = 37.9886 \dots$$

Angle ABD =  $180^\circ - (100 + A)^\circ = 42.01 \dots^\circ$

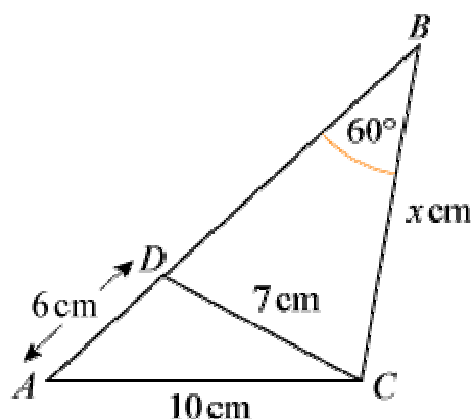
Using  $\frac{b}{\sin B} = \frac{d}{\sin D}$

$$\frac{x}{\sin B} = \frac{8}{\sin 100}$$

$$x = \frac{8 \sin B}{\sin 100^\circ} = 5.436 \dots$$

$$x = 5.44 \text{ (3 s.f.)}$$

(b)



In  $\triangle ADC$ , using  $\cos A = \frac{c^2 + d^2 - a^2}{2cd}$

$$\cos A = \frac{6^2 + 10^2 - 7^2}{2 \times 6 \times 10} = 0.725$$

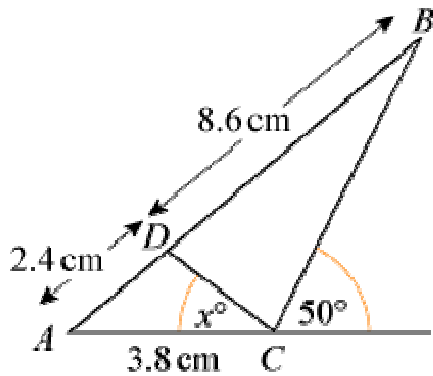
$$\text{So } A = 43.53 \dots^\circ$$

Using the sine rule in  $\triangle ABC$ :  $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\text{So } \frac{x}{\sin A} = \frac{10}{\sin 60^\circ}$$

$$\Rightarrow x = \frac{10 \sin A}{\sin 60^\circ} = 7.95 \text{ (3 s.f.)}$$

(c)



In  $\triangle ABC$ ,  $c = 11$  cm,  $b = 3.8$  cm,  $\angle ACB = 130^\circ$

Using  $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\sin B = \frac{3.8 \sin 130^\circ}{11} = 0.2646 \dots$$

$$B = 15.345 \dots^\circ$$

$$\text{So } A = 180^\circ - (130 + B)^\circ = 34.654 \dots^\circ$$

In  $\triangle ADC$ ,  $c = 2.4$  cm,  $d = 3.8$  cm,  $A = 34.654 \dots^\circ$

Using the cosine rule:  $a^2 = c^2 + d^2 - 2cd \cos A$

$$\text{So } DC^2 = 2.4^2 + 3.8^2 - 2 \times 2.4 \times 3.8 \times \cos A = 5.1959 \dots$$

$$\Rightarrow DC = 2.279 \dots \text{ cm.}$$

Using the sine rule:  $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\sin x^\circ = \frac{2.4 \sin A}{DC} = 0.59869 \dots$$

$$x = 36.8 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

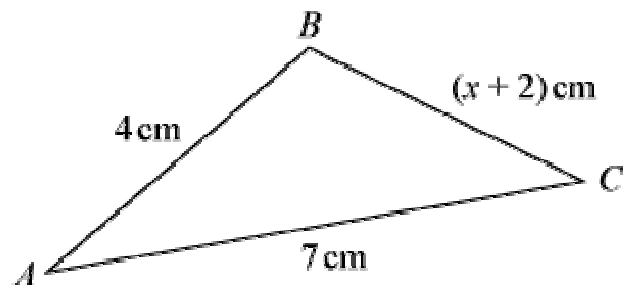
### The sine and cosine rule

#### Exercise F, Question 7

#### Question:

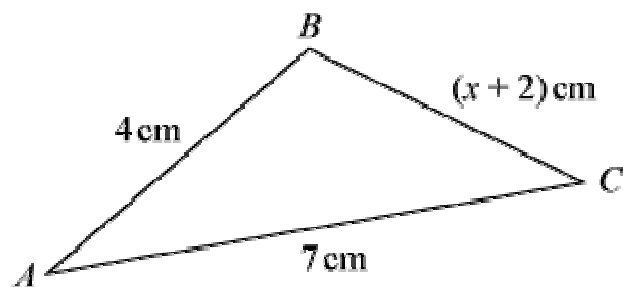
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

In  $\triangle ABC$ , shown below,  $AB = 4$  cm,  $BC = (x + 2)$  cm and  $AC = 7$  cm.



- (a) Explain how you know that  $1 < x < 9$ .
- (b) Work out the value of  $x$  for the cases when
- $\angle ABC = 60^\circ$  and
  - $\angle ABC = 45^\circ$ , giving your answers to 3 significant figures.

#### Solution:



- (a) As  $AB + BC > AC$

$$4 + (x + 2) > 7$$

$$\Rightarrow x + 2 > 3$$

$$\Rightarrow x > 1$$

- As  $AB + AC > BC$

$$4 + 7 > x + 2$$

$$\Rightarrow 9 > x$$

So  $1 < x < 9$

- (b) Using  $b^2 = a^2 + c^2 - 2ac \cos B$

(i)  $7^2 = (x + 2)^2 + 4^2 - 2 \times (x + 2) \times 4 \times \cos 60^\circ$

$$49 = x^2 + 4x + 4 + 16 - 4(x + 2)$$

$$49 = x^2 + 4x + 4 + 16 - 4x - 8$$

$$\text{So } x^2 = 37$$

$$\Rightarrow x = 6.08 \text{ (3 s.f.)}$$

(ii)  $7^2 = (x + 2)^2 + 4^2 - 2 \times (x + 2) \times 4 \times \cos 45^\circ$

$$49 = x^2 + 4x + 4 + 16 - (8 \cos 45^\circ)x - 16 \cos 45^\circ$$

$$\text{So } x^2 + (4 - 8 \cos 45^\circ) x - (29 + 16 \cos 45^\circ) = 0$$

$$\text{or } x^2 + 4(1 - \sqrt{2}) x - (29 + 8\sqrt{2}) = 0$$

Use the quadratic equation formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  with

$$a = 1$$

$$b = 4 - 8 \cos 45^\circ = 4(1 - \sqrt{2}) = -1.6568 \dots$$

$$c = -(29 + 16 \cos 45^\circ) = -(29 + 8\sqrt{2}) = -40.313 \dots$$

$$x = 7.23 \text{ (3 s.f.) (The other value of } x \text{ is less than } -2.)$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

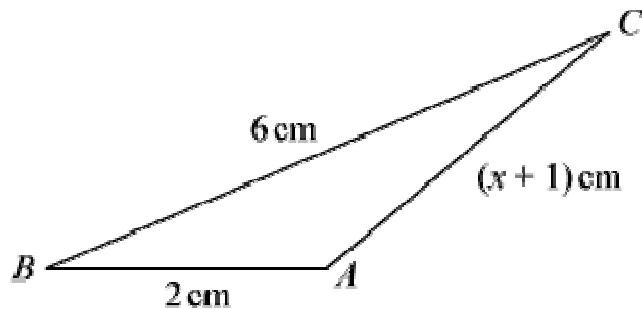
#### Exercise F, Question 8

#### Question:

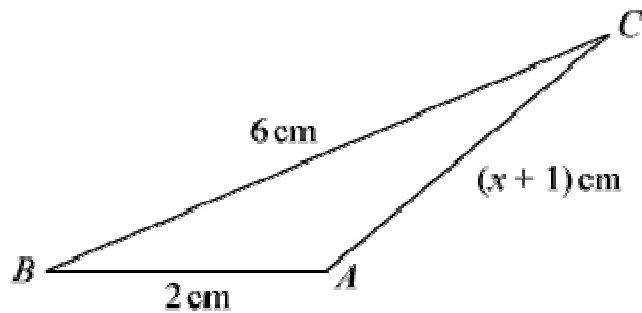
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

In the triangle shown below,  $\cos \angle ABC = \frac{5}{8}$ .

Calculate the value of  $x$ .



#### Solution:



Using  $b^2 = a^2 + c^2 - 2ac \cos B$  where  $\cos B = \frac{5}{8}$

$$(x + 1)^2 = 6^2 + 2^2 - 2 \times 6 \times 2 \times \frac{5}{8}$$

$$x^2 + 2x + 1 = 36 + 4 - 15$$

$$x^2 + 2x - 24 = 0$$

$$(x + 6)(x - 4) = 0$$

$$\text{So } x = 4 \quad (x > -1)$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

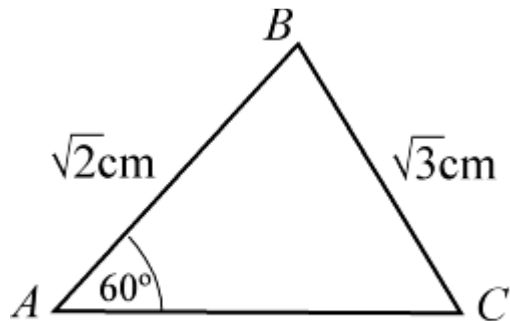
#### Exercise F, Question 9

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

In  $\triangle ABC$ ,  $AB = \sqrt{2}$  cm,  $BC = \sqrt{3}$  cm and  $\angle BAC = 60^\circ$ . Show that  $\angle ACB = 45^\circ$  and find  $AC$ .

#### Solution:



$$\text{Using } \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{\sqrt{2} \sin 60^\circ}{\sqrt{3}} = 0.7071 \dots$$

$$C = \sin^{-1} \left( \frac{\sqrt{2} \sin 60^\circ}{\sqrt{3}} \right) = 45^\circ$$

$$B = 180^\circ - (60 + 45)^\circ = 75^\circ$$

$$\text{Using } \frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{AC}{\sin 75^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\text{So } AC = \frac{\sqrt{3} \sin 75^\circ}{\sin 60^\circ} = 1.93 \text{ cm (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise F, Question 10

#### Question:

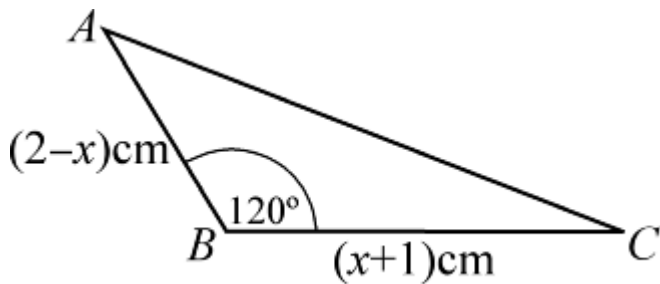
(Note: Try to use the neatest method, and give answers to 3 significant figures.)

In  $\triangle ABC$ ,  $AB = (2 - x)$  cm,  $BC = (x + 1)$  cm and  $\angle ABC = 120^\circ$ :

(a) Show that  $AC^2 = x^2 - x + 7$ .

(b) Find the value of  $x$  for which  $AC$  has a minimum value.

#### Solution:



(a) Using the cosine rule:  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $AC^2 = (x + 1)^2 + (2 - x)^2 - 2(x + 1)(2 - x) \cos 120^\circ$   
 $AC^2 = (x^2 + 2x + 1) + (4 - 4x + x^2) + (x + 1)(2 - x)$   
 $AC^2 = x^2 + 2x + 1 + 4 - 4x + x^2 - x^2 + 2x - x + 2$   
 $AC^2 = x^2 - x + 7$

(b) Using the method of completing the square:

$$x^2 - x + 7 \equiv \left(x - \frac{1}{2}\right)^2 + 7 - \frac{1}{4} \equiv \left(x - \frac{1}{2}\right)^2 + 6\frac{3}{4}$$

This is a minimum when  $x - \frac{1}{2} = 0$ , i.e.  $x = \frac{1}{2}$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

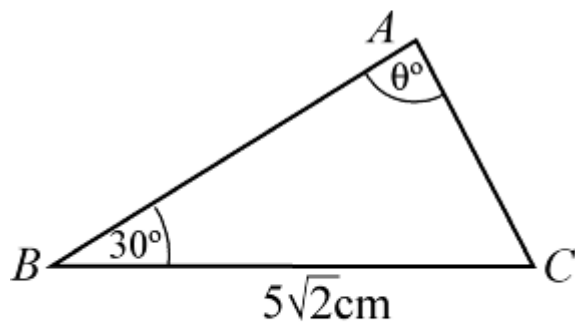
#### Exercise F, Question 11

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

Triangle  $ABC$  is such that  $BC = 5\sqrt{2}$  cm,  $\angle ABC = 30^\circ$  and  $\angle BAC = \theta$ , where  $\sin \theta = \frac{\sqrt{5}}{8}$ . Work out the length of  $AC$ , giving your answer in the form  $a\sqrt{b}$ , where  $a$  and  $b$  are integers.

#### Solution:



$$\text{Using } \frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{AC}{\sin 30^\circ} = \frac{5\sqrt{2}}{\sin \theta^\circ}$$

$$AC = \frac{5\sqrt{2} \sin 30^\circ}{\left(\frac{\sqrt{5}}{8}\right)}$$

$$AC = \frac{5\sqrt{2} \sin 30^\circ \times 8}{\sqrt{5}} = \left(\sqrt{5}\sqrt{2}\right) \left(8 \sin 30^\circ\right) = 4\sqrt{10}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

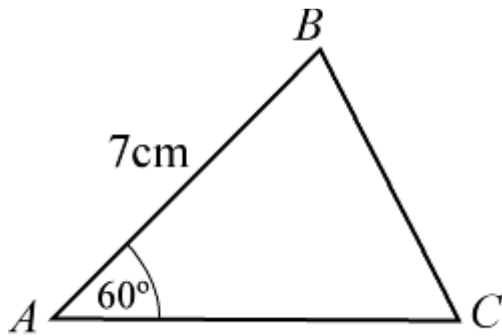
#### Exercise F, Question 12

#### Question:

(Note: Try to use the neatest method, and give answers to 3 significant figures.)

The perimeter of  $\triangle ABC = 15$  cm. Given that  $AB = 7$  cm and  $\angle BAC = 60^\circ$ , find the lengths  $AC$  and  $BC$ .

#### Solution:



Using the cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$

with  $a = x$ ,  $b = (8 - x)$ ,  $c = 7$  and  $A = 60^\circ$

$$x^2 = (8 - x)^2 + 7^2 - 2(8 - x) \times 7 \times \cos 60^\circ$$

$$x^2 = 64 - 16x + x^2 + 49 - 7(8 - x)$$

$$x^2 = 64 - 16x + x^2 + 49 - 56 + 7x$$

$$\Rightarrow 9x = 57$$

$$\Rightarrow x = \frac{57}{9} = \frac{19}{3} = 6\frac{1}{3}$$

So  $BC = 6\frac{1}{3}$  cm and  $AC = \left(8 - 6\frac{1}{3}\right)$  cm  $= 1\frac{2}{3}$  cm.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

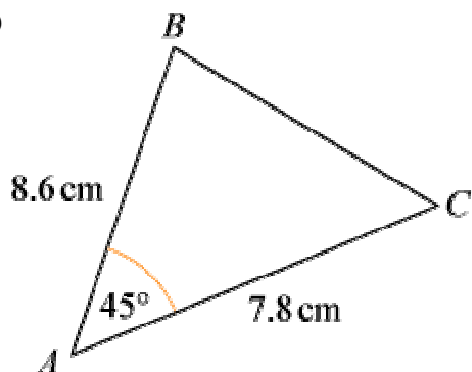
### The sine and cosine rule

#### Exercise G, Question 1

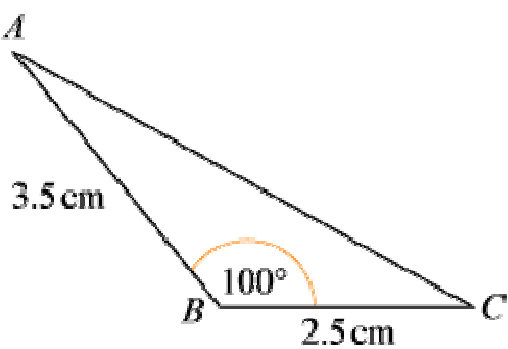
#### Question:

Calculate the area of the following triangles:

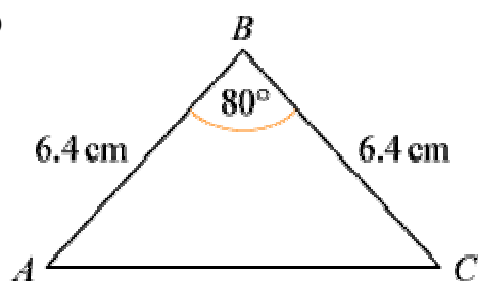
(a)



(b)

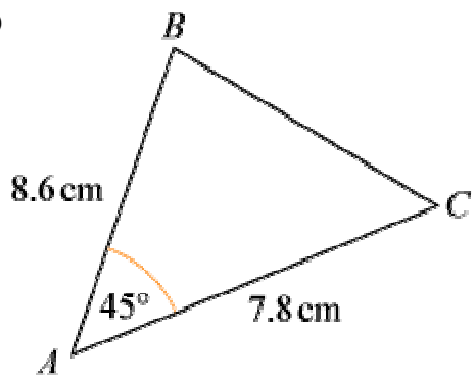


(c)

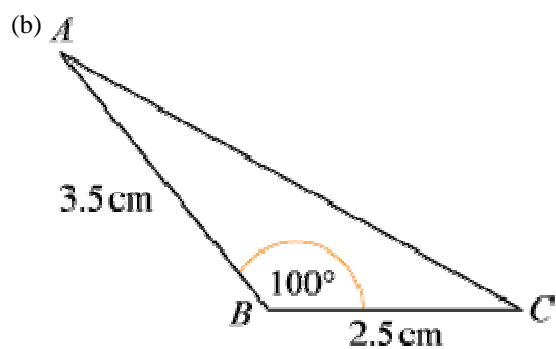


#### Solution:

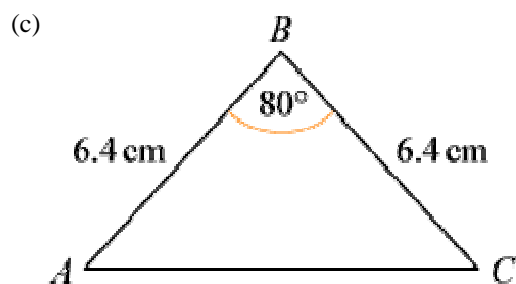
(a)



$$\text{Area} = \frac{1}{2} \times 7.8 \times 8.6 \times \sin 45^\circ = 23.71 \dots = 23.7 \text{ cm}^2 \text{ (3 s.f.)}$$



$$\text{Area} = \frac{1}{2} \times 2.5 \times 3.5 \times \sin 100^\circ = 4.308 \dots = 4.31 \text{ cm}^2 \text{ (3 s.f.)}$$



$$\text{Area} = \frac{1}{2} \times 6.4 \times 6.4 \times \sin 80^\circ = 20.16 \dots = 20.2 \text{ cm}^2 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

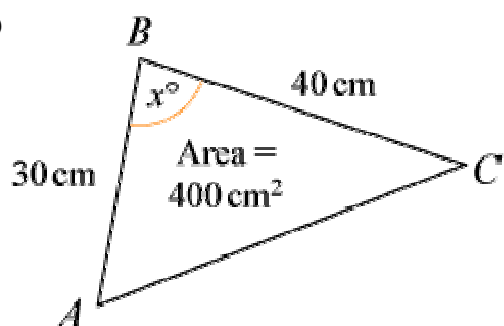
### The sine and cosine rule

#### Exercise G, Question 2

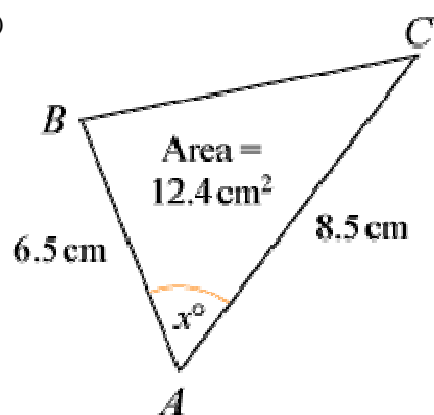
#### Question:

Work out the possible values of  $x$  in the following triangles:

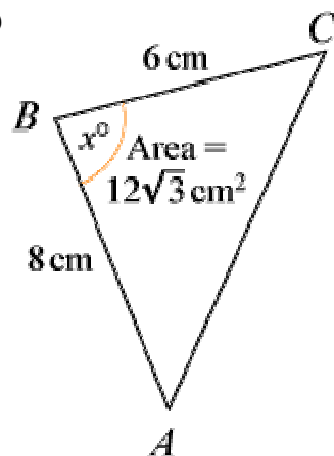
(a)



(b)

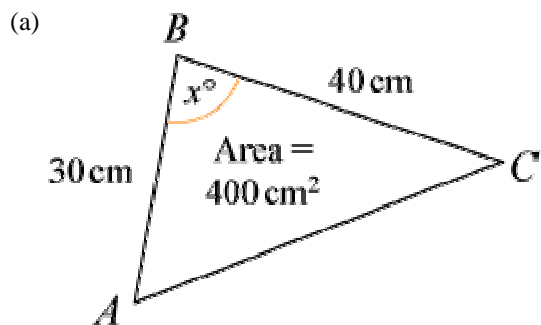


(c)



#### Solution:





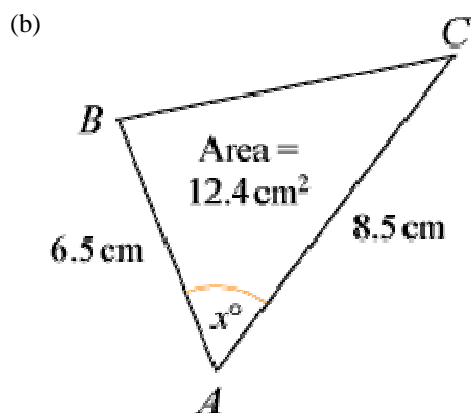
Using area =  $\frac{1}{2}ac \sin B$

$$400 = \frac{1}{2} \times 40 \times 30 \times \sin x^\circ$$

$$\text{So } \sin x^\circ = \frac{400}{600} = \frac{2}{3}$$

$$x^\circ = \sin^{-1} \left( \frac{2}{3} \right) \text{ or } 180^\circ - \sin^{-1} \left( \frac{2}{3} \right)$$

$$x = 41.8 \text{ (3 s.f.) or } 138 \text{ (3 s.f.)}$$



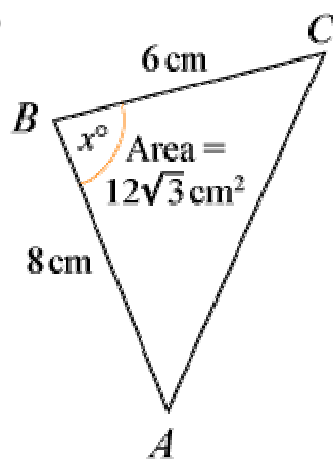
Using area =  $\frac{1}{2}bc \sin A$

$$12.4 = \frac{1}{2} \times 8.5 \times 6.5 \times \sin x^\circ$$

$$\text{So } \sin x^\circ = \frac{12.4}{27.625} = 0.4488 \dots$$

$$x = 26.7 \text{ (3 s.f.) or } 153 \text{ (3 s.f.)}$$

(c)



Using area =  $\frac{1}{2}ac \sin B$

$$12\sqrt{3} = \frac{1}{2} \times 6 \times 8 \sin x^\circ$$

$$\text{So } \sin x^\circ = \frac{12\sqrt{3}}{24} = \frac{\sqrt{3}}{2}$$

$$x = 60 \text{ or } 120$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

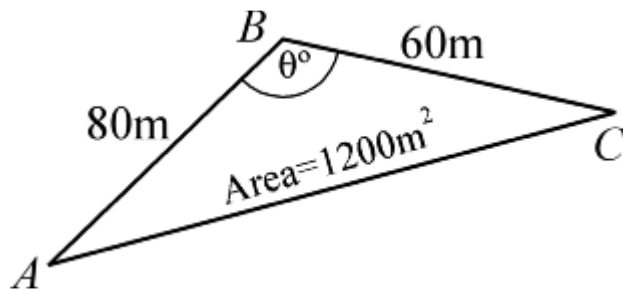
### The sine and cosine rule

#### Exercise G, Question 3

#### Question:

A fenced triangular plot of ground has area  $1200 \text{ m}^2$ . The fences along the two smaller sides are  $60 \text{ m}$  and  $80 \text{ m}$  respectively and the angle between them is  $\theta^\circ$ . Show that  $\theta = 150$ , and work out the total length of fencing.

#### Solution:



Using area =  $\frac{1}{2}ac \sin B$

$$1200 = \frac{1}{2} \times 60 \times 80 \times \sin \theta^\circ$$

$$\sin \theta^\circ = \frac{1200}{2400} = \frac{1}{2}$$

$$\theta = 30 \text{ or } 150$$

but as AC is the largest side,  $\theta$  must be the largest angle.

$$\text{So } \theta = 150$$

Using the cosine rule:  $b^2 = a^2 + c^2 - 2ac \cos B$  to find AC

$$AC^2 = 60^2 + 80^2 - 2 \times 60 \times 80 \times \cos 150^\circ = 18313.84 \dots$$

$$AC = 135.3 \dots$$

$$AC = 135 \text{ m (3 s.f.)}$$

$$\text{So perimeter} = 60 + 80 + 135 = 275 \text{ m (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

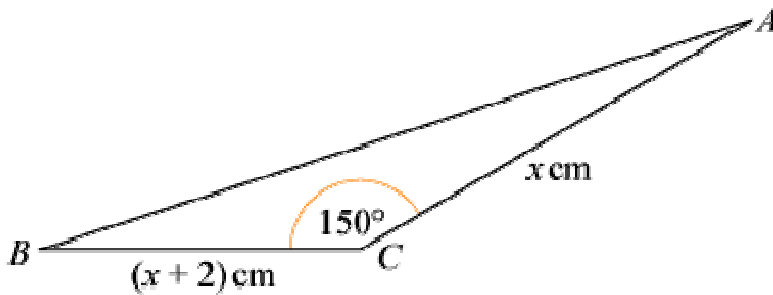
### The sine and cosine rule

#### Exercise G, Question 4

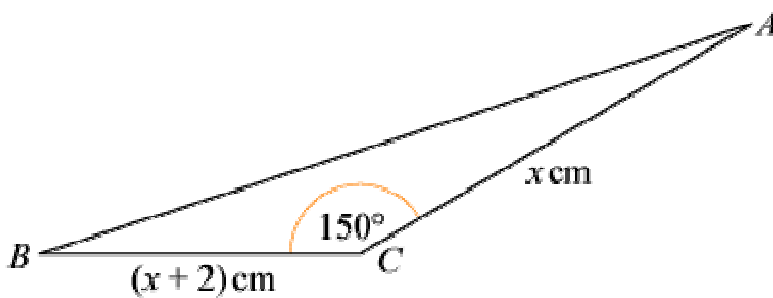
#### Question:

In triangle  $ABC$ , shown below,  $BC = (x + 2)$  cm,  $AC = x$  cm and  $\angle BCA = 150^\circ$ .

Given that the area of the triangle is  $5 \text{ cm}^2$ , work out the value of  $x$ , giving your answer to 3 significant figures.



#### Solution:



$$\text{Area of } \triangle ABC = \frac{1}{2}x \left( x + 2 \right) \sin 150^\circ \text{ cm}^2$$

$$\text{So } 5 = \frac{1}{2}x \left( x + 2 \right) \times \frac{1}{2}$$

$$\text{So } 20 = x(x + 2)$$

$$\text{or } x^2 + 2x - 20 = 0$$

$$\text{Using the quadratic equation formula } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{84}}{2} = 3.582 \dots \quad \text{or} \quad -5.582 \dots$$

As  $x > 0$ ,  $x = 3.58$  (3 s.f.)

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise G, Question 5

#### Question:

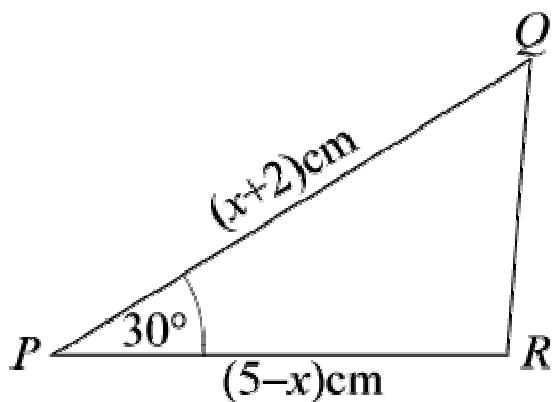
In  $\triangle PQR$ ,  $PQ = (x + 2)$  cm,  $PR = (5 - x)$  cm and  $\angle QPR = 30^\circ$ .

The area of the triangle is  $A$  cm<sup>2</sup>:

(a) Show that  $A = \frac{1}{4} (10 + 3x - x^2)$ .

(b) Use the method of completing the square, or otherwise, to find the maximum value of  $A$  and give the corresponding value of  $x$ .

#### Solution:



(a) Using area of  $\triangle PQR = \frac{1}{2}qr \sin P$

$$A \text{ cm}^2 = \frac{1}{2} (5 - x) (x + 2) \sin 30^\circ \text{ cm}^2$$

$$\Rightarrow A = \frac{1}{2} (5x - 2x + 10 - x^2) \times \frac{1}{2}$$

$$\Rightarrow A = \frac{1}{4} (10 + 3x - x^2)$$

(b)  $10 + 3x - x^2$

$$= - (x^2 - 3x - 10)$$

$$= - \left[ \left( x - 1\frac{1}{2} \right)^2 - 2\frac{1}{4} - 10 \right] \text{ (completing the square)}$$

$$= - \left[ \left( x - 1\frac{1}{2} \right)^2 - 12\frac{1}{4} \right]$$

$$= 12\frac{1}{4} - \left( x - 1\frac{1}{2} \right)^2$$

The maximum value of  $10 + 3x - x^2 = 12 \frac{1}{4}$ , when  $x = 1 \frac{1}{2}$ .

The maximum value of  $A$  is  $\frac{1}{4} \left( 12 \frac{1}{4} \right) = 3 \frac{1}{16}$ , when  $x = 1 \frac{1}{2}$ .

(You could find the maximum using differentiation.)

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

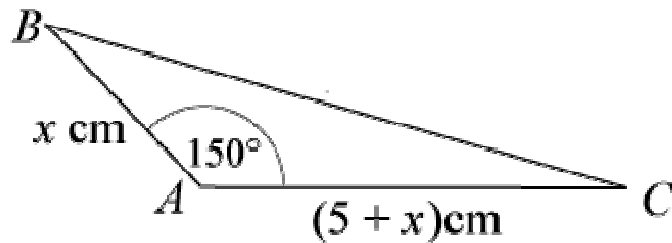
#### Exercise G, Question 6

#### Question:

In  $\triangle ABC$ ,  $AB = x$  cm,  $AC = (5 + x)$  cm and  $\angle BAC = 150^\circ$ . Given that the area of the triangle is  $3\frac{3}{4}$  cm<sup>2</sup>:

- (a) Show that  $x$  satisfies the equation  $x^2 + 5x - 15 = 0$ .
- (b) Calculate the value of  $x$ , giving your answer to 3 significant figures.

#### Solution:



(a) Using area of  $\triangle BAC = \frac{1}{2}bc \sin A$

$$3\frac{3}{4} \text{ cm}^2 = \frac{1}{2}x(5+x) \sin 150^\circ \text{ cm}^2$$

$$3\frac{3}{4} = \frac{1}{2} \left[ 5x + x^2 \right] \times \frac{1}{2}$$

$$\Rightarrow 15 = 5x + x^2$$

$$\Rightarrow x^2 + 5x - 15 = 0$$

(b) Using the quadratic equation formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{85}}{2} = 2.109 \dots \quad \text{or} \quad -7.109 \dots$$

As  $x > 0$ ,  $x = 2.11$  (3 s.f.)

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise H, Question 1

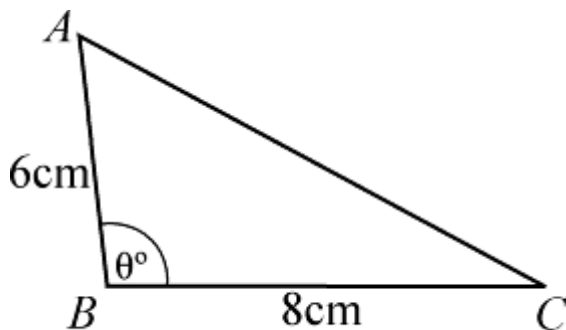
#### Question:

(Given non-exact answers to 3 significant figures.)

The area of a triangle is  $10 \text{ cm}^2$ . The angle between two of the sides, of length 6 cm and 8 cm respectively, is obtuse. Work out:

- The size of this angle.
- The length of the third side.

#### Solution:



(a) Using area of  $\triangle ABC = \frac{1}{2}ac \sin B$

$$10 \text{ cm}^2 = \frac{1}{2} \times 6 \times 8 \times \sin \theta^\circ \text{ cm}^2$$

$$\text{So } 10 = 24 \sin \theta^\circ$$

$$\text{So } \sin \theta^\circ = \frac{10}{24} = \frac{5}{12}$$

$$\Rightarrow \theta = 24.6 \text{ or } 155 \text{ (3 s.f.)}$$

As  $\theta$  is obtuse,  $\angle ABC = 155^\circ$  (3 s.f.)

(b) Using the cosine rule:  $b^2 = a^2 + c^2 - 2ac \cos B$

$$AC^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos B = 187.26 \dots$$

$$AC = 13.68 \dots$$

The third side has length 13.7m (3 s.f.)



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

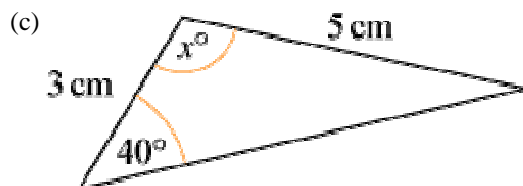
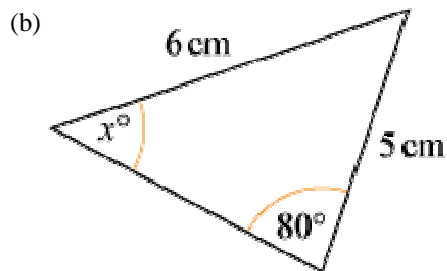
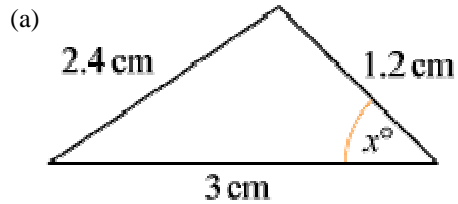
### The sine and cosine rule

#### Exercise H, Question 2

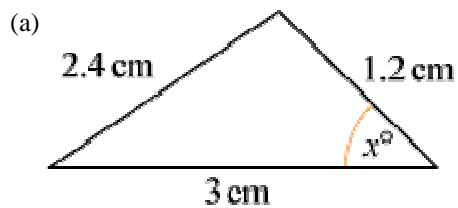
#### Question:

(Give non-exact answers to 3 significant figures.)

In each triangle below, find the value of  $x$  and the area of the triangle:



#### Solution:



Using the cosine rule:

$$\cos x^\circ = \frac{3^2 + 1.2^2 - 2.4^2}{2 \times 3 \times 1.2} = 0.65$$

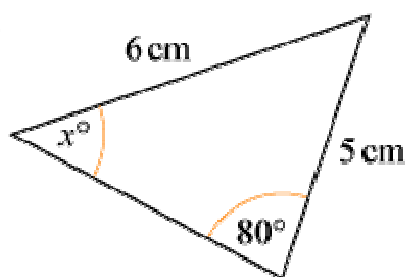
$$x = \cos^{-1}(0.65) = 49.458 \dots$$

$$x = 49.5 \text{ (3 s.f.)}$$

Using the area of a triangle formula:

$$\text{area} = \frac{1}{2} \times 1.2 \times 3 \times \sin x^\circ \text{ cm}^2 = 1.367 \dots \text{ cm}^2 = 1.37 \text{ cm}^2 \text{ (3 s.f.)}$$

(b)



Using the sine rule:

$$\frac{\sin x^\circ}{5} = \frac{\sin 80^\circ}{6}$$

$$\sin x^\circ = \frac{5 \sin 80^\circ}{6} = 0.8206 \dots$$

$$x = 55.152 \dots$$

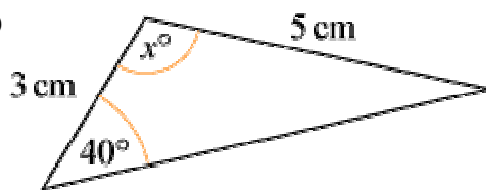
$$x = 55.2 \text{ (3 s.f.)}$$

The angle between 5 cm and 6 cm sides is  $180^\circ - (80 + x)^\circ = (100 - x)^\circ$ .

Using the area of a triangle formula:

$$\text{area} = \frac{1}{2} \times 5 \times 6 \times \sin \left( 100 - x \right)^\circ \text{ cm}^2 = 10.6 \text{ cm}^2 \text{ (3 s.f.)}$$

(c)

Using the sine rule to find angle opposite 3 cm. Call this  $y^\circ$ .

$$\frac{\sin y^\circ}{3} = \frac{\sin 40^\circ}{5}$$

$$\sin y^\circ = \frac{3 \sin 40^\circ}{5}$$

$$\Rightarrow y = 22.68 \dots$$

$$\text{So } x = 180 - (40 + y) = 117.3 \dots = 117 \text{ (3 s.f.)}$$

$$\text{Area of triangle} = \frac{1}{2} \times 3 \times 5 \times \sin x^\circ = 6.66 \text{ cm}^2 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

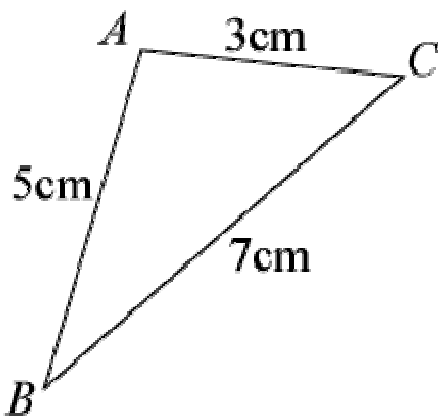
#### Exercise H, Question 3

#### Question:

(Give non-exact answers to 3 significant figures.)

The sides of a triangle are 3 cm, 5 cm and 7 cm respectively. Show that the largest angle is  $120^\circ$ , and find the area of the triangle.

#### Solution:



Using cosine rule to find angle A

$$\cos A = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5} = -0.5$$

$$A = \cos^{-1}(-0.5) = 120^\circ$$

$$\text{Area of triangle} = \frac{1}{2} \times 3 \times 5 \times \sin A \text{ cm}^2 = 6.495 \dots \text{ cm}^2 = 6.50 \text{ cm}^2 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

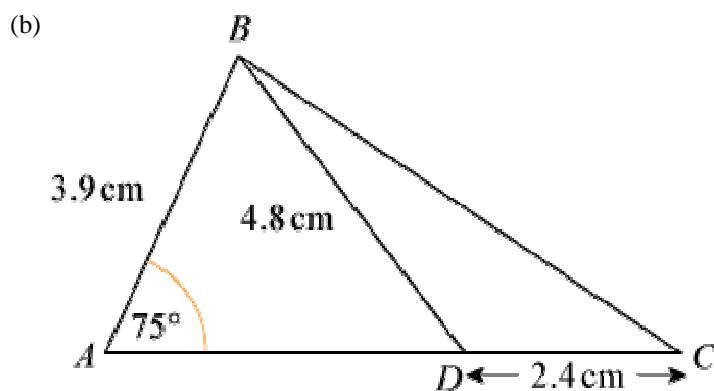
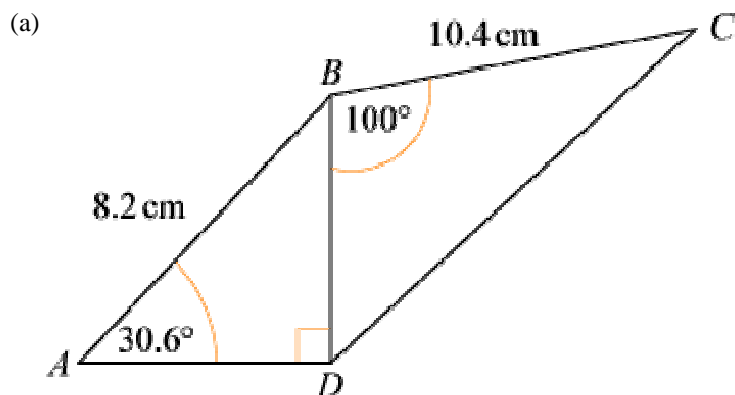
### The sine and cosine rule

#### Exercise H, Question 4

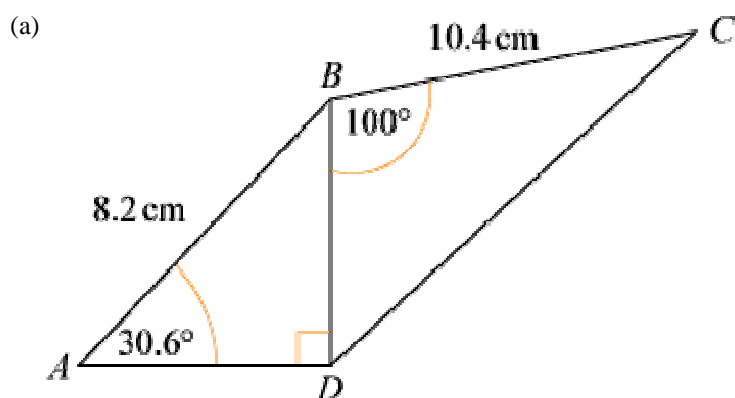
#### Question:

(Give non-exact answers to 3 significant figures.)

In each of the figures below calculate the total area:



#### Solution:



$$\text{In } \triangle BDA: \frac{BD}{8.2} = \sin 30.6^\circ$$

$$\Rightarrow BD = 8.2 \sin 30.6^\circ = 4.174 \dots$$

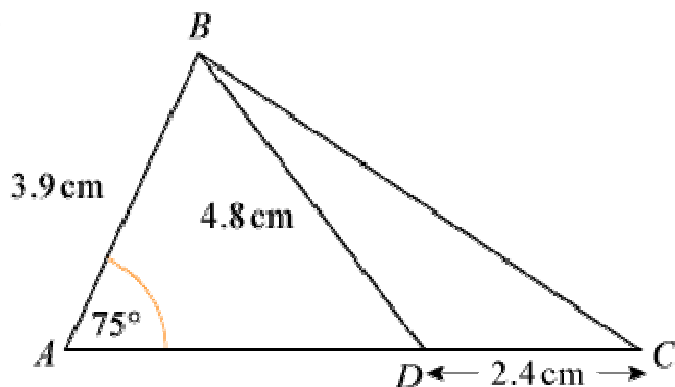
$$\text{Angle } ABD = 90^\circ - 30.6^\circ = 59.4^\circ$$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times 8.2 \times BD \times \sin 59.4^\circ = 14.7307 \dots \text{ cm}^2$$

$$\text{Area of } \triangle BDC = \frac{1}{2} \times 10.4 \times BD \times \sin 100^\circ = 21.375 \dots \text{ cm}^2$$

$$\text{Total area} = \text{area of } \triangle ABD + \text{area } \triangle BDC = 36.1 \text{ cm}^2 \text{ (3 s.f.)}$$

(b)



In  $\triangle ABD$ , using the sine rule to find  $\angle ADB$ ,

$$\frac{\sin \angle ADB}{3.9} = \frac{\sin 75^\circ}{4.8}$$

$$\sin \angle ADB = \frac{3.9 \sin 75^\circ}{4.8}$$

$$\angle ADB = \sin^{-1} \left( \frac{3.9 \sin 75^\circ}{4.8} \right) = 51.7035 \dots^\circ$$

$$\text{So } \angle ABD = 180^\circ - (75^\circ + \angle ADB)^\circ = 53.296 \dots^\circ$$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times 3.9 \times 4.8 \times \sin \angle ABD \text{ cm}^2 = 7.504 \dots \text{ cm}^2$$

$$\text{In } \triangle BDC, \angle BDC = 180^\circ - \angle BDA = 128.29 \dots^\circ$$

$$\text{Area of } \triangle BDC = \frac{1}{2} \times 2.4 \times 4.8 \times \sin \angle BDC \text{ cm}^2 = 4.520 \dots \text{ cm}^2$$

$$\text{Total area} = \text{area of } \triangle ABD + \text{area of } \triangle BDC = 12.0 \text{ cm}^2 \text{ (3 s.f.)}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise H, Question 5

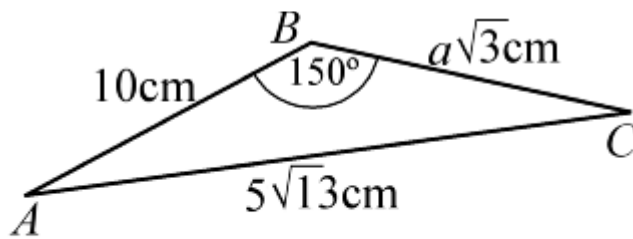
#### Question:

(Give non-exact answers to 3 significant figures.)

In  $\triangle ABC$ ,  $AB = 10$  cm,  $BC = a\sqrt{3}$  cm,  $AC = 5\sqrt{13}$  cm and  $\angle ABC = 150^\circ$ . Calculate:

- The value of  $a$ .
- The exact area of  $\triangle ABC$ .

#### Solution:



(a) Using the cosine rule:  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $(5\sqrt{13})^2 = (a\sqrt{3})^2 + 10^2 - 2 \times a\sqrt{3} \times 10 \times \cos 150^\circ$   
 $325 = 3a^2 + 100 + 30a$   
 $3a^2 + 30a - 225 = 0$   
 $a^2 + 10a - 75 = 0$   
 $(a + 15)(a - 5) = 0$   
 $\Rightarrow a = 5$  as  $a > 0$

(b) Area of  $\triangle ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} \times \sin 150^\circ \text{ cm}^2 = 12.5\sqrt{3} \text{ cm}^2$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

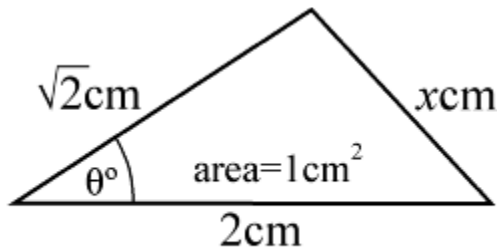
#### Exercise H, Question 6

#### Question:

(Give non-exact answers to 3 significant figures.)

In a triangle, the largest side has length 2 cm and one of the other sides has length  $\sqrt{2}$  cm. Given that the area of the triangle is  $1 \text{ cm}^2$ , show that the triangle is right-angled and isosceles.

#### Solution:



Using the area formula:

$$1 = \frac{1}{2} \times 2 \times \sqrt{2} \times \sin \theta^\circ$$

$$\Rightarrow \sin \theta^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 45 \text{ or } 135$$

but as  $\theta$  is not the largest angle,  $\theta$  must be 45.

Using the cosine rule to find  $x$ :

$$x^2 = 2^2 + (\sqrt{2})^2 - 2 \times 2 \times \sqrt{2} \times \cos 45^\circ$$

$$x^2 = 4 + 2 - 4 = 2$$

$$\text{So } x = \sqrt{2}$$

So the triangle is isosceles with two angles of  $45^\circ$ .

It is a right-angled isosceles triangle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise H, Question 7

#### Question:

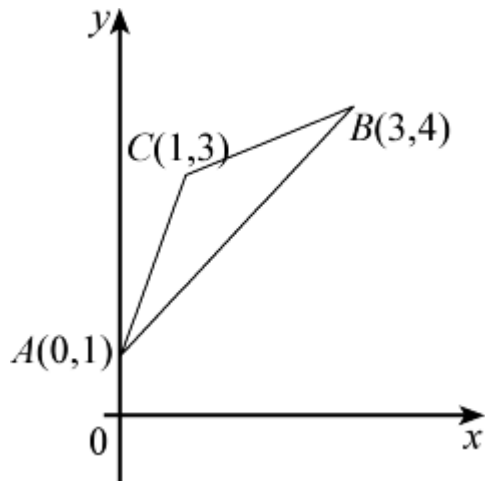
(Give non-exact answers to 3 significant figures.)

The three points  $A$ ,  $B$  and  $C$ , with coordinates  $A(0, 1)$ ,  $B(3, 4)$  and  $C(1, 3)$  respectively, are joined to form a triangle:

(a) Show that  $\cos \angle ACB = -\frac{4}{5}$ .

(b) Calculate the area of  $\triangle ABC$ .

#### Solution:



$$(a) AC = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5} = b$$

$$BC = \sqrt{(3-1)^2 + (4-3)^2} = \sqrt{5} = a$$

$$AB = \sqrt{(3-0)^2 + (4-1)^2} = \sqrt{18} = c$$

$$\text{Using the cosine rule: } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}} = \frac{-8}{10} = \frac{-4}{5}$$

(b) Using the area formula:

$$\text{area of } \triangle ABC = \frac{1}{2} ab \sin C = \frac{1}{2} \times \sqrt{5} \times \sqrt{5} \times \sin C = 1.5 \text{ cm}^2$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### The sine and cosine rule

#### Exercise H, Question 8

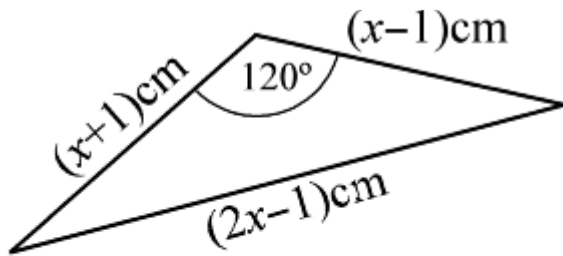
#### Question:

(Give non-exact answers to 3 significant figures.)

The longest side of a triangle has length  $(2x - 1)$  cm. The other sides have lengths  $(x - 1)$  cm and  $(x + 1)$  cm. Given that the largest angle is  $120^\circ$ , work out:

(a) the value of  $x$  and (b) the area of the triangle.

#### Solution:



(a) Using the cosine rule:

$$(2x - 1)^2 = (x + 1)^2 + (x - 1)^2 - 2(x + 1)(x - 1) \cos 120^\circ$$

$$4x^2 - 4x + 1 = (x^2 + 2x + 1) + (x^2 - 2x + 1) + (x^2 - 1)$$

$$4x^2 - 4x + 1 = 3x^2 + 1$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$\Rightarrow x = 4 \text{ as } x > 1$$

(b) Area of triangle

$$= \frac{1}{2} \times \left( x + 1 \right) \times \left( x - 1 \right) \times \sin 120^\circ \text{ cm}^2$$

$$= \frac{1}{2} \times 5 \times 3 \times \sin 120^\circ \text{ cm}^2$$

$$= \frac{1}{2} \times 5 \times 3 \times \frac{\sqrt{3}}{2} \text{ cm}^2$$

$$= \frac{15\sqrt{3}}{4} \text{ cm}^2$$

$$= 6.50 \text{ cm}^2 \text{ (3 s.f.)}$$