

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

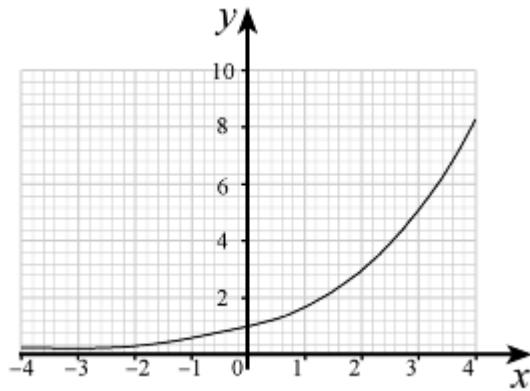
Exercise A, Question 1

Question:

(a) Draw an accurate graph of $y = (1.7)^x$, for $-4 \leq x \leq 4$.

(b) Use your graph to solve the equation $(1.7)^x = 4$.

Solution:



(a)

(b) Where $y = 4$, $x \approx 2.6$

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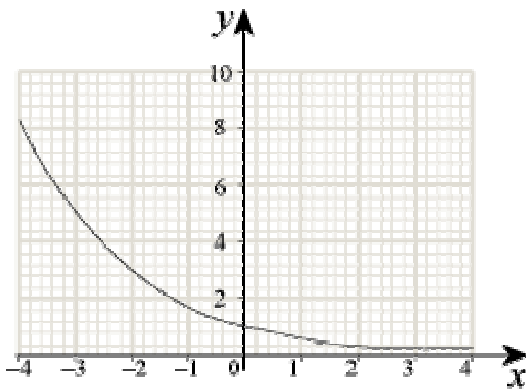
Exponentials and logarithms

Exercise A, Question 2

Question:

- (a) Draw an accurate graph of $y = (0.6)^x$, for $-4 \leq x \leq 4$.
- (b) Use your graph to solve the equation $(0.6)^x = 2$.

Solution:



(a)

- (b) Where $y = 2$, $x \approx -1.4$

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Edexcel Modular Mathematics for AS and A-Level

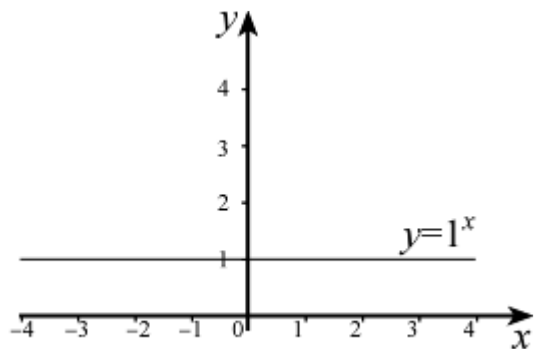
Exponentials and logarithms

Exercise A, Question 3

Question:

Sketch the graph of $y = 1^x$.

Solution:



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Exponentials and logarithms

Exercise B, Question 1

Question:

Rewrite as a logarithm:

(a) $4^4 = 256$

(b) $3^{-2} = \frac{1}{9}$

(c) $10^6 = 1\ 000\ 000$

(d) $11^1 = 11$

(e) $(0.2)^3 = 0.008$

Solution:

(a) $\log_4 256 = 4$

(b) $\log_3 \left(\frac{1}{9} \right) = -2$

(c) $\log_{10} 1\ 000\ 000 = 6$

(d) $\log_{11} 11 = 1$

(e) $\log_{0.2} 0.008 = 3$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise B, Question 2

Question:

Rewrite using a power:

(a) $\log_2 16 = 4$

(b) $\log_5 25 = 2$

(c) $\log_9 3 = \frac{1}{2}$

(d) $\log_5 0.2 = -1$

(e) $\log_{10} 100\ 000 = 5$

Solution:

(a) $2^4 = 16$

(b) $5^2 = 25$

(c) $9^{\frac{1}{2}} = 3$

(d) $5^{-1} = 0.2$

(e) $10^5 = 100\ 000$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise B, Question 3

Question:

Find the value of:

(a) $\log_2 8$

(b) $\log_5 25$

(c) $\log_{10} 10\,000\,000$

(d) $\log_{12} 12$

(e) $\log_3 729$

(f) $\log_{10} \sqrt{10}$

(g) $\log_4 (0.25)$

(h) $\log_{0.25} 16$

(i) $\log_a (a^{10})$

(j) $\log \left(\frac{2}{3} \right) \left(\frac{9}{4} \right)$

Solution:

(a) If $\log_2 8 = x$ then $2^x = 8$, so $x = 3$

(b) If $\log_5 25 = x$ then $5^x = 25$, so $x = 2$

(c) If $\log_{10} 10\,000\,000 = x$ then $10^x = 10\,000\,000$, so $x = 7$

(d) If $\log_{12} 12 = x$ then $12^x = 12$, so $x = 1$

(e) If $\log_3 729 = x$ then $3^x = 729$, so $x = 6$

(f) If $\log_{10} \sqrt{10} = x$ then $10^x = \sqrt{10}$, so $x = \frac{1}{2}$

(Power $\frac{1}{2}$ means 'square root'.)

(g) If $\log_4 (0.25) = x$ then $4^x = 0.25 = \frac{1}{4}$, so $x = -1$

(Negative power means 'reciprocal'.)

$$(h) \log_{0.25} 16 = x$$

$$\Rightarrow 0.25^x = 16$$

$$\Rightarrow \left(\frac{1}{4}\right)^x = 16, \text{ so } x = -2$$

$$\left[\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16 \right]$$

$$(i) \log_a (a^{10}) = x$$

$$\Rightarrow a^x = a^{10}, \text{ so } x = 10$$

$$(j) \log \left(\frac{2}{3}\right) \left(\frac{9}{4}\right) = x$$

$$\Rightarrow \left(\frac{2}{3}\right)^x = \frac{9}{4}, \text{ so } x = -2$$

$$\left[\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\left(\frac{4}{9}\right)} = \frac{9}{4} \right]$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise B, Question 4

Question:

Find the value of x for which:

(a) $\log_5 x = 4$

(b) $\log_x 81 = 2$

(c) $\log_7 x = 1$

(d) $\log_x (2x) = 2$

Solution:

(a) Using a power, $5^4 = x$
So $x = 625$

(b) Using a power, $x^2 = 81$
So $x = 9$
(The base of a logarithm cannot be negative, so $x = -9$ is not possible.)

(c) Using a power, $7^1 = x$
So $x = 7$

(d) Using a power,
 $x^2 = 2x$
 $x^2 - 2x = 0$
 $x(x - 2) = 0$
 $x = 2$
(The base of a logarithm cannot be zero, so $x = 0$ is not possible.)

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise C, Question 1

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 20$$

Solution:

$$\log_{10} 20 = 1.3010 \dots = 1.30 \text{ (3 s.f.)}$$

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Exponentials and logarithms

Exercise C, Question 2

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 4$$

Solution:

$$\log_{10} 4 = 0.6020 \dots = 0.602 \text{ (3 s.f.)}$$

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Exponentials and logarithms

Exercise C, Question 3

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 7000$$

Solution:

$$\log_{10} 7000 = 3.8450 \dots = 3.85 \text{ (3 s.f.)}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise C, Question 4

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 0.786$$

Solution:

$$\log_{10} 0.786 = -0.1045 \dots = -0.105 \text{ (3 s.f.)}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise C, Question 5

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 11$$

Solution:

$$\log_{10} 11 = 1.0413 \dots = 1.04 \text{ (3 s.f.)}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise C, Question 6

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 35.3$$

Solution:

$$\log_{10} 35.3 = 1.5477 \dots = 1.55 \text{ (3 s.f.)}$$

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Exponentials and logarithms

Exercise C, Question 7

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 0.3$$

Solution:

$$\log_{10} 0.3 = -0.5228 \dots = -0.523 \text{ (3 s.f.)}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise C, Question 8

Question:

Find from your calculator the value to 3 s.f. of:

$$\log_{10} 999$$

Solution:

$$\log_{10} 999 = 2.9995 \dots = 3.00 \text{ (3 s.f.)}$$

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Exponentials and logarithms

Exercise D, Question 1

Question:

Write as a single logarithm:

(a) $\log_2 7 + \log_2 3$

(b) $\log_2 36 - \log_2 4$

(c) $3 \log_5 2 + \log_5 10$

(d) $2 \log_6 8 - 4 \log_6 3$

(e) $\log_{10} 5 + \log_{10} 6 - \log_{10} \left(\frac{1}{4} \right)$

Solution:

(a) $\log_2 (7 \times 3) = \log_2 21$

(b) $\log_2 \left(\frac{36}{4} \right) = \log_2 9$

(c) $3 \log_5 2 = \log_5 2^3 = \log_5 8$
 $\log_5 8 + \log_5 10 = \log_5 (8 \times 10) = \log_5 80$

(d) $2 \log_6 8 = \log_6 8^2 = \log_6 64$
 $4 \log_6 3 = \log_6 3^4 = \log_6 81$

$\log_6 64 - \log_6 81 = \log_6 \left(\frac{64}{81} \right)$

(e) $\log_{10} 5 + \log_{10} 6 = \log_{10} (5 \times 6) = \log_{10} 30$

$\log_{10} 30 - \log_{10} \left(\frac{1}{4} \right) = \log_{10} \left[\frac{30}{\left(\frac{1}{4} \right)} \right] = \log_{10} 120$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise D, Question 2

Question:

Write as a single logarithm, then simplify your answer:

(a) $\log_2 40 - \log_2 5$

(b) $\log_6 4 + \log_6 9$

(c) $2 \log_{12} 3 + 4 \log_{12} 2$

(d) $\log_8 25 + \log_8 10 - 3 \log_8 5$

(e) $2 \log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

Solution:

$$(a) \log_2 \left(\frac{40}{5} \right) = \log_2 8 = 3 \quad \left(2^3 = 8 \right)$$

$$(b) \log_6 (4 \times 9) = \log_6 36 = 2 \quad (6^2 = 36)$$

$$\begin{aligned} (c) \log_{12} (3^2) + \log_{12} (2^4) \\ &= \log_{12} 9 + \log_{12} 16 \\ &= \log_{12} (9 \times 16) \\ &= \log_{12} 144 \\ &= 2 \quad (12^2 = 144) \end{aligned}$$

$$\begin{aligned} (d) \log_8 (25 \times 10) - \log_8 (5^3) \\ &= \log_8 250 - \log_8 125 \\ &= \log_8 \left(\frac{250}{125} \right) \\ &= \log_8 2 \\ &= \frac{1}{3} \quad \left(8^{\frac{1}{3}} = 2 \right) \end{aligned}$$

$$\begin{aligned} (e) \log_{10} (20^2) - \log_{10} (5 \times 8) \\ &= \log_{10} 400 - \log_{10} 40 \\ &= \log_{10} \left(\frac{400}{40} \right) \\ &= \log_{10} 10 \\ &= 1 \quad (10^1 = 10) \end{aligned}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise D, Question 3

Question:

Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:

(a) $\log_a (x^3y^4z)$

(b) $\log_a \left(\frac{x^5}{y^2} \right)$

(c) $\log_a (a^2x^2)$

(d) $\log_a \left(\frac{x\sqrt{y}}{z} \right)$

(e) $\log_a \sqrt{ax}$

Solution:

(a) $\log_a x^3 + \log_a y^4 + \log_a z$
 $= 3 \log_a x + 4 \log_a y + \log_a z$

(b) $\log_a x^5 - \log_a y^2$
 $= 5 \log_a x - 2 \log_a y$

(c) $\log_a a^2 + \log_a x^2$
 $= 2 \log_a a + 2 \log_a x$
 $= 2 + 2 \log_a x \quad (\log_a a = 1)$

(d) $\log_a x + \log_a y^{\frac{1}{2}} - \log_a z$
 $= \log_a x + \frac{1}{2} \log_a y - \log_a z$

(e) $\log_a (ax)^{\frac{1}{2}}$
 $= \frac{1}{2} \log_a (ax)$
 $= \frac{1}{2} \log_a a + \frac{1}{2} \log_a x$
 $= \frac{1}{2} + \frac{1}{2} \log_a x$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise E, Question 1

Question:

Solve, giving your answer to 3 significant figures:

(a) $2^x = 75$

(b) $3^x = 10$

(c) $5^x = 2$

(d) $4^{2x} = 100$

(e) $9^{x+5} = 50$

(f) $7^{2x-1} = 23$

(g) $3^{x-1} = 8^{x+1}$

(h) $2^{2x+3} = 3^{3x+2}$

(i) $8^{3-x} = 10^x$

(j) $3^{4-3x} = 4^{x+5}$

Solution:

(a) $2^x = 75$

$$\log 2^x = \log 75$$

$$x \log 2 = \log 75$$

$$x = \frac{\log 75}{\log 2}$$

$$x = 6.23 \text{ (3 s.f.)}$$

(b) $3^x = 10$

$$\log 3^x = \log 10$$

$$x \log 3 = \log 10$$

$$x = \frac{\log 10}{\log 3}$$

$$x = 2.10 \text{ (3 s.f.)}$$

(c) $5^x = 2$

$$\log 5^x = \log 2$$

$$x \log 5 = \log 2$$

$$x = \frac{\log 2}{\log 5}$$

$$x = 0.431 \text{ (3 s.f.)}$$

(d) $4^{2x} = 100$

$$\log 4^{2x} = \log 100$$

$$2x \log 4 = \log 100$$

$$x = \frac{\log 100}{2 \log 4}$$

$$x = 1.66 \text{ (3 s.f.)}$$

$$\text{(e) } 9^{x+5} = 50$$

$$\log 9^{x+5} = \log 50$$

$$(x+5) \log 9 = \log 50$$

$$x \log 9 + 5 \log 9 = \log 50$$

$$x \log 9 = \log 50 - 5 \log 9$$

$$x = \frac{\log 50 - 5 \log 9}{\log 9}$$

$$x = -3.22 \text{ (3 s.f.)}$$

$$\text{(f) } 7^{2x-1} = 23$$

$$\log 7^{2x-1} = \log 23$$

$$(2x-1) \log 7 = \log 23$$

$$2x \log 7 - \log 7 = \log 23$$

$$2x \log 7 = \log 23 + \log 7$$

$$x = \frac{\log 23 + \log 7}{2 \log 7}$$

$$x = 1.31 \text{ (3 s.f.)}$$

$$\text{(g) } 3^{x-1} = 8^{x+1}$$

$$\log 3^{x-1} = \log 8^{x+1}$$

$$(x-1) \log 3 = (x+1) \log 8$$

$$x \log 3 - \log 3 = x \log 8 + \log 8$$

$$x (\log 3 - \log 8) = \log 3 + \log 8$$

$$x = \frac{\log 3 + \log 8}{\log 3 - \log 8}$$

$$x = -3.24 \text{ (3 s.f.)}$$

$$\text{(h) } 2^{2x+3} = 3^{3x+2}$$

$$\log 2^{2x+3} = \log 3^{3x+2}$$

$$(2x+3) \log 2 = (3x+2) \log 3$$

$$2x \log 2 + 3 \log 2 = 3x \log 3 + 2 \log 3$$

$$2x \log 2 - 3x \log 3 = 2 \log 3 - 3 \log 2$$

$$x (2 \log 2 - 3 \log 3) = 2 \log 3 - 3 \log 2$$

$$x = \frac{2 \log 3 - 3 \log 2}{2 \log 2 - 3 \log 3}$$

$$x = -0.0617 \text{ (3 s.f.)}$$

$$\text{(i) } 8^{3-x} = 10^x$$

$$\log 8^{3-x} = \log 10^x$$

$$(3-x) \log 8 = x \log 10$$

$$3 \log 8 - x \log 8 = x \log 10$$

$$3 \log 8 = x (\log 10 + \log 8)$$

$$x = \frac{3 \log 8}{\log 10 + \log 8}$$

$$x = 1.42 \text{ (3 s.f.)}$$

$$\text{(j) } 3^{4-3x} = 4^{x+5}$$

$$\log 3^{4-3x} = \log 4^{x+5}$$

$$(4-3x) \log 3 = (x+5) \log 4$$

$$4 \log 3 - 3x \log 3 = x \log 4 + 5 \log 4$$

$$4 \log 3 - 5 \log 4 = x \log 4 + 3x \log 3$$

$$4 \log 3 - 5 \log 4 = x (\log 4 + 3 \log 3)$$

$$x = \frac{4 \log 3 - 5 \log 4}{\log 4 + 3 \log 3}$$

$$x = -0.542 \text{ (3 s.f.)}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise E, Question 2

Question:

Solve, giving your answer to 3 significant figures:

(a) $2^{2x} - 6(2^x) + 5 = 0$

(b) $3^{2x} - 15(3^x) + 44 = 0$

(c) $5^{2x} - 6(5^x) - 7 = 0$

(d) $3^{2x} + 3^{x+1} - 10 = 0$

(e) $7^{2x} + 12 = 7^{x+1}$

(f) $2^{2x} + 3(2^x) - 4 = 0$

(g) $3^{2x+1} - 26(3^x) - 9 = 0$

(h) $4(3^{2x+1}) + 17(3^x) - 7 = 0$

Solution:

(a) Let $y = 2^x$

$$y^2 - 6y + 5 = 0$$

$$(y - 1)(y - 5) = 0$$

So $y = 1$ or $y = 5$

If $y = 1$, $2^x = 1$, $x = 0$

If $y = 5$, $2^x = 5$

$$\log 2^x = \log 5$$

$$x \log 2 = \log 5$$

$$x = \frac{\log 5}{\log 2}$$

$$x = 2.32 \text{ (3 s.f.)}$$

So $x = 0$ or $x = 2.32$

(b) Let $y = 3^x$

$$y^2 - 15y + 44 = 0$$

$$(y - 4)(y - 11) = 0$$

So $y = 4$ or $y = 11$

If $y = 4$, $3^x = 4$

$$\log 3^x = \log 4$$

$$x \log 3 = \log 4$$

$$x = \frac{\log 4}{\log 3}$$

$$x = 1.26 \text{ (3 s.f.)}$$

If $y = 11$, $3^x = 11$

$$\log 3^x = \log 11$$

$$x \log 3 = \log 11$$

$$x = \frac{\log 11}{\log 3}$$

$$x = 2.18 \text{ (3 s.f.)}$$

So $x = 1.26$ or $x = 2.18$

(c) Let $y = 5^x$

$$y^2 - 6y - 7 = 0$$

$$(y + 1)(y - 7) = 0$$

So $y = -1$ or $y = 7$

If $y = -1$, $5^x = -1$. No solution.

If $y = 7$, $5^x = 7$

$$\log 5^x = \log 7$$

$$x \log 5 = \log 7$$

$$x = \frac{\log 7}{\log 5}$$

$x = 1.21$ (3 s.f.)

(d) Let $y = 3^x$

$$(3^x)^2 + (3^x \times 3) - 10 = 0$$

$$y^2 + 3y - 10 = 0$$

$$(y + 5)(y - 2) = 0$$

So $y = -5$ or $y = 2$

If $y = -5$, $3^x = -5$. No solution.

If $y = 2$, $3^x = 2$

$$\log 3^x = \log 2$$

$$x \log 3 = \log 2$$

$$x = \frac{\log 2}{\log 3}$$

$x = 0.631$ (3 s.f.)

(e) Let $y = 7^x$

$$(7^x)^2 + 12 = 7^x \times 7$$

$$y^2 + 12 = 7y$$

$$y^2 - 7y + 12 = 0$$

$$(y - 3)(y - 4) = 0$$

So $y = 3$ or $y = 4$

If $y = 3$, $7^x = 3$

$$x \log 7 = \log 3$$

$$x = \frac{\log 3}{\log 7}$$

$x = 0.565$ (3 s.f.)

If $y = 4$, $7^x = 4$

$$x \log 7 = \log 4$$

$$x = \frac{\log 4}{\log 7}$$

$x = 0.712$ (3 s.f.)

So $x = 0.565$ or $x = 0.712$

$$(f) 2^{2x} + 3(2^x) - 4 = 0$$

Let $y = 2^x$

$$\text{Then } y^2 + 3y - 4 = 0$$

$$\text{So } (y + 4)(y - 1) = 0$$

So $y = -4$ or $y = 1$

$2^x = -4$ has no solution

Therefore $2^x = 1$

So $x = 0$ is the only solution

$$(g) 3^{2x+1} - 26(3^x) - 9 = 0$$

Let $y = 3^x$

$$\text{Then } 3y^2 - 26y - 9 = 0$$

$$\text{So } (3y + 1)(y - 9) = 0$$

$$\text{So } y = -\frac{1}{3} \text{ or } y = 9$$

$$3^x = -\frac{1}{3} \text{ has no solution}$$

$$\text{Therefore } 3^x = 9$$

So $x = 2$ is the only solution

$$\text{(h) } 4(3^{2x+1}) + 17(3^x) - 7 = 0$$

$$12(3^{2x}) + 17(3^x) - 7 = 0$$

$$\text{Let } y = 3^x$$

$$\text{So } 12y^2 + 17y - 7 = 0$$

$$\text{So } (3y - 1)(4y + 7) = 0$$

$$\text{So } y = \frac{1}{3} \text{ or } y = -\frac{7}{4}$$

$$3^x = -\frac{7}{4} \text{ has no solution}$$

$$\text{Therefore } 3^x = \frac{1}{3}$$

So $x = -1$ is the only solution

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise F, Question 1

Question:

Find, to 3 decimal places:

(a) $\log_7 120$

(b) $\log_3 45$

(c) $\log_2 19$

(d) $\log_{11} 3$

(e) $\log_6 4$

Solution:

$$(a) \log_7 120 = \frac{\log_{10} 120}{\log_{10} 7} = 2.460 \text{ (3 d.p.)}$$

$$(b) \log_3 45 = \frac{\log_{10} 45}{\log_{10} 3} = 3.465 \text{ (3 d.p.)}$$

$$(c) \log_2 19 = \frac{\log_{10} 19}{\log_{10} 2} = 4.248 \text{ (3 d.p.)}$$

$$(d) \log_{11} 3 = \frac{\log_{10} 3}{\log_{10} 11} = 0.458 \text{ (3 d.p.)}$$

$$(e) \log_6 4 = \frac{\log_{10} 4}{\log_{10} 6} = 0.774 \text{ (3 d.p.)}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise F, Question 2

Question:

Solve, giving your answer to 3 significant figures:

(a) $8^x = 14$

(b) $9^x = 99$

(c) $12^x = 6$

Solution:

(a) $\log 8^x = \log 14$

$x \log 8 = \log 14$

$$x = \frac{\log_{10} 14}{\log_{10} 8}$$

$x = 1.27$ (3 s.f.)

(b) $\log 9^x = \log 99$

$x \log 9 = \log 99$

$$x = \frac{\log_{10} 99}{\log_{10} 9}$$

$x = 2.09$ (3 s.f.)

(c) $\log 12^x = \log 6$

$x \log 12 = \log 6$

$$x = \frac{\log_{10} 6}{\log_{10} 12}$$

$x = 0.721$ (3 s.f.)

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise F, Question 3

Question:

Solve, giving your answer to 3 significant figures:

(a) $\log_2 x = 8 + 9 \log_x 2$

(b) $\log_4 x + 2 \log_x 4 + 3 = 0$

(c) $\log_2 x + \log_4 x = 2$

Solution:

(a) $\log_2 x = 8 + 9 \log_x 2$

$$\log_2 x = 8 + \frac{9}{\log_2 x}$$

Let $\log_2 x = y$

$$y = 8 + \frac{9}{y}$$

$$y^2 = 8y + 9$$

$$y^2 - 8y - 9 = 0$$

$$(y + 1)(y - 9) = 0$$

So $y = -1$ or $y = 9$

If $y = -1$, $\log_2 x = -1$

$$\Rightarrow x = 2^{-1} = \frac{1}{2}$$

If $y = 9$, $\log_2 x = 9$

$$\Rightarrow x = 2^9 = 512$$

So $x = \frac{1}{2}$ or $x = 512$

(b) $\log_4 x + 2 \log_x 4 + 3 = 0$

$$\log_4 x + \frac{2}{\log_4 x} + 3 = 0$$

Let $\log_4 x = y$

$$y + \frac{2}{y} + 3 = 0$$

$$y^2 + 2 + 3y = 0$$

$$y^2 + 3y + 2 = 0$$

$$(y + 1)(y + 2) = 0$$

So $y = -1$ or $y = -2$

If $y = -1$, $\log_4 x = -1$

$$\Rightarrow x = 4^{-1} = \frac{1}{4}$$

If $y = -2$, $\log_4 x = -2$

$$\Rightarrow x = 4^{-2} = \frac{1}{16}$$

$$\text{So } x = \frac{1}{4} \text{ or } x = \frac{1}{16}$$

$$(c) \log_2 x + \log_4 x = 2$$

$$\log_2 x + \frac{\log_2 x}{\log_2 4} = 2$$

But $\log_2 4 = 2$ (because $2^2 = 4$), so

$$\log_2 x + \frac{\log_2 x}{2} = 2$$

$$\frac{3}{2} \log_2 x = 2$$

$$\log_2 x = \frac{4}{3}$$

$$x = 2^{\frac{4}{3}}$$

$$x = 2.52 \text{ (3 s.f.)}$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 1

Question:

Find the possible values of x for which $2^{2x+1} = 3(2^x) - 1$. **[E]**

Solution:

$$2^{2x+1} = 3(2^x) - 1$$

$$2^{2x} \times 2^1 = 3(2^x) - 1$$

$$\text{Let } 2^x = y$$

$$2y^2 = 3y - 1$$

$$2y^2 - 3y + 1 = 0$$

$$(2y - 1)(y - 1) = 0$$

$$\text{So } y = \frac{1}{2} \text{ or } y = 1$$

$$\text{If } y = \frac{1}{2}, 2^x = \frac{1}{2}, x = -1$$

$$\text{If } y = 1, 2^x = 1, x = 0$$

$$\text{So } x = 0 \text{ or } x = -1$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 2

Question:

- (a) Express $\log_a (p^2q)$ in terms of $\log_a p$ and $\log_a q$.
- (b) Given that $\log_a (pq) = 5$ and $\log_a (p^2q) = 9$, find the values of $\log_a p$ and $\log_a q$. **[E]**

Solution:

(a) $\log_a (p^2q) = \log_a (p^2) + \log_a q = 2 \log_a p + \log_a q$

(b) $\log_a (pq) = \log_a p + \log_a q$

So

$$\log_a p + \log_a q = 5 \quad \text{①}$$

$$2 \log_a p + \log_a q = 9 \quad \text{②}$$

Subtracting equation ① from equation ②:

$$\log_a p = 4$$

So $\log_a q = 1$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 3

Question:

Given that $p = \log_q 16$, express in terms of p ,

(a) $\log_q 2$,

(b) $\log_q (8q)$. **[E]**

Solution:

(a) $p = \log_q 16$

$$p = \log_q (2^4)$$

$$p = 4 \log_q 2$$

$$\log_q 2 = \frac{p}{4}$$

(b) $\log_q (8q) = \log_q 8 + \log_q q$

$$= \log_q (2^3) + \log_q q$$

$$= 3 \log_q 2 + \log_q q$$

$$= \frac{3p}{4} + 1$$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 4

Question:

- (a) Given that $\log_3 x = 2$, determine the value of x .
- (b) Calculate the value of y for which $2 \log_3 y - \log_3 (y + 4) = 2$.
- (c) Calculate the values of z for which $\log_3 z = 4 \log_z 3$.

[E]

Solution:

(a) $\log_3 x = 2$

$$x = 3^2 = 9$$

(b) $2 \log_3 y - \log_3 (y + 4) = 2$

$$\log_3 (y^2) - \log_3 (y + 4) = 2$$

$$\log_3 \left(\frac{y^2}{y + 4} \right) = 2$$

$$\frac{y^2}{y + 4} = 9$$

$$y^2 = 9y + 36$$

$$y^2 - 9y - 36 = 0$$

$$(y + 3)(y - 12) = 0$$

$$y = -3 \text{ or } y = 12$$

But $\log_3 (-3)$ is not defined,

So $y = 12$

(c) $\log_3 z = 4 \log_z 3$

$$\log_3 z = \frac{4}{\log_3 z}$$

$$(\log_3 z)^2 = 4$$

Either $\log_3 z = 2$ or $\log_3 z = -2$

$$z = 3^2 \text{ or } z = 3^{-2}$$

$$z = 9 \text{ or } z = \frac{1}{9}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 5

Question:

(a) Using the substitution $u = 2^x$, show that the equation $4^x - 2^{(x+1)} - 15 = 0$ can be written in the form $u^2 - 2u - 15 = 0$.

(b) Hence solve the equation $4^x - 2^{(x+1)} - 15 = 0$, giving your answer to 2 decimal places. **[E]**

Solution:

$$(a) 4^x - 2^{(x+1)} - 15 = 0$$

$$4^x = (2^2)^x = (2^x)^2$$

$$2^{x+1} = 2^x \times 2^1$$

$$\text{Let } u = 2^x$$

$$u^2 - 2u - 15 = 0$$

$$(b) (u + 3)(u - 5) = 0$$

$$\text{So } u = -3 \text{ or } u = 5$$

$$\text{If } u = -3, 2^x = -3. \text{ No solution.}$$

$$\text{If } u = 5, 2^x = 5$$

$$\log 2^x = \log 5$$

$$x \log 2 = \log 5$$

$$x = \frac{\log 5}{\log 2}$$

$$x = 2.32 \text{ (2 d.p.)}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 6

Question:

Solve, giving your answers as exact fractions, the simultaneous equations:

$$8^y = 4^{2x+3}$$

$$\log_2 y = \log_2 x + 4. \quad \text{[E]}$$

Solution:

$$8^y = 4^{2x+3}$$

$$(2^3)^y = (2^2)^{2x+3}$$

$$2^{3y} = 2^{2(2x+3)}$$

$$3y = 4x + 6 \quad \text{①}$$

$$\log_2 y - \log_2 x = 4$$

$$\log_2 \left(\frac{y}{x} \right) = 4$$

$$\frac{y}{x} = 2^4 = 16$$

$$y = 16x \quad \text{②}$$

Substitute ② into ①:

$$48x = 4x + 6$$

$$44x = 6$$

$$x = \frac{3}{22}$$

$$y = 16x = \frac{48}{22} = 2 \frac{2}{11}$$

$$\text{So } x = \frac{3}{22}, y = 2 \frac{2}{11}$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 7

Question:

Find the values of x for which $\log_3 x - 2 \log_x 3 = 1$. **[E]**

Solution:

$$\log_3 x - 2 \log_x 3 = 1$$

$$\log_3 x - \frac{2}{\log_3 x} = 1$$

$$\text{Let } \log_3 x = y$$

$$y - \frac{2}{y} = 1$$

$$y^2 - 2 = y$$

$$y^2 - y - 2 = 0$$

$$(y + 1)(y - 2) = 0$$

$$\text{So } y = -1 \text{ or } y = 2$$

$$\text{If } y = -1, \log_3 x = -1$$

$$\Rightarrow x = 3^{-1} = \frac{1}{3}$$

$$\text{If } y = 2, \log_3 x = 2$$

$$\Rightarrow x = 3^2 = 9$$

$$\text{So } x = \frac{1}{3} \text{ or } x = 9$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 8

Question:

Solve the equation

$$\log_3 (2 - 3x) = \log_9 (6x^2 - 19x + 2) . \quad \text{[E]}$$

Solution:

$$\log_3 (2 - 3x) = \log_9 (6x^2 - 19x + 2)$$

$$\log_9 \left(6x^2 - 19x + 2 \right) = \frac{\log_3 (6x^2 - 19x + 2)}{\log_3 9} = \frac{\log_3 (6x^2 - 19x + 2)}{2}$$

So

$$2 \log_3 (2 - 3x) = \log_3 (6x^2 - 19x + 2)$$

$$\log_3 (2 - 3x)^2 = \log_3 (6x^2 - 19x + 2)$$

$$(2 - 3x)^2 = 6x^2 - 19x + 2$$

$$4 - 12x + 9x^2 = 6x^2 - 19x + 2$$

$$3x^2 + 7x + 2 = 0$$

$$(3x + 1)(x + 2) = 0$$

$$x = -\frac{1}{3} \text{ or } x = -2$$

(Both solutions are valid, since they give logs of positive numbers in the original equation.)

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 9

Question:

If $xy = 64$ and $\log_x y + \log_y x = \frac{5}{2}$, find x and y . [E]

Solution:

$$\log_x y + \log_y x = \frac{5}{2}$$

$$\log_x y + \frac{1}{\log_x y} = \frac{5}{2}$$

Let $\log_x y = u$

$$u + \frac{1}{u} = \frac{5}{2}$$

$$2u^2 + 2 = 5u$$

$$2u^2 - 5u + 2 = 0$$

$$(2u - 1)(u - 2) = 0$$

$$u = \frac{1}{2} \text{ or } u = 2$$

$$\text{If } u = \frac{1}{2}, \log_x y = \frac{1}{2}$$

$$\Rightarrow y = x^{\frac{1}{2}} = \sqrt{x}$$

Since $xy = 64$,

$$x\sqrt{x} = 64 \quad \left(x^{\frac{3}{2}} = 64 \right)$$

$$x = 16$$

$$y = \sqrt{x} = 4$$

If $u = 2, \log_x y = 2$

$$\Rightarrow y = x^2$$

Since $xy = 64$,

$$x^3 = 64$$

$$x = 4$$

$$y = x^2 = 16$$

So $x = 16, y = 4$ or $x = 4, y = 16$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 10

Question:

Prove that if $a^x = b^y = (ab)^{xy}$, then $x + y = 1$. [E]

Solution:

Given that $a^x = b^y = (ab)^{xy}$

Take logs to base a for $a^x = b^y$:

$$\log_a (a^x) = \log_a (b^y)$$

$$x \log_a a = y \log_a b$$

$$x = y \log_a b \quad \textcircled{1}$$

Take logs to base a for $a^x = (ab)^{xy}$

$$x = \log_a (ab)^{xy}$$

$$x = xy \log_a (ab)$$

$$x = xy (\log_a a + \log_a b)$$

$$x = xy (1 + \log_a b)$$

$$1 = y (1 + \log_a b) \quad \textcircled{2}$$

But, from $\textcircled{1}$, $\log_a b = \frac{x}{y}$

Substitute into $\textcircled{2}$:

$$1 = y \left(1 + \frac{x}{y} \right)$$

$$1 = y + x$$

$$x + y = 1$$

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 11

Question:

(a) Show that $\log_4 3 = \log_2 \sqrt{3}$.

(b) Hence or otherwise solve the simultaneous equations:

$$2 \log_2 y = \log_4 3 + \log_2 x,$$

$$3^y = 9^x,$$

given that x and y are positive. **[E]**

Solution:

$$(a) \log_4 3 = \frac{\log_2 3}{\log_2 4} = \frac{\log_2 3}{2}$$

$$\log_4 3 = \frac{1}{2} \log_2 3 = \log_2 3^{\frac{1}{2}} = \log_2 \sqrt{3}$$

(b) $3^y = 9^x$

$$3^y = (3^2)^x = 3^{2x}$$

$$\text{So } y = 2x$$

$$2 \log_2 y = \log_4 3 + \log_2 x$$

$$\log_2 (y^2) = \log_2 \sqrt{3} + \log_2 x = \log_2 (x \sqrt{3})$$

$$\text{So } y^2 = x \sqrt{3}$$

$$\text{Since } y = 2x, (2x)^2 = x \sqrt{3}$$

$$\Rightarrow 4x^2 = x \sqrt{3}$$

$$x \text{ is positive, so } x \neq 0, x = \frac{\sqrt{3}}{4}$$

$$\Rightarrow y = 2x = \frac{\sqrt{3}}{2}$$

$$\text{So } x = \frac{\sqrt{3}}{4}, y = \frac{\sqrt{3}}{2}$$

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Exponentials and logarithms

Exercise G, Question 12

Question:

- (a) Given that $3 + 2 \log_2 x = \log_2 y$, show that $y = 8x^2$.
- (b) Hence, or otherwise, find the roots α and β , where $\alpha < \beta$, of the equation $3 + 2 \log_2 x = \log_2 (14x - 3)$.
- (c) Show that $\log_2 \alpha = -2$.
- (d) Calculate $\log_2 \beta$, giving your answer to 3 significant figures. **[E]**

Solution:

$$(a) 3 + 2 \log_2 x = \log_2 y$$

$$\log_2 y - 2 \log_2 x = 3$$

$$\log_2 y - \log_2 x^2 = 3$$

$$\log_2 \left(\frac{y}{x^2} \right) = 3$$

$$\frac{y}{x^2} = 2^3 = 8$$

$$y = 8x^2$$

$$(b) \text{ Comparing equations,}$$

$$y = 14x - 3$$

$$8x^2 = 14x - 3$$

$$8x^2 - 14x + 3 = 0$$

$$(4x - 1)(2x - 3) = 0$$

$$x = \frac{1}{4} \text{ or } x = \frac{3}{2}$$

$$\alpha = \frac{1}{4}, \beta = \frac{3}{2}$$

$$(c) \log_2 \alpha = \log_2 \left(\frac{1}{4} \right) = -2,$$

$$\text{since } 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$(d) \log_2 \beta = \log_2 \left(\frac{3}{2} \right)$$

$$\log_2 1.5 = \frac{\log_{10} 1.5}{\log_{10} 2} = 0.585 \text{ (3 s.f.)}$$