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Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise A, Question 1

Question:

Find the mid-point of the line joining these pairs of points:

(a) $(4, 2)$, $(6, 8)$

(b) $(0, 6)$, $(12, 2)$

(c) $(2, 2)$, $(-4, 6)$

(d) $(-6, 4)$, $(6, -4)$

(e) $(-5, 3)$, $(7, 5)$

(f) $(7, -4)$, $(-3, 6)$

(g) $(-5, -5)$, $(-11, 8)$

(h) $(6a, 4b)$, $(2a, -4b)$

(i) $(2p, -q)$, $(4p, 5q)$

(j) $(-2s, -7t)$, $(5s, t)$

(k) $(-4u, 0)$, $(3u, -2v)$

(l) $(a+b, 2a-b)$, $(3a-b, -b)$

(m) $(4\sqrt{2}, 1)$, $(2\sqrt{2}, 7)$

(n) $(-\sqrt{3}, 3\sqrt{5})$, $(5\sqrt{3}, 2\sqrt{5})$

(o) $(\sqrt{2}-\sqrt{3}, 3\sqrt{2}+4\sqrt{3})$, $(3\sqrt{2}+\sqrt{3}, -\sqrt{2}+2\sqrt{3})$

Solution:

(a) $(x_1, y_1) = (4, 2)$, $(x_2, y_2) = (6, 8)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4+6}{2}, \frac{2+8}{2}\right) = \left(\frac{10}{2}, \frac{10}{2}\right) = (5, 5)$

(b) $(x_1, y_1) = (0, 6)$, $(x_2, y_2) = (12, 2)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{0+12}{2}, \frac{6+2}{2}\right) = \left(\frac{12}{2}, \frac{8}{2}\right) = (6, 4)$

(c) $(x_1, y_1) = (2, 2)$, $(x_2, y_2) = (-4, 6)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+(-4)}{2}, \frac{2+6}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$

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(d) $(x_1, y_1) = (-6, 4)$, $(x_2, y_2) = (6, -4)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-6+6}{2}, \frac{4+(-4)}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = \left(0, 0\right)$

(e) $(x_1, y_1) = (-5, 3)$, $(x_2, y_2) = (7, 5)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+7}{2}, \frac{3+5}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = \left(1, 4\right)$

(f) $(x_1, y_1) = (7, -4)$, $(x_2, y_2) = (-3, 6)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{7+(-3)}{2}, \frac{-4+6}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = \left(2, 1\right)$

(g) $(x_1, y_1) = (-5, -5)$, $(x_2, y_2) = (-11, 8)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+(-11)}{2}, \frac{-5+8}{2}\right) = \left(\frac{-16}{2}, \frac{3}{2}\right) = \left(-8, \frac{3}{2}\right)$

(h) $(x_1, y_1) = (6a, 4b)$, $(x_2, y_2) = (2a, -4b)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6a+2a}{2}, \frac{4b+(-4b)}{2}\right) = \left(\frac{8a}{2}, \frac{0}{2}\right) = \left(4a, 0\right)$

(i) $(x_1, y_1) = (2p, -q)$, $(x_2, y_2) = (4p, 5q)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2p+4p}{2}, \frac{-q+5q}{2}\right) = \left(\frac{6p}{2}, \frac{4q}{2}\right) = \left(3p, 2q\right)$

(j) $(x_1, y_1) = (-2s, -7t)$, $(x_2, y_2) = (5s, t)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2s+5s}{2}, \frac{-7t+t}{2}\right) = \left(\frac{3s}{2}, \frac{-6t}{2}\right) = \left(\frac{3s}{2}, -3t\right)$

(k) $(x_1, y_1) = (-4u, 0)$, $(x_2, y_2) = (3u, -2v)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4u+3u}{2}, \frac{0+(-2v)}{2}\right) = \left(\frac{-u}{2}, -v\right)$

(l) $(x_1, y_1) = (a+b, 2a-b)$, $(x_2, y_2) = (3a-b, -b)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a+b+3a-b}{2}, \frac{2a-b+(-b)}{2}\right) = \left(\frac{4a}{2}, \frac{2a-2b}{2}\right) = \left(2a, a-b\right)$

(m) $(x_1, y_1) = (4\sqrt{2}, 1)$, $(x_2, y_2) = (2\sqrt{2}, 7)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4\sqrt{2}+2\sqrt{2}}{2}, \frac{1+7}{2}\right) = \left(\frac{6\sqrt{2}}{2}, \frac{8}{2}\right) = \left(3\sqrt{2}, 4\right)$

(n) $(x_1, y_1) = (-\sqrt{3}, 3\sqrt{5})$, $(x_2, y_2) = (5\sqrt{3}, 2\sqrt{5})$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-\sqrt{3}+5\sqrt{3}}{2}, \frac{3\sqrt{5}+2\sqrt{5}}{2}\right) = \left(\frac{4\sqrt{3}}{2}, \frac{5\sqrt{5}}{2}\right) = \left(2\sqrt{3}, \frac{5\sqrt{5}}{2}\right)$

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(o) $(x_1, y_1) = (\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3})$, $(x_2, y_2) = (3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} + (-\sqrt{2} + 2\sqrt{3})}{2}\right)$

$= \left(\frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} - \sqrt{2} + 2\sqrt{3}}{2}\right)$

$= \left(\frac{4\sqrt{2}}{2}, \frac{2\sqrt{2} + 6\sqrt{3}}{2}\right)$

$= (2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$

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Coordinate geometry in the (x,y) plane
Exercise A, Question 2

Question:

The line PQ is a diameter of a circle, where P and Q are $(-4, 6)$ and $(7, 8)$ respectively. Find the coordinates of the centre of the circle.

Solution:

$(x_1, y_1) = (-4, 6)$, $(x_2, y_2) = (7, 8)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 7}{2}, \frac{6 + 8}{2}\right) = \left(\frac{3}{2}, \frac{14}{2}\right) = \left(\frac{3}{2}, 7\right)$

The centre is $\left(\frac{3}{2}, 7\right)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 3

Question:

The line *RS* is a diameter of a circle, where *R* and *S* are $\left(\frac{4a}{5} , - \frac{3b}{4} \right)$ and $\left(\frac{2a}{5} , \frac{5b}{4} \right)$ respectively. Find the coordinates of the centre of the circle.

Solution:

$$\left(x_1 , y_1 \right) = \left(\frac{4a}{5} , - \frac{3b}{4} \right) , \left(x_2 , y_2 \right) = \left(\frac{2a}{5} , \frac{5b}{4} \right)$$

So $\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{\frac{4a}{5} + \frac{2a}{5}}{2} , \frac{\frac{-3b}{4} + \frac{5b}{4}}{2} \right) = \left(\frac{\frac{6a}{5}}{2} , \frac{\frac{2b}{4}}{2} \right) = \left(\frac{3a}{5} , \frac{b}{4} \right)$

The centre is $\left(\frac{3a}{5} , \frac{b}{4} \right)$.

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Coordinate geometry in the (x,y) plane

Exercise A, Question 4

Question:

The line AB is a diameter of a circle, where A and B are $(-3, -4)$ and $(6, 10)$ respectively. Show that the centre of the circle lies on the line $y = 2x$.

Solution:

$(x_1, y_1) = (-3, -4)$, $(x_2, y_2) = (6, 10)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 6}{2}, \frac{-4 + 10}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = \left(\frac{3}{2}, 3\right)$

Substitute $x = \frac{3}{2}$ into $y = 2x$:

$y = 2\left(\frac{3}{2}\right) = 3$ ✓

So the centre is on the line $y = 2x$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 5

Question:

The line *JK* is a diameter of a circle, where *J* and *K* are $\left(\frac{3}{4}, \frac{4}{3}\right)$ and $\left(-\frac{1}{2}, 2\right)$ respectively. Show that the centre of the circle lies on the line $y = 8x + \frac{2}{3}$.

Solution:

$$\left(x_1, y_1\right) = \left(\frac{3}{4}, \frac{4}{3}\right), \left(x_2, y_2\right) = \left(-\frac{1}{2}, 2\right)$$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\frac{3}{4} + \left(-\frac{1}{2}\right)}{2}, \frac{\frac{4}{3} + 2}{2}\right) = \left(\frac{\frac{1}{4}}{2}, \frac{\frac{10}{3}}{2}\right) = \left(\frac{1}{8}, \frac{5}{3}\right)$

Substitute $x = \frac{1}{8}$ into $y = 8x + \frac{2}{3}$:

$y = 8\left(\frac{1}{8}\right) + \frac{2}{3} = 1 + \frac{2}{3} = \frac{5}{3}$ ✓

So the centre is on the line $y = 8x + \frac{2}{3}$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 6

Question:

The line AB is a diameter of a circle, where A and B are $(0, -2)$ and $(6, -5)$ respectively. Show that the centre of the circle lies on the line $x - 2y - 10 = 0$.

Solution:

$(x_1, y_1) = (0, -2)$, $(x_2, y_2) = (6, -5)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 6}{2}, \frac{-2 + (-5)}{2} \right) = \left(\frac{6}{2}, \frac{-7}{2} \right) = \left(3, \frac{-7}{2} \right)$

Substitute $x = 3$ and $y = \frac{-7}{2}$ into $x - 2y - 10 = 0$:

$(3) - 2\left(\frac{-7}{2} \right) - 10 = 3 + 7 - 10 = 0$ ✓

So the centre is on the line $x - 2y - 10 = 0$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 7

Question:

The line FG is a diameter of the circle centre $(6, 1)$. Given F is $(2, -3)$, find the coordinates of G.

Solution:

$(x_1, y_1) = (a, b)$, $(x_2, y_2) = (2, -3)$
The centre is $(6, 1)$ so

$$\left(\frac{a+2}{2}, \frac{b+(-3)}{2}\right) = \left(6, 1\right)$$

$$\frac{a+2}{2} = 6$$

$a + 2 = 12$
$a = 10$

$$\frac{b+(-3)}{2} = 1$$

$$\frac{b-3}{2} = 1$$

$b - 3 = 2$
$b = 5$

The coordinates of G are $(10, 5)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 8

Question:

The line *CD* is a diameter of the circle centre $(-2a, 5a)$. Given *D* has coordinates $(3a, -7a)$, find the coordinates of *C*.

Solution:

$(x_1, y_1) = (p, q)$, $(x_2, y_2) = (3a, -7a)$
The centre is $(-2a, 5a)$ so

$$\left(\frac{p+3a}{2}, \frac{q+(-7a)}{2}\right) = \left(-2a, 5a\right)$$

$$\frac{p+3a}{2} = -2a$$

$p + 3a = -4a$
$p = -7a$

$$\frac{q+(-7a)}{2} = 5a$$

$$\frac{q-7a}{2} = 5a$$

$q - 7a = 10a$
$q = 17a$

The coordinates of *C* are $(-7a, 17a)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 9

Question:

The points $M(3, p)$ and $N(q, 4)$ lie on the circle centre $(5, 6)$. The line *MN* is a diameter of the circle. Find the value of *p* and *q*.

Solution:

$(x_1, y_1) = (3, p)$, $(x_2, y_2) = (q, 4)$ so

$$\left(\frac{3+q}{2}, \frac{p+4}{2}\right) = \left(5, 6\right)$$

$$\frac{3+q}{2} = 5$$

$3 + q = 10$
$q = 7$

$$\frac{p+4}{2} = 6$$

$p + 4 = 12$
$p = 8$

So $p = 8$, $q = 7$

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Coordinate geometry in the (x,y) plane
Exercise A, Question 10

Question:

The points $V(-4, 2a)$ and $W(3b, -4)$ lie on the circle centre $(b, 2a)$. The line VW is a diameter of the circle. Find the value of a and b.

Solution:

$(x_1, y_1) = (-4, 2a)$, $(x_2, y_2) = (3b, -4)$ so

$$\left(\frac{-4+3b}{2}, \frac{2a-4}{2} \right) = \left(b, 2a \right)$$

$$\frac{-4+3b}{2} = b$$

$-4 + 3b = 2b$
$-4 = -b$
$b = 4$

$$\frac{2a-4}{2} = 2a$$

$2a - 4 = 4a$
$-4 = 2a$
$a = -2$

So $a = -2$, $b = 4$

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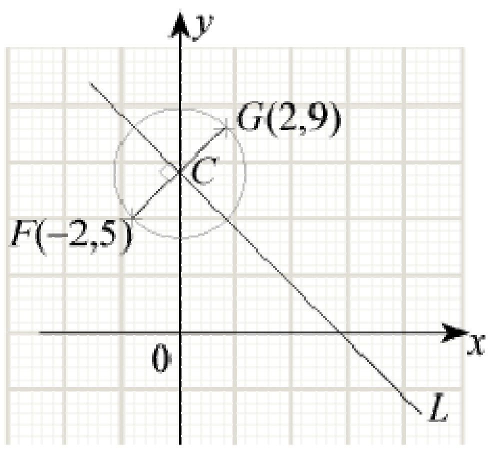
Coordinate geometry in the (x,y) plane
Exercise B, Question 1

Question:

The line *FG* is a diameter of the circle centre *C*, where *F* and *G* are $(-2 , 5)$ and $(2 , 9)$ respectively. The line *l* passes through *C* and is perpendicular to *FG*. Find the equation of *l*.

Solution:

(1)



(2) The gradient of *FG* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{2 - (-2)} = \frac{4}{4} = 1$$

(3) The gradient of a line perpendicular to *FG* is $\frac{-1}{(1)} = -1$.

(4) *C* is the mid-point of *FG*, so the coordinates of *C* are

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 2}{2} , \frac{5 + 9}{2} \right) = \left(\frac{0}{2} , \frac{14}{2} \right) = \left(0 , 7 \right)$$

(5) The equation of *l* is

$y - y_1 = m (x - x_1)$
$y - 7 = -1 (x - 0)$
$y - 7 = -x$
$y = -x + 7$

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Coordinate geometry in the (x,y) plane

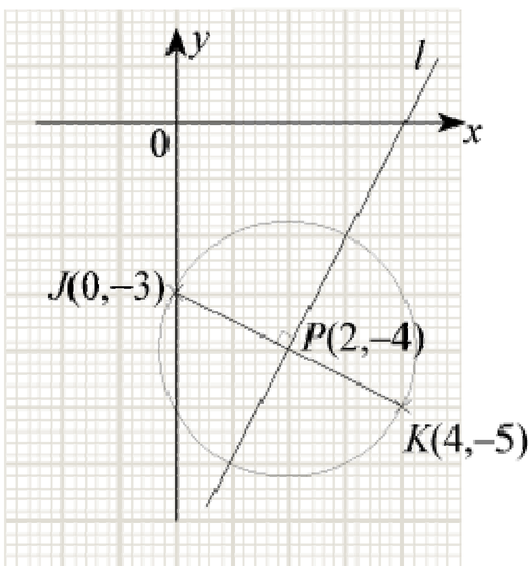
Exercise B, Question 2

Question:

The line *JK* is a diameter of the circle centre *P*, where *J* and *K* are $(0, -3)$ and $(4, -5)$ respectively. The line *l* passes through *P* and is perpendicular to *JK*. Find the equation of *l*. Write your answer in the form ax + by + *c* = 0, where *a*, *b* and *c* are integers.

Solution:

(1)



(2) The gradient of *JK* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{4 - 0} = \frac{-5 + 3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

(3) The gradient of a line perpendicular to *JK* is $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$

(4) *P* is the mid-point of *JK*, so the coordinates of *P* are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{-3 + (-5)}{2}\right) = \left(\frac{4}{2}, \frac{-8}{2}\right) = \left(2, -4\right)$$

(5) The equation of *l* is

$y - y_1 = m(x - x_1)$

$y - (-4) = 2(x - 2)$

$y + 4 = 2x - 4$

$0 = 2x - y - 4 - 4$

$2x - y - 8 = 0$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 3

Question:

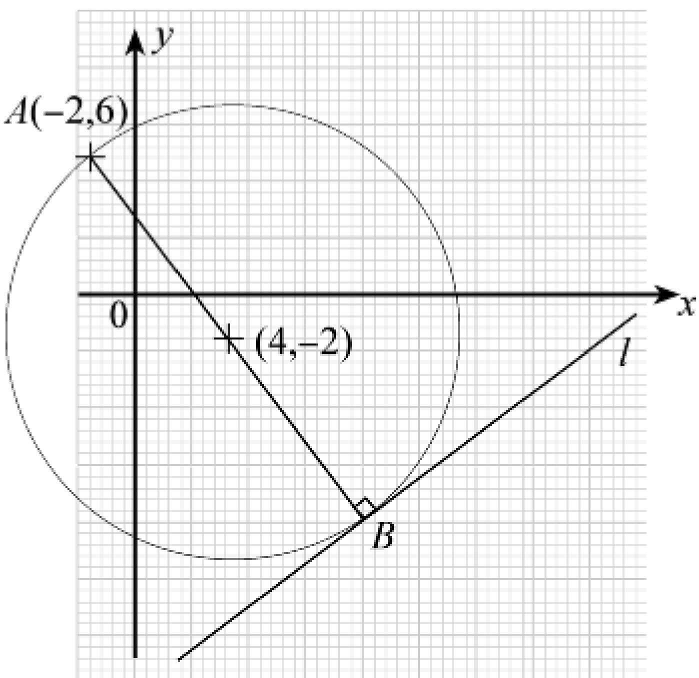
The line *AB* is a diameter of the circle centre $(4 , - 2)$. The line *l* passes through *B* and is perpendicular to *AB*. Given that *A* is $(- 2 , 6)$,

(a) find the coordinates of *B*.

(b) Hence, find the equation of *l*.

Solution:

(1)



(2) Let the coordinates of *B* be (a , b).
$(4 , - 2)$ is the mid-point of *AB* so

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(4 , - 2 \right)$$

i.e. $$\left(\frac{- 2 + a}{2} , \frac{6 + b}{2} \right) = \left(4 , - 2 \right)$$

So

$$\frac{- 2 + a}{2} = 4$$

$- 2 + a = 8$
$a = 10$
and

$$\frac{6 + b}{2} = - 2$$

$6 + b = - 4$
$b = - 10$

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(a) The coordinates of B are $(10, -10)$.

(3) Using $(-2, 6)$ and $(4, -2)$, the gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{4-(-2)} = \frac{-8}{6} = \frac{-4}{3}$$

(4) The gradient of a line perpendicular to AB is $\dfrac{-1}{\left(\frac{-4}{3}\right)} = \dfrac{3}{4}$

(5) The equation of l is

$$y - y_1 = m(x - x_1)$$

$$y - \left(-10\right) = \frac{3}{4}\left(x - 10\right)$$

$$y + 10 = \frac{3x}{4} - \frac{30}{4}$$

$$y = \frac{3x}{4} - \frac{30}{4} - 10$$

$$y = \frac{3x}{4} - \frac{70}{4}$$

$$y = \frac{3x}{4} - \frac{35}{2}$$

(b) The equation of l is $y = \frac{3}{4}x - \frac{35}{2}$.

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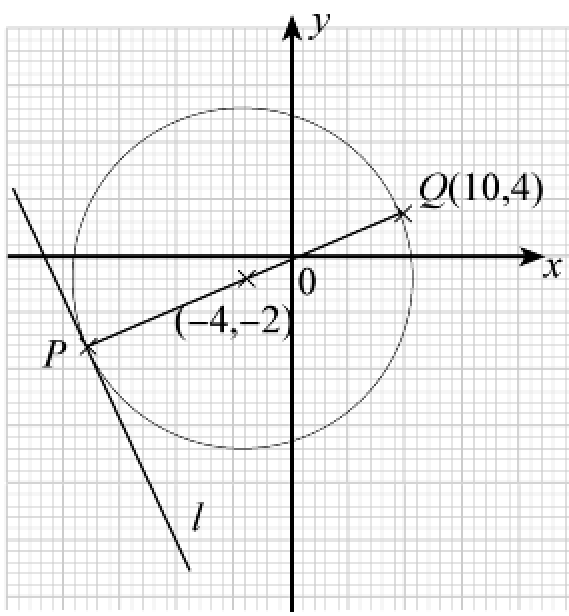
Coordinate geometry in the (x,y) plane
Exercise B, Question 4

Question:

The line *PQ* is a diameter of the circle centre $(-4, -2)$. The line *l* passes through *P* and is perpendicular to *PQ*. Given that *Q* is $(10, 4)$, find the equation of *l*.

Solution:

(1)



(2) Let the coordinates of *P* be (a, b).
$(-4, -2)$ is the mid-point of *PQ* so

$$\left(\frac{10+a}{2}, \frac{4+b}{2}\right) = \left(-4, -2\right)$$

$$\frac{10+a}{2} = -4$$

$10 + a = -8$
$a = -18$

$$\frac{4+b}{2} = -2$$

$4 + b = -4$
$b = -8$
The coordinates of *P* are $(-18, -8)$.

(3) The gradient of *PQ* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{10-(-4)} = \frac{6}{14} = \frac{3}{7}$$

(4) The gradient of a line perpendicular to *PQ* is $\frac{-1}{\left(\frac{3}{7}\right)} = \frac{-7}{3}$.

(5) The equation of *l* is

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$y - y_1 = m (x - x_1)$

$y - \left(-8 \right) = \frac{-7}{3} \left[x - \left(-18 \right) \right]$

$y + 8 = \frac{-7}{3} \left(x + 18 \right)$

$y + 8 = \frac{-7}{3} x - 42$

$y = \frac{-7}{3} x - 50$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 5

Question:

The line *RS* is a chord of the circle centre $(5, -2)$, where *R* and *S* are $(2, 3)$ and $(10, 1)$ respectively. The line l is perpendicular to *RS* and bisects it. Show that l passes through the centre of the circle.

Solution:

(1) The gradient of *RS* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{10 - 2} = \frac{-2}{8} = \frac{-1}{4}$$

(2) The gradient of a line perpendicular to *RS* is $\frac{-1}{\left(\frac{-1}{4}\right)} = 4$.

(3) The mid-point of *RS* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 10}{2}, \frac{3 + 1}{2}\right) = \left(\frac{12}{2}, \frac{4}{2}\right) = \left(6, 2\right)$$

(4) The equation of l is

$y - y_1 = m(x - x_1)$
$y - 2 = 4(x - 6)$
$y - 2 = 4x - 24$
$y = 4x - 22$

(5) Substitute $x = 5$ into $y = 4x - 22$:
$y = 4(5) - 22 = 20 - 22 = -2$ ✓
So l passes through the centre of the circle.

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Coordinate geometry in the (x,y) plane

Exercise B, Question 6

Question:

The line MN is a chord of the circle centre $\left(1 , - \frac{1}{2} \right)$, where M and N are $(- 5 , - 5)$ and $(7 , 4)$ respectively. The line l is perpendicular to MN and bisects it. Find the equation of l. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(1) The gradient of MN is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (- 5)}{7 - (- 5)} = \frac{4 + 5}{7 + 5} = \frac{9}{12} = \frac{3}{4}$$

(2) The gradient of a line perpendicular to MN is $\frac{-1}{\left(\frac{3}{4} \right)} = \frac{-4}{3}$.

(3) The coordinates of the mid-point of MN are

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{-5 + 7}{2} , \frac{-5 + 4}{2} \right) = \left(\frac{2}{2} , \frac{-1}{2} \right) = \left(1 , \frac{-1}{2} \right)$$

(4) The equation of l is

$y - y_1 = m (x - x_1)$

$$y - \left(\frac{-1}{2} \right) = \frac{-4}{3} \left(x - 1 \right)$$

$$y + \frac{1}{2} = \frac{-4}{3} \left(x - 1 \right)$$

$$y + \frac{1}{2} = \frac{-4}{3} x + \frac{4}{3}$$

$(\times 6)$
$6y + 3 = - 8x + 8$
$8x + 6y + 3 = 8$
$8x + 6y - 5 = 0$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 7

Question:

The lines AB and CD are chords of a circle. The line $y = 2x + 8$ is the perpendicular bisector of AB. The line $y = -2x - 4$ is the perpendicular bisector of CD. Find the coordinates of the centre of the circle.

Solution:

$y = 2x + 8$
$\underline{y = -2x - 4}$
$2y = 4$
$y = 2$
Substitute $y = 2$ into $y = 2x + 8$:
$2 = 2x + 8$
$-6 = 2x$
$x = -3$
Check.
Substitute $x = -3$ and $y = 2$ into $y = -2x - 4$:
$(2) = -2 (-3) - 4$
$2 = 6 - 4$
$2 = 2$ ✓
The coordinates of the centre of the circle are $(-3 , 2)$.

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Coordinate geometry in the (x,y) plane

Exercise B, Question 8

Question:

The lines EF and GH are chords of a circle. The line $y = 3x - 24$ is the perpendicular bisector of EF. Given G and F are $(-2, 4)$ and $(4, 10)$ respectively, find the coordinates of the centre of the circle.

Solution:

(1) The gradient of GF is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{4 - (-2)} = \frac{6}{6} = 1$$

(2) The gradient of a line perpendicular to GF is $-\frac{1}{(1)} = -1$.

(3) The mid-point of GF is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 4}{2}, \frac{4 + 10}{2} \right) = \left(\frac{2}{2}, \frac{14}{2} \right) = \left(1, 7 \right)$$

(4) The equation of the perpendicular bisector is

$y - y_1 = m(x - x_1)$
$y - 7 = -1(x - 1)$
$y - 7 = -x + 1$
$y = -x + 8$

(5) Solving $y = -x + 8$ and $y = 3x - 24$ simultaneously:
$-x + 8 = 3x - 24$
$-4x = -32$
$x = \frac{-32}{-4}$
$x = 8$
Substitute $x = 8$ into $y = -x + 8$:
$y = -(8) + 8$
$y = -8 + 8$
$y = 0$
So the centre of the circle is $(8, 0)$.

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Coordinate geometry in the (x,y) plane

Exercise B, Question 9

Question:

The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle.

(a) Find the equation of the perpendicular bisector of
(i) *PQ*
(ii) *PR*.

(b) Hence, find the coordinates of the centre of the circle.

Solution:

(a) (i) The gradient *PQ* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 16}{11 - 3} = \frac{-4}{8} = \frac{-1}{2}$$

The gradient of a line perpendicular to *PQ* is $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$.

The mid-point of *PQ* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 11}{2}, \frac{16 + 12}{2}\right) = \left(7, 14\right)$$

The equation of the perpendicular bisector of *PQ* is
$y - y_1 = m(x - x_1)$
$y - 14 = 2(x - 7)$
$y - 14 = 2x - 14$
$y = 2x$

(ii) The gradient of *PR* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 16}{-7 - 3} = \frac{-10}{-10} = 1$$

The gradient of a line perpendicular to *PR* is $-\frac{1}{(1)} = -1$.

The mid-point of *PR* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + (-7)}{2}, \frac{16 + 6}{2}\right) = \left(\frac{3 - 7}{2}, \frac{22}{2}\right) = \left(-2, 11\right)$$

The equation of the perpendicular bisector of *PR* is
$y - y_1 = m(x - x_1)$
$y - 11 = -1[x - (-2)]$
$y - 11 = -1(x + 2)$
$y - 11 = -x - 2$
$y = -x + 9$

(b) Solving $y = 2x$ and $y = -x + 9$ simultaneously:
$2x = -x + 9$
$3x = 9$
$x = 3$
Substitute $x = 3$ in $y = 2x$:
$y = 2(3)$

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$y = 6$

Check.

Substitute $x = 3$ and $y = 6$ into $y = -x + 9$:

$(6) = -(3) + 9$

$6 = -3 + 9$

$6 = 6$ ✓

The coordinates of the centre are $(3, 6)$.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 10

Question:

The points $A(-3, 19)$, $B(9, 11)$ and $C(-15, 1)$ lie on the circumference of a circle. Find the coordinates of the centre of the circle.

Solution:

(1) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 19}{9 - (-3)} = \frac{-8}{12} = \frac{-2}{3}$$

The gradient of a line perpendicular to AB is $\dfrac{-1}{\left(\frac{-2}{3}\right)} = \dfrac{3}{2}$.

The mid-point of AB is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 9}{2}, \frac{19 + 11}{2}\right) = \left(\frac{6}{2}, \frac{30}{2}\right) = (3, 15)$$

The equation of the perpendicular bisector of AB is

$y - y_1 = m(x - x_1)$

$$y - 15 = \frac{3}{2}\left(x - 3\right)$$

$$y - 15 = \frac{3}{2}x - \frac{9}{2}$$

$$y = \frac{3}{2}x + \frac{21}{2}$$

(2) The gradient of BC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-15 - 9} = \frac{-10}{-24} = \frac{5}{12}$$

The gradient of a line perpendicular to BC is $\dfrac{-1}{\left(\frac{5}{12}\right)} = \dfrac{-12}{5}$

The mid-point of BC is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{9 + (-15)}{2}, \frac{11 + 1}{2}\right) = \left(\frac{9 - 15}{2}, \frac{11 + 1}{2}\right) = \left(\frac{-6}{2}, \frac{12}{2}\right) = \left(-3, 6\right)$$

The equation of the perpendicular bisector of BC is

$y - y_1 = m(x - x_1)$

$$y - 6 = \frac{-12}{5}\left[x - \left(-3\right)\right]$$

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$y - 6 = \frac{-12}{5}\left(x + 3\right)$

$y - 6 = \frac{-12}{5}x - \frac{36}{5}$

$y = \frac{-12}{5}x - \frac{6}{5}$

(3) Solving $y = \frac{-12}{5}x - \frac{6}{5}$ and $y = \frac{3}{2}x + \frac{21}{2}$ simultaneously:

$\frac{3}{2}x + \frac{21}{2} = \frac{-12}{5}x - \frac{6}{5}$

$\frac{3}{2}x + \frac{12}{5}x = \frac{-6}{5} - \frac{21}{2}$

$\frac{39}{10}x = - \frac{117}{10}$

$39x = -117$

$x = -3$

Substitute $x = -3$ into $y = \frac{3}{2}x + \frac{21}{2}$:

$y = \frac{3}{2}\left(-3\right) + \frac{21}{2}$

$y = \frac{-9}{2} + \frac{21}{2}$

$y = \frac{12}{2}$

$y = 6$

Check.

Substitute $x = -3$ and $y = 6$ into $y = \frac{-12}{5}x - \frac{6}{5}$:

$\left(6\right) = \frac{-12}{5}\left(-3\right) - \frac{6}{5}$

$6 = \frac{36}{5} - \frac{6}{5}$

$6 = \frac{30}{5}$

$6 = 6$ ✓

The centre of the circle is $\left(-3, 6\right)$

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Coordinate geometry in the (x,y) plane
Exercise C, Question 1

Question:

Find the distance between these pairs of points:

(a) $(0, 1), (6, 9)$

(b) $(4, -6), (9, 6)$

(c) $(3, 1), (-1, 4)$

(d) $(3, 5), (4, 7)$

(e) $(2, 9), (4, 3)$

(f) $(0, -4), (5, 5)$

(g) $(-2, -7), (5, 1)$

(h) $(-4a, 0), (3a, -2a)$

(i) $(-b, 4b), (-4b, -2b)$

(j) $(2c, c), (6c, 4c)$

(k) $(-4d, d), (2d, -4d)$

(l) $(-e, -e), (-3e, -5e)$

(m) $(3\sqrt{2}, 6\sqrt{2}), (2\sqrt{2}, 4\sqrt{2})$

(n) $(-\sqrt{3}, 2\sqrt{3}), (3\sqrt{3}, 5\sqrt{3})$

(o) $(2\sqrt{3}-\sqrt{2}, \sqrt{5}+\sqrt{3}), (4\sqrt{3}-\sqrt{2}, 3\sqrt{5}+\sqrt{3})$

Solution:

(a) $(x_1, y_1) = (0, 1), (x_2, y_2) = (6, 9)$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(6-0)^2+(9-1)^2}$
$= \sqrt{6^2+8^2}$
$= \sqrt{36+64}$
$= \sqrt{100}$
$= 10$

(b) $(x_1, y_1) = (4, -6), (x_2, y_2) = (9, 6)$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(9-4)^2+[6-(-6)]^2}$
$= \sqrt{5^2+12^2}$
$= \sqrt{25+144}$
$= \sqrt{169}$
$= 13$

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(c) $(x_1, y_1) = (3, 1)$, $(x_2, y_2) = (-1, 4)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-3)^2 + (4-1)^2}$

$= \sqrt{(-4)^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

(d) $(x_1, y_1) = (3, 5)$, $(x_2, y_2) = (4, 7)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-3)^2 + (7-5)^2}$

$= \sqrt{1^2 + 2^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$

(e) $(x_1, y_1) = (2, 9)$, $(x_2, y_2) = (4, 3)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (3-9)^2}$

$= \sqrt{2^2 + (-6)^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$= \sqrt{4 \times 10}$

$= \sqrt{4} \times \sqrt{10}$

$= 2\sqrt{10}$

(f) $(x_1, y_1) = (0, -4)$, $(x_2, y_2) = (5, 5)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-0)^2 + [5-(-4)]^2}$

$= \sqrt{5^2 + 9^2}$

$= \sqrt{25 + 81}$

$= \sqrt{106}$

(g) $(x_1, y_1) = (-2, -7)$, $(x_2, y_2) = (5, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[5-(-2)]^2 + [1-(-7)]^2}$

$= \sqrt{(5+2)^2 + (1+7)^2}$

$= \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$

$= \sqrt{113}$

(h) $(x_1, y_1) = (-4a, 0)$, $(x_2, y_2) = (3a, -2a)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[3a-(-4a)]^2 + (-2a-0)^2}$

$= \sqrt{(3a+4a)^2 + (-2a)^2}$

$= \sqrt{(7a)^2 + (-2a)^2}$

$= \sqrt{49a^2 + 4a^2}$

$= \sqrt{53a^2}$

$= \sqrt{53}\sqrt{a^2}$

$= a\sqrt{53}$

(i) $(x_1, y_1) = (-b, 4b)$, $(x_2, y_2) = (-4b, -2b)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[-4b-(-b)]^2 + (-2b-4b)^2}$

$= \sqrt{(-4b+b)^2 + (-6b)^2}$

$= \sqrt{(-3b)^2 + (-6b)^2}$

$= \sqrt{9b^2 + 36b^2}$

$= \sqrt{45b^2}$

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$= \sqrt{9 \times 5 \times b^2}$
$= \sqrt{9}\sqrt{5}\sqrt{b^2}$
$= 3b\sqrt{5}$

(j) $(x_1, y_1) = (2c, c)$, $(x_2, y_2) = (6c, 4c)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6c - 2c)^2 + (4c - c)^2}$
$= \sqrt{(4c)^2 + (3c)^2}$
$= \sqrt{16c^2 + 9c^2}$
$= \sqrt{25c^2}$
$= \sqrt{25}\sqrt{c^2}$
$= 5c$

(k) $(x_1, y_1) = (-4d, d)$, $(x_2, y_2) = (2d, -4d)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[2d - (-4d)]^2 + (-4d - d)^2}$
$= \sqrt{(2d + 4d)^2 + (-5d)^2}$
$= \sqrt{(6d)^2 + (-5d)^2}$
$= \sqrt{36d^2 + 25d^2}$
$= \sqrt{61d^2}$
$= \sqrt{61}\sqrt{d^2}$
$= d\sqrt{61}$

(l) $(x_1, y_1) = (-e, -e)$, $(x_2, y_2) = (-3e, -5e)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-3e - (-e)]^2 + [-5e - (-e)]^2}$
$= \sqrt{(-3e + e)^2 + (-5e + e)^2}$
$= \sqrt{(-2e)^2 + (-4e)^2}$
$= \sqrt{4e^2 + 16e^2}$
$= \sqrt{20e^2}$
$= \sqrt{4 \times 5 \times e^2}$
$= \sqrt{4} \times \sqrt{5} \times \sqrt{e^2}$
$= 2\sqrt{5}e$

(m) $(x_1, y_1) = (3\sqrt{2}, 6\sqrt{2})$, $(x_2, y_2) = (2\sqrt{2}, 4\sqrt{2})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(2\sqrt{2} - 3\sqrt{2})^2 + (4\sqrt{2} - 6\sqrt{2})^2}$
$= \sqrt{(-\sqrt{2})^2 + (-2\sqrt{2})^2}$
$= \sqrt{2 + 8}$
$= \sqrt{10}$

(n) $(x_1, y_1) = (-\sqrt{3}, 2\sqrt{3})$, $(x_2, y_2) = (3\sqrt{3}, 5\sqrt{3})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3\sqrt{3} - (-\sqrt{3})]^2 + (5\sqrt{3} - 2\sqrt{3})^2}$
$= \sqrt{(3\sqrt{3} + \sqrt{3})^2 + (3\sqrt{3})^2}$
$= \sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2}$
$= \sqrt{48 + 27}$
$= \sqrt{75}$
$= \sqrt{25 \times 3}$
$= \sqrt{25} \times \sqrt{3}$
$= 5\sqrt{3}$

(o) $(x_1, y_1) = (2\sqrt{3} - \sqrt{2}, \sqrt{5} + \sqrt{3})$, $(x_2, y_2) = (4\sqrt{3} - \sqrt{2}, 3\sqrt{5} + \sqrt{3})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[4\sqrt{3} - \sqrt{2} - (2\sqrt{3} - \sqrt{2})]^2 + [3\sqrt{5} + \sqrt{3} - (\sqrt{5} + \sqrt{3})]^2}$
$= \sqrt{(4\sqrt{3} - \sqrt{2} - 2\sqrt{3} + \sqrt{2})^2 + (3\sqrt{5} + \sqrt{3} - \sqrt{5} - \sqrt{3})^2}$
$= \sqrt{(2\sqrt{3})^2 + (2\sqrt{5})^2}$

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$= \sqrt{12 + 20}$
$= \sqrt{32}$
$= \sqrt{16 \times 2}$
$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$

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Coordinate geometry in the (x,y) plane

Exercise C, Question 2

Question:

The point $(4, -3)$ lies on the circle centre $(-2, 5)$. Find the radius of the circle.

Solution:

$(x_1, y_1) = (4, -3), (x_2, y_2) = (-2, 5)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2 - 4)^2 + [5 - (-3)]^2}$

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

Radius of circle $= 10$.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 3

Question:

The point $(14, 9)$ is the centre of the circle radius 25. Show that $(-10, 2)$ lies on the circle.

Solution:

$(x_1, y_1) = (-10, 2)$, $(x_2, y_2) = (14, 9)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[14 - (-10)]^2 + (9 - 2)^2}$

$= \sqrt{24^2 + 7^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25$

So $(-10, 2)$ is on the circle.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 4

Question:

The line *MN* is a diameter of a circle, where *M* and *N* are $(6, -4)$ and $(0, -2)$ respectively. Find the radius of the circle.

Solution:

$(x_1, y_1) = (6, -4)$, $(x_2, y_2) = (0, -2)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-6)^2 + [-2-(-4)]^2}$

$= \sqrt{(-6)^2 + (-2+4)^2}$

$= \sqrt{(-6)^2 + (2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= \sqrt{4 \times 10}$

$= \sqrt{4} \times \sqrt{10}$

$= 2\sqrt{10}$

The diameter has length $2\sqrt{10}$.

So the radius has length $\frac{2\sqrt{10}}{2} = \sqrt{10}$.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 5

Question:

The line *QR* is a diameter of the circle centre *C*, where *Q* and *R* have coordinates $(11, 12)$ and $(-5, 0)$ respectively. The point *P* is $(13, 6)$.

(a) Find the coordinates of *C*.

(b) Show that *P* lies on the circle.

Solution:

(a) The mid-point of *QR* is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{11+(-5)}{2}, \frac{12+0}{2}\right) = \left(\frac{11-5}{2}, \frac{12}{2}\right) = \left(\frac{6}{2}, \frac{12}{2}\right) = (3, 6)$$

(b) The radius of the circle is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(11-3)^2+(12-6)^2}$$
$$=\sqrt{8^2+6^2}$$
$$=\sqrt{64+36}$$
$$=\sqrt{100}$$
$$=10$$

The distance between *C* and *P* is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(13-3)^2+(6-6)^2}$$
$$=\sqrt{10^2+0^2}$$
$$=\sqrt{10^2}$$
$$=10$$

So *P* is on the circle.

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Coordinate geometry in the (x,y) plane

Exercise C, Question 6

Question:

The points $(-3, 19)$, $(-15, 1)$ and $(9, 1)$ are vertices of a triangle. Show that a circle centre $(-3, 6)$ can be drawn through the vertices of the triangle.

Solution:

(1) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (-3, 19)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[(-3) - (-3)]^2 + (19 - 6)^2}$
$= \sqrt{(-3 + 3)^2 + (13)^2}$
$= \sqrt{0^2 + 13^2}$
$= \sqrt{13^2}$
$= 13$

(2) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (-15, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-15 - (-3)]^2 + (1 - 6)^2}$
$= \sqrt{(-12)^2 + (-5)^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

(3) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (9, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[9 - (-3)]^2 + (1 - 6)^2}$
$= \sqrt{(12)^2 + (-5)^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

The distance of each vertex of the triangle to $(-3, 6)$ is 13. So a circle centre $(-3, 6)$ and radius 13 can be drawn through the vertices of the triangle.

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Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise C, Question 7

Question:

The line ST is a diameter of the circle c_1, where S and T are $(5, 3)$ and $(-3, 7)$ respectively.
The line UV is a diameter of the circle c_2 centre $(4, 4)$. The point U is $(1, 8)$.

(a) Find the radius of (i) c_1 (ii) c_2.

(b) Find the distance between the centres of c_1 and c_2.

Solution:

(a) (i) The centre of c_1 is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+(-3)}{2}, \frac{3+7}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1, 5)$$

The radius of c_1 is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5-1)^2+(3-5)^2}$$
$$= \sqrt{4^2+(-2)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= \sqrt{4\times 5}$$
$$= \sqrt{4}\times\sqrt{5}$$
$$= 2\sqrt{5}$$

(ii) The radius of c_2 is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(4-1)^2+(4-8)^2}$$
$$= \sqrt{3^2+(-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

(b) The distance between the centres is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1-4)^2+(5-4)^2}$$
$$= \sqrt{(-3)^2+(1)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

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Coordinate geometry in the (x,y) plane

Exercise C, Question 8

Question:

The points $U(-2, 8)$, $V(7, 7)$ and $W(-3, -1)$ lie on a circle.

(a) Show that $\triangle UVW$ has a right angle.

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (1) The distance *UV* is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[7 - (-2)]^2 + (7 - 8)^2}$

$= \sqrt{(7 + 2)^2 + (-1)^2}$

$= \sqrt{9^2 + (-1)^2}$

$= \sqrt{81 + 1}$

$= \sqrt{82}$

(2) The distance *VW* is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 7)^2 + (-1 - 7)^2}$

$= \sqrt{(-10)^2 + (-8)^2}$

$= \sqrt{100 + 64}$

$= \sqrt{164}$

(3) The distance *UW* is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[-3 - (-2)]^2 + (-1 - 8)^2}$

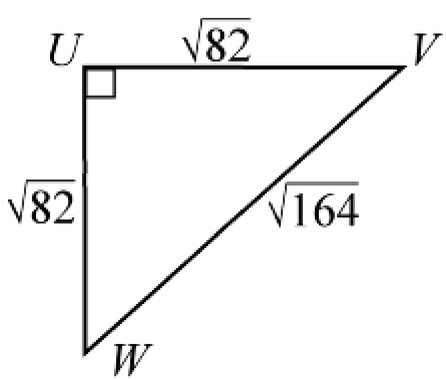
$= \sqrt{(-3 + 2)^2 + (-9)^2}$

$= \sqrt{(-1)^2 + (-9)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

Now $(\sqrt{82})^2 + (\sqrt{82})^2 = (\sqrt{164})^2$



i.e. $UV^2 + UW^2 = VW^2$

So, by Pythagoras' theorem, $\triangle UVW$ has a right angle at *U*.

(b) The angle in a semicircle is a right angle. So *VW* is a diameter of the circle.

The mid-point of *VW* is

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$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{7 + (- 3)}{2} , \frac{7 + (- 1)}{2} \right) = \left(\frac{7 - 3}{2} , \frac{7 - 1}{2} \right) = \left(2 , 3 \right)$$

The centre of the circle is $(2 , 3)$.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 9

Question:

The points $A(2, 6)$, $B(5, 7)$ and $C(8, -2)$ lie on a circle.

(a) Show that $\triangle ABC$ has a right angle.

(b) Find the area of the triangle.

Solution:

(a) (1) The distance AB is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 2)^2 + (7 - 6)^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

(2) The distance BC is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 5)^2 + (-2 - 7)^2}$

$= \sqrt{3^2 + (-9)^2}$

$= \sqrt{9 + 81}$

$= \sqrt{90}$

(3) The distance AC is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 2)^2 + (-2 - 6)^2}$

$= \sqrt{6^2 + (-8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

Now $(\sqrt{10})^2 + (\sqrt{90})^2 = (\sqrt{100})^2$

i.e. $AB^2 + BC^2 = AC^2$

So, by Pythagoras' theorem, there is a right angle at B.

(b) The area of the triangle is

$\frac{1}{2} \times \text{ base} \times \text{height } = \frac{1}{2} \times AB \times BC = \frac{1}{2} \sqrt{10} \sqrt{90} = \frac{1}{2} \sqrt{900} = \frac{1}{2} \times 30 = 15$

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Coordinate geometry in the (x,y) plane

Exercise C, Question 10

Question:

The points $A(-1,9)$, $B(6,10)$, $C(7,3)$ and $D(0,2)$ lie on a circle.

(a) Show that *ABCD* is a square.

(b) Find the area of *ABCD*.

(c) Find the centre of the circle.

Solution:

(a) (1) The length of *AB* is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{[6-(-1)]^2+(10-9)^2}$

$=\sqrt{7^2+1^2}$

$=\sqrt{49+1}$

$=\sqrt{50}$

(2) The length of *BC* is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-6)^2+(3-10)^2}$

$=\sqrt{1^2+(-7)^2}$

$=\sqrt{1+49}$

$=\sqrt{50}$

(3) The length of *CD* is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(0-7)^2+(2-3)^2}$

$=\sqrt{(-7)^2+(-1)^2}$

$=\sqrt{49+1}$

$=\sqrt{50}$

(4) The length of *DA* is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-1-0)^2+(9-2)^2}$

$=\sqrt{(-1)^2+7^2}$

$=\sqrt{1+49}$

$=\sqrt{50}$

The sides of the quadrilateral are equal.

(5) The gradient of *AB* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{10-9}{6-(-1)}=\frac{1}{7}$$

The gradient of *BC* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{3-10}{7-6}=\frac{-7}{1}=-7$$

The product of the gradients $=-1$ $\left(\frac{1}{7}\times-7=-1\right)$.

So the line *AB* is perpendicular to *BC*.
So the quadrilateral *ABCD* is a square.

(b) The area $=\sqrt{50}\times\sqrt{50}=50$

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(c) The mid-point of *AC* is

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{-1 + 7}{2} , \frac{9 + 3}{2} \right) = \left(\frac{6}{2} , \frac{12}{2} \right) = \left(3 , 6 \right)$$

So the centre of the circle is $(3 , 6)$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 1

Question:

Write down the equation of these circles:

(a) Centre $(3, 2)$, radius 4

(b) Centre $(-4, 5)$, radius 6

(c) Centre $(5, -6)$, radius $2\sqrt{3}$

(d) Centre $(2a, 7a)$, radius $5a$

(e) Centre $(-2\sqrt{2}, -3\sqrt{2})$, radius 1

Solution:

(a) $(x_1, y_1) = (3, 2)$, $r = 4$
So $(x-3)^2 + (y-2)^2 = 4^2$
or $(x-3)^2 + (y-2)^2 = 16$

(b) $(x_1, y_1) = (-4, 5)$, $r = 6$
So $[x-(-4)]^2 + (y-5)^2 = 6^2$
or $(x+4)^2 + (y-5)^2 = 36$

(c) $(x_1, y_1) = (5, -6)$, $r = 2\sqrt{3}$
So $(x-5)^2 + [y-(-6)]^2 = (2\sqrt{3})^2$
$(x-5)^2 + (y+6)^2 = 2^2(\sqrt{3})^2$
$(x-5)^2 + (y+6)^2 = 4 \times 3$
$(x-5)^2 + (y+6)^2 = 12$

(d) $(x_1, y_1) = (2a, 7a)$, $r = 5a$
So $(x-2a)^2 + (y-7a)^2 = (5a)^2$
or $(x-2a)^2 + (y-7a)^2 = 25a^2$

(e) $(x_1, y_1) = (-2\sqrt{2}, -3\sqrt{2})$, $r = 1$
So $[x-(-2\sqrt{2})]^2 + [y-(-3\sqrt{2})]^2 = 1^2$
or $(x+2\sqrt{2})^2 + (y+3\sqrt{2})^2 = 1$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 2

Question:

Write down the coordinates of the centre and the radius of these circles:

(a) $(x+5)^2+(y-4)^2=9^2$

(b) $(x-7)^2+(y-1)^2=16$

(c) $(x+4)^2+y^2=25$

(d) $(x+4a)^2+(y+a)^2=144a^2$

(e) $(x-3\sqrt{5})^2+(y+\sqrt{5})^2=27$

Solution:

(a) $(x+5)^2+(y-4)^2=9^2$
or $[x-(-5)]^2+(y-4)^2=9^2$
The centre of the circle is $(-5, 4)$ and the radius is 9.

(b) $(x-7)^2+(y-1)^2=16$
or $(x-7)^2+(y-1)^2=4^2$
The centre of the circle is $(7, 1)$ and the radius is 4.

(c) $(x+4)^2+y^2=25$
or $[x-(-4)]^2+(y-0)^2=5^2$
The centre of the circle is $(-4, 0)$ and the radius is 5.

(d) $(x+4a)^2+(y+a)^2=144a^2$
or $[x-(-4a)]^2+[y-(-a)]^2=(12a)^2$
The centre of the circle is $(-4a, -a)$ and the radius is $12a$.

(e) $(x-3\sqrt{5})^2+(y+\sqrt{5})^2=27$
or $(x-3\sqrt{5})^2+[y-(-\sqrt{5})]^2=(\sqrt{27})^2$
Now $\sqrt{27}=\sqrt{9\times 3}=\sqrt{9}\times\sqrt{3}=3\sqrt{3}$
The centre of the circle is $(3\sqrt{5}, -\sqrt{5})$ and the radius is $3\sqrt{3}$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 3

Question:

Find the centre and radius of these circles by first writing in the form $(x-a)^2+(y-b)^2=r^2$

(a) $x^2+y^2+4x+9y+3=0$

(b) $x^2+y^2+5x-3y-8=0$

(c) $2x^2+2y^2+8x+15y-1=0$

(d) $2x^2+2y^2-8x+8y+3=0$

Solution:

(a) $x^2+y^2+4x+9y+3=0$
$x^2+4x+y^2+9y=-3$

$$(x+2)^2-4\left(y+\frac{9}{2}\right)^2-\frac{81}{4}=-3$$

$$(x+2)^2+\left(y+\frac{9}{2}\right)^2=\frac{85}{4}$$

So the centre is $(-2,-4.5)$ and the radius is 4.61 (2 d.p.)

(b) $x^2+y^2+5x-3y-8=0$
$x^2+5x+y^2-3y=8$

$$\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+\left(y-\frac{3}{2}\right)^2-\frac{9}{4}=8$$

$$\left(x+\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=16.5$$

So the centre is $(-2.5, 1.5)$ and the radius is 4.06 (2 d.p.)

(c) $2x^2+2y^2+8x+15y-1=0$

$x^2+y^2+4x+\frac{15}{2}y-½=0$

$x^2+4x+y^2+\frac{15}{2}y=½$

$$(x+2)^2-4+\left(y+\frac{15}{4}\right)^2-\frac{225}{16}=½$$

$$(x+2)^2+\left(y+\frac{15}{4}\right)^2=18\frac{9}{16}$$

So the centre is $(-2,-3.75)$ and the radius is 4.31 (2 d.p.)

(d) $2x^2+2y^2-8x+8y+3=0$

$x^2+y^2-4x+4y+\frac{3}{2}=0$

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$x^2 - 4x + y^2 + 4y = -\frac{3}{2}$

$(x - 2)^2 - 4 + (y + 2)^2 - 4 = -\frac{3}{2}$

$(x - 2)^2 + (y + 2)^2 = \frac{13}{2}$

So the centre is $(2, -2)$ and the radius is 2.55 (2 d.p.)

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Coordinate geometry in the (x,y) plane
Exercise D, Question 4

Question:

In each case, show that the circle passes through the given point:

(a) $(x-2)^2+(y-5)^2=13$, $(4,8)$

(b) $(x+7)^2+(y-2)^2=65$, $(0,-2)$

(c) $x^2+y^2=25^2$, $(7,-24)$

(d) $(x-2a)^2+(y+5a)^2=20a^2$, $(6a,-3a)$

(e) $(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(2\sqrt{10})^2$, $(\sqrt{5},-\sqrt{5})$

Solution:

(a) Substitute $x=4$, $y=8$ into $(x-2)^2+(y-5)^2=13$
$(x-2)^2+(y-5)^2=(4-2)^2+(8-5)^2=2^2+3^2=4+9=13$ ✓
So the circle passes through $(4,8)$.

(b) Substitute $x=0$, $y=-2$ into $(x+7)^2+(y-2)^2=65$
$(x+7)^2+(y-2)^2=(0+7)^2+(-2-2)^2=7^2+(-4)^2=49+16=65$ ✓
So the circle passes through $(0,-2)$.

(c) Substitute $x=7$ and $y=-24$ into $x^2+y^2=25^2$
$x^2+y^2=7^2+(-24)^2=49+576=625=25^2$ ✓
So the circle passes through $(7,-24)$.

(d) Substitute $x=6a$, $y=-3a$ into $(x-2a)^2+(y+5a)^2=20a^2$
$(x-2a)^2+(y+5a)^2=(6a-2a)^2+(-3a+5a)^2=(4a)^2+(2a)^2=16a^2+4a^2=20a^2$ ✓
So the circle passes through $(6a,-3a)$.

(e) Substitute $x=\sqrt{5}$, $y=-\sqrt{5}$ into $(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(2\sqrt{10})^2$
$(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(\sqrt{5}-3\sqrt{5})^2+(-\sqrt{5}-\sqrt{5})^2=(-2\sqrt{5})^2+(-2\sqrt{5})^2=4\times5+4\times5=20+20=40=(\sqrt{40})^2$
Now $\sqrt{40}=\sqrt{4\times10}=\sqrt{4}\times\sqrt{10}=2\sqrt{10}$ ✓
So the circle passes through $(\sqrt{5},-\sqrt{5})$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 5

Question:

The point $(4, -2)$ lies on the circle centre $(8, 1)$. Find the equation of the circle.

Solution:

The radius of the circle is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8-4)^2 + [1-(-2)]^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

The centre of the circle is $(8, 1)$ and the radius is 5.

So $(x-8)^2 + (y-1)^2 = 5^2$

or $(x-8)^2 + (y-1)^2 = 25$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 6

Question:

The line PQ is the diameter of the circle, where P and Q are $(5, 6)$ and $(-2, 2)$ respectively. Find the equation of the circle.

Solution:

(1) The centre of the circle is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+(-2)}{2}, \frac{6+2}{2}\right) = \left(\frac{3}{2}, \frac{8}{2}\right) = \left(\frac{3}{2}, 4\right)$$

(2) The radius of the circle is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{\left(5-\frac{3}{2}\right)^2+(6-4)^2}$$

$$= \sqrt{\left(\frac{7}{2}\right)^2+(2)^2}$$

$$= \sqrt{\frac{49}{4}+4}$$

$$= \sqrt{\frac{49}{4}+\frac{16}{4}}$$

$$= \sqrt{\frac{65}{4}}$$

So the equation of the circle is

$$\left(x-\frac{3}{2}\right)^2+(y-4)^2 = \left(\sqrt{\frac{65}{4}}\right)^2$$

or $\left(x-\frac{3}{2}\right)^2+(y-4)^2 = \frac{65}{4}$

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Coordinate geometry in the (x,y) plane

Exercise D, Question 7

Question:

The point $(1, -3)$ lies on the circle $(x-3)^2 + (y+4)^2 = r^2$. Find the value of r.

Solution:

Substitute $x = 1$, $y = -3$ into $(x-3)^2 + (y+4)^2 = r^2$

$(1-3)^2 + (-3+4)^2 = r^2$

$(-2)^2 + (1)^2 = r^2$

$4 + 1 = r^2$

$5 = r^2$

So $r = \sqrt{5}$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 8

Question:

The line $y = 2x + 13$ touches the circle $x^2 + (y - 3)^2 = 20$ at $(-4, 5)$. Show that the radius at $(-4, 5)$ is perpendicular to the line.

Solution:

(1) The centre of the circle $x^2 + (y - 3)^2 = 20$ is $(0, 3)$.
(2) The gradient of the line joining $(0, 3)$ and $(-4, 5)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-4 - 0} = \frac{2}{-4} = -\frac{1}{2}$$

(3) The gradient of $y = 2x + 13$ is 2.
(4) The product of the gradients is

$$-\frac{1}{2} \times 2 = -1$$

So the radius is perpendicular to the line.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 9

Question:

The line $x + 3y - 11 = 0$ touches the circle $(x + 1)^2 + (y + 6)^2 = 90$ at $(2 , 3)$.

(a) Find the radius of the circle.

(b) Show that the radius at $(2 , 3)$ is perpendicular to the line.

Solution:

(a) The radius of the circle $(x + 1)^2 + (y + 6)^2 = 90$ is $\sqrt{90}$.
$\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$

(b) (1) The centre of the circle $(x + 1)^2 + (y + 6)^2 = 90$ is $(-1 , -6)$.
(2) The gradient of the line joining $(-1 , -6)$ and $(2 , 3)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-6)}{2 - (-1)} = \frac{3 + 6}{2 + 1} = \frac{9}{3} = 3$$

(3) Rearrange $x + 3y - 11 = 0$ into the form $y = mx + c$
$x + 3y - 11 = 0$
$3y - 11 = -x$
$3y = -x + 11$
$y = -\frac{1}{3}x + \frac{11}{3}$

So the gradient of $x + 3y - 11 = 0$ is $-\frac{1}{3}$.

(4) The product of the gradients is

$3 \times -\frac{1}{3} = -1$

So the radius is perpendicular to the line.

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Coordinate geometry in the (x,y) plane

Exercise D, Question 10

Question:

The point $P(1, -2)$ lies on the circle centre $(4, 6)$.

(a) Find the equation of the circle.

(b) Find the equation of the tangent to the circle at P.

Solution:

(a) (1) The radius of the circle is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-1)^2 + [6-(-2)]^2} = \sqrt{3^2 + 8^2} = \sqrt{9+64} = \sqrt{73}$

(2) The equation of the circle is

$(x-4)^2 + (y-6)^2 = (\sqrt{73})^2$

or $(x-4)^2 + (y-6)^2 = 73$

(b) (1) The gradient of the line joining $(1, -2)$ and $(4, 6)$ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-2)}{4-1} = \frac{6+2}{3} = \frac{8}{3}$

(2) The gradient of the tangent is $\frac{-1}{\left(\frac{8}{3}\right)} = -\frac{3}{8}$.

(3) The equation of the tangent to the circle at $(1, -2)$ is

$y - y_1 = m(x - x_1)$

$y - \left(-2\right) = -\frac{3}{8}\left(x-1\right)$

$y + 2 = -\frac{3}{8}\left(x-1\right)$

$8y + 16 = -3(x-1)$

$8y + 16 = -3x + 3$

$3x + 8y + 16 = 3$

$3x + 8y + 13 = 0$

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Coordinate geometry in the (x,y) plane
Exercise E, Question 1

Question:

Find where the circle $(x - 1)^2 + (y - 3)^2 = 45$ meets the x-axis.

Solution:

Substitute $y = 0$ into $(x - 1)^2 + (y - 3)^2 = 45$
$(x - 1)^2 + (- 3)^2 = 45$
$(x - 1)^2 + 9 = 45$
$(x - 1)^2 = 36$
$x - 1 = \pm \sqrt{36}$
$x - 1 = \pm 6$
So $x - 1 = 6 \Rightarrow x = 7$
and $x - 1 = - 6 \Rightarrow x = - 5$
The circle meets the x-axis at $(7 , 0)$ and $(- 5 , 0)$.

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Coordinate geometry in the (x,y) plane

Exercise E, Question 2

Question:

Find where the circle $(x - 2)^2 + (y + 3)^2 = 29$ meets the y-axis.

Solution:

Substitute $x = 0$ into $(x - 2)^2 + (y + 3)^2 = 29$

$(- 2)^2 + (y + 3)^2 = 29$

$4 + (y + 3)^2 = 29$

$(y + 3)^2 = 25$

$y + 3 = \pm \sqrt{25}$

$y + 3 = \pm 5$

So $y + 3 = 5 \quad \Rightarrow \quad y = 2$

and $y + 3 = - 5 \quad \Rightarrow \quad y = - 8$

The circle meets the y-axis at $(0 , 2)$ and $(0 , - 8)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 3

Question:

The circle $(x-3)^2+(y+3)^2=34$ meets the x-axis at $(a, 0)$ and the y-axis at $(0, b)$. Find the possible values of a and b.

Solution:

(1) Substitute $x=a$, $y=0$ into $(x-3)^2+(y+3)^2=34$
$(a-3)^2+(3)^2=34$
$(a-3)^2+9=34$
$(a-3)^2=25$
$a-3=\pm\sqrt{25}$
$a-3=\pm 5$
So $a-3=5 \Rightarrow a=8$
and $a-3=-5 \Rightarrow a=-2$
The circle meets the x-axis at $(8, 0)$ and $(-2, 0)$.
(2) Substitute $x=0$, $y=b$ into $(x-3)^2+(y+3)^2=34$
$(-3)^2+(b+3)^2=34$
$9+(b+3)^2=34$
$(b+3)^2=25$
$b+3=\pm\sqrt{25}$
$b+3=\pm 5$
So $b+3=5 \Rightarrow b=2$
and $b+3=-5 \Rightarrow b=-8$
The circle meets the y-axis at $(0, 2)$ and $(0, -8)$.

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Coordinate geometry in the (x,y) plane

Exercise E, Question 4

Question:

The line $y = x + 4$ meets the circle $(x-3)^2 + (y-5)^2 = 34$ at A and B. Find the coordinates of A and B.

Solution:

Substitute $y = x + 4$ into $(x-3)^2 + (y-5)^2 = 34$
$(x-3)^2 + [(x+4)-5]^2 = 34$
$(x-3)^2 + (x+4-5)^2 = 34$
$(x-3)^2 + (x-1)^2 = 34$
$x^2 - 6x + 9 + x^2 - 2x + 1 = 34$
$2x^2 - 8x + 10 = 34$
$2x^2 - 8x - 24 = 0$
$x^2 - 4x - 12 = 0$
$(x-6)(x+2) = 0$
So $x = 6$ and $x = -2$

Substitute $x = 6$ into $y = x + 4$
$y = 6 + 4$
$y = 10$
Substitute $x = -2$ into $y = x + 4$
$y = -2 + 4$
$y = 2$

The coordinates of A and B are $(6, 10)$ and $(-2, 2)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 5

Question:

Find where the line $x + y + 5 = 0$ meets the circle $(x + 3)^2 + (y + 5)^2 = 65$.

Solution:

Rearranging $x + y + 5 = 0$
$y + 5 = - x$
$y = - x - 5$
Substitute $y = - x - 5$ into $(x + 3)^2 + (y + 5)^2 = 65$
$(x + 3)^2 + [(- x - 5) + 5]^2 = 65$
$(x + 3)^2 + (- x - 5 + 5)^2 = 65$
$(x + 3)^2 + (- x)^2 = 65$
$x^2 + 6x + 9 + x^2 = 65$
$2x^2 + 6x + 9 = 65$
$2x^2 + 6x - 56 = 0$
$x^2 + 3x - 28 = 0$
$(x + 7) (x - 4) = 0$
So $x = - 7$ and $x = 4$

Substitute $x = - 7$ into $y = - x - 5$
$y = - (- 7) - 5$
$y = 7 - 5$
$y = 2$
Substitute $x = 4$ into $y = - x - 5$
$y = - (4) - 5$
$y = - 4 - 5$
$y = - 9$

So the line meets the circle at $(- 7 , 2)$ and $(4 , - 9)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 6

Question:

Show that the line $y = x - 10$ does not meet the circle $(x - 2)^2 + y^2 = 25$.

Solution:

Substitute $y = x - 10$ into $(x - 2)^2 + y^2 = 25$
$(x - 2)^2 + (x - 10)^2 = 25$
$x^2 - 4x + 4 + x^2 - 20x + 100 = 25$
$2x^2 - 24x + 104 = 25$
$2x^2 - 24x + 79 = 0$
Now $b^2 - 4ac = (-24)^2 - 4(2)(79) = 576 - 632 = -56$
As $b^2 - 4ac < 0$ then $2x^2 - 24x + 79 = 0$ has no real roots.
So the line does not meet the circle.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 7

Question:

Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$.

Solution:

Rearranging $x + y = 11$
$y = 11 - x$
Substitute $y = 11 - x$ into $x^2 + (y - 3)^2 = 32$
$x^2 + [(11 - x) - 3]^2 = 32$
$x^2 + (11 - x - 3)^2 = 32$
$x^2 + (8 - x)^2 = 32$
$x^2 + 64 - 16x + x^2 = 32$
$2x^2 - 16x + 64 = 32$
$2x^2 - 16x + 32 = 0$
$x^2 - 8x + 16 = 0$
$(x - 4)(x - 4) = 0$
The line meets the circle at $x = 4$ (only).
So the line is a tangent.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 8

Question:

Show that the line $3x - 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$.

Solution:

Rearrange $3x - 4y + 25 = 0$
$3x + 25 = 4y$
$4y = 3x + 25$
$y = \frac{3}{4}x + \frac{25}{4}$

Substitute $y = \frac{3}{4}x + \frac{25}{4}$ into $x^2 + y^2 = 25$

$$x^2 + \left(\frac{3}{4}x + \frac{25}{4} \right)^2 = 25$$

$$x^2 + \frac{9}{16}x^2 + \frac{150}{16}x + \frac{625}{16} = 25$$

$$\frac{25}{16}x^2 + \frac{150}{16}x + \frac{225}{16} = 0$$

$25x^2 + 150x + 225 = 0$
$x^2 + 6x + 9 = 0$
$(x + 3)(x + 3) = 0$
The line meets the circle at $x = -3$ (only).
So the line is a tangent.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 9

Question:

The line $y = 2x - 2$ meets the circle $(x-2)^2 + (y-2)^2 = 20$ at A and B.

(a) Find the coordinates of A and B.

(b) Show that AB is a diameter of the circle.

Solution:

(a) Substitute $y = 2x - 2$ into $(x-2)^2 + (y-2)^2 = 20$
$(x-2)^2 + [(2x-2) - 2]^2 = 20$
$(x-2)^2 + (2x-4)^2 = 20$
$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$
$5x^2 - 20x + 20 = 20$
$5x^2 - 20x = 0$
$5x(x-4) = 0$
So $x = 0$ and $x = 4$

Substitute $x = 0$ into $y = 2x - 2$
$y = 2(0) - 2$
$y = 0 - 2$
$y = -2$
Substitute $x = 4$ into $y = 2x - 2$
$y = 2(4) - 2$
$y = 8 - 2$
$y = 6$

So the coordinates of A and B are $(0, -2)$ and $(4, 6)$.

(b) (1) The length of AB is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4-0)^2 + [6 - (-2)]^2}$
$= \sqrt{4^2 + (6+2)^2}$
$= \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64}$
$= \sqrt{80}$
$= \sqrt{4 \times 20}$
$= \sqrt{4} \times \sqrt{20}$
$= 2\sqrt{20}$
The radius of the circle $(x-2)^2 + (y-2)^2 = 20$ is $\sqrt{20}$.
So the length of the chord AB is twice the length of the radius.
AB is a diameter of the circle.
(2) Substitute $x = 2$, $y = 2$ into $y = 2x - 2$
$2 = 2(2) - 2 = 4 - 2 = 2$ ✓
So the line $y = 2x - 2$ joining A and B passes through the centre $(2, 2)$ of the circle.
So AB is a diameter of the circle.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 10

Question:

The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3 , 10)$, where a and p are constants.

(a) Work out the value of a.

(b) Work out the two possible values of p.

Solution:

(a) Substitute $x = 3$, $y = 10$ into $x + y = a$
$(3) + (10) = a$
So $a = 13$

(b) Substitute $x = 3$, $y = 10$ into $(x - p)^2 + (y - 6)^2 = 20$
$(3 - p)^2 + (10 - 6)^2 = 20$
$(3 - p)^2 + 4^2 = 20$
$(3 - p)^2 + 16 = 20$
$(3 - p)^2 = 4$
$(3 - p) = \sqrt{4}$
$3 - p = \pm 2$
So $3 - p = 2 \Rightarrow p = 1$
and $3 - p = - 2 \Rightarrow p = 5$

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Coordinate geometry in the (x,y) plane

Exercise F, Question 1

Question:

The line $y = 2x - 8$ meets the coordinate axes at A and B. The line AB is a diameter of the circle. Find the equation of the circle.

Solution:

Substitute $x = 0$ into $y = 2x - 8$
$y = 2(0) - 8$
$y = -8$
Substitute $y = 0$ into $y = 2x - 8$
$0 = 2x - 8$
$2x = 8$
$x = 4$
The line meets the coordinate axes at $(0, -8)$ and $(4, 0)$
The coordinates of the centre of the circle is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 4}{2}, \frac{-8 + 0}{2} \right) = \left(\frac{4}{2}, -\frac{8}{2} \right) = \left(2, -4 \right)$$

The length of the diameter is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4 - 0)^2 + [0 - (-8)]^2}$
$= \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64}$
$= \sqrt{80}$
$= \sqrt{16 \times 5}$
$= \sqrt{16} \times \sqrt{5}$
$= 4\sqrt{5}$

So the length of the radius is $\frac{4\sqrt{5}}{2} = 2\sqrt{5}$.

The centre of the circle is $(2, -4)$ and the radius is $2\sqrt{5}$.
So the equation is
$(x - x_1)^2 + (y - y_1)^2 = r^2$
$(x - 2)^2 + [y - (-4)]^2 = (2\sqrt{5})^2$
$(x - 2)^2 + (y + 4)^2 = 20$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 2

Question:

The circle centre $(8, 10)$ meets the x-axis at $(4, 0)$ and $(a, 0)$.

(a) Find the radius of the circle.

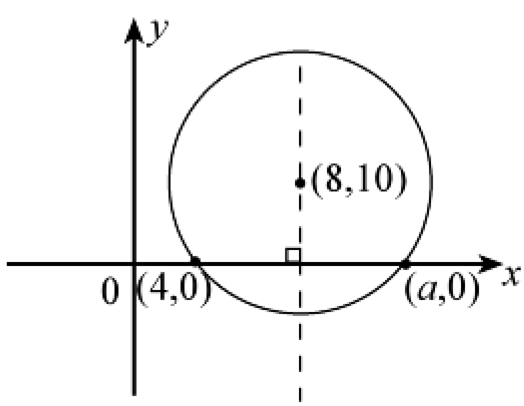
(b) Find the value of a.

Solution:

(a) The radius is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(8 - 4)^2 + (10 - 0)^2}$
$= \sqrt{4^2 + 10^2}$
$= \sqrt{16 + 100}$
$= \sqrt{116}$
$= 2\sqrt{29}$

(b)



The centre is on the perpendicular bisector of $(4, 0)$ and $(a, 0)$. So

$\frac{4 + a}{2} = 8$

$4 + a = 16$
$a = 12$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 3

Question:

The circle $(x - 5)^2 + y^2 = 36$ meets the x-axis at P and Q. Find the coordinates of P and Q.

Solution:

Substitute $y = 0$ into $(x - 5)^2 + y^2 = 36$
$(x - 5)^2 = 36$
$x - 5 = \sqrt{36}$
$x - 5 = \pm 6$
So $x - 5 = 6 \quad \Rightarrow \quad x = 11$
and $x - 5 = -6 \quad \Rightarrow \quad x = -1$
The coordinates of P and Q are $(-1 , 0)$ and $(11 , 0)$.

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Coordinate geometry in the (x,y) plane

Exercise F, Question 4

Question:

The circle $(x+4)^2+(y-7)^2=121$ meets the y-axis at $(0,m)$ and $(0,n)$. Find the value of m and n.

Solution:

Substitute $x=0$ into $(x+4)^2+(y-7)^2=121$
$4^2+(y-7)^2=121$
$16+(y-7)^2=121$
$(y-7)^2=105$
$y-7=\pm\sqrt{105}$
So $y=7\pm\sqrt{105}$
The values of m and n are $7+\sqrt{105}$ and $7-\sqrt{105}$.

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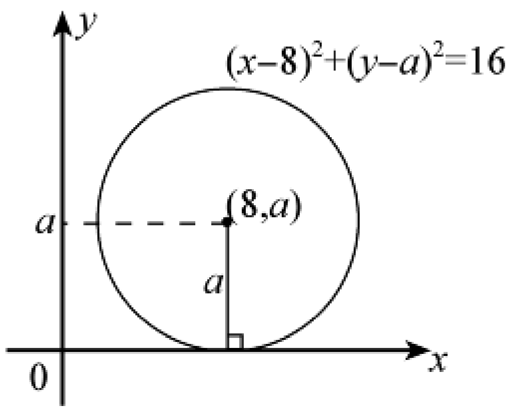
Coordinate geometry in the (x,y) plane
Exercise F, Question 5

Question:

The line $y = 0$ is a tangent to the circle $(x-8)^2 + (y-a)^2 = 16$. Find the value of a.

Solution:



The radius of the circle is $\sqrt{16} = 4$.
So $a = 4$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 6

Question:

The point $A(-3, -7)$ lies on the circle centre $(5, 1)$.
Find the equation of the tangent to the circle at A.

Solution:

The gradient of the line joining $(-3, -7)$ and $(5, 1)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{5 - (-3)} = \frac{1 + 7}{5 + 3} = \frac{8}{8} = 1$$

So the gradient of the tangent is $-\frac{1}{(1)} = -1$.

The equation of the tangent is

$y - y_1 = m(x - x_1)$

$y - (-7) = -1[x - (-3)]$

$y + 7 = -1(x + 3)$

$y + 7 = -x - 3$

$y = -x - 10$ or $x + y + 10 = 0$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 7

Question:

The circle $(x+3)^2+(y+8)^2=100$ meets the positive coordinate axes at $A(a, 0)$ and $B(0, b)$.

(a) Find the value of a and b.

(b) Find the equation of the line AB.

Solution:

(a) Substitute $y=0$ into $(x+3)^2+(y+8)^2=100$
$(x+3)^2+8^2=100$
$(x+3)^2+64=100$
$(x+3)^2=36$
$x+3=\pm\sqrt{36}$
$x+3=\pm 6$
So $x+3=6 \Rightarrow x=3$
and $x+3=-6 \Rightarrow x=-9$
As $a>0$, $a=3$.

Substitute $x=0$ into $(x+3)^2+(y+8)^2=100$
$3^2+(y+8)^2=100$
$9+(y+8)^2=100$
$(y+8)^2=91$
$y+8=\pm\sqrt{91}$
So $y+8=\sqrt{91} \Rightarrow y=\sqrt{91}-8$
and $y+8=-\sqrt{91} \Rightarrow y=-\sqrt{91}-8$
As $b>0$, $b=\sqrt{91}-8$.

(b) The equation of the line joining $(3, 0)$ and $(0, \sqrt{91}-8)$ is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{(\sqrt{91}-8)-0}=\frac{x-3}{0-3}$$

$$\frac{y}{\sqrt{91}-8}=\frac{x-3}{-3}$$

$$y=\left(\sqrt{91}-8\right)\times\left(\frac{x-3}{-3}\right)$$

$$y=\left(\frac{\sqrt{91}-8}{-3}\right)\left(x-3\right)$$

$$y=\left(\frac{8-\sqrt{91}}{3}\right)\left(x-3\right)$$

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Coordinate geometry in the (x,y) plane

Exercise F, Question 8

Question:

The circle $(x+2)^2+(y-5)^2=169$ meets the positive coordinate axes at $C(c,0)$ and $D(0,d)$.

(a) Find the value of c and d.

(b) Find the area of $\triangle OCD$, where O is the origin.

Solution:

(a) Substitute $y=0$ into $(x+2)^2+(y-5)^2=169$

$(x+2)^2+(-5)^2=169$

$(x+2)^2+25=169$

$(x+2)^2=144$

$x+2=\pm\sqrt{144}$

$x+2=\pm 12$

So $x+2=12 \Rightarrow x=10$

and $x+2=-12 \Rightarrow x=-14$

As $c>0$, $c=10$.

Substitute $x=0$ into $(x+2)^2+(y-5)^2=169$

$2^2+(y-5)^2=169$

$4+(y-5)^2=169$

$(y-5)^2=165$

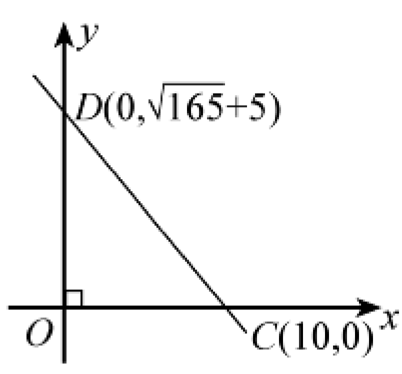
$y-5=\pm\sqrt{165}$

So $y-5=\sqrt{165} \Rightarrow y=\sqrt{165}+5$

and $y-5=-\sqrt{165} \Rightarrow y=-\sqrt{165}+5$

As $d>0$, $d=\sqrt{165}+5$.

(b)



The area of $\triangle OCD$ is

$$\frac{1}{2}\times 10\times\left(\sqrt{165}+5\right)=5\left(\sqrt{165}+5\right)$$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 9

Question:

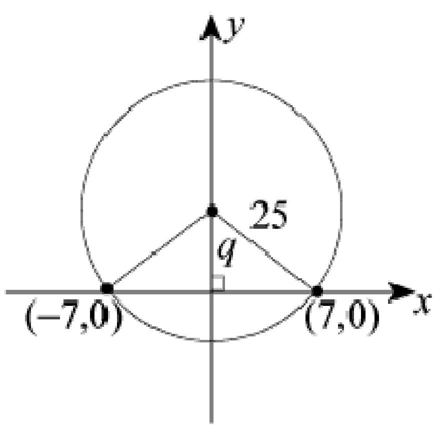
The circle, centre (p, q) radius 25, meets the x-axis at $(-7, 0)$ and $(7, 0)$, where $q > 0$.

(a) Find the value of p and q.

(b) Find the coordinates of the points where the circle meets the y-axis.

Solution:

(a) By symmetry $p = 0$.



Using Pythagoras' theorem
$q^2 + 7^2 = 25^2$
$q^2 + 49 = 625$
$q^2 = 576$
$q = \pm \sqrt{576}$
$q = \pm 24$
As $q > 0$, $q = 24$.

(b) The circle meets the y-axis at $q \pm r$; i.e.
at $24 + 25 = 49$
and $24 - 25 = -1$
So the coordinates are $(0, 49)$ and $(0, -1)$.

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Coordinate geometry in the (x,y) plane

Exercise F, Question 10

Question:

Show that $(0, 0)$ lies inside the circle $(x-5)^2 + (y+2)^2 = 30$.

Solution:

The distance between $(0, 0)$ and $(5, -2)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-0)^2 + (-2-0)^2}$

$= \sqrt{5^2 + (-2)^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$

The radius of the circle is $\sqrt{30}$.

As $\sqrt{29} < \sqrt{30}$ $(0, 0)$ lies inside the circle.

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Coordinate geometry in the (x,y) plane

Exercise F, Question 11

Question:

The points $A(-4, 0)$, $B(4, 8)$ and $C(6, 0)$ lie on a circle. The lines AB and BC are chords of the circle. Find the coordinates of the centre of the circle.

Solution:

(1) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{4 - (-4)} = \frac{8}{4 + 4} = \frac{8}{8} = 1$$

(2) The gradient of a line perpendicular to AB is $\frac{-1}{(1)} = -1$.

(3) The mid-point of AB is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-4 + 4}{2}, \frac{0 + 8}{2} \right) = \left(\frac{0}{2}, \frac{8}{2} \right) = \left(0, 4 \right)$$

(4) The equation of the perpendicular bisector of AB is

$y - y_1 = m(x - x_1)$

$y - 4 = -1(x - 0)$

$y - 4 = -x$

$y = -x + 4$

(5) The gradient of BC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 8}{6 - 4} = \frac{-8}{2} = -4$$

(6) The gradient of a line perpendicular to BC is $-\frac{1}{(-4)} = \frac{1}{4}$.

(7) The mid-point of BC is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{4 + 6}{2}, \frac{8 + 0}{2} \right) = \left(\frac{10}{2}, \frac{8}{2} \right) = \left(5, 4 \right)$$

(8) The equation of the perpendicular bisector of BC is

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{4}\left(x - 5 \right)$

$y - 4 = \frac{1}{4}x - \frac{5}{4}$

$y = \frac{1}{4}x + \frac{11}{4}$

(9) Solving $y = -x + 4$ and $y = \frac{1}{4}x + \frac{11}{4}$ simultaneously

$\frac{1}{4}x + \frac{11}{4} = -x + 4$

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$\frac{5}{4}x + \frac{11}{4} = 4$

$\frac{5}{4}x = \frac{5}{4}$

$x = 1$
Substitute $x = 1$ into $y = -x + 4$
$y = -1 + 4$
$y = 3$
So coordinates of the centre of the circle are $(1, 3)$.

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Coordinate geometry in the (x,y) plane

Exercise F, Question 12

Question:

The points $R(-4, 3)$, $S(7, 4)$ and $T(8, -7)$ lie on a circle.

(a) Show that $\triangle RST$ has a right angle.

(b) Find the equation of the circle.

Solution:

(a) (1) The distance between R and S is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[7 - (-4)]^2 + (4 - 3)^2}$

$= \sqrt{(7 + 4)^2 + 1^2}$

$= \sqrt{11^2 + 1^2}$

$= \sqrt{121 + 1}$

$= \sqrt{122}$

(2) The distance between S and T is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 7)^2 + (-7 - 4)^2}$

$= \sqrt{1^2 + (-11)^2}$

$= \sqrt{1 + 121}$

$= \sqrt{122}$

(3) The distance between R and T is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[8 - (-4)]^2 + (-7 - 3)^2}$

$= \sqrt{(8 + 4)^2 + (-10)^2}$

$= \sqrt{12^2 + (-10)^2}$

$= \sqrt{144 + 100}$

$= \sqrt{244}$

By Pythagoras' theorem

$(\sqrt{122})^2 + (\sqrt{122})^2 = (\sqrt{244})^2$

So $\triangle RST$ has a right angle (at S).

(b) (1) The radius of the circle is

$\frac{1}{2} \times \text{diameter} = \frac{1}{2}\sqrt{244} = \frac{1}{2}\sqrt{4 \times 61} = \frac{1}{2}\sqrt{4} \times \sqrt{61} = \frac{1}{2} \times 2\sqrt{61} = \sqrt{61}$

(2) The centre of the circle is the mid-point of RT:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{-4 + 8}{2}, \frac{3 + (-7)}{2} \right) = \left(\frac{4}{2}, -\frac{4}{2} \right) = \left(2, -2 \right)$

So the equation of the circle is

$(x - 2)^2 + (y + 2)^2 = (\sqrt{61})^2$

or $(x - 2)^2 + (y + 2)^2 = 61$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 13

Question:

The points $A(-7,7)$, $B(1,9)$, $C(3,1)$ and $D(-7,1)$ lie on a circle. The lines *AB* and *CD* are chords of the circle.

(a) Find the equation of the perpendicular bisector of (i) *AB* (ii) *CD*.

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (i) (1) The gradient of the line joining *A* and *B* is
$$\frac{y_2-y_1}{x_2-x_1}=\frac{9-7}{1-(-7)}=\frac{2}{1+7}=\frac{2}{8}=\frac{1}{4}$$

(2) The gradient of a line perpendicular to *AB* is $-\frac{1}{m}=\frac{-1}{\left(\frac{1}{4}\right)}=-4$

(3) The mid-point of *AB* is
$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-7+1}{2},\frac{7+9}{2}\right)=\left(\frac{-6}{2},\frac{16}{2}\right)=\left(-3,8\right)$$
(4) The equation of the perpendicular bisector of *AB* is
$y-y_1=m(x-x_1)$
$y-8=-4[x-(-3)]$
$y-8=-4(x+3)$
$y-8=-4x-12$
$y=-4x-4$

(ii) (1) The gradient of the line joining *C* and *D* is
$$\frac{y_2-y_1}{x_2-x_1}=\frac{1-1}{-7-3}=\frac{0}{-10}=0$$
So the line is horizontal.

(2) The mid-point of *CD* is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+(-7)}{2},\frac{1+1}{2}\right)=\left(\frac{-4}{2},\frac{2}{2}\right)=\left(-2,1\right)$

(3) The equation of the perpendicular bisector of *CD* is $x=-2$
i.e. the vertical line through $(-2,1)$

(b) Solving $y=-4x-4$ and $x=-2$ simultaneously,
substitute $x=-2$ into $y=-4x-4$
$y=-4(-2)-4=8-4=4$
So the centre of the circle is $(-2,4)$.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 14

Question:

The centres of the circles $(x - 8)^2 + (y - 8)^2 = 117$ and $(x + 1)^2 + (y - 3)^2 = 106$ are *P* and *Q* respectively.

(a) Show that *P* lies on $(x + 1)^2 + (y - 3)^2 = 106$.

(b) Find the length of *PQ*.

Solution:

(a) The centre of $(x - 8)^2 + (y - 8)^2 = 117$ is $(8 , 8)$.
Substitute $(8 , 8)$ into $(x + 1)^2 + (y - 3)^2 = 106$
$(8 + 1)^2 + (8 - 3)^2 = 9^2 + 5^2 = 81 + 25 = 106$ ✓
So $(8 , 8)$ lies on the circle $(x + 1)^2 + (y - 3)^2 = 106$.

(b) As *Q* is the centre of the circle $(x + 1)^2 + (y - 3)^2 = 106$ and *P* lies on this circle, the length *PQ* must equal the radius.
So PQ = $\sqrt{106}$

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Coordinate geometry in the (x,y) plane

Exercise F, Question 15

Question:

The line $y = -3x + 12$ meets the coordinate axes at A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the mid-point of AB.

(c) Find the equation of the circle that passes through A, B and O, where O is the origin.

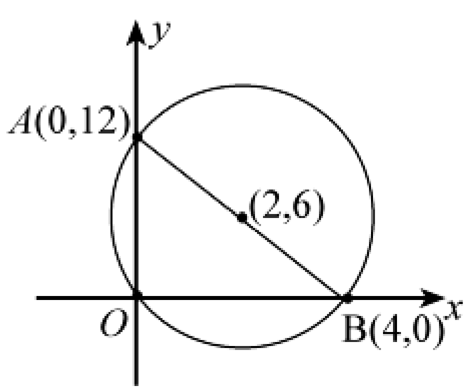
Solution:

(a) $y = -3x + 12$
(1) Substitute $x = 0$ into $y = -3x + 12$
$y = -3(0) + 12 = 12$
So A is $(0, 12)$.
(2) Substitute $y = 0$ into $y = -3x + 12$
$0 = -3x + 12$
$3x = 12$
$x = 4$
So B is $(4, 0)$.

(b) The mid-point of AB is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 4}{2}, \frac{12 + 0}{2} \right) = \left(2, 6 \right)$$

(c)



$\angle AOB = 90°$, so AB is a diameter of the circle.
The centre of the circle is the mid-point of AB, i.e. $(2, 6)$.
The length of the diameter AB is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 0)^2 + (0 - 12)^2}$$
$$= \sqrt{4^2 + (-12)^2}$$
$$= \sqrt{16 + 144}$$
$$= \sqrt{160}$$

So the radius of the circle is $\frac{\sqrt{160}}{2}$.

The equation of the circle is

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$$(x-2)^2+(y-6)^2=\left(\frac{\sqrt{160}}{2}\right)^2$$

$$(x-2)^2+(y-6)^2=\frac{160}{4}$$

$$(x-2)^2+(y-6)^2=40$$

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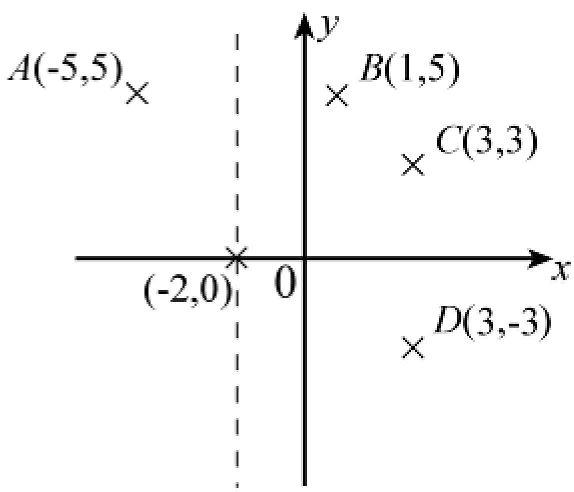
Coordinate geometry in the (x,y) plane
Exercise F, Question 16

Question:

The points $A(-5,5)$, $B(1,5)$, $C(3,3)$ and $D(3,-3)$ lie on a circle. Find the equation of the circle.

Solution:

(1)



(2) The mid-point of AB is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-5+1}{2},\frac{5+5}{2}\right)=\left(\frac{-4}{2},\frac{10}{2}\right)=\left(-2,5\right)$$

So the equation of the perpendicular bisector of AB is $x=-2$.

(3) The mid-point of CD is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+3}{2},\frac{3+(-3)}{2}\right)=\left(\frac{6}{2},\frac{3-3}{2}\right)=\left(3,\frac{0}{2}\right)=\left(3,0\right)$$

So the equation of the perpendicular bisector of CD is $y=0$.

(4) The perpendicular bisectors intersect at $(-2,0)$.

(5) The radius is the distance between $(-2,0)$ and $(-5,5)$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{[-5-(-2)]^2+(5-0)^2}$

$=\sqrt{(-5+2)^2+(5)^2}$

$=\sqrt{(-3)^2+(5)^2}$

$=\sqrt{9+25}$

$=\sqrt{34}$

(6) So the equation of the circle centre $(-2,0)$ and radius $\sqrt{34}$ is

$[x-(-2)]^2+(y-0)^2=(\sqrt{34})^2$

$(x+2)^2+y^2=34$

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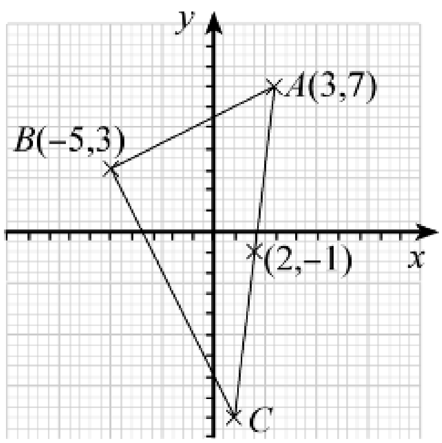
Coordinate geometry in the (x,y) plane
Exercise F, Question 17

Question:

The line AB is a chord of a circle centre $(2, -1)$, where A and B are $(3, 7)$ and $(-5, 3)$ respectively. AC is a diameter of the circle. Find the area of $\triangle ABC$.

Solution:

(1)



(2) Let the coordinates of C be (p, q).

$(2, -1)$ is the mid-point of $(3, 7)$ and (p, q)

So $\frac{3+p}{2} = 2$ and $\frac{7+q}{2} = -1$

$\frac{3+p}{2} = 2$

$3 + p = 4$

$p = 1$

$\frac{7+q}{2} = -1$

$7 + q = -2$

$q = -9$

So the coordinates of C are $(1, -9)$.

(3) The length of AB is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-3)^2 + (3-7)^2}$

$= \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

The length of BC is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-1)^2 + [3-(-9)]^2}$

$= \sqrt{(-6)^2 + (3+9)^2}$

$= \sqrt{(-6)^2 + (12)^2}$

$= \sqrt{36 + 144}$

$= \sqrt{180}$

(4) The area of $\triangle ABC$ is

$\frac{1}{2}\sqrt{180}\sqrt{80} = \frac{1}{2}\sqrt{14400} = \frac{1}{2}\sqrt{144 \times 100} = \frac{1}{2}\sqrt{144} \times \sqrt{100} = \frac{1}{2} \times 12 \times 10 = 60$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 18

Question:

The points $A(-1, 0)$, $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $C\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are the vertices of a triangle.

(a) Show that the circle $x^2 + y^2 = 1$ passes through the vertices of the triangle.

(b) Show that $\triangle ABC$ is equilateral.

Solution:

(a) (1) Substitute $(-1, 0)$ into $x^2 + y^2 = 1$
$(-1)^2 + (0)^2 = 1 + 0 = 1$ ✓
So $(-1, 0)$ is on the circle.

(2) Substitute $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ into $x^2 + y^2 = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$ ✓

So $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the circle.

(3) Substitute $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ into $x^2 + y^2 = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$ ✓

So $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is on the circle.

(b) (1) The distance between $(-1, 0)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[\frac{1}{2} - \left(-1\right)\right]^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2}$

$= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{3}{4}}$

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$= \sqrt{\frac{12}{4}}$

$= \sqrt{3}$

(2) The distance between $(-1, 0)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[\frac{1}{2} - \left(-1\right)\right]^2 + \left(\frac{-\sqrt{3}}{2} - 0\right)^2}$

$= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{3}{4}}$

$= \sqrt{\frac{12}{4}}$

$= \sqrt{3}$

(3) The distance between $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{0^2 + (-\sqrt{3})^2}$

$= \sqrt{0 + 3}$

$= \sqrt{3}$

So AB, BC and AC all equal $\sqrt{3}$.

$\triangle ABC$ is equilateral.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 19

Question:

The points $P(2, 2)$, $Q(2+\sqrt{3}, 5)$ and $R(2-\sqrt{3}, 5)$ lie on the circle $(x-2)^2+(y-4)^2=r^2$.

(a) Find the value of r.

(b) Show that $\triangle PQR$ is equilateral.

Solution:

(a) Substitute $(2, 2)$ into $(x-2)^2+(y-4)^2=r^2$
$(2-2)^2+(2-4)^2=r^2$
$0^2+(-2)^2=r^2$
$r^2=4$
$r=2$

(b) (1) The distance between $(2, 2)$ and $(2+\sqrt{3}, 5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(2+\sqrt{3}-2)^2+(5-2)^2}$
$=\sqrt{(\sqrt{3})^2+3^2}$
$=\sqrt{3+9}$
$=\sqrt{12}$
(2) The distance between $(2, 2)$ and $(2-\sqrt{3}, 5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(2-\sqrt{3}-2)^2+(5-2)^2}$
$=\sqrt{(-\sqrt{3})^2+(3)^2}$
$=\sqrt{3+9}$
$=\sqrt{12}$
(3) The distance between $(2+\sqrt{3}, 5)$ and $(2-\sqrt{3}, 5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{[(2-\sqrt{3})-(2+\sqrt{3})]^2+(5-5)^2}$
$=\sqrt{(2-\sqrt{3}-2-\sqrt{3})^2+0^2}$
$=\sqrt{(-2\sqrt{3})^2}$
$=\sqrt{(-2)^2\times(\sqrt{3})^2}$
$=\sqrt{4\times 3}$
$=\sqrt{12}$
So PQ, QR and PR all equal $\sqrt{12}$.
$\triangle PQR$ is equilateral.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 20

Question:

The points $A(-3, -2)$, $B(-6, 0)$ and $C(p, q)$ lie on a circle centre $\left(-\frac{5}{2}, 2\right)$. The line BC is a diameter of the circle.

(a) Find the value of p and q.

(b) Find the gradient of (i) AB (ii) AC.

(c) Show that AB is perpendicular to AC.

Solution:

(a) The mid-point of $(-6, 0)$ and (p, q) is $\left(-\frac{5}{2}, 2\right)$.

So $\left(\frac{-6+p}{2}, \frac{0+q}{2}\right) = \left(-\frac{5}{2}, 2\right)$

$\frac{-6+p}{2} = -\frac{5}{2}$

$-6 + p = -5$

$p = -5 + 6$

$p = 1$

$\frac{0+q}{2} = 2$

$\frac{q}{2} = 2$

$q = 4$

(b) (i) The gradient of the line joining $(-3, -2)$ and $(-6, 0)$ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2)}{-6-(-3)} = \frac{2}{-6+3} = \frac{2}{-3} = -\frac{2}{3}$

(ii) The gradient of the line joining $(-3, -2)$ and $(1, 4)$ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{1-(-3)} = \frac{4+2}{1+3} = \frac{6}{4} = \frac{3}{2}$

(c) Two lines are perpendicular if $m_1 \times m_2 = -1$.

Now $-\frac{2}{3} \times \frac{3}{2} = -1$ ✓

So AB is perpendicular to AC.