Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Radian measure and its applications

Exercise A, Question 1

Question:

Convert the following angles in radians to degrees:

(a) $\frac{\pi}{20}$

(b) $\frac{\pi}{15}$

(c) $\frac{5\pi}{12}$

(d) $\frac{\pi}{2}$

(e) $\frac{7\pi}{9}$

(f) $\frac{7\pi}{6}$

(g) $\frac{5\pi}{4}$

(h) $\frac{3\pi}{2}$

(i) 3π

Solution:

(a) $\frac{\pi}{20}$ rad $= \frac{180°}{20} = 9°$

(b) $\frac{\pi}{15}$ rad $= \frac{180°}{15} = 12°$

(c) $\frac{5\pi}{12}$ rad $= \frac{5\times \overset{15°}{\cancel{180°}}}{\cancel{12}} = 75°$

(d) $\frac{\pi}{2}$ rad $= \frac{180°}{2} = 90°$

(e) $\frac{7\pi}{9}$ rad $= \frac{7\times \overset{20°}{\cancel{180°}}}{\cancel{9}} = 140°$

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(f) $\frac{7\pi}{6}$ rad = $\frac{7 \times \overset{30°}{\cancel{180°}}}{\cancel{6}}$ = 210 °

(g) $\frac{5\pi}{4}$ rad = $\frac{5 \times \overset{45°}{\cancel{180°}}}{\cancel{4}}$ = 225 °

(h) $\frac{3\pi}{2}$ rad = 3 × 90 ° = 270 °

(i) 3π rad = 3 × 180 ° = 540 °

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Radian measure and its applications
Exercise A, Question 2

Question:

Use your calculator to convert the following angles to degrees, giving your answer to the nearest 0.1 ° :

(a) 0.46^c

(b) 1^c

(c) 1.135^c

(d) $\sqrt{3}^c$

(e) 2.5^c

(f) 3.14^c

(g) 3.49^c

Solution:

(a) $0.46^c = 26.356 \ldots° = 26.4°$ (nearest 0.1 °)

(b) $1^c = 57.295 \ldots° = 57.3°$ (nearest 0.1 °)

(c) $1.135^c = 65.030 \ldots° = 65.0°$ (nearest 0.1 °)

(d) $\sqrt{3}^c = 99.239 \ldots° = 99.2°$ (nearest 0.1 °)

(e) $2.5^c = 143.239 \ldots° = 143.2°$ (nearest 0.1 °)

(f) $3.14^c = 179.908 \ldots° = 179.9°$ (nearest 0.1 °)

(g) $3.49^c = 199.96 \ldots° = 200.0°$ (nearest 0.1 °)

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Radian measure and its applications

Exercise A, Question 3

Question:

Use your calculator to write down the value, to 3 significant figures, of the following trigonometric functions.

(a) $\sin\ 0.5^c$

(b) $\cos\ \sqrt{2}^c$

(c) $\tan\ 1.05^c$

(d) $\sin\ 2^c$

(e) $\cos\ 3.6^c$

Solution:

(a) $\sin\ 0.5^c = 0.47942 \ldots = 0.479$ (3 s.f.)

(b) $\cos\ \sqrt{2}^c = 0.1559 \ldots = 0.156$ (3 s.f.)

(c) $\tan\ 1.05^c = 1.7433 \ldots = 1.74$ (3 s.f.)

(d) $\sin\ 2^c = 0.90929 \ldots = 0.909$ (3 s.f.)

(e) $\cos\ 3.6^c = -0.8967 \ldots = -0.897$ (3 s.f.)

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Radian measure and its applications
Exercise A, Question 4

Question:

Convert the following angles to radians, giving your answers as multiples of π.

(a) 8°

(b) 10°

(c) 22.5°

(d) 30°

(e) 45°

(f) 60°

(g) 75°

(h) 80°

(i) 112.5°

(j) 120°

(k) 135°

(l) 200°

(m) 240°

(n) 270°

(o) 315°

(p) 330°

Solution:

(a) $8° = \overset{2}{\cancel{8}} \times \frac{\pi}{\underset{45}{\cancel{180}}} \text{ rad} = \frac{2\pi}{45} \text{ rad}$

(b) $10° = 10 \times \frac{\pi}{180} \text{ rad} = \frac{\pi}{18} \text{ rad}$

(c) $22.5° = \cancel{22.5} \times \frac{\pi}{\underset{8}{\cancel{180}}} \text{ rad} = \frac{\pi}{8} \text{ rad}$

(d) $30° = 30 \times \frac{\pi}{180}$ rad $= \frac{\pi}{6}$ rad

(e) $45° = 45 \times \frac{\pi}{180}$ rad $= \frac{\pi}{4}$ rad

(f) $60° = 2 \times$ answer to (d) $= \frac{\pi}{3}$ rad

(g) $75° = \overset{5}{\cancel{75}} \times \frac{\pi}{\underset{12}{\cancel{180}}}$ rad $= \frac{5\pi}{12}$ rad

(h) $80° = 8\cancel{0} \times \frac{\pi}{18\cancel{0}}$ rad $= \frac{4\pi}{9}$ rad

(i) $112.5° = 5 \times$ answer to (c) $= \frac{5\pi}{8}$ rad

(j) $120° = 2 \times$ answer to (f) $= \frac{2\pi}{3}$ rad

(k) $135° = 3 \times$ answer to (e) $= \frac{3\pi}{4}$ rad

(l) $200° = 20\cancel{0} \times \frac{\pi}{18\cancel{0}}$ rad $= \frac{10\pi}{9}$ rad

(m) $240° = 2 \times$ answer to (j) $= \frac{4\pi}{3}$ rad

(n) $270° = 3 \times 90° = \frac{3\pi}{2}$ rad

(o) $315° = 180° + 135° = \pi + \frac{3\pi}{4} = \frac{7\pi}{4}$ rad

(p) $330° = 11 \times 30° = \frac{11\pi}{6}$ rad

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Radian measure and its applications
Exercise A, Question 5

Question:

Use your calculator to convert the following angles to radians, giving your answers to 3 significant figures:

(a) 50°

(b) 75°

(c) 100°

(d) 160°

(e) 230°

(f) 320°

Solution:

(a) $50° = 0.8726 \ldots^c = 0.873^c$ (3 s.f.)

(b) $75° = 1.3089 \ldots^c = 1.31^c$ (3 s.f.)

(c) $100° = 1.7453 \ldots^c = 1.75^c$ (3 s.f.)

(d) $160° = 2.7925 \ldots^c = 2.79^c$ (3 s.f.)

(e) $230° = 4.01425 \ldots^c = 4.01^c$ (3 s.f.)

(f) $320° = 5.585 \ldots^c = 5.59^c$ (3 s.f.)

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Radian measure and its applications
Exercise B, Question 1

Question:

An arc AB of a circle, centre O and radius r cm, subtends an angle θ radians at O. The length of AB is l cm.

(a) Find l when
(i) $r = 6$, $\theta = 0.45$
(ii) $r = 4.5$, $\theta = 0.45$
(iii) $r = 20$, $\theta = \frac{3}{8}\pi$

(b) Find r when
(i) $l = 10$, $\theta = 0.6$
(ii) $l = 1.26$, $\theta = 0.7$
(iii) $l = 1.5\pi$, $\theta = \frac{5}{12}\pi$

(c) Find θ when
(i) $l = 10$, $r = 7.5$
(ii) $l = 4.5$, $r = 5.625$
(iii) $l = \sqrt{12}$, $r = \sqrt{3}$

Solution:

(a) Using $l = r\theta$
(i) $l = 6 \times 0.45 = 2.7$
(ii) $l = 4.5 \times 0.45 = 2.025$
(iii) $l = 20 \times \frac{3}{8}\pi = 7.5\pi$ (23.6 3 s.f.)

(b) Using $r = \frac{l}{\theta}$

(i) $r = \frac{10}{0.6} = 16\frac{2}{3}$

(ii) $r = \frac{1.26}{0.7} = 1.8$

(iii) $r = \frac{1.5\pi}{\frac{5}{12}\pi} = 1.5 \times \frac{12}{5} = \frac{18}{5} = 3\frac{3}{5}$

(c) Using $\theta = \frac{l}{r}$

(i) $\theta = \frac{10}{7.5} = 1\frac{1}{3}$

(ii) $\theta = \frac{4.5}{5.625} = 0.8$

(iii) $\theta = \frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

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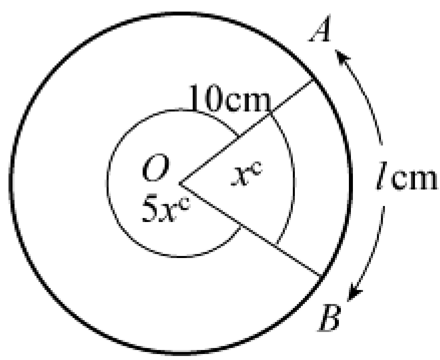
Radian measure and its applications
Exercise B, Question 2

Question:

A minor arc *AB* of a circle, centre *O* and radius 10 cm, subtends an angle x at *O*. The major arc *AB* subtends an angle $5x$ at *O*. Find, in terms of π, the length of the minor arc *AB*.

Solution:



The total angle at the centre is $6x^c$ so

$6x = 2\pi$

$x = \frac{\pi}{3}$

Using $l = r\theta$ to find minor arc *AB*

$l = 10 \times \frac{\pi}{3} = \frac{10\pi}{3}$ cm

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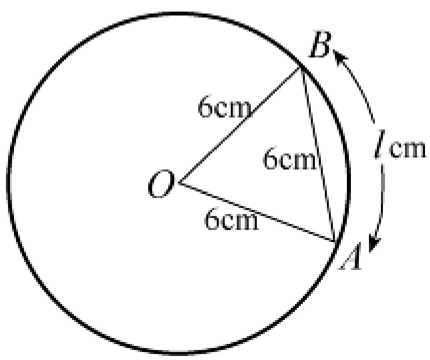
Radian measure and its applications
Exercise B, Question 3

Question:

An arc *AB* of a circle, centre *O* and radius 6 cm, has length *l* cm. Given that the chord *AB* has length 6 cm, find the value of *l*, giving your answer in terms of *π*.

Solution:



$\triangle$OAB is equilateral, so $\angle$ AOB = $\frac{\pi}{3}$ rad.

Using $l = r\theta$

$l = 6 \times \frac{\pi}{3} = 2\pi$

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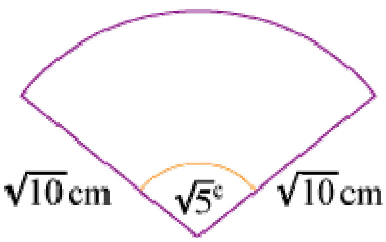
Radian measure and its applications

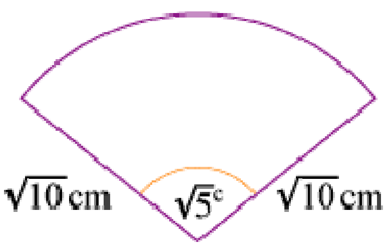
Exercise B, Question 4

Question:

The sector of a circle of radius $\sqrt{10}$ cm contains an angle of $\sqrt{5}$ radians, as shown in the diagram. Find the length of the arc, giving your answer in the form $p\sqrt{q}$ cm, where p and q are integers.



Solution:



Using $l = r\theta$ with $r = \sqrt{10}$ cm and $\theta = \sqrt{5}^c$

$l = \sqrt{10} \times \sqrt{5} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ cm

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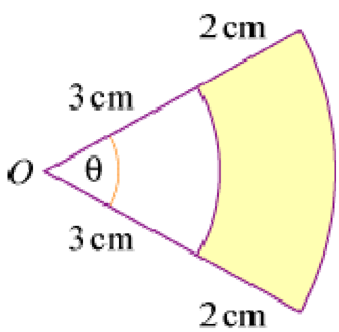
Radian measure and its applications
Exercise B, Question 5

Question:

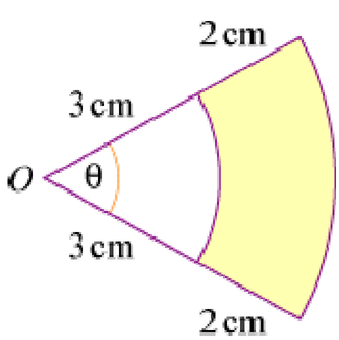
Referring to the diagram, find:



(a) The perimeter of the shaded region when $\theta = 0.8$ radians.

(b) The value of θ when the perimeter of the shaded region is 14 cm.

Solution:



(a) Using $l = r\theta$,
the smaller arc $= 3 \times 0.8 = 2.4$ cm
the larger arc $= (3 + 2) \times 0.8 = 4$ cm
Perimeter $= 2.4$ cm $+ 2$ cm $+ 4$ cm $+ 2$ cm $= 10.4$ cm

(b) The smaller arc $= 3\theta$ cm, the larger arc $= 5\theta$ cm.
So perimeter $= (3\theta + 5\theta + 2 + 2)$ cm .
As perimeter is 14 cm,
$8\theta + 4 = 14$
$8\theta = 10$
$\theta = \frac{10}{8} = 1\frac{1}{4}$

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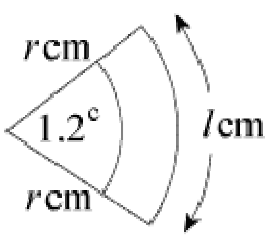
Radian measure and its applications
Exercise B, Question 6

Question:

A sector of a circle of radius r cm contains an angle of 1.2 radians. Given that the sector has the same perimeter as a square of area 36 cm^2, find the value of r.

Solution:

Using $l = r\theta$, the arc length $= 1.2r$ cm.
The area of the square $= 36$ cm^2, so each side $= 6$ cm and the perimeter is, therefore, 24 cm.
The perimeter of the sector = arc length $+ 2r$ cm $= (1.2r + 2r)$ cm $= 3.2r$ cm .
The perimeter of square = perimeter of sector so
$24 = 3.2r$
$r = \frac{24}{3.2} = 7.5$



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Radian measure and its applications
Exercise B, Question 7

Question:

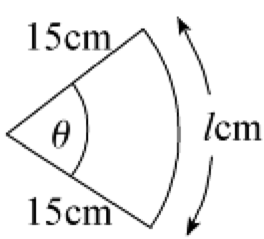
A sector of a circle of radius 15 cm contains an angle of θ radians. Given that the perimeter of the sector is 42 cm, find the value of θ.

Solution:

Using $l = r\theta$, the arc length of the sector $= 15\theta$ cm.
So the perimeter $= (15\theta + 30)$ cm .
As the perimeter $= 42$ cm
$15\theta + 30 = 42$

$\Rightarrow \quad 15\theta = 12$

$\Rightarrow \quad \theta = \frac{12}{15} = \frac{4}{5}$



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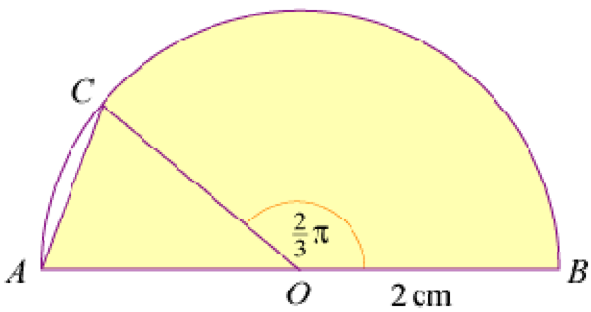
Radian measure and its applications
Exercise B, Question 8

Question:

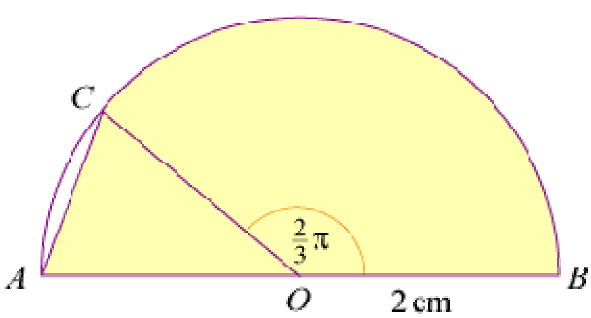
In the diagram *AB* is the diameter of a circle, centre *O* and radius 2 cm. The point *C* is on the circumference such that $\angle$ COB = $\frac{2}{3}\pi$ radians.



(a) State the value, in radians, of $\angle$ COA.
The shaded region enclosed by the chord *AC*, arc *CB* and *AB* is the template for a brooch.

(b) Find the exact value of the perimeter of the brooch.

Solution:



(a) $\angle$ COA $= \pi - \frac{2}{3}\pi = \frac{\pi}{3}$ rad

(b) The perimeter of the brooch = AB + arc BC + chord AC.
AB = 4 cm

arc BC = $r\theta$ with $r = 2$ cm and $\theta = \frac{2}{3}\pi$ so

arc BC $= 2 \times \frac{2}{3}\pi = \frac{4}{3}\pi$ cm

As $\angle$ COA $= \frac{\pi}{3}$ (60 °), △COA is equilateral, so

chord AC = 2 cm

The perimeter $= 4$ cm $+ \frac{4}{3}\pi$ cm $+ 2$ cm $= \left(6 + \frac{4}{3}\pi \right)$ cm

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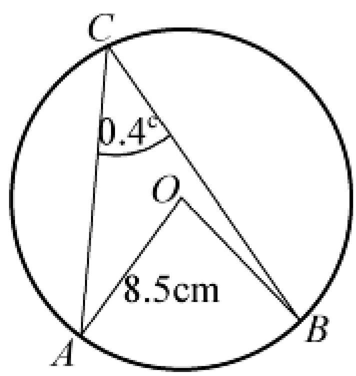
Radian measure and its applications

Exercise B, Question 9

Question:

The points *A* and *B* lie on the circumference of a circle with centre *O* and radius 8.5 cm. The point *C* lies on the major arc *AB*. Given that $\angle$ ACB = 0.4 radians, calculate the length of the minor arc *AB*.

Solution:



Using the circle theorem:
Angle subtended at the centre of the circle = 2 × angle subtended at the circumference
$\angle$ AOB = 2 $\angle$ ACB = 0.8^c
Using $l = r\theta$
length of minor arc AB = 8.5 × 0.8 cm = 6.8 cm

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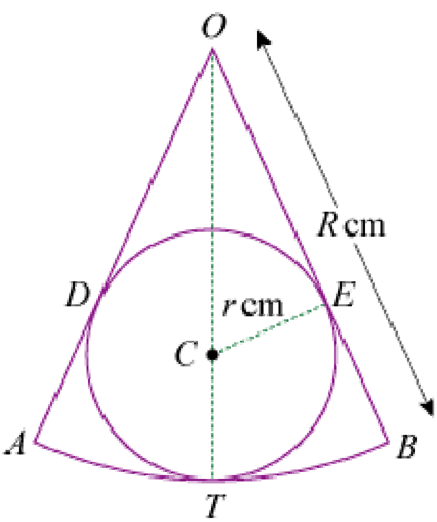
Radian measure and its applications
Exercise B, Question 10

Question:

In the diagram *OAB* is a sector of a circle, centre *O* and radius *R* cm, and $\angle AOB = 2\theta$ radians. A circle, centre *C* and radius *r* cm, touches the arc *AB* at *T*, and touches *OA* and *OB* at *D* and *E* respectively, as shown.

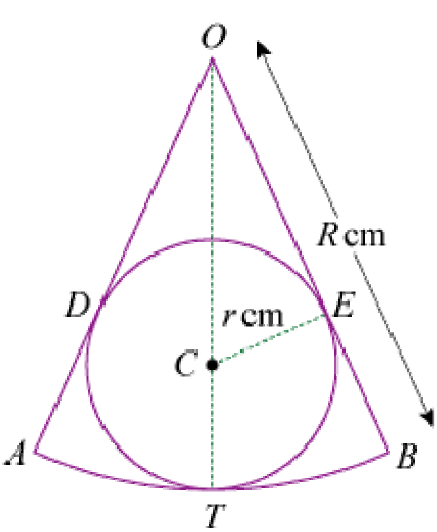


(a) Write down, in terms of *R* and *r*, the length of *OC*.

(b) Using $\triangle OCE$, show that $R \sin \theta = r (1 + \sin \theta)$.

(c) Given that $\sin \theta = \frac{3}{4}$ and that the perimeter of the sector *OAB* is 21 cm, find *r*, giving your answer to 3 significant figures.

Solution:



(a) $OC = OT - CT = R \text{ cm} - r \text{ cm} = (R - r) \text{ cm}$

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(b) In $\triangle OCE$, $\angle CEO = 90°$ (radius perpendicular to tangent)

and $\angle COE = \theta$ (OT bisects $\angle AOB$)

Using $\sin \angle COE = \frac{CE}{OC}$

$\sin \theta = \frac{r}{R - r}$

$(R - r) \sin \theta = r$

$R \sin \theta - r \sin \theta = r$

$R \sin \theta = r + r \sin \theta$

$R \sin \theta = r (1 + \sin \theta)$

(c) As $\sin \theta = \frac{3}{4}$, $\frac{3}{4}R = \frac{7}{4}r \Rightarrow R = \frac{7}{3}r$

and $\theta = \sin^{-1} \frac{3}{4} = 0.84806 \ldots^{c}$

The perimeter of the sector $= 2R + 2R\theta = 2R\left(1 + \theta\right) = \frac{14}{3}r\left(1.84806 \ldots\right)$

So $21 = \frac{14}{3}r\left(1.84806 \ldots\right)$

$\Rightarrow r = \frac{21 \times 3}{14(1.84806 \ldots)} = \frac{9}{2(1.84806 \ldots)} = 2.43$ (3 s.f.)

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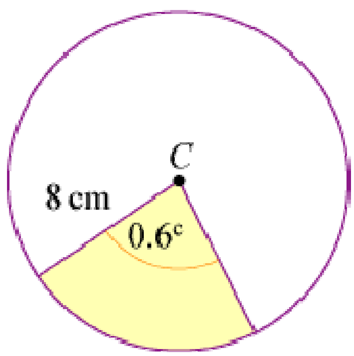
Radian measure and its applications
Exercise C, Question 1

Question:

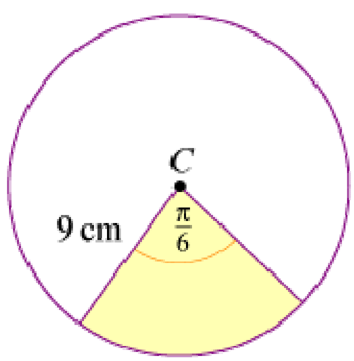
(*Note:* give non-exact answers to 3 significant figures.)

Find the area of the shaded sector in each of the following circles with centre C. Leave your answer in terms of π, where appropriate.

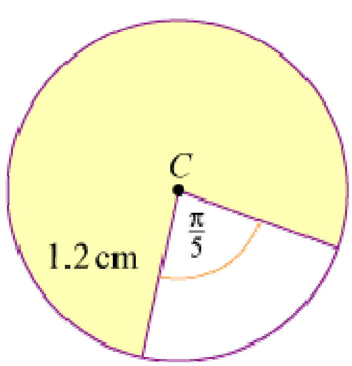
(a)



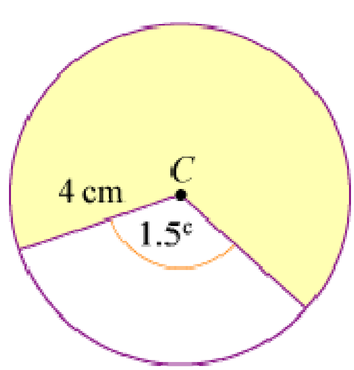
(b)



(c)



(d)



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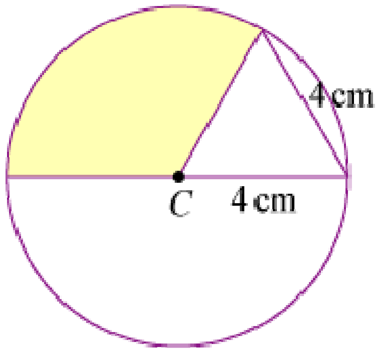
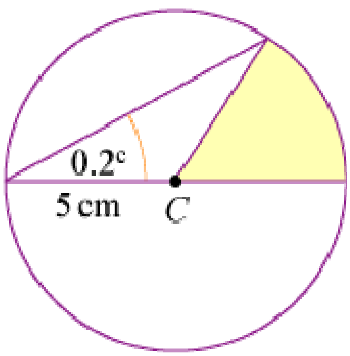
(e)

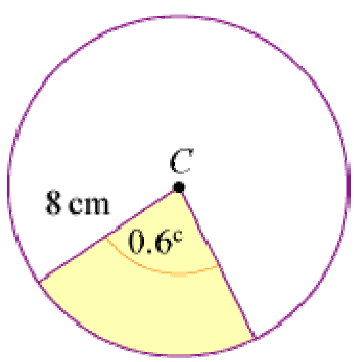


(f)



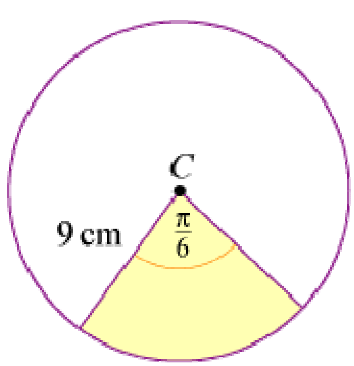
Solution:

(a)



Area of shaded sector $= \frac{1}{2} \times 8^2 \times 0.6 = 19.2 \text{ cm}^2$

(b)



Area of shaded sector $= \frac{1}{2} \times 9^2 \times \frac{\pi}{6} = \frac{27\pi}{4} \text{ cm}^2 = 6.75\pi \text{ cm}^2$

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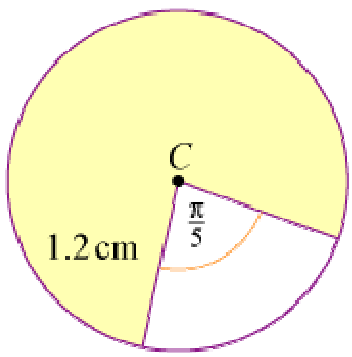
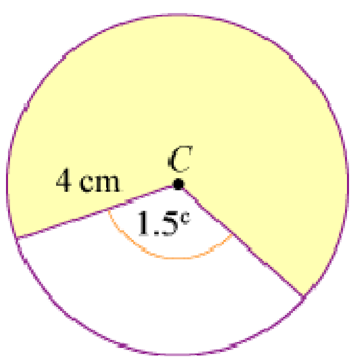
(c)



Angle subtended at C by major arc $= 2\pi - \frac{\pi}{5} = \frac{9\pi}{5}$ rad

Area of shaded sector $= \frac{1}{2} \times 1.2^2 \times \frac{9\pi}{5} = 1.296\pi\ \text{cm}^2$

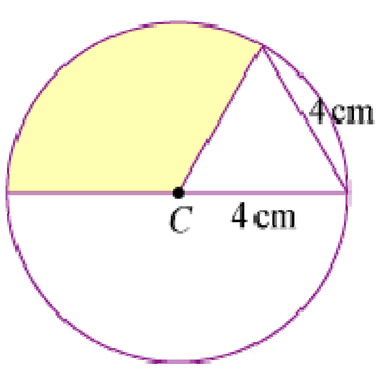
(d)



Angle subtended at C by major arc $= (2\pi - 1.5)$ rad

Area of shaded sector $= \frac{1}{2} \times 4^2 \times \left(2\pi - 1.5\right) = 38.3\ \text{cm}^2$ (3 s.f.)

(e)



The triangle is equilateral so angle at C in the triangle is $\frac{\pi}{3}$ rad.

Angle subtended at C by shaded sector $= \pi - \frac{\pi}{3}$ rad $= \frac{2\pi}{3}$ rad

Area of shaded sector $= \frac{1}{2} \times 4^2 \times \frac{2\pi}{3} = \frac{16}{3}\pi\ \text{cm}^2$

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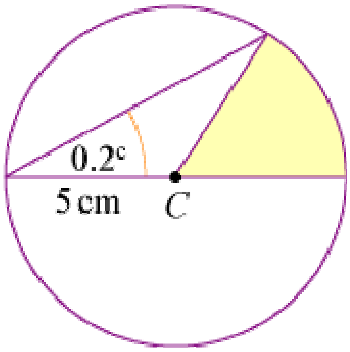
(f)



As triangle is isosceles, angle at *C* in shaded sector is 0.4^c.

Area of shaded sector $= \frac{1}{2} \times 5^2 \times 0.4 = 5 \text{ cm}^2$

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Radian measure and its applications

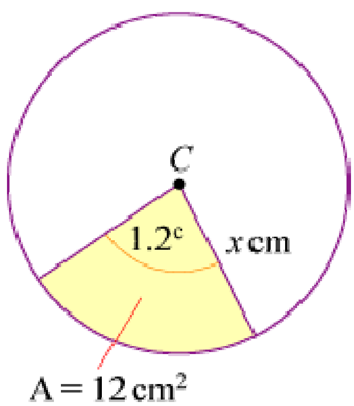
Exercise C, Question 2

Question:

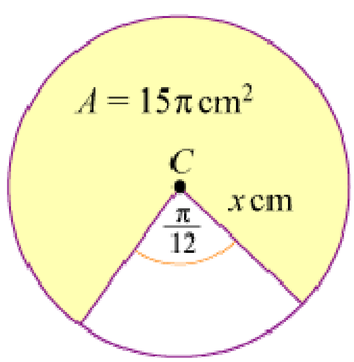
(*Note:* give non-exact answers to 3 significant figures.)

For the following circles with centre *C*, the area *A* of the shaded sector is given. Find the value of *x* in each case.

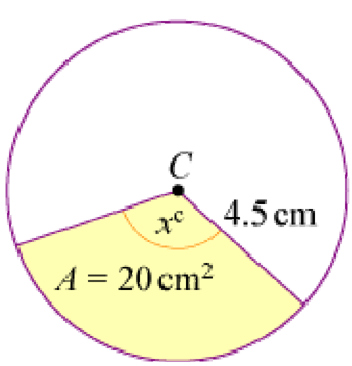
(a)



(b)



(c)



Solution:

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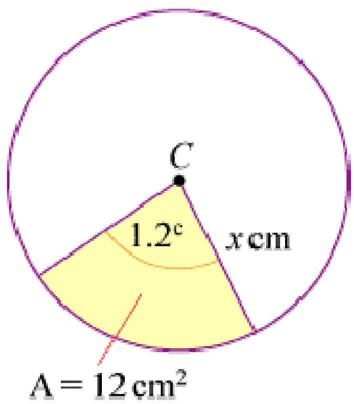
(a)



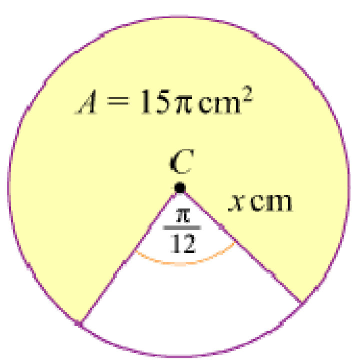
Area of shaded sector $= \frac{1}{2} \times x^2 \times 1.2 = 0.6x^2$ cm^2

So $0.6x^2 = 12$

$\Rightarrow$ $x^2 = 20$

$\Rightarrow$ $x = 4.47$ (3 s.f.)

(b)



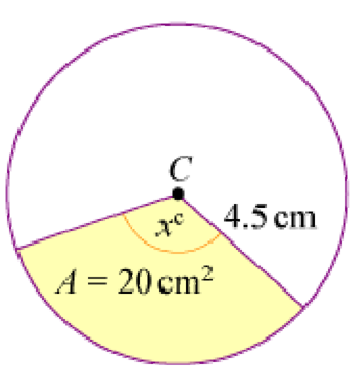
Area of shaded sector $= \frac{1}{2} \times x^2 \times \left(2\pi - \frac{\pi}{12} \right) = \frac{1}{2} x^2 \times \frac{23\pi}{12}$ cm^2

So $15\pi = \frac{23}{24}\pi x^2$

$\Rightarrow$ $x^2 = \frac{24 \times 15}{23}$

$\Rightarrow$ $x = 3.96$ (3 s.f.)

(c)



Area of shaded sector $= \frac{1}{2} \times 4.5^2 \times x$ cm^2

So $20 = \frac{1}{2} \times 4.5^2 x$

$\Rightarrow \quad x = \frac{40}{4.5^2} = 1.98$ (3 s.f.)

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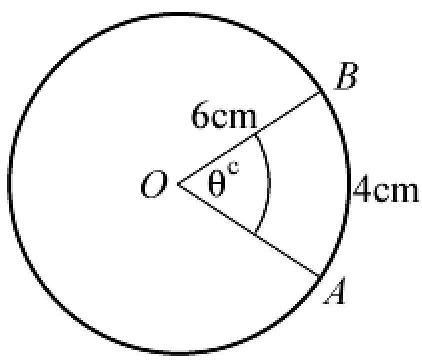
Radian measure and its applications
Exercise C, Question 3

Question:

(*Note:* give non-exact answers to 3 significant figures.)

The arc *AB* of a circle, centre *O* and radius 6 cm, has length 4 cm.
Find the area of the minor sector *AOB*.

Solution:



Using $l = r\theta$
$4 = 6\theta$
$\theta = \frac{2}{3}$

So area of sector $= \frac{1}{2} \times 6^2 \times \frac{2}{3} = 12 \text{ cm}^2$

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Radian measure and its applications
Exercise C, Question 4

Question:

(*Note:* give non-exact answers to 3 significant figures.)

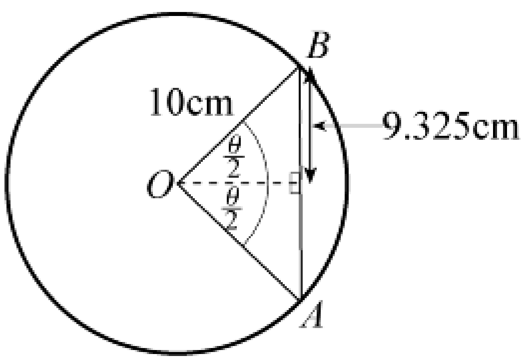
The chord *AB* of a circle, centre *O* and radius 10 cm, has length 18.65 cm and subtends an angle of θ radians at *O*.

(a) Show that $\theta = 2.40$ (to 3 significant figures).

(b) Find the area of the minor sector *AOB.*

Solution:

(a)



Using the line of symmetry in the isosceles triangle *OAB*

$\sin \frac{\theta}{2} = \frac{9.325}{10}$

$\frac{\theta}{2} = \sin^{-1}\left(\frac{9.325}{10}\right)$ (Use radian mode)

$\theta = 2 \sin^{-1}\left(\frac{9.325}{10}\right) = 2.4025 \quad \ldots \quad = 2.40$ (3 s.f.)

(b) Area of minor sector *AOB* = $\frac{1}{2} \times 10^2 \times \theta = 120 \text{ cm}^2$ (3 s.f.)

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Radian measure and its applications

Exercise C, Question 5

Question:

(*Note:* give non-exact answers to 3 significant figures.)

The area of a sector of a circle of radius 12 cm is 100 cm^2.
Find the perimeter of the sector.

Solution:

Using area of sector $= \frac{1}{2}r^2\theta$

$100 = \frac{1}{2} \times 12^2\theta$

$\Rightarrow \quad \theta = \frac{100}{72} = \frac{25}{18}^c$

The perimeter of the sector $= 12 + 12 + 12\theta = 12\left(2 + \theta\right) = 12 \times \frac{61}{18} = \frac{122}{3} = 40\frac{2}{3}$ cm

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Radian measure and its applications
Exercise C, Question 6

Question:

(*Note:* give non-exact answers to 3 significant figures.)

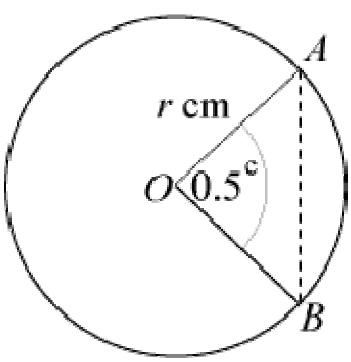
The arc *AB* of a circle, centre *O* and radius *r* cm, is such that $\angle AOB = 0.5$ radians. Given that the perimeter of the minor sector *AOB* is 30 cm:

(a) Calculate the value of *r*.

(b) Show that the area of the minor sector *AOB* is 36 cm^2.

(c) Calculate the area of the segment enclosed by the chord *AB* and the minor arc *AB*.

Solution:



(a) The perimeter of minor sector AOB $= r + r + 0.5r = 2.5r$ cm
So $30 = 2.5r$

$\Rightarrow \quad r = \frac{30}{2.5} = 12$

(b) Area of minor sector $= \frac{1}{2} \times r^2 \times \theta = \frac{1}{2} \times 12^2 \times 0.5 = 36$ cm^2

(c) Area of segment

$= \frac{1}{2} r^2 \left(\theta - \sin \theta \right)$

$= \frac{1}{2} \times 12^2 \left(0.5 - \sin 0.5 \right)$

$= 72 (0.5 - \sin 0.5)$
$= 1.48$ cm^2 (3 s.f.)

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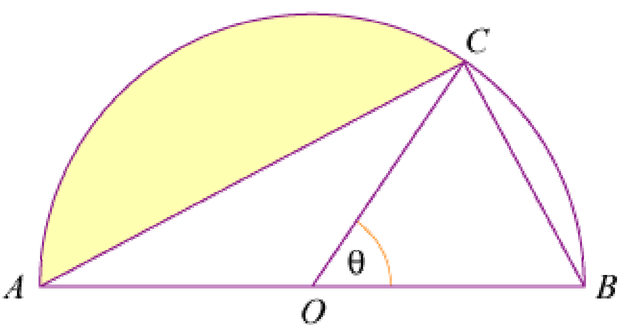
Radian measure and its applications

Exercise C, Question 7

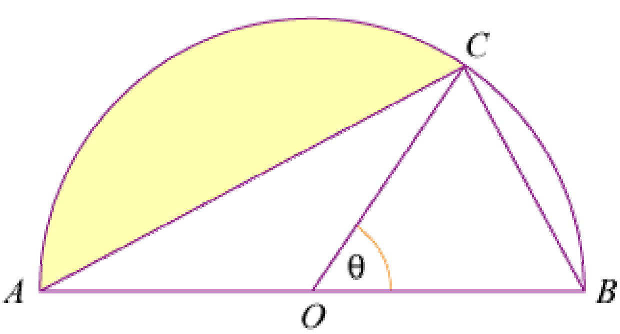
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *AB* is the diameter of a circle of radius *r* cm and $\angle$ BOC = θ radians. Given that the area of $\triangle$COB is equal to that of the shaded segment, show that $\theta + 2 \sin \theta = \pi$.



Solution:



Using the formula

area of a triangle $= \frac{1}{2} \text{ab} \sin C$

area of $\triangle$COB $= \frac{1}{2} r^2 \sin \theta$ ①

$\angle$ AOC $= (\pi - \theta)$ rad

Area of shaded segment $= \frac{1}{2} r^2 \left[\left(\pi - \theta \right) - \sin \left(\pi - \theta \right) \right]$ ②

As ① and ② are equal

$\frac{1}{2} r^2 \sin \theta = \frac{1}{2} r^2 \left[\pi - \theta - \sin \left(\pi - \theta \right) \right]$

$\sin \theta = \pi - \theta - \sin (\pi - \theta)$
and as $\sin (\pi - \theta) = \sin \theta$
$\sin \theta = \pi - \theta - \sin \theta$
So $\theta + 2 \sin \theta = \pi$

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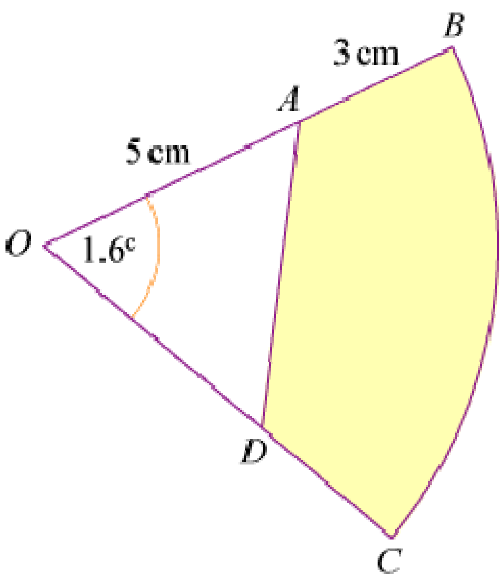
Radian measure and its applications
Exercise C, Question 8

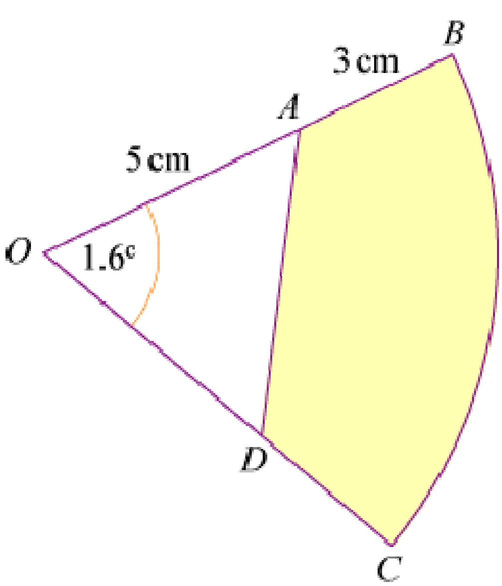
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *BC* is the arc of a circle, centre *O* and radius 8 cm. The points *A* and *D* are such that OA = OD = 5 cm. Given that ∠ BOC = 1.6 radians, calculate the area of the shaded region.



Solution:



Area of sector OBC = $\frac{1}{2}r^2\theta$ with $r = 8$ cm and $\theta = 1.6^c$

Area of sector OBC = $\frac{1}{2} \times 8^2 \times 1.6 = 51.2$ cm^2

Using area of triangle formula

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Area of $\triangle$OAD = $\frac{1}{2} \times 5 \times 5 \times \sin 1.6^c = 12.495 \text{ cm}^2$

Area of shaded region $= 51.2 - 12.495 = 38.7 \text{ cm}^2$ (3 s.f.)

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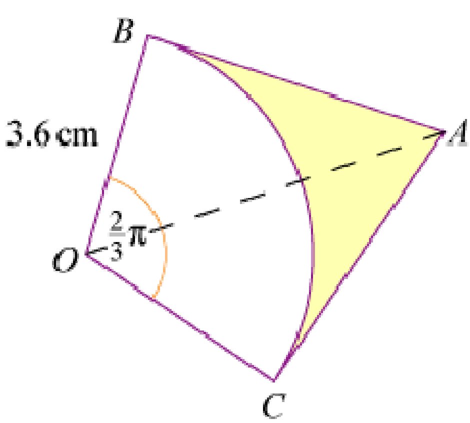
Radian measure and its applications
Exercise C, Question 9

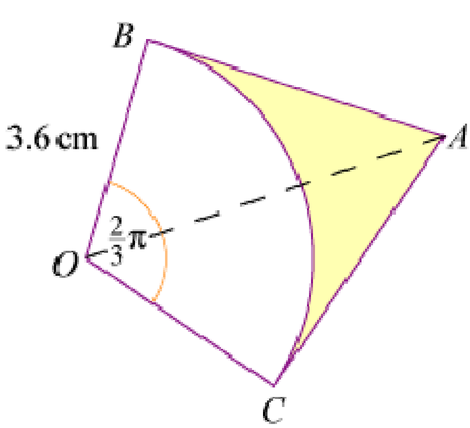
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *AB* and *AC* are tangents to a circle, centre *O* and radius 3.6 cm. Calculate the area of the shaded region, given that $\angle BOC = \frac{2}{3}\pi$ radians.



Solution:



In right-angled $\triangle OBA$: $\tan\frac{\pi}{3} = \frac{AB}{3.6}$

$\Rightarrow \quad AB = 3.6 \tan\frac{\pi}{3}$

Area of $\triangle OBA = \frac{1}{2} \times 3.6 \times 3.6 \times \tan\frac{\pi}{3}$

So area of quadrilateral OBAC $= 3.6^2 \times \tan\frac{\pi}{3} = 22.447 \ldots \text{ cm}^2$

Area of sector $= \frac{1}{2} \times 3.6^2 \times \frac{2}{3}\pi = 13.57 \ldots \text{ cm}^2$

Area of shaded region

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= area of quadrilateral *OBAC* − area of sector *OBC*
= 8.88 cm^2 (3 s.f.)

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Radian measure and its applications
Exercise C, Question 10

Question:

(*Note:* give non-exact answers to 3 significant figures.)

A chord *AB* subtends an angle of θ radians at the centre *O* of a circle of radius 6.5 cm. Find the area of the segment enclosed by the chord *AB* and the minor arc *AB*, when:

(a) $\theta = 0.8$

(b) $\theta = \frac{2}{3}\pi$

(c) $\theta = \frac{4}{3}\pi$

Solution:

(a) Area of sector OAB = $\frac{1}{2} \times 6.5^2 \times 0.8$

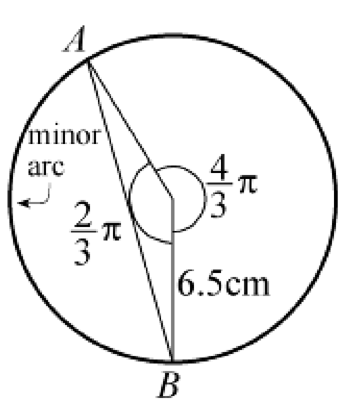
Area of $\triangle$OAB = $\frac{1}{2} \times 6.5^2 \times \sin\ 0.8$

Area of segment $= \frac{1}{2} \times 6.5^2 \times 0.8 - \frac{1}{2} \times 6.5^2 \times \sin\ 0.8 = 1.75\ \text{cm}^2$ (3 s.f.)

(b) Area of segment $= \frac{1}{2} \times 6.5^2 \left(\frac{2}{3}\pi - \sin \frac{2}{3}\pi \right) = 25.9\ \text{cm}^2$ (3 s.f.)

(c) Area of segment $= \frac{1}{2} \times 6.5^2 \left(\frac{2}{3}\pi - \sin \frac{2}{3}\pi \right) = 25.9\ \text{cm}^2$ (3 s.f.)

Diagram shows why $\frac{2}{3}\pi$ is required.



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Radian measure and its applications

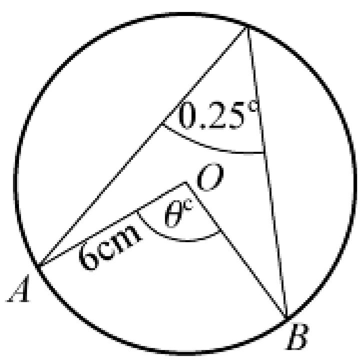
Exercise C, Question 11

Question:

(*Note:* give non-exact answers to 3 significant figures.)

An arc *AB* subtends an angle of 0.25 radians at the *circumference* of a circle, centre *O* and radius 6 cm. Calculate the area of the minor sector *OAB*.

Solution:



Using the circle theorem: angle at the centre $= 2 \times$ angle at circumference

$\angle AOB = 0.5^c$

Area of minor sector AOB $= \frac{1}{2} \times 6^2 \times 0.5 = 9 \text{ cm}^2$

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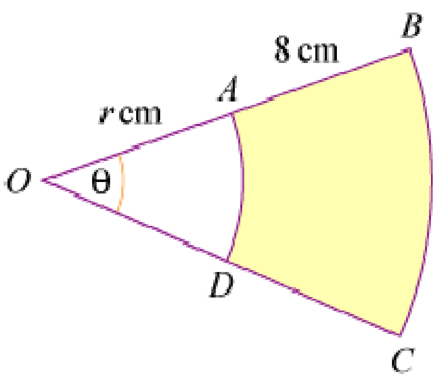
Radian measure and its applications

Exercise C, Question 12

Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *AD* and *BC* are arcs of circles with centre *O*, such that $OA = OD = r$ cm, $AB = DC = 8$ cm and $\angle BOC = \theta$ radians.



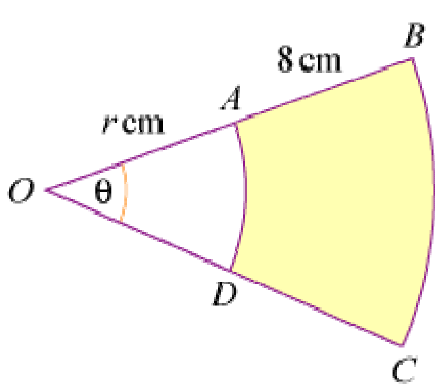
(a) Given that the area of the shaded region is 48 cm^2, show that

$r = \frac{6}{\theta} - 4$.

(b) Given also that $r = 10\theta$, calculate the perimeter of the shaded region.

Solution:

(a)



Area of larger sector $= \frac{1}{2}(r+8)^2\theta$ cm^2

Area of smaller sector $= \frac{1}{2}r^2\theta$ cm^2

Area of shaded region

$= \frac{1}{2}(r+8)^2\theta - \frac{1}{2}r^2\theta$ cm^2

$= \frac{1}{2}\theta\left[\left(r^2 + 16r + 64\right) - r^2\right]$ cm^2

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$= \frac{1}{2}\theta \left(16r + 64 \right)$ cm^2

$= 8\theta (r + 4)$ cm^2

So $48 = 8\theta (r + 4)$

$\Rightarrow \quad 6 = r\theta + 4\theta \quad *$

$\Rightarrow \quad r\theta = 6 - 4\theta$

$\Rightarrow \quad r = \frac{6}{\theta} - 4$

(b) As $r = 10\theta$, using *

$10\theta^2 + 4\theta - 6 = 0$

$5\theta^2 + 2\theta - 3 = 0$

$(5\theta - 3) (\theta + 1) = 0$

So $\theta = \frac{3}{5}$ and $r = 10\theta = 6$

Perimeter of shaded region $= [r\theta + 8 + (r + 8) \theta + 8]$ cm

So perimeter $= \frac{18}{5} + 8 + \frac{42}{5} + 8 = 28$ cm

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Radian measure and its applications

Exercise C, Question 13

Question:

(*Note:* give non-exact answers to 3 significant figures.)

A sector of a circle of radius 28 cm has perimeter P cm and area A cm^2.
Given that $A = 4P$, find the value of P.

Solution:

The area of the sector $= \frac{1}{2} \times 28^2 \times \theta = 392\theta$ cm$^2 = A$ cm^2

The perimeter of the sector $= (28\theta + 56)$ cm $= P$ cm
As $A = 4P$
$392\theta = 4(28\theta + 56)$
$98\theta = 28\theta + 56$
$70\theta = 56$
$\theta = \frac{56}{70} = 0.8$

$P = 28\theta + 56 = 28(0.8) + 56 = 78.4$

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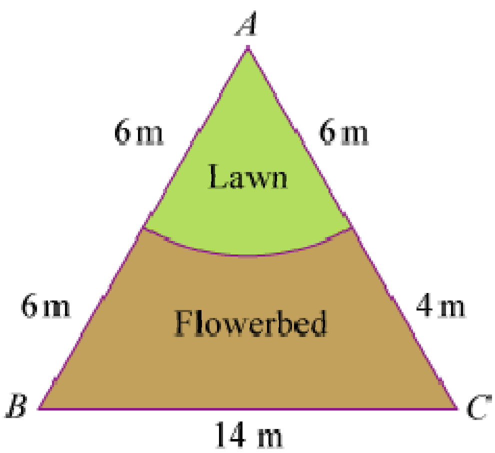
Radian measure and its applications

Exercise C, Question 14

Question:

(*Note:* give non-exact answers to 3 significant figures.)

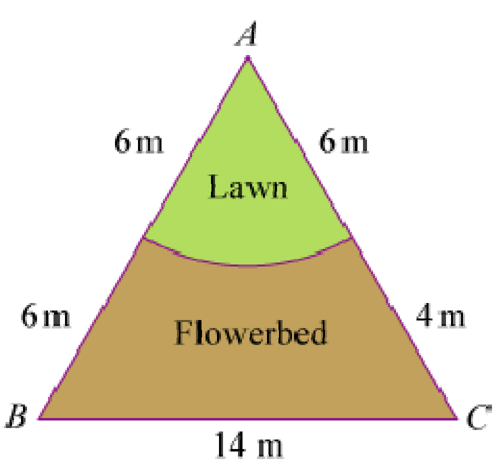
The diagram shows a triangular plot of land. The sides *AB*, *BC* and *CA* have lengths 12 m, 14 m and 10 m respectively. The lawn is a sector of a circle, centre *A* and radius 6 m.



(a) Show that ∠BAC = 1.37 radians, correct to 3 significant figures.

(b) Calculate the area of the flowerbed.

Solution:



(a) Using cosine rule

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{10^2 + 12^2 - 14^2}{2 \times 10 \times 12} = 0.2$

$A = \cos^{-1}(0.2)$ (use in radian mode)

$A = 1.369 \ldots = 1.37$ (3 s.f.)

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(b) Area of $\triangle$ABC = $\frac{1}{2} \times 12 \times 10 \times \sin A = 58.787 \ldots \text{ m}^2$

Area of sector (lawn) = $\frac{1}{2} \times 6^2 \times A = 24.649 \ldots \text{ m}^2$

Area of flowerbed = area of $\triangle ABC$ − area of sector = 34.1m^2 (3 s.f.)

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Radian measure and its applications
Exercise D, Question 1

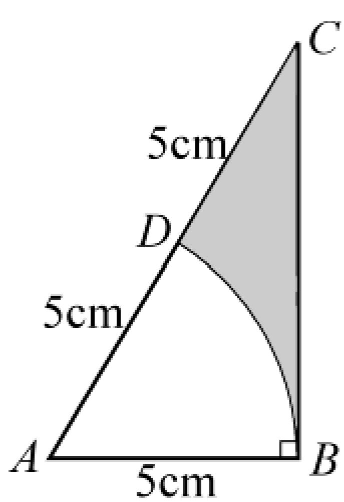
Question:

Triangle *ABC* is such that AB = 5 cm, AC = 10 cm and $\angle$ ABC = 90 ° . An arc of a circle, centre *A* and radius 5 cm, cuts *AC* at *D*.

(a) State, in radians, the value of $\angle$ BAC.

(b) Calculate the area of the region enclosed by *BC*, *DC* and the arc *BD*.

Solution:



(a) In the right-angled $\triangle$ABC

cos $\angle$ BAC = $\frac{5}{10}$ = $\frac{1}{2}$

$\angle$ BAC = $\frac{\pi}{3}$

(b) Area of $\triangle$ABC = $\frac{1}{2} \times 5 \times 10 \times \sin \frac{\pi}{3} = 21.650 \ldots$ cm^2

Area of sector DAB = $\frac{1}{2} \times 5^2 \times \frac{\pi}{3} = 13.089 \ldots$ cm^2

Area of shaded region = area of $\triangle$ABC − area of sector *DAB* = 8.56 cm^2 (3 s.f.)

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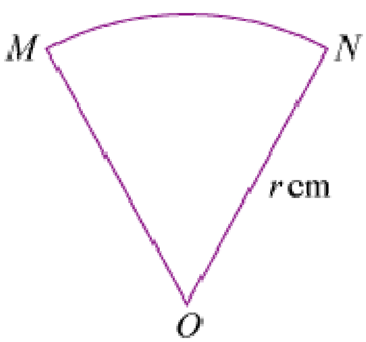
Radian measure and its applications

Exercise D, Question 2

Question:

The diagram shows a minor sector *OMN* of a circle centre *O* and radius *r* cm. The perimeter of the sector is 100 cm and the area of the sector is *A* cm^2.

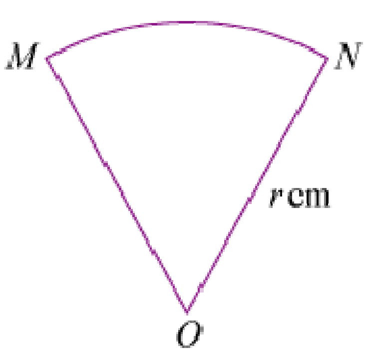


(a) Show that $A = 50r - r^2$.

(b) Given that *r* varies, find:
(i) The value of *r* for which *A* is a maximum and show that *A* is a maximum.
(ii) The value of $\angle$ MON for this maximum area.
(iii) The maximum area of the sector *OMN*.

[E]

Solution:



(a) Let $\angle MON = \theta^c$

Perimeter of sector $= (2r + r\theta)$ cm

So $100 = 2r + r\theta$

$\Rightarrow \quad r\theta = 100 - 2r$

$\Rightarrow \quad \theta = \frac{100}{r} - 2$ *

The area of the sector $= A$ $cm^2 = \frac{1}{2}r^2\theta$ cm^2

So $A = \frac{1}{2}r^2\left(\frac{100}{r} - 2\right)$

$\Rightarrow \quad A = 50r - r^2$

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(b) (i) $A = -\left(r^2 - 50r\right) = -\left[\left(r - 25\right)^2 - 625\right] = 625 - \left(r - 25\right)^2$
The maximum value occurs when $r = 25$, as for all other values of r something is subtracted from 625.

(ii) Using *, when $r = 25$, $\theta = \frac{100}{25} - 2 = 2^c$

(iii) Maximum area = 625 cm^2

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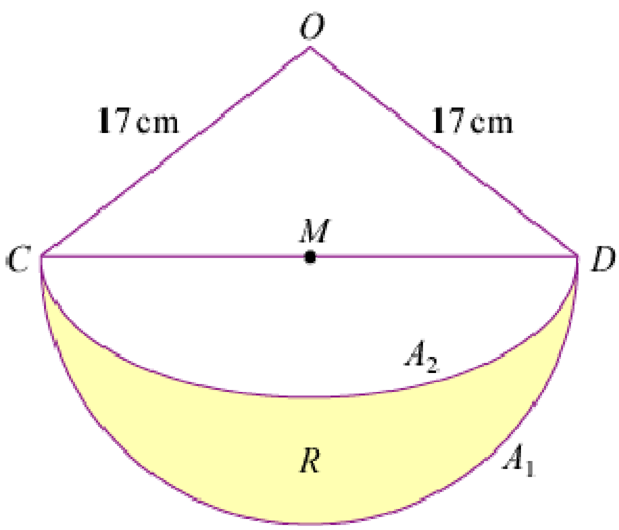
Radian measure and its applications
Exercise D, Question 3

Question:

The diagram shows the triangle *OCD* with OC = OD = 17 cm and CD = 30 cm. The mid-point of *CD* is *M*. With centre *M*, a semicircular arc A_1 is drawn on *CD* as diameter. With centre *O* and radius 17 cm, a circular arc A_2 is drawn from *C* to *D*. The shaded region *R* is bounded by the arcs A_1 and A_2. Calculate, giving answers to 2 decimal places:



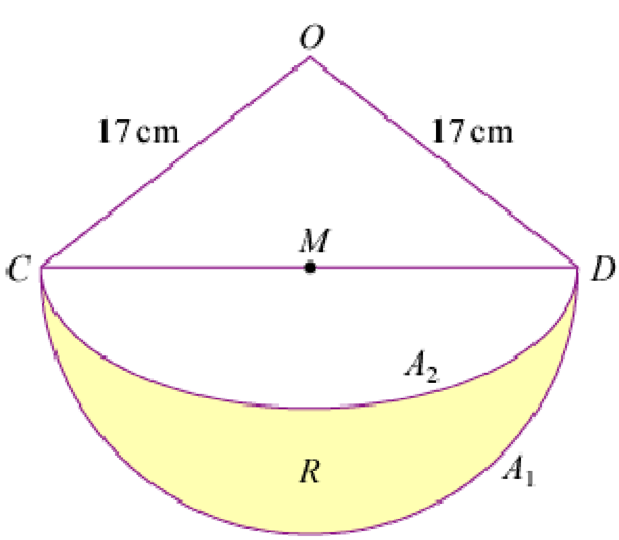
(a) The area of the triangle *OCD*.

(b) The angle *COD* in radians.

(c) The area of the shaded region *R*.

[E]

Solution:



(a) Using Pythagoras' theorem to find *OM*:
$OM^2 = 17^2 - 15^2 = 64$

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$\Rightarrow$ OM = 8 cm

Area of $\triangle$OCD = $\frac{1}{2}$CD × OM = $\frac{1}{2}$ × 30 × 8 = 120 cm^2

(b) In $\triangle$OCM: sin $\angle$ COM = $\frac{15}{17}$ $\Rightarrow$ $\angle$ COM = 1.0808 … ^{c}

So $\angle$ COD = 2 × $\angle$ COM = 2.16^c (2 d.p.)

(c) Area of shaded region R = area of semicircle − area of segment CDA_2

Area of segment = area of sector OCD – area of sector $\triangle$ OCD

$= \frac{1}{2} \times 17^2 \left(\angle \text{COD} - \sin \angle \text{COD} \right)$ (angles in radians)

= 192.362 … cm^2 (use at least 3 d.p.)

Area of semicircle = $\frac{1}{2} \times \pi \times 15^2$ = 353.429 … cm^2

So area of shaded region R = 353.429 … − 192.362 … = 161.07 cm^2 (2 d.p.)

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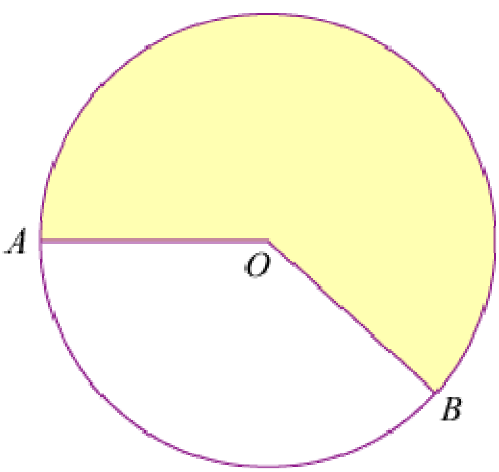
Radian measure and its applications

Exercise D, Question 4

Question:

The diagram shows a circle, centre O, of radius 6 cm. The points A and B are on the circumference of the circle. The area of the shaded major sector is 80 cm^2. Given that $\angle AOB = \theta$ radians, where $0 < \theta < \pi$, calculate:

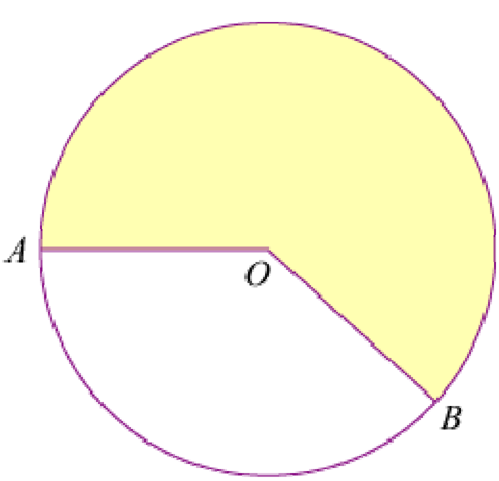


(a) The value, to 3 decimal places, of θ.

(b) The length in cm, to 2 decimal places, of the minor arc AB.

[E]

Solution:



(a) Reflex angle AOB = $(2\pi - \theta)$ rad

Area of shaded sector $= \frac{1}{2} \times 6^2 \times \left(2\pi - \theta\right) = 36\pi - 18\theta \text{ cm}^2$

So $80 = 36\pi - 18\theta$

$\Rightarrow \quad 18\theta = 36\pi - 80$

$\Rightarrow \quad \theta = \frac{36\pi - 80}{18} = 1.839$ (3 d.p.)

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(b) Length of minor arc AB = $6\theta = 11.03$ cm (2 d.p.)

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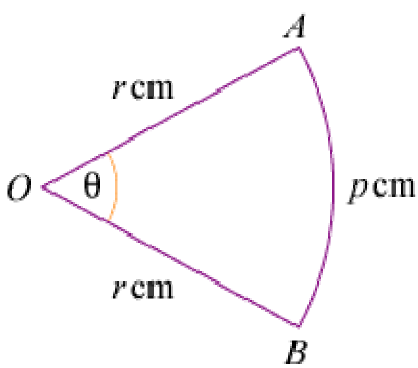
Radian measure and its applications
Exercise D, Question 5

Question:

The diagram shows a sector *OAB* of a circle, centre *O* and radius *r* cm. The length of the arc *AB* is *p* cm and ∠ AOB is θ radians.



(a) Find θ in terms of *p* and *r*.

(b) Deduce that the area of the sector is $\frac{1}{2}pr$ cm^2.

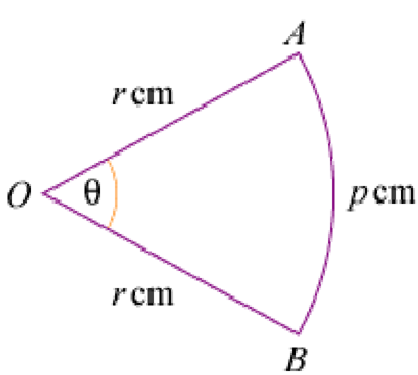
Given that $r = 4.7$ and $p = 5.3$, where each has been measured to 1 decimal place, find, giving your answer to 3 decimal places:

(c) The least possible value of the area of the sector.

(d) The range of possible values of θ.

[E]

Solution:



(a) Using $l = r\theta \quad \Rightarrow \quad p = r\theta$

So $\theta = \frac{p}{r}$

(b) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times \frac{p}{r} = \frac{1}{2}pr$ cm^2

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(c) $4.65 \leq r < 4.75$, $5.25 \leq p < 5.35$

Least value for area of sector $= \frac{1}{2} \times 5.25 \times 4.65 = 12.207 \text{ cm}^2$ (3 d.p.)

(**Note**: Lowest is 12.20625, so 12.207 should be given.)

(d) Max value of $\theta = \frac{\max\ p}{\min\ r} = \frac{5.35}{4.65} = 1.1505 \ldots$

So give 1.150 (3 d.p.)

Min value of $\theta = \frac{\min\ p}{\max\ r} = \frac{5.25}{4.75} = 1.10526 \ldots$

So give 1.106 (3 d.p.)

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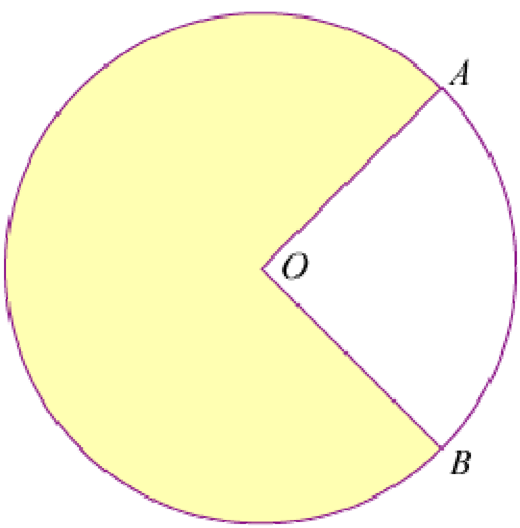
Radian measure and its applications

Exercise D, Question 6

Question:

The diagram shows a circle centre *O* and radius 5 cm. The length of the minor arc *AB* is 6.4 cm.



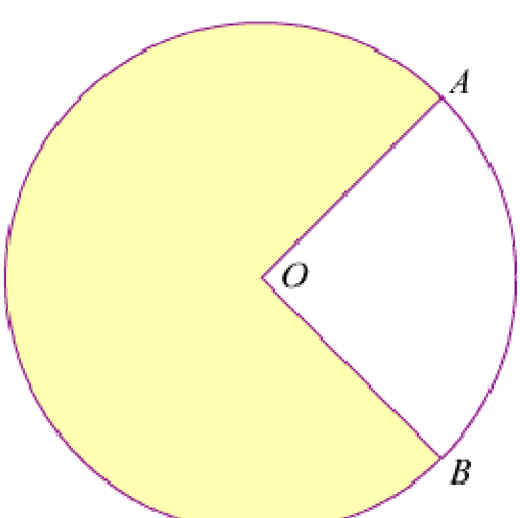
(a) Calculate, in radians, the size of the acute angle *AOB*.
The area of the minor sector *AOB* is R_1 cm^2 and the area of the shaded major sector *AOB* is R_2 cm^2.

(b) Calculate the value of R_1.

(c) Calculate $R_1 : R_2$ in the form 1: *p*, giving the value of *p* to 3 significant figures.

[E]

Solution:



(a) Using $l = r\theta$, $6.4 = 5\theta$

$\Rightarrow \quad \theta = \frac{6.4}{5} = 1.28^c$

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(b) Using area of sector $= \frac{1}{2}r^2\theta$

$R_1 = \frac{1}{2} \times 5^2 \times 1.28 = 16$

(c) $R_2 =$ area of circle $- R_1 = \pi 5^2 - 16 = 62.5398\ \ldots$

So $\frac{R_1}{R_2} = \frac{16}{62.5398\ \ldots} = \frac{1}{3.908\ \ldots} = \frac{1}{p}$

$\Rightarrow \quad p = 3.91$ (3 s.f.)

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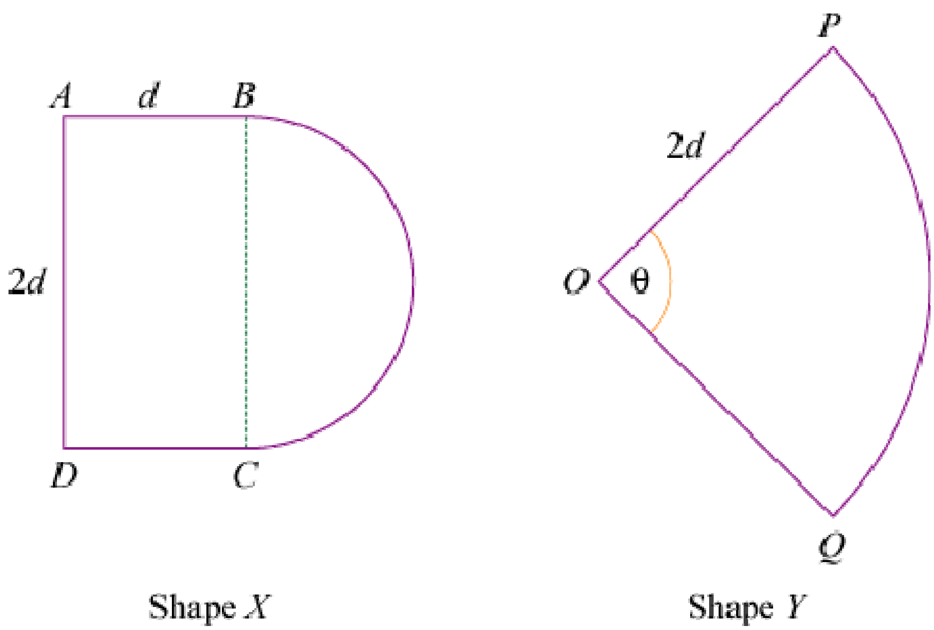
Radian measure and its applications
Exercise D, Question 7

Question:



The diagrams show the cross-sections of two drawer handles.
Shape *X* is a rectangle *ABCD* joined to a semicircle with *BC* as diameter. The length AB = *d* cm and BC = 2*d* cm. Shape *Y* is a sector *OPQ* of a circle with centre *O* and radius 2*d* cm. Angle *POQ* is θ radians.
Given that the areas of shapes *X* and *Y* are equal:

(a) Prove that $\theta = 1 + \frac{1}{4}\pi$.

Using this value of θ, and given that $d = 3$, find in terms of π:

(b) The perimeter of shape *X*.

(c) The perimeter of shape *Y*.

(d) Hence find the difference, in mm, between the perimeters of shapes *X* and *Y*. **[E]**

Solution:

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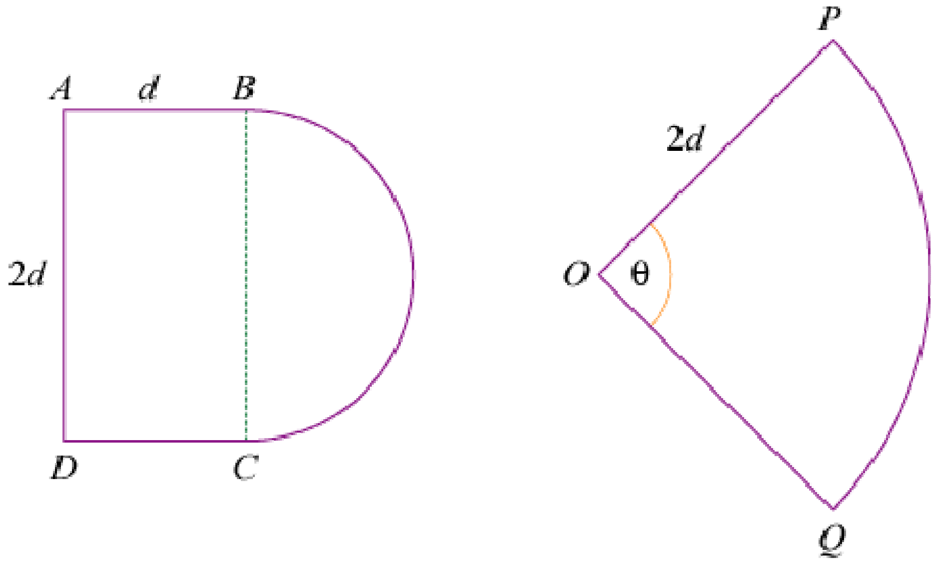

Shape X Shape Y

(a) Area of shape X
= area of rectangle + area of semicircle
$= 2d^2 + \frac{1}{2}\pi d^2 \text{ cm}^2$

Area of shape $Y = \frac{1}{2}(2d)^2\theta = 2d^2\theta \text{ cm}^2$

As $X = Y$: $2d^2 + \frac{1}{2}\pi d^2 = 2d^2\theta$

Divide by $2d^2$: $1 + \frac{\pi}{4} = \theta$

(b) Perimeter of X
$= (d + 2d + d + \pi d)$ cm with $d = 3$
$= (3\pi + 12)$ cm

(c) Perimeter of Y

$= (2d + 2d + 2d\theta)$ cm with $d = 3$ and $\theta = 1 + \frac{\pi}{4}$

$= 12 + 6\left(1 + \frac{\pi}{4}\right)$

$= \left(18 + \frac{3\pi}{2}\right)$ cm

(d) Difference (in mm)

$= \left[\left(18 + \frac{3\pi}{2}\right) - \left(3\pi + 12\right)\right] \times 10$

$= 10\left(6 - \frac{3\pi}{2}\right)$

$= 12.87 \ldots$
$= 12.9$ (3 s.f.)

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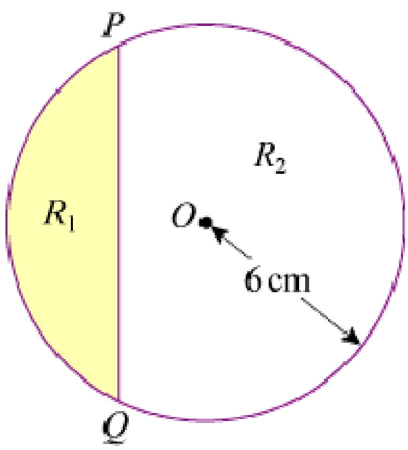
Radian measure and its applications

Exercise D, Question 8

Question:

The diagram shows a circle with centre O and radius 6 cm. The chord PQ divides the circle into a minor segment R_1 of area A_1 cm^2 and a major segment R_2 of area A_2 cm^2. The chord PQ subtends an angle θ radians at O.

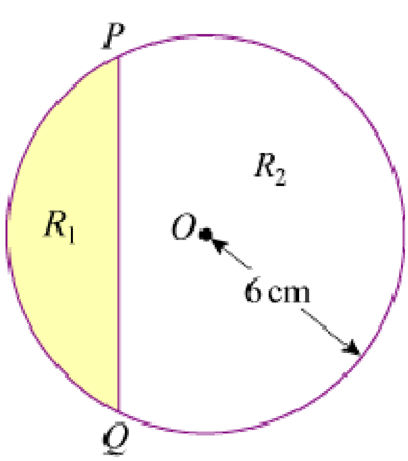


(a) Show that $A_1 = 18 (\theta - \sin \theta)$.
Given that $A_2 = 3A_1$ and $f (\theta) = 2\theta - 2 \sin \theta - \pi$:

(b) Prove that $f (\theta) = 0$.

(c) Evaluate f(2.3) and f(2.32) and deduce that $2.3 < \theta < 2.32$. **[E]**

Solution:



(a) Area of segment R_1 = area of sector OPQ − area of triangle OPQ

$\Rightarrow A_1 = \frac{1}{2} \times 6^2 \times \theta - \frac{1}{2} \times 6^2 \times \sin \theta$

$\Rightarrow A_1 = 18 (\theta - \sin \theta)$

(b) Area of segment R_2 = area of circle − area of segment R_1

$\Rightarrow A_2 = \pi 6^2 - 18 (\theta - \sin \theta)$

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$\Rightarrow \quad A_2 = 36\pi - 18\theta + 18 \sin \theta$

As $A_2 = 3A_1$

$36\pi - 18\theta + 18 \sin \theta = 3(18\theta - 18 \sin \theta) = 54\theta - 54 \sin \theta$

So $72\theta - 72 \sin \theta - 36\pi = 0$

$\Rightarrow \quad 36(2\theta - 2 \sin \theta - \pi) = 0$

$\Rightarrow \quad 2\theta - 2 \sin \theta - \pi = 0$

So $f(\theta) = 0$

(c) $f(2.3) = -0.0330 \quad \ldots$

$f(2.32) = +0.0339 \quad \ldots$

As there is a change of sign θ lies between 2.3 and 2.32.

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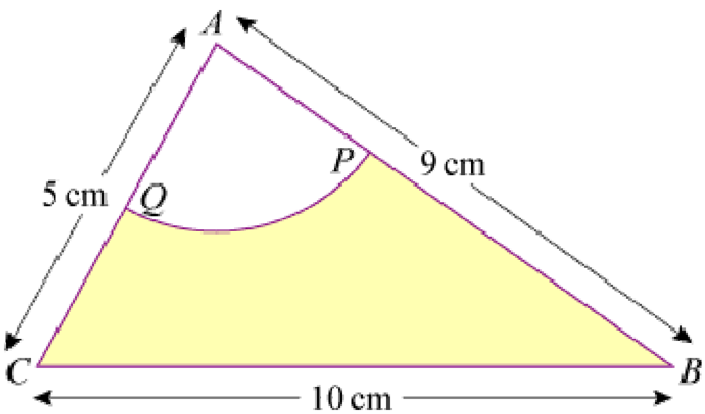
Radian measure and its applications
Exercise D, Question 9

Question:

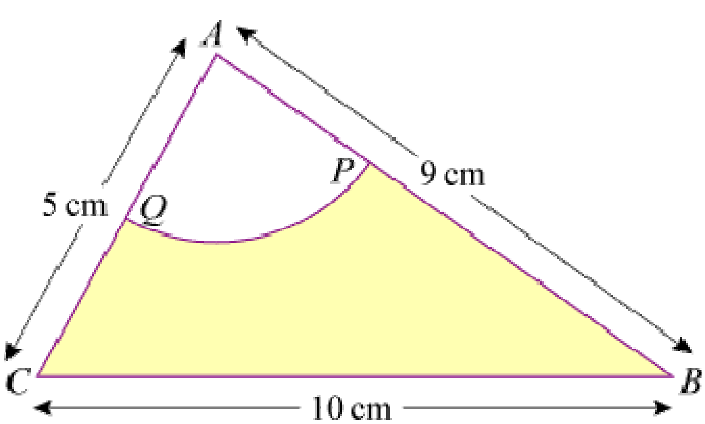
Triangle *ABC* has AB = 9 cm, BC = 10 cm and CA = 5 cm. A circle, centre *A* and radius 3 cm, intersects *AB* and *AC* at *P* and *Q* respectively, as shown in the diagram.



(a) Show that, to 3 decimal places, ∠ BAC = 1.504 radians.

(b) Calculate:
(i) The area, in cm^2, of the sector *APQ*.
(ii) The area, in cm^2, of the shaded region *BPQC*.
(iii) The perimeter, in cm, of the shaded region *BPQC*. **[E]**

Solution:



(a) In △*ABC* using the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \quad \cos \angle BAC = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9} = 0.06$$

$$\Rightarrow \quad \angle BAC = 1.50408 \ldots \text{ radians} = 1.504^c \text{ (3 d.p.)}$$

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(b) (i) Using the sector area formula: area of sector $= \frac{1}{2}r^2\theta$

$\Rightarrow$ area of sector APQ $= \frac{1}{2} \times 3^2 \times 1.504 = 6.77$ cm^2 (3 s.f.)

(ii) Area of shaded region *BPQC*
= area of $\triangle ABC -$ area of sector *APQ*

$= \frac{1}{2} \times 5 \times 9 \times \sin 1.504^c - \frac{1}{2} \times 3^2 \times 1.504$ cm^2

= 15.681 … cm^2
= 15.7 cm^2 (3 s.f.)

(iii) Perimeter of shaded region *BPQC*
= QC + CB + BP + arc *PQ*
= 2 + 10 + 6 + (3 × 1.504) cm
= 22.51 … cm
= 22.5 cm (3 s.f.)

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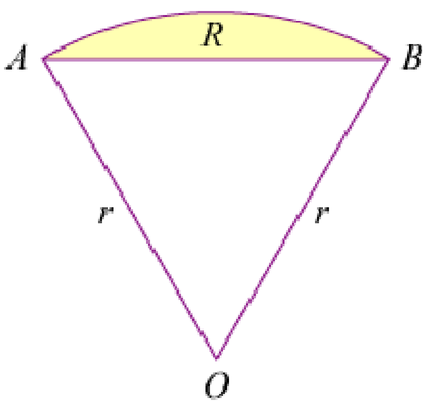
Radian measure and its applications

Exercise D, Question 10

Question:

The diagram shows the sector *OAB* of a circle of radius r cm. The area of the sector is 15 cm^2 and $\angle AOB = 1.5$ radians.



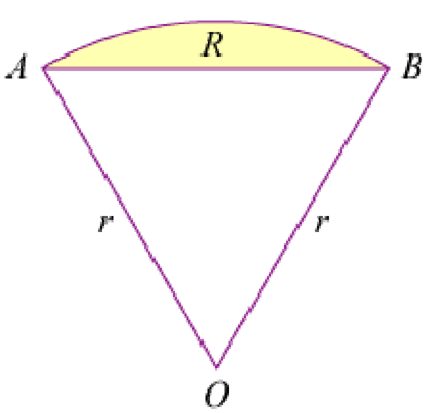
(a) Prove that $r = 2\sqrt{5}$.

(b) Find, in cm, the perimeter of the sector *OAB*.
The segment *R*, shaded in the diagram, is enclosed by the arc *AB* and the straight line *AB*.

(c) Calculate, to 3 decimal places, the area of *R*.

[E]

Solution:



(a) Area of sector $= \frac{1}{2}r^2\left(1.5\right)$ cm^2

So $\frac{3}{4}r^2 = 15$

$\Rightarrow \quad r^2 = \frac{60}{3} = 20$

$\Rightarrow \quad r = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

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(b) Arc length AB = $r\left(1.5\right) = 3\sqrt{5}$ cm
Perimeter of sector
$= AO + OB + \text{arc } AB$
$= \left(2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}\right)$ cm
$= 7\sqrt{5}$ cm
= 15.7 cm (3 s.f.)

(c) Area of segment R
= area of sector − area of triangle
$= 15 - \frac{1}{2}r^2 \sin 1.5^c \text{ cm}^2$

$= \left(15 - 10 \sin 1.5^c\right) \text{ cm}^2$
$= 5.025 \text{ cm}^2$ (3 d.p.)

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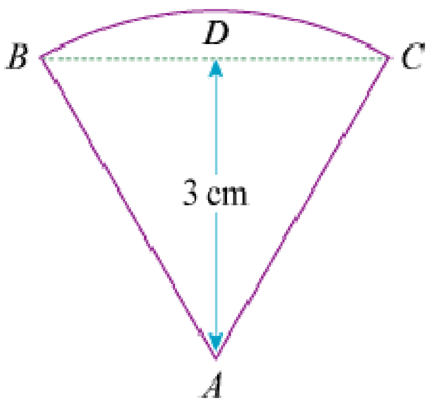
Radian measure and its applications
Exercise D, Question 11

Question:

The shape of a badge is a sector *ABC* of a circle with centre *A* and radius *AB*, as shown in the diagram. The triangle *ABC* is equilateral and has perpendicular height 3 cm.



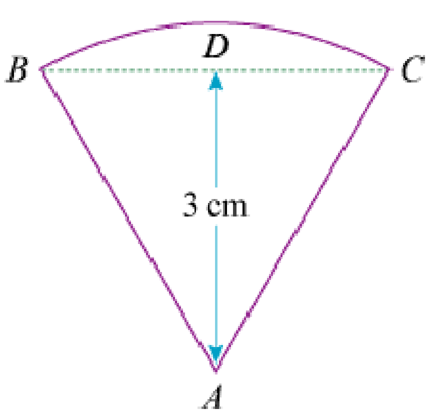
(a) Find, in surd form, the length of *AB*.

(b) Find, in terms of π, the area of the badge.

(c) Prove that the perimeter of the badge is $\frac{2\sqrt{3}}{3}\left(\pi+6\right)$ cm.

[E]

Solution:



(a) Using the right-angled $\triangle ABD$, with $\angle ABD = 60°$,

$\sin 60° = \frac{3}{AB}$

$$\Rightarrow AB = \frac{3}{\sin 60°} = \frac{3}{\frac{\sqrt{3}}{2}} = 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$

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(b) Area of badge
= area of sector

$= \frac{1}{2} \times (2\sqrt{3})^2\theta$ where $\theta = \frac{\pi}{3}$

$= \frac{1}{2} \times 12 \times \frac{\pi}{3}$

$= 2\pi$ cm^2

(c) Perimeter of badge
= AB + AC + arc *BC*

$= \left(2\sqrt{3} + 2\sqrt{3} + 2\sqrt{3}\frac{\pi}{3} \right)$ cm

$= 2\sqrt{3}\left(2 + \frac{\pi}{3} \right)$ cm

$= \frac{2\sqrt{3}}{3}\left(6 + \pi \right)$ cm

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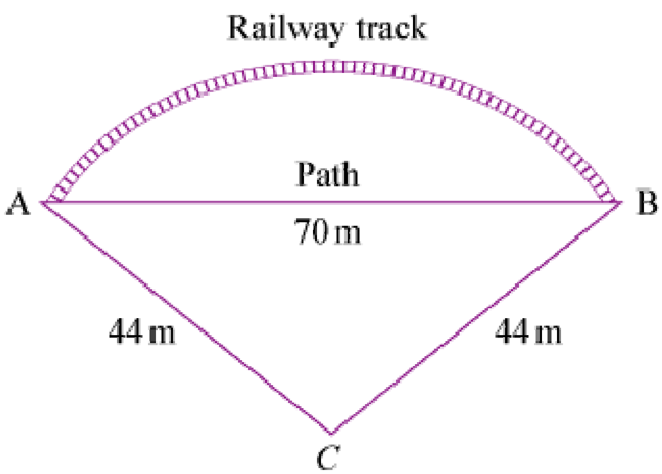
Radian measure and its applications
Exercise D, Question 12

Question:

There is a straight path of length 70 m from the point A to the point B. The points are joined also by a railway track in the form of an arc of the circle whose centre is C and whose radius is 44 m, as shown in the diagram.

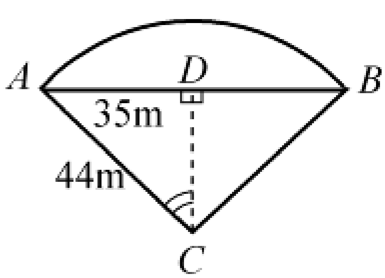


(a) Show that the size, to 2 decimal places, of $\angle ACB$ is 1.84 radians.

(b) Calculate:
(i) The length of the railway track.
(ii) The shortest distance from C to the path.
(iii) The area of the region bounded by the railway track and the path.

[E]

Solution:



(a) Using right-angled $\triangle ADC$

$\sin \angle ACD = \frac{35}{44}$

So $\angle ACD = \sin^{-1}\left(\frac{35}{44}\right)$

and $\angle ACB = 2 \sin^{-1}\left(\frac{35}{44}\right)$ (work in radian mode)

$\Rightarrow \angle ACB = 1.8395 \ldots = 1.84^c$ (2 d.p.)

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(b) (i) Length of railway track = length of arc AB = $44 \times 1.8395 \ldots = 80.9$ m (3 s.f.)
(ii) Shortest distance from *C* to *AB* is *DC*.
Using Pythagoras' theorem:
$DC^2 = 44^2 - 35^2$
$DC = \sqrt{44^2 - 35^2} = 26.7$ m (3 s.f.)
(iii) Area of region = area of segment
= area of sector *ABC* − area of $\triangle ABC$

$= \frac{1}{2} \times 44^2 \times 1.8395 \ldots - \frac{1}{2} \times 70 \times DC \quad \left(\text{or } \frac{1}{2} \times 44^2 \times \sin 1.8395 \ldots^{c}\right)$

$= 847 \text{ m}^2$ (3 s.f.)

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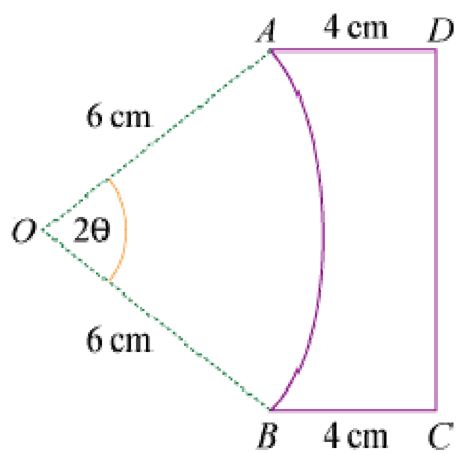
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Radian measure and its applications
Exercise D, Question 13

Question:



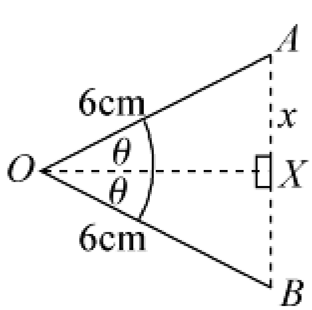
The diagram shows the cross-section *ABCD* of a glass prism. AD = BC = 4 cm and both are at right angles to *DC*. *AB* is the arc of a circle, centre *O* and radius 6 cm. Given that $\angle AOB = 2\theta$ radians, and that the perimeter of the cross-section is $2(7+\pi)$ cm:

(a) Show that $\left(2\theta + 2\sin\theta - 1\right) = \frac{\pi}{3}$.

(b) Verify that $\theta = \frac{\pi}{6}$.

(c) Find the area of the cross-section.

Solution:



(a) In △OAX (see diagram)

$\frac{x}{6} = \sin\theta$

$\Rightarrow \quad x = 6\sin\theta$

So AB = $2x = 12\sin\theta$ (AB = DC)

The perimeter of cross-section

$= \text{arc } AB + AD + DC + BC$

$= [6(2\theta) + 4 + 12\sin\theta + 4]$ cm

$= (8 + 12\theta + 12\sin\theta)$ cm

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So $2(7+\pi) = 8 + 12\theta + 12 \sin \theta$

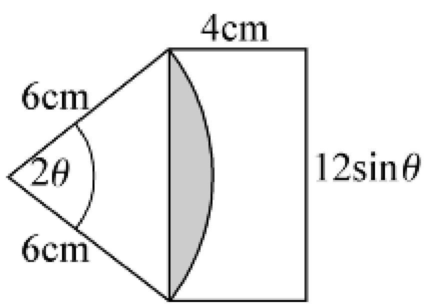
$\Rightarrow \quad 14 + 2\pi = 8 + 12\theta + 12 \sin \theta$

$\Rightarrow \quad 12\theta + 12 \sin \theta - 6 = 2\pi$

Divide by 6: $2\theta + 2 \sin \theta - 1 = \frac{\pi}{3}$

(b) When $\theta = \frac{\pi}{6}$, $2\theta + 2 \sin \theta - 1 = \frac{\pi}{3} + \left(2 \times \frac{1}{2} \right) - 1 = \frac{\pi}{3}$ ✓

(c)



The area of cross-section = area of rectangle *ABCD* − area of shaded segment

Area of rectangle $= 4 \times \left(12 \sin \frac{\pi}{6} \right) = 24 \text{ cm}^2$

Area of shaded segment
= area of sector − area of triangle

$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6^2 \sin \frac{\pi}{3}$

$= 3.261 \ldots \text{ cm}^2$

So area of cross-section = 20.7 cm^2 (3 s.f.)

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Solutionbank C2
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Radian measure and its applications
Exercise D, Question 14

Question:

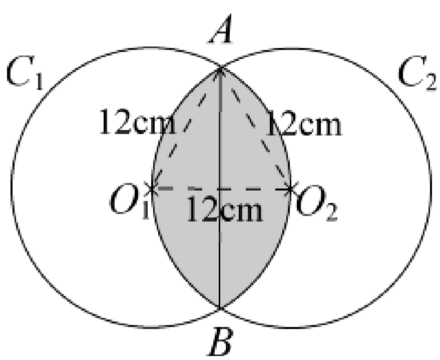
Two circles C_1 and C_2, both of radius 12 cm, have centres O_1 and O_2 respectively. O_1 lies on the circumference of C_2; O_2 lies on the circumference of C_1. The circles intersect at A and B, and enclose the region R.

(a) Show that $\angle AO_1B = \frac{2}{3}\pi$ radians.

(b) Hence write down, in terms of π, the perimeter of R.

(c) Find the area of R, giving your answer to 3 significant figures.

Solution:



(a) $\triangle AO_1O_2$ is equilateral.

So $\angle AO_1O_2 = \frac{\pi}{3}$ radians

$\angle AO_1B = 2\angle AO_1O_2 = \frac{2\pi}{3}$ radians

(b) Consider arc AO_2B in circle C_1.
Using arc length $= r\theta$

arc $AO_2B = 12 \times \frac{2\pi}{3} = 8\pi$ cm

Perimeter of $R =$ arc $AO_2B +$ arc $AO_1B = 2 \times 8\pi = 16\pi$ cm

(c) Consider the segment AO_2B in circle C_1.
Area of segment AO_2B
= area of sector $O_1AB -$ area of $\triangle O_1AB$

$= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 12^2 \times \sin \frac{2\pi}{3}$

$= 88.442 \ldots$ cm^2
Area of region R
= area of segment $AO_2B +$ area of segment AO_1B
$= 2 \times 88.442 \ldots$ cm^2
$= 177$ cm^2 (3 s.f.)