

1. Simplify

(a) $(2\sqrt{5})^2$

(1)

(b) $\frac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$ giving your answer in the form $a + \sqrt{b}$, where a and b are integers.

(4)

2. Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

3. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$

(3)

(b) $\int y dx$

(3)

4. (i) A sequence U_1, U_2, U_3, \dots is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

- (ii) Another sequence V_1, V_2, V_3, \dots is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

- (a) Find V_3 and V_4 in terms of k .

(2)

Given that $\sum_{n=1}^5 V_n = 165$,

- (b) find the value of k .

(3)

5. The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p .

(4)

6. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(5)

7. Given that $y = 2^x$,

(a) express 4^x in terms of y .

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

8. (a) Factorise completely $9x - 4x^3$ (3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x -axis. (3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found. (4)

9. Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1 500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32 000 in year k . Her annual salary then remained at £32 000.

(a) Find the value of the constant k .

(2)

(b) Calculate the total amount that Jess has earned in the 20 years.

(5)

10. A curve with equation $y = f(x)$ passes through the point $(4, 9)$.

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

(b) Find the x coordinate of P .

(5)