Examination style paper

1 Write in the form $k\sqrt{3}$, stating the value of k in each case.

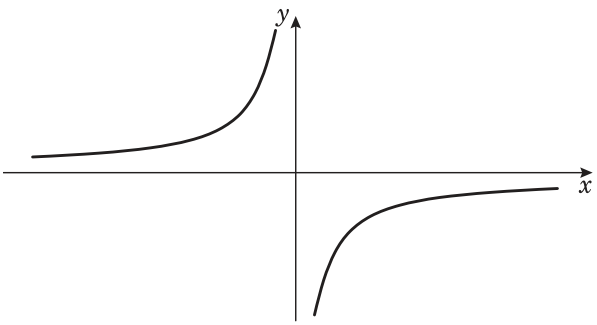
a $\sqrt{75}$ (1)

b $\sqrt{12} + \sqrt{147} - \sqrt{27}$ (2)

2 **a** Find the value of $27^{\frac{2}{3}}$. (2)

b Simplify $\dfrac{16x^{\frac{2}{3}}}{2x}$. (2)

3



The diagram shows a sketch of the curve with equation

$$y = -\frac{6}{x}, \quad x \neq 0.$$

a On a separate diagram sketch the curve with equation

$$y = 2 - \frac{6}{x}, \quad x \neq 0.$$

showing clearly the coordinates of any point where the curve crosses the coordinate axes. (3)

b Write down the equations of the asymptotes to the curve with equation

$$y = 2 - \frac{6}{x}, \quad x \neq 0.$$ (2)

4 An arithmetic series has 1st term 49 and 15th term 7.

a Find the value of the common difference. (3)

b Find the value of the sum of the first 15 terms of the series. (3)

5 The equation $kx^2 + kx + 3 - k = 0$, where k is a constant, has no real roots.

a Show that $5k^2 - 12k < 0$. (2)

b Find the set of possible values of k. (4)

6 A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2$$
$$a_{n+1} = 7 - 3a_n, \; n \geqslant 1$$

a Find a_2 and a_3. (2)

b Find $\sum_{r=1}^{5} a_r$ and show that this sum is divisible by 12. (4)

7 Given that $y = 8x^3 + \frac{3}{\sqrt{x}} + 5, \; x > 0$

find

a $\frac{dy}{dx}$ (3)

b $\frac{d^2y}{dx^2}$ (2)

c $\int y \, dx$ (3)

8 The line l_1 has equation $2y = x - 3$ and the line l_2 has equation $5y + 2x - 18 = 0$.

a Find the gradient of l_2. (2)

The point of intersection of l_1 and l_2 is P.

b Find the coordinates of P. (3)

The lines l_1 and l_2 cross the x-axis as the points A and B respectively.

c Find the area of triangle APB. (4)

9 The curve C with equation $y = f(x)$ passes through the point $(2, 4)$ and $f'(x) = 3(x - 1)(x + 1)$.

a Use integration to find $f(x)$. (5)

b Show that $(x - 1)^2(x + 2) = f(x)$ (3)

c Sketch C, showing the coordinates of the point where C crosses the x-axis. (3)

10 The curve C has equation

$$y = 8x + x^2 + \frac{9}{x}, \quad x \neq 0$$

The points P and Q lie on C and have x-coordinates -3 and 1 respectively.

a Find an equation of the chord PQ. (6)

b Show that the tangents to C at the points P and Q are parallel. (4)

The tangent to C at P and the normal to C at Q intersect at the point $R(17, 2)$.

c Show that $PR = 20\sqrt{2}$ (2)

d Find QR. (2)

e Explain why angle PRQ is a right angle and find the area of triangle PQR. (3)