

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6664/01)

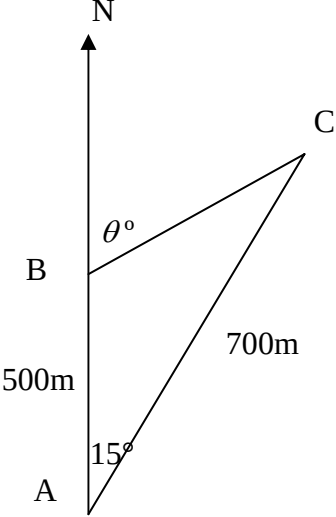
January 2008
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1.	a)i) $f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8 \quad ; = 5$ ii) $f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact) M1 is for attempt at either $f(3)$ or $f(-3)$ in (i) or $f(-2)$ or $f(2)$ in (ii). (b) $\frac{[(x+2)](x^2 - 4x + 4)}{(x+2)(x-2)^2} \quad (= 0 \text{ not required}) \text{ [must be seen or used in (b)]}$ $\quad \quad \quad (= 0) \quad \quad \quad (\text{can imply previous 2 marks})$ Solutions: $x = 2$ or -2 (both) or $(-2, 2, 2)$ A1 (4)	M1; A1 A1 (3) M1 A1 M1 [7]
Notes: (a)	No working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate. <u>Alternative (Long division)</u> Divide by $(x-3)$ OR $(x+2)$ to get $x^2 + ax + b$, a may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1] (b) First M1 requires division by a found factor ; e.g $(x+2)$, $(x-2)$ or what candidate thinks is a factor to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x+2)](x^2 - 4x + 4)$ or $(x-2)(x^2 - 4)$ Second M1: attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x+c)(x+d)$, where $cd = b$.] N.B. Second A1 is for solutions, not factors <u>Alternative (first two marks)</u> $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1] $b = -4$, $c = 4$ [A1] <u>Method of grouping</u> $x^3 - 2x^2 - 4x + 8 = x^2(x-2) + 4(x-2)$ M1; $= x^2(x-2) - 4(x-2)$ A1 $[= (x^2 - 4)(x-2)] = (x+2)(x-2)^2$ M1 Solutions: $x=2$, $x=-2$ both A1	
2.	(a) Complete method, using terms of form ar^k, to find r [e.g. Dividing $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$ (b) Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.]	M1 A1 (2) M1

(c)	$(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25) A1 (2) Substituting their values of a and r into correct formula for sum. M1 $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this) A1 (2) [6]
Notes:	(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.
3. (a)	$\left(1 + \frac{1}{2}x\right)^{10} = 1 + \frac{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}{\quad}$ M1 A1 $= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified) A1; A1 (4) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients] (b) $\left(1 + \frac{1}{2} \times 0.01\right)^{10} = 1 + 5(0.01) + \frac{45}{4} \text{ or } 11.25(0.01)^2 + 15(0.01)^3$ M1 A1✓ $= 1 + 0.05 + 0.001125 + 0.000015$ $= 1.05114 \quad \text{cao}$ A1 (3) [7]
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x, (iii) May be listed, need not be added; <i>this applies for all marks.</i> First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1 for $1 + 5x$ (b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)

4.	(a) $3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (M1: Use of $\sin^2 \theta + \cos^2 \theta = 1$) $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3$ cso AG (b) $\sin^2 \theta = \frac{3}{5}$, so $\sin \theta = (\pm)\sqrt{0.6}$ Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -$ (may be implied by later work) M1 $\theta = 50.7685^\circ$ awrt $\theta = 50.8^\circ$ (dependent on first M1 only) A1 $\theta (= 180^\circ - 50.7685^\circ); = 129.23...^\circ$ awrt 129.2° M1; A1 ✓ [f.t. dependent on first M and 3rd M] $\sin \theta = -\sqrt{0.6}$ $\theta = 230.785^\circ$ and 309.23152° awrt $230.8^\circ, 309.2^\circ$ (both) M1A1 (7)	(2) [9]
Notes:	(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$. (b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ Second M1: Considering the $-$ value for $\sin \theta$. (usually later) First A1: Given for awrt 50.8°. Not dependent on second M. Third M1: For $(180 - 50.8_c)^\circ$, need not see written down Final M1: Dependent on second M (but may be implied by answers) For $(180 + \text{candidate's } 50.8)^\circ$ or $(360 - 50.8_c)^\circ$ or equiv. Final A1: Requires both values. (no follow through) [Finds $\cos^2 \theta = k$ ($k = 2/5$) and so $\cos \theta = (\pm)...$M1, then mark equivalently]	

5.	<u>Method 1</u> (Substituting $a = 3b$ into second equation at some stage) Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$ M1 Substitution of $3b$ for a (or $a/3$ for b) e.g. $\log_3 3b^2 = 2$ M1 Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$ M1 First correct value $b = \sqrt{3}$ (allow $3^{1/2}$) A1 Correct method to find other value (dep. on at least first M mark) M1 Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$ A1 <u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$) “ Taking logs” of first equation and “ separating” $\log_3 a = \log_3 3 + \log_3 b$ M1 $(= 1 + \log_3 b)$ Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ M1 $[\log_3 a = 1\frac{1}{2}, \log_3 b = \frac{1}{2}]$ Using base correctly to find a or b M1 Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$ A1 Correct method for second answer, dep. on first M; correct second answer M1;A1[6] [Ignore negative values]	
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on Method 1, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of $a = 3b$, (iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$	

6.	 $BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$ $(= 63851.92...)$ $BC = 253 \quad \text{awrt}$ (a) $\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$ $\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an obtuse B (134.2°) dep (b) $\theta = 180^\circ - \text{candidate's angle } B$ (Dep. on first M only, B can be acute) M1 $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) [46 needs to be from correct working]	M1 A1 A1 (3) M1 M1 M1 A1 (4) [7]
Notes:	(a) If use $\cos 15^\circ = \dots$, then A1 not scored until written as $BC^2 = \dots$ correctly <i>Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC</i> Finding value for BX and CX and using Pythagoras M1 $BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2$ A1 $BC = 253 \quad \text{awrt}$ A1 (b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)) (i) $\cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$ or $700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$ M1 Finding angle B M1, then M1 as above (ii) 2 triangle approach, as defined in notes for (a) $\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX}$ M1 Finding value for $\angle CBX$ ($\approx 59^\circ$) M1 $\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)]$ M1 (iii) Using sine rule (or cos rule) to find C first: Correct use of sine or cos rule for C M1, Finding value for C M1 Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$ M1 (iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ M2 {first two Ms earned in this case} Solving for θ; $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9 M1; A1	

7	(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$] (b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = ..$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$, (c) (Area =) $\int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3} (+ c)$ Correct use of correct limits on their result above (see notes on limits) $[\frac{3}{2}x^2 - \frac{x^3}{3}]^4 - [\frac{3}{2}x^2 - \frac{x^3}{3}]_0$ with limits substituted [= $48 - 21\frac{1}{3} = 26\frac{2}{3}$] Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = $\pm$ (area under curve – area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)	B1 (1) M1 A1 A1 (3) M1 A1 M1 A1 M1 A1 (6)[10]
Notes	(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned ; accept “ clear from diagram” (c) Alternative Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3} (+ c)]$ A1, Correct use of correct limits on their result second M1, Totally correct, unsimplified $\pm$ expression (may be implied by correct ans.) A1 10$\frac{2}{3}$ A1 [Allow this if, having given - 10$\frac{2}{3}$, they correct it] M1 for correct use of correct limits. Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Use of trapezium rule: M0A0MA0,possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

8	(a) $(x-6)^2 + (y-4)^2 = ; 3^2$ (b) Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$ $= \sqrt{40} \quad (= 6.325)$ [These first two marks can be scored if seen as part of solution for (c)] Complete method for $\cos \theta$, $\sin \theta$ or $\tan \theta$ e.g. $\cos \theta = \frac{MT}{MP} = \frac{3}{\text{candidate's } \sqrt{40}} \quad (= 0.4743) \quad (\theta = 61.6835^\circ)$ [If $TP = 6$ is used, then M0] $\theta = 1.0766 \text{ rad} \quad \mathbf{AG}$ (c) Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ $= \frac{3}{2} \sqrt{31} \quad (= 8.3516..)$ allow awrt 8.35 Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766 \quad (= 4.8446..)$ Area $TPQ = \text{candidate's } (8.3516.. - 4.8446..)$ $= 3.507 \quad \text{awrt}$ [Note: 3.51 is A0]	B1; B1 (2) M1 A1 M1 A1 (4) M1 A1 M1 M1 A1 (5) [11]
Notes	(a) Allow 9 for 3^2. (b) First M1 can be implied by $\sqrt{40}$ For second M1: May find $TP = \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin \theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}} \quad (= 0.8803..)$ or $\tan \theta = \frac{\sqrt{31}}{3} \quad (1.8859..)$ or cos rule NB. Answer is given, but allow final A1 if all previous work is correct. (c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40 - 9}$	

9	(a) (Total area) = $3xy + 2x^2$ (Vol:) $x^2y = 100$ $(y = \frac{100}{x^2}, xy = \frac{100}{x})$ Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area) (Area =) $\frac{300}{x} + 2x^2$ AG (b) $\frac{dA}{dx} = -\frac{300}{x^2} + 4x$ Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$) (c) $\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}$ therefore minimum (d) Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) [$y = \frac{100}{4.2172^2} = 5.6228$] Area = 106.707 awrt 107	B1 B1 M1 A1 cso (4) M1A1 A1 (4) M1A1 (2) M1 A1 (2) [12]
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive” A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, $\checkmark$). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with “107” A1: Both values greater than “$x = 107$” and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused.	

--	--