

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Core Mathematics C1 (6663)

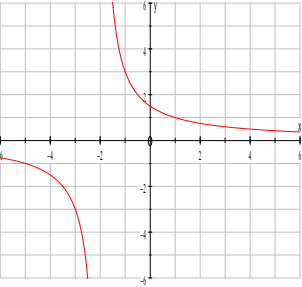
June 2007
6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$9 - 5$ or $3^2 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{5} \times \sqrt{5}$ or $3^2 - \sqrt{5} \times \sqrt{5}$ or $3^2 - (\sqrt{5})^2$ $= 4$	M1 A1cso (2) 2
	<p>M1 for an attempt to multiply out. There must be at least 3 correct terms. Allow one sign slip only, no arithmetic errors.</p> <p>e.g. $3^2 + 3\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2$ is M1A0</p> <p>$3^2 + 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2$ is M1A0 as indeed is $9 \pm 6\sqrt{5} - 5$</p> <p>BUT $9 + \sqrt{15} - \sqrt{15} - 5 (= 4)$ is M0A0 since there is more than a sign error.</p> <p>$6 + 3\sqrt{5} - 3\sqrt{5} - 5$ is M0A0 since there is an arithmetic error.</p> <p>If all you see is 9 ± 5 that is M1 but please check it has not come from incorrect working.</p> <p>Expansion of $(3 + \sqrt{5})(3 + \sqrt{5})$ is M0A0</p> <p>A1cso for 4 only. Please check that no incorrect working is seen.</p> <p>Correct answer only scores both marks.</p>	

Question number	Scheme	Marks
2.	<p>(a) Attempt $\sqrt[3]{8}$ or $\sqrt[3]{(8^4)}$</p> <p>$= \underline{16}$</p> <p>(b) $\underline{5x^{\frac{1}{3}}}$</p> <p style="text-align: right;">$5, x^{\frac{1}{3}}$</p>	<p>M1</p> <p>A1 (2)</p> <p>B1, B1 (2)</p> <p>4</p>
(a)	<p>M1 for: 2 (on its own) or $(2^3)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $(\sqrt[3]{8})^4$ or 2^4 or $\sqrt[3]{8^4}$ or $\sqrt[3]{4096}$</p> <p>8^3 or 512 or $(4096)^{\frac{1}{3}}$ is M0</p> <p>A1 for 16 only</p>	
(b)	<p>1st B1 for 5 on its own or \times something.</p> <p>So e.g. $\frac{5x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0</p> <p>An expression showing cancelling is not sufficient (see first expression of QC0184500123945 the mark is scored for the second expression)</p> <p>2nd B1 for $x^{\frac{1}{3}}$</p> <p>Can use ISW (incorrect subsequent working)</p> <p>e.g $5x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5x^4}$ which we ignore as ISW.</p> <p>Correct answers only score full marks in both parts.</p>	

Question number	Scheme	Marks
3.	<p>(a) $\left(\frac{dy}{dx}\right) = 6x^1 + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$</p> <p>(b) $6 + -x^{-\frac{3}{2}}$ or $6 + -1 \times x^{-\frac{3}{2}}$</p> <p>(c) $x^3 + \frac{8}{3}x^{\frac{3}{2}} + C$ A1: $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ A1: both, simplified and + C</p>	<p>M1 A1 (2)</p> <p>M1 A1ft (2)</p> <p>M1 A1 A1 (3)</p> <p>7</p>
(a)	<p>M1 for <u>some</u> attempt to differentiate: $x^n \rightarrow x^{n-1}$ Condone missing $\frac{dy}{dx}$ or $y = \dots$</p> <p>A1 for both terms correct, as written or better. No + C here. Of course $\frac{2}{\sqrt{x}}$ is acceptable.</p>	
(b)	<p>M1 for some attempt to differentiate again. Follow through their $\frac{dy}{dx}$, at least one term correct or correct follow through.</p> <p>A1ft. as written or better, follow through must have 2 <u>distinct</u> terms and simplified e.g. $\frac{4}{4} = 1$.</p>	
(c)	<p>M1 for some attempt to integrate: $x^n \rightarrow x^{n+1}$. Condone misreading $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ for y. (+C alone is not sufficient)</p> <p>1st A1 for either $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ (or better) $\frac{2}{3} \times 4x^{\frac{3}{2}}$ is OK here too but not for 2nd A1.</p> <p>2nd A1 for <u>both</u> x^3 and $\frac{8}{3}x^{\frac{3}{2}}$ or $\frac{8}{3}x\sqrt{x}$ i.e. simplified terms <u>and</u> +C all on one line. $2\frac{2}{3}$ instead of $\frac{8}{3}$ is OK</p>	

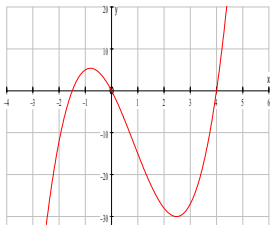
Question number	Scheme	Marks
4.	<p>(a) Identify $a = 5$ and $d = 2$ (May be implied)</p> $(u_{200} =) a + (200 - 1)d \quad (= 5 + (200 - 1) \times 2)$ $= 403(\text{p}) \text{ or } (\text{£}) 4.03$ <p>(b) $(S_{200} =) \frac{200}{2} [2a + (200 - 1)d]$ or $\frac{200}{2} (a + \text{"their 403"})$</p> $= \frac{200}{2} [2 \times 5 + (200 - 1) \times 2] \text{ or } \frac{200}{2} (5 + \text{"their 403"})$ $= 40\,800 \text{ or } \text{£}408$	<p>B1</p> <p>M1</p> <p>A1 (3)</p> <p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>6</p>
(a)	<p>B1 can be implied if the correct answer is obtained. If 403 is <u>not</u> obtained then the values of a and d must be clearly identified as $a = 5$ and $d = 2$.</p> <p>This mark can be awarded at any point.</p> <p>M1 for attempt to use nth term formula with $n = 200$. Follow through their a and d. Must have use of $n = 200$ and one of a or d correct or correct follow through. Must be 199 not 200.</p> <p>A1 for 403 or 4.03 (i.e. condone missing £ sign here). Condone £403 here.</p>	
N.B.	<p>$a = 3, d = 2$ is B0 and $a + 200d$ is M0 <u>BUT</u> $3 + 200 \times 2$ is B1M1 and A1 if it leads to 403. Answer only of 403 (or 4.03) scores 3/3.</p>	
(b)	<p>M1 for use of correct sum formula with $n = 200$. Follow through their a and d and their 403. Must have <u>some</u> use of $n = 200$, and some of a, d or l correct or correct follow through.</p> <p>1st A1 for any correct expression (i.e. must have $a = 5$ and $d = 2$) but can f.t. their 403 still.</p> <p>2nd A1 for 40800 or £408 (i.e. the £ sign is required before we accept 408 this time). 40800p is fine for A1 but £40800 is A0.</p>	
ALT	<p><u>Listing</u></p>	
(a)	They might score B1 if $a = 5$ and $d = 2$ are clearly identified. Then award M1A1 together for 403.	
(b)	$\sum_{r=1}^{200} (2r + 3)$ <p>. Give M1 for $2 \times \frac{200}{2} \times (201) + 3k$ (with $k > 1$), A1 for $k = 200$ and A1 for 40800.</p>	

Question number	Scheme	Marks
5.	<p>(a)</p>  <p>Translation parallel to x-axis Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap</p> <p>$\left(0, \frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y-axis</p> <p>(b) $x = -2, y = 0$</p> <p>S.C. [Allow ft on first B1 for $x = 2$ when translated “the wrong way” but must be compatible with their sketch.]</p>	<p>M1</p> <p>A1</p> <p>B1 (3)</p> <p>B1, B1 (2)</p> <p>5</p>
	<p>(a)</p> <p>M1 for a horizontal translation – two branches with one branch cutting y – axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. $x=0$ and $y = 1.5$ in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere.</p> <p>(b)</p> <p>1st B1 for $x = -2$. NB $x \neq -2$ is B0. Can accept $x = +2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2nd B1 for $y = 0$.</p> <p>S.C.</p> <p>If $x = -2$ and $y = 0$ and some other asymptotes are also given award B1B0</p> <p>The asymptote equations should be clearly stated in part (b). Simply marking $x = -2$ or $y = 0$ on the sketch is insufficient <u>unless</u> they are clearly marked “asymptote $x = -2$” etc.</p>	

Question number	Scheme	Marks
6.	<p>(a) $2x^2 - x(x - 4) = 8$</p> <p>$x^2 + 4x - 8 = 0$ (*)</p> <p>(b) $x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times -8)}}{2}$ or $(x + 2)^2 \pm 4 - 8 = 0$</p> <p>$x = -2 \pm$ (any correct expression)</p> <p>$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ or $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$</p> <p>$y = (-2 \pm 2\sqrt{3}) - 4$ M: Attempt at least one y value</p> <p><u>$x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3}$</u> <u>$x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}$</u></p>	<p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p> <p>7</p>
(a)	<p>M1 for correct attempt to form an equation in x only. Condone sign errors/slips but attempt at this line must be seen. E.g. $2x^2 - x^2 \pm 4x = 8$ is OK for M1.</p> <p>A1cso for correctly simplifying to printed form. No incorrect working seen. The $= 0$ is required.</p> <p>These two marks can be scored in part (b). For multiple attempts pick best.</p>	
(b)	<p>1st M1 for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing $x =$ or just $+$ or $-$ instead of \pm for M1.</p> <p>For completing the square must have as printed or better.</p> <p>If they have $x^2 - 4x - 8 = 0$ then M1 can be given for $(x - 2)^2 \pm 4 - 8 = 0$.</p> <p>1st A1 for $-2 \pm$ any correct expression. (The \pm is required but $x =$ is not)</p> <p>B1 for simplifying the surd e.g. $\sqrt{48} = 4\sqrt{3}$. Must reduce to $b\sqrt{3}$ so $\sqrt{16}\sqrt{3}$ or $\sqrt{4}\sqrt{3}$ are OK.</p> <p>2nd M1 for attempting to find at least one y value. Substitution into one of the given equations and an attempt to solve for y.</p> <p>2nd A1 for correct y answers. Pairings need <u>not</u> be explicit but they must say which is x and which y. Mis-labelling x and y loses final A1 only.</p>	

Question number	Scheme	Marks
7.	<p>(a) Attempt to use discriminant $b^2 - 4ac$</p> $k^2 - 4(k+3) > 0 \Rightarrow k^2 - 4k - 12 > 0 \quad (*)$ <p>(b) $k^2 - 4k - 12 = 0 \Rightarrow$</p> $(k \pm a)(k \pm b), \text{ with } ab = 12 \text{ or } (k =) \frac{4 \pm \sqrt{4^2 - 4 \times 12}}{2} \text{ or } (k - 2)^2 \pm 2^2 - 12$ <p>$k = -2 \text{ and } 6$ (both)</p> <p><u>$k < -2, k > 6$</u> or <u>$(-\infty, -2); (6, \infty)$</u> M: choosing "outside"</p>	<p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>A1</p> <p>M1 A1ft (4)</p> <p>6</p>
(a)	<p>M1 for use of $b^2 - 4ac$, one of b or c must be correct. Or full attempt using completing the square that leads to a 3TQ in k</p> <p>e.g. $\left(\left[x + \frac{k}{2} \right]^2 = \right) \frac{k^2}{4} - (k+3)$</p> <p>A1cso Correct argument to printed result. Need to state (or imply) that $b^2 - 4ac > 0$ and no incorrect working seen. Must have >0. If >0 just appears with $k^2 - 4(k+3) > 0$ that is OK. If >0 appears on last line only with no explanation give A0. $b^2 - 4ac$ followed by $k^2 - 4k - 12 > 0$ only is insufficient so M0A0</p> <p>e.g. $k^2 - 4 \times 1 \times k + 3$ (missing brackets) can get M1A0 but $k^2 + 4(k+3)$ is M0A0 (wrong formula) Using $\sqrt{b^2 - 4ac} > 0$ is M0.</p>	
(b)	<p>1st M1 for attempting to find critical regions. Factors, formula or completing the square. 1st A1 for $k = 6$ and -2 only 2nd M1 for choosing the outside regions 2nd A1f.t. as printed or f.t. their (non identical) critical values</p> <p>$6 < k < -2$ is M1A0 but ignore if it follows a correct version $-2 < k < 6$ is M0A0 whatever their diagram looks like</p> <p>Condone use of x instead of k for critical values and final answers in (b).</p>	
Treat this question as 3 two mark parts. If part (a) is seen in (b) or vice versa marks can be awarded.		

Question number	Scheme	Marks
8.	<p>(a) $(a_2 =) \underline{3k + 5}$ [must be seen in part (a) or labelled $a_2 =$]</p> <p>(b) $(a_3 =) 3(3k + 5) + 5$ $= \underline{9k + 20}$ (*)</p> <p>(c)(i) $a_4 = 3(9k + 20) + 5$ ($= 27k + 65$)</p> $\sum_{r=1}^4 a_r = k + (3k + 5) + (9k + 20) + (27k + 65)$ <p>(ii) $= 40k + 90$ $= \underline{10(4k + 9)}$ (or explain why divisible by 10)</p>	<p>B1 (1)</p> <p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1ft (4)</p> <p>7</p>
(b)	<p>M1 for attempting to find a_3, follow through their $a_2 \neq k$.</p> <p>A1cso for simplifying to printed result with no incorrect working seen.</p>	
(c)	<p>1st M1 for attempting to find a_4. Can allow a slip here e.g. $3(9k + 20)$ [i.e. forgot +5]</p> <p>2nd M1 for attempting sum of 4 relevant terms, follow through their (a) and (b). Must have 4 terms starting with k. Use of arithmetic series formulae at this point is M0A0A0</p> <p>1st A1 for simplifying to $40k + 90$ or better</p> <p>2nd A1ft for taking out a factor of 10 or dividing by 10 or an explanation in words true $\forall k$. Follow through their sum of 4 terms provided that both Ms are scored and their sum <u>is</u> divisible by 10. A comment is <u>not</u> required. e.g. $\frac{40k + 90}{10} = 4k + 9$ is OK for this final A1.</p>	
S.C.	$\sum_{r=2}^5 a_r = 120k + 290 = 10(12k + 29)$ can have M1M0A0A1ft.	

Question number	Scheme	Marks
9.	<p>(a) $f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x (+C)$</p> <p>$x = 5: \quad 250 - 125 - 60 + C = 65 \quad C = 0$</p> <p>(b) $x(2x^2 - 5x - 12)$ or $(2x^2 + 3x)(x - 4)$ or $(2x + 3)(x^2 - 4x)$</p> <p>$= x(2x + 3)(x - 4)$ (*)</p> <p>(c) </p> <p>Shape</p> <p>Through origin</p> <p>$\left(-\frac{3}{2}, 0\right)$ and $(4, 0)$</p>	<p>M1 A1</p> <p>M1 A1 (4)</p> <p>M1</p> <p>A1cso (2)</p> <p>B1</p> <p>B1</p> <p>B1 (3)</p> <p>9</p>
(a)	<p>1st M1 for attempting to integrate, $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for all x terms correct, need not be simplified. Ignore $+C$ here.</p> <p>2nd M1 for some use of $x = 5$ and $f(5)=65$ to form an equation in C based on their integration.</p> <p>There must be some visible attempt to use $x = 5$ and $f(5)=65$. No $+C$ is M0.</p> <p>2nd A1 for $C = 0$. This mark cannot be scored unless a suitable equation is seen.</p>	
(b)	<p>M1 for attempting to take out a correct factor or to verify. Allow usual errors on signs.</p> <p>They must get to the equivalent of one of the given partially factorised expressions or, if verifying, $x(2x^2 + 3x - 8x - 12)$ i.e. with no errors in signs.</p> <p>A1cso for proceeding to printed answer with no incorrect working seen. Comment <u>not</u> required.</p> <p>This mark is <u>dependent upon a fully correct solution to part (a)</u> so M1A1M0A0M1A0 for (a) & (b).</p> <p>Will be common or M1A1M1A0M1A0. To score 2 in (b) they must score 4 in (a).</p>	
(c)	<p>1st B1 for positive x^3 shaped curve (with a max and a min) positioned anywhere.</p> <p>2nd B1 for any curve that passes through the origin (B0 if it only touches at the origin)</p> <p>3rd B1 for the two points <u>clearly</u> given as coords or values marked in appropriate places on x axis.</p> <p>Ignore any extra crossing points (they should have lost first B1).</p> <p>Condone $(1.5, 0)$ if clearly marked on $-ve$ x-axis. Condone $(0, 4)$ etc if marked on $+ve$ x axis.</p> <p>Curve can <u>stop</u> (i.e. not pass through) at $(-1.5, 0)$ and $(4, 0)$.</p> <p>A point on the graph overrides coordinates given elsewhere.</p>	

Question number	Scheme	Marks
10.	<p>(a) $x = 1: y = -5 + 4 = \underline{-1}$, $x = 2: y = -16 + 2 = \underline{-14}$ (can be given in (b) or (c))</p> $PQ = \sqrt{(2-1)^2 + (-14 - (-1))^2} = \sqrt{170} \quad (*)$ <p>(b) $y = x^3 - 6x^2 + 4x^{-1}$</p> $\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$ <p>$x = 1: \frac{dy}{dx} = 3 - 12 - 4 = -13$ M: Evaluate at one of the points</p> <p>$x = 2: \frac{dy}{dx} = 12 - 24 - 1 = -13$ \therefore Parallel A: Both correct + conclusion</p> <p>(c) Finding gradient of normal $\left(m = \frac{1}{13}\right)$</p> $y - -1 = \frac{1}{13}(x - 1)$ $\underline{x - 13y - 14 = 0} \quad \text{o.e.}$	<p>1st B1 for - 1 2nd B1 for - 14</p> <p>M1 A1cso (4)</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>M1 A1ft</p> <p>A1cso (4)</p> <p>13</p>
(a)	<p>M1 for attempting PQ or PQ^2 using their P and their Q. Usual rules about quoting formulae. We must see attempt at $1^2 + (y_p - y_q)^2$ for M1. $PQ^2 = \sqrt{\dots}$ etc could be M1A0.</p> <p>A1cso for proceeding to the correct answer with no incorrect working seen.</p>	
(b)	<p>1st M1 for multiplying by x^2, the x^3 or $-6x^2$ must be correct.</p> <p>2nd M1 for some correct differentiation, at least one term must be correct as printed.</p> <p>1st A1 for a fully correct derivative.</p> <p>These 3 marks can be awarded anywhere when first seen.</p> <p>3rd M1 for attempting to substitute $x = 1$ or $x = 2$ in their derivative. Substituting in y is M0.</p> <p>2nd A1 for -13 from both substitutions <u>and</u> a brief comment.</p> <p>The - 13 must come from their derivative.</p>	
(c)	<p>1st M1 for use of the perpendicular gradient rule. Follow through their - 13.</p> <p>2nd M1 for full method to find the equation of the normal or tangent at P. If formula is quoted allow slips in substitution, otherwise a correct substitution is required.</p> <p>1st A1ft for a correct expression. Follow through their - 1 and their changed gradient.</p> <p>2nd A1cso for a correct equation with = 0 and integer coefficients.</p> <p>This mark is dependent upon the - 13 coming from their derivative in (b) hence cso.</p> <p>Tangent can get M0M1A0A0, changed gradient can get M0M1A1A0 or M1M1A1A0.</p> <p>Condone confusion over terminology of tangent and normal, mark gradient and equation.</p>	
MR	<p>Allow for $-\frac{4}{x}$ or $(x+6)$ but not omitting $4x^{-1}$ or treating it as $4x$.</p>	

Question number	Scheme	Marks
11.	<p>(a) $y = -\frac{3}{2}x(+4)$ Gradient = $-\frac{3}{2}$</p> <p>(b) $3x + 2 = -\frac{3}{2}x + 4$ $x = \dots, \frac{4}{9}$</p> <p>$y = 3\left(\frac{4}{9}\right) + 2 = \frac{10}{3} \left(= 3\frac{1}{3}\right)$</p> <p>(c) Where $y = 1$, $l_1 : x_A = -\frac{1}{3}$ $l_2 : x_B = 2$ M: Attempt one of these</p> <p>Area = $\frac{1}{2}(x_B - x_A)(y_P - 1)$</p> <p>$= \frac{1}{2} \times \frac{7}{3} \times \frac{7}{3} = \frac{49}{18} = 2\frac{13}{18}$ o.e.</p>	<p>M1 A1 (2)</p> <p>M1, A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>9</p>
(a)	<p>M1 for an attempt to write $3x + 2y - 8 = 0$ in the form $y = mx + c$</p> <p>or a full method that leads to $m =$, e.g find 2 points, and attempt gradient using $\frac{y_2 - y_1}{x_2 - x_1}$</p> <p>e.g. finding $y = -1.5x + 4$ alone can score M1 (even if they go on to say $m = 4$)</p> <p>A1 for $m = -\frac{3}{2}$ (can ignore the $+c$) or $\frac{dy}{dx} = -\frac{3}{2}$</p>	
(b)	<p>M1 for forming a suitable equation in one variable and attempting to solve leading to $x =$..or $y =$</p> <p>1st A1 for any exact correct value for x</p> <p>2nd A1 for any exact correct value for y</p> <p>(These 3 marks can be scored anywhere, they may treat (a) and (b) as a single part)</p>	
(c)	<p>1st M1 for attempting the x coordinate of A or B. One correct value seen scores M1.</p> <p>1st A1 for $x_A = -\frac{1}{3}$ and $x_B = 2$</p> <p>2nd M1 for a full method for the area of the triangle – follow through their x_A, x_B, y_P.</p> <p>e.g. determinant approach $\frac{1}{2} \begin{vmatrix} 2 & -\frac{1}{3} & \frac{4}{9} & 2 \\ 1 & 1 & \frac{10}{3} & 1 \end{vmatrix} = \frac{1}{2} \left 2 - \dots - \left(-\frac{1}{3}\dots\right) \right$</p> <p>2nd A1 for $\frac{49}{18}$ or an exact equivalent.</p> <p>All accuracy marks require answers as single fractions or mixed numbers not necessarily in lowest terms.</p>	