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# Mark Scheme (Results)

# Summer 2013

# GCE Core Mathematics 2 (6664/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

## General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

**Method mark for solving 3 term quadratic:**

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to x =}$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \quad \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to x =...}$$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

| Question Number | Scheme | Marks |
|---|---|---|
| **1. (a)** | $\{r =\}\ \frac{2}{3}$ | B1 **(1)** |
| **(b)** | $\{p =\}\ 8$ | B1 cao **(1)** |
| **(c)** | $\{S_{15} =\}\ \frac{18\left(1-(\frac{2}{3})^{15}\right)}{1-\frac{2}{3}}$ | M1 |
| | $\{S_{15} = 53.87668...\} \Rightarrow S_{15} = \text{awrt } 53.877$ | A1 **(2)** **[4]** |
| | **Notes for Question 1** | |
| **(a)** | B1: Accept $\frac{12}{18}$, $0.\dot{6}$ or 0.6 recurring, or even 0.667 (3sf) but not 0.6 or 0.67 | |
| **(b)** | B1: accept 8 only | |
| **(c)** | M1: Applies this formula $S_{15} = \frac{18\left(1-(\text{their } r)^{15}\right)}{1-(\text{their } r)}$, can be implied by their answer. For this mark they may use any value for $r$ except $r$ = 1 or $r$ = 0 (even 3/2 or -6 may be used)<br>A1: Answers which round to 53.877 | |
| **Alternative method for (c)** | M1: (Adding terms is an unlikely method for this question) Need to see 15 terms listed as 18+12+……0.06165877 or can be implied by correct answer<br>A1: awrt 53.877<br>**Answer only** : 53.9 is M0A0 with no working, but 53.877 with no working is M1A1 | |

| Question Number | Scheme | Marks |
|---|---|---|
| **2. (a)** | $(2+3x)^4$ **-** Mark (a) and (b) together<br>$2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$<br>First term of 16<br>$\left({}^4C_1 \times \ldots \times x\right) + \left({}^4C_2 \times \ldots \times x^2\right) + \left({}^4C_3 \times \ldots \times x^3\right) + \left({}^4C_4 \times \ldots \times x^4\right)$<br>$= (16+)\ 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0 | <br><br>B1<br>M1<br>A1 A1<br><br>**(4)** |
| **(b)** | $(2-3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$ | B1ft<br>**(1)**<br>**5** |
| **Alternative method (a)** | $(2+3x)^4 = 2^4(1+\frac{3x}{2})^4$<br>$2^4\,(1 + {}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$<br>Scheme is applied exactly as before | |
| | **Notes for Question 2** | |
| **(a)** | B1: The constant term should be 16 in their expansion<br>M1: Two binomial coefficients must be correct and must be with the correct power of *x*. Accept ${}^4C_1$ or $\binom{4}{1}$ or $4$ **as a coefficient, and** ${}^4C_2$ or $\binom{4}{2}$ or $6$ **as another……..** Pascal's triangle may be used to establish coefficients.<br>A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method.<br>A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines) | |
| **(b)** | B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five terms** ft and must be **subtracting the** $x$ **and** $x^3$ **terms**<br>Allow terms in (b) to be in descending order and allow +-96*x* and +-216$x^3$ in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines) | |
| | e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $= (16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5<br>**Fully correct answer with no working can score B1 in part (a) and B1 in part (b).** The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned.<br>**Squaring the bracket and squaring again** may also earn **B1 M0 A0 A0 B1 if correct**<br>**Omitting the final term** but otherwise correct is B1 M1 A1 A0 B0ft so 3/5<br>If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **3. (a)** | Either (Way 1) : Attempt $f(3)$ or $f(-3)$<br>$f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ *<br>Or (Way 2): Assume $a$ = -9 and attempt $f(3)$ or $f(-3)$<br>f(3) = 0 so ($x$ -3) is factor | M1<br>A1 * cso<br>**(2)** |
| | Or (Way 3): $\left(2x^3 - 5x^2 + ax + 18\right) \div \left(x - 3\right) = 2x^2 + px + q$ where $p$ is a number and $q$ is an expression in terms of $a$<br>Sets the remainder 18+3$a$+9 = 0 and solves to give $a = -9$ | M1<br>A1* cso<br>**(2)** |
| **(b)** | Either (Way 1): $f(x) = (x-3)(2x^2 + x - 6)$<br>$= (x-3)(2x-3)(x+2)$ | M1A1<br>M1A1<br>**(4)** |
| | Or (Way 2) Uses trial or factor theorem to obtain $x$ = -2 **or** $x$ = 3/2<br>Uses trial or factor theorem to obtain both $x$ = -2 **and** $x$ = 3/2<br>Puts three factors together (see notes below)<br>Correct factorisation : $(x-3)(2x-3)(x+2)$ or $(3-x)(3-2x)(x+2)$ or<br>$2(x-3)(x-\frac{3}{2})(x+2)$ oe | M1<br>A1<br>M1<br>A1<br>**(4)** |
| | Or (Way 3) No working three factors $(x-3)(2x-3)(x+2)$ otherwise need working | M1A1M1A1 |
| **(c)** | $\{3^y = 3 \Rightarrow\}$ $\underline{y = 1}$ or g(1) = 0<br>$\{3^y = 1.5 \Rightarrow\} \log\left(3^y\right) = \log 1.5$ or $y = \log_3 1.5$<br>$\{y = 0.3690702...\} \Rightarrow y =$ awrt 0.37 | B1<br>M1<br>A1 **(3)**<br>**[9]** |
| | **Notes for Question 3** | |
| **(a)** | M1 for attempting either $f(3)$ or $f(-3)$ – with **numbers substituted into expression**<br>A1 for applying $f(3)$ **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept ***x*** = -9) Note that the answer is given in part (a).<br>If they **assume** $a$ = -9 and **verify** by factor theorem or division they must state ($x$ – 3) **is a factor** for A1 (or equivalent such as QED or a tick). | |
| **(b)** | 1st M1: attempting to divide by $(x-3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by (3 – $x$) , in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc.<br>1st A1: usually for $2x^2 + x - 6$ … Credit when seen and use isw if miscopied<br>2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1)<br>2nd A1 is cao and needs all three factors together.<br>Ignore subsequent work (such as a solution to a quadratic equation.)<br>NB: $(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M0A0, $(x-3)(x-\frac{3}{2})(2x+4)$ is M1A1M1A0, but $2(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M1A1. | |
| **(c)** | B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0.<br>M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of f($x$) = 0 (ft their factorization)<br>A1: for an answer that rounds to 0.37. If a third answer is included (and not “rejected”) such as ln(-2) lose final A mark | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** | $x$: 0, 0.5, 1, 1.5, 2, 2.5, 3 <br> $y$: 5, 4, 2.5, 1.538, 1, 0.690, 0.5 | |
| **(a)** | $\{\text{At } x = 1.5,\}\ y = 1.538$ (only) | B1 **cao** |
| | | **[1]** |
| **(b)** | $\frac{1}{2}\times 0.5$ ; | B1 oe |
| | $\{5 + 0.5 + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}$    For structure of $\{\ldots\ldots\ldots\ldots\}$ ; | M1A1ft |
| | $\frac{1}{2}\times 0.5\times\{(5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}$ $\{= \frac{1}{4}(24.956) = 6.239\}$ = awrt 6.24 | A1 |
| | | **[4]** |
| **(c)** | Adds Area of Rectangle or first integral $= 3\times 4$ or $[4x]_0^3$ to **previous answer** | M1 |
| | So required estimate = $\{\text{"}6.239\text{"} + 12 = \text{"}18.239\text{"}\} = \text{"awrt } 18.24\text{"}$ (or 12 + previous answer). | A1ft |
| | N.B. $7\times 4$ + previous answer is M0A0 (added 4 seven times because 7 numbers in table) | **[2]** |
| | | **7** |

**Notes for Question 4**

| | | |
|---|---|---|
| **(a)** | B1: 1.538 |
| **(b)** | B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent. <br> M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first $y$ value **plus** last $y$ value and the second bracket to be multiplied by 2 and to be the summation of the remaining $y$ values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed ( An extra repeated term forfeits the M mark however). M0 if values used in brackets are $x$ values instead of $y$ values <br> A1ft: for the correct bracket $\{\ldots\ldots\}$ following through candidate's $y$ value found in part (a). <br> A1: for answer which rounds to 6.24. <br> NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 $h(a + b)$ used 5 or 6 times (and A1ft if it is all correct ) Then A1 as before. <br> Special case: Bracketing mistake $0.25\times(5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 20.831 usually indicates this error. |
| **(c)** | M1: Relates **previous answer ( not integral of previous answer**) to this question by integrating 4 between limits, and adding, or by using geometry to find rectangle and adding. <br> A1ft: for 12 + answer to (b) |
| Alternative **method (c)** | Those who do a trapezium rule for part (b)- using the table from (a) with 4 added to each cell of the table <br> Get: M1 for "*their* $\frac{1}{4}$"$\times\{9 + 4.5 + 2(8 + 6.5 + \text{their } 5.538 + 5 + 4.690)\}$ = (structure must be correct – allow one copying error only) <br> And A1ft: for awrt 18.24 (or 12 + previous answer). |

| Question Number | Scheme | Marks |
|---|---|---|
| **5. (a)** | Mark (a) and (b) together. | |
| | **Usually answered in radians: Uses** either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{1}{2}(12)^2(\text{angle})$ or both | M1 |
| | Area $= \frac{1}{2}(23)(12)\sin 0.64$ **or** $\frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091...$ **or** $180.1146711...\}$ | A1 |
| | Area $= \frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091... + 180.1146711...\}$ | A1 |
| | $\{\text{Area} = 262.527642...\}$ = awrt 262.5 ($m^2$) or 262.4($m^2$) or 262.6 ($m^2$) | A1 |
| | | **(4)** |
| **(b)** | $CDE = 12 \times (angle)$, $= 12(\pi - 0.64)\{\Rightarrow CDE = 30.01911...\}$ | M1, A1 |
| | $AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ $\{AE = 15.17376...\}$ | M1 |
| | Perimeter $= 23 + 12 + 15.17376... + 30.01911...$ | M1 |
| | $= 80.19287... =$ awrt 80.2 (m) | A1 |
| | | **(5)** |
| | | **[9]** |
| | **Notes for Question 5** | |
| **(a)** | M1: uses either area of triangle formula as given in mark scheme, or area of sector or both (may be implied by answer)<br>A1: one correct area expression (with **correct angle** used) $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ or see awrt 82.4 **or** awrt 180 (180 may be split as 226.2(semicircle) minus 46.1(small sector))<br>A1: two correct area expressions (with correct angles) **added together (**allow 2.5 as implying $\pi - 0.64$) or see awrt 82.4 **+** awrt 180 ( or 226 - 46 )<br>A1: answers which round to 262.5 or 262.4 or 262.6 | |
| **(b)** | 1st M1 for attempt to use $s = r\theta$ (any angle)<br>1st A1 for $\pi - 0.64$ in the formula (or 2.5)<br>2nd M1: Uses correct cosine rule to obtain $AE$ or $AE^2$(this may appear in part (a))<br>3rd M1(**independent)**: Perimeter = $23 + 12 +$ their $AE$ + their $CDE$<br>2nd A1: awrt 80.2 (ignore units – even incorrect units) | |
| **Degrees (a)** | Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{\text{angle in degrees}}{360} \times \pi(12)^2$ or both for M1<br>Area $= \frac{1}{2}(23)(12)\sin 36.7$ **or** $\frac{(180\text{- }36.7)}{360} \times \pi(12)^2 \{= awrt\ 82.4...$ **or** $180\}$ A1<br>Area $= \frac{1}{2}(23)(12)\sin 36.7 + \frac{(180\text{- }36.7)}{360} \times \pi(12)^2$ $\{= awrt\ 82.4... + 180\}$ A1<br>Final mark as before | |
| **(b)** | $CDE = \frac{\text{Angle in degrees}}{360} \times 24\pi, = \frac{180 - 36.7}{360} \times 24\pi \{\Rightarrow CDE = 30.01268...\}$ M1, A1<br>Final three marks as before | |

| Question Number | Scheme | Marks |
|---|---|---|
| **6. (a)** | Seeing $-4$ and 2. | B1 **(1)** |
| **(b)** | $x(x+4)(x-2) = \underline{x^3 + 2x^2 - 8x}$ or $\underline{x^3 - 2x^2 + 4x^2 - 8x}$ ( without simplifying) | B1 |
| | $\int (x^3 + 2x^2 - 8x)\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\}$ or $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$ | M1A1ft |
| | $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0} = (0) - \left(64 - \frac{128}{3} - 64\right)$ **or** $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{0}^{2} = \left(4 + \frac{16}{3} - 16\right) - (0)$ | dM1 |
| | One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7 ) **or** other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7) | A1 |
| | Hence Area $= "their\,42\frac{2}{3}" + "their\,6\frac{2}{3}"$ or Area $= "their\,42\frac{2}{3}" - "-their\,6\frac{2}{3}"$ | dM1 |
| | $= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$ ) | A1 |
| | (An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully) | **(7)** **[8]** |
| | **Notes for Question 6** | |
| **(a)** | B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or $A$ = -4, $B$ = 2 or indeed any indication of -4 and 2 – check graph also | |
| **(b)** | B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here)<br>M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms<br>A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0)<br>dM1: (dependent on previous M) substituting EITHER -$a$ and 0 and subtracting either way round OR similarly for 0 and $b$. **If their limits –*a* and *b* are used in ONE integral, apply the Special Case below**.<br>A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ **or** $\frac{20}{3}$ ) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0.<br>dM1 (depends on first method mark) **Correct** method **to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**.<br>A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen.<br>For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen.<br>**(**Trapezium rule **gets no marks after first two B marks)** | |
| **(b)** | **Special Case: one integral only from –*a* to *b*:** B1M1A1 available as before, then<br>$\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{2} = (4 + \frac{16}{3} - 16) - \left(64 - \frac{128}{3} - 64\right) = -6\tfrac{2}{3} + 42\frac{2}{3}$ **=….** **dM1** for correct use of their limits –$a$ and $b$ and **subtracting** either way round**.**<br>A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **7. (i) Method 1** | $\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2) | M1 |
| | $\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$ | M1 |
| | $16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent) | dM1 |
| | $x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work | A1 **cso** **(4)** |
| **7(i) Method 2** | $\log_2(2x) + 3 = \log_2(5x+4)$ | |
| | So $\log_2(2x) + \log_2(8) = \log_2(5x+4)$ (3 replaced by $\log_2 8$) | 2nd M1 |
| | Then $\log_2(16x) = \log_2(5x+4)$ (addition law of logs) | 1st M1 |
| | Then final M1 A1 as before | dM1A1 |
| **(ii)** | $\log_a y + \log_a 2^3 = 5$ | M1 |
| | $\log_a 8y = 5$ Applies product law of logarithms. | dM1 |
| | $y = \frac{1}{8}a^5$ $y = \frac{1}{8}a^5$ | A1cao |
| | | **(3)** **[7]** |
| | **Notes for Question 7** | |
| **(i)** | 1st M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms in *x* into one** log term in *x*<br>2nd M1: For RHS of either $2^{-3}$, $2^3$, $2^4$ or $\log_2\left(\frac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. **Use of** $3^2$ **is M0**<br>3rd dM1: Obtains **correct** linear equation in *x*. usually the one in the scheme and attempts *x* =<br>A1: cso Answer of 4/11 with **no** suspect log work preceding this. | |
| **(ii)** | M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ *or* $\log_a 8$<br>dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$ | |
| **(i)** | **Special case 1:** $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or<br>$\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \log_2\frac{2x}{5x+4} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each attempt scores M0M1M1A0 – special case | |
| | **Special case 2:**<br>$\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x+4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1<br>Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **8. (i)** | $(\lvert\alpha\rvert = 56.3099...)$ | |
| | $x = \{\alpha + 40 = 96.309993...\} =$ **awrt 96.3** | B1 |
| | $x - 40^\circ = -180 + "56.3099"...$ or $x - 40^\circ = -\pi + "0.983"...$ | M1 |
| | $x = \{-180 + 56.3099... + 40 = -83.6901...\} =$ **awrt -83.7** | A1 |
| | | **(3)** |
| **(ii)(a)** | $\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$ | M1 |
| | $\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$ | dM1 |
| | $1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ * | A1 cso * |
| | | **(3)** |
| **(b)** | $\cos\theta = \dfrac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ | M1 |
| | **or** $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos\theta = ...$ | |
| | One solution is $72^\circ$ or $144^\circ$, Two solutions are $72^\circ$ and $144^\circ$ | A1, A1 |
| | $\theta = \{72, 144, 216, 288\}$ | M1 A1 |
| | | **(5)** **[11]** |
| | **Notes for Question 8** | |
| **(i)** | B1: 96.3 by any or no method<br>M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360).<br>A1: awrt –83.7<br>Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned)<br>Working in radians – could earn M1 for $x - 40^\circ = -\pi + "0.983"...$ so B0M1A0 | |
| **(ii) (a)** | M1: uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \dfrac{\sin}{\cos}$, with no argument)<br>dM1: uses $\sin^2\theta = 1-\cos^2\theta$ (quoted correctly) in equation<br>A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0" | |
| **(b)** | M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0.<br>1st A1: Either 72 **or** 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.)<br>M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M)<br>A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)<br>(**Premature approximation**: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles)<br>Do **not** require degrees symbol for the marks<br>**Special case: Working in radians**<br>M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both<br>M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5 | |

| Question Number | Scheme | Marks |
|---|---|---|
| **9. (a)** | $\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$ | M1 A1 |
| | $2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ ,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ <br> [or $2x-16x^{-\frac{1}{2}}=0\ \Rightarrow x=4$ (no wrong work seen)] | M1 |
| | $(x^{\frac{3}{2}}=8\ \Rightarrow)\ x=\ 4$ | A1 |
| | $x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore $y$ = 100 as second answer) | M1 A1 |
| | | **(6)** |
| **(b)** | $\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$ | M1 A1 |
| | $(\frac{d^2y}{dx^2}>0\ \Rightarrow)\ y$ is a minimum ( there should be no wrong reasoning) | A1 |
| | | **(3)** <br> **[9]** |
| **(b)** | **<u>Alternative Method: Gradient Test:</u>** <br> M1 for finding the gradient either side of their $x$-value from part (a). <br> A1 for <u>both gradients calculated correctly to 1 significant figure</u>, then <u>using < 0 and > 0 respectively *maybe by use of sketch or table*</u>. (See appendix for gradient values. This is **not ft their *x***) <br> A1 states minimum needs M1A1 to have been awarded. | |
| | **Notes for Question 9** | |
| **(a)** | 1st M1: At least one term differentiated correctly, so $x^2\to 2x$, or $32\sqrt{x}\to 16x^{-\frac{1}{2}}$, or $20\to 0$ <br> A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ <br> 2nd M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ *or* $x^3=$ after correct squaring or spots $x$ = 4 <br> (NB $\left\{\frac{d^2y}{dx^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ **is M0 )** <br> N.B. Common error: Putting derivative = 0 and merely obtaining $x$ = 0 is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) <br> A1: $x$ = 4 cao [ $x$ = -4 is A0 and $x=\pm 4$ is also A0 ] <br> 3rd M1: Substitutes **their positive** found $x$ **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ \ x>0$ .**Should follow attempting to set** $\frac{dy}{dx}$ **= 0 and not setting** $\frac{d^2y}{dx^2}=0$ <br> A1: -28 cao (Does not need to be written as coordinates) | |
| **(b)** | M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. <br> A1: Answer in scheme or equivalent <br> A1: States minimum (Second derivative should be correct- can follow incorrect positive $x$. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2}>0$ but should not have said $\frac{d^2y}{dx^2}=0$ for example ) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **10. (a)** | Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$ , $k > 0$ | M1 |
| | Equation of form $(x - a)^2 + (y - b)^2 = 5^2$ , with values for *a* and *b* | M1 |
| | $(x + 5)^2 + (y - 9)^2 = 25 = 5^2$ | A1 |
| | | **(3)** |
| | $P(8, -7)$. Let centre of circle $= X(-5, 9)$ | |
| **(b)** | $PX^2 = (8 - "-5")^2 + (-7 - "9")^2$ or $PX = \sqrt{(8 - -5)^2 + (-7 - 9)^2}$ | M1 |
| | ( $PX = \sqrt{425}$ or $5\sqrt{17}$ ) $PT^2 = (PX)^2 - 5^2$ with numerical $PX$ | dM1 |
| | $PT \{= \sqrt{400}\} = 20$ (allow 20.0) | A1 **cso** |
| | | **(3)** **[6]** |
| **Alternative 2 for (a)** | Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$ | M1 |
| | Uses $a^2 + b^2 - 5^2 = c$ with their *a* and *b* or substitutes (0, 9) giving $+9^2 \pm 2b \times 9 + c = 0$ | M1 |
| | $x^2 + y^2 + 10x - 18y + 81 = 0$ | A1 |
| | | **(3)** |
| **Alternative 2 for (b)** | An attempt to find the point *T* may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first) | M1 |
| | M1: Use either of the correct points with *P* (8, -7) and distance between two points formula | dM1 |
| | A1: 20 | A1cso |
| | | **(3)** |
| **Alternative 3 for (b)** | Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$ | M1 |
| | Square roots to give $PT \{= \sqrt{400}\} = 20$ | dM1A1 |
| | | **(3)** |
| | **Notes for Question 10** | |
| **(a)** | **The three marks in (a) each require a circle equation – (see special cases which are not circles)**<br>M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be $r^2$ or $k > 0$ or a positive value)<br>M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or $5^2$<br>A1: correct circle equation in any equivalent form<br>**Special cases** $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is **not a circle** equation so M0M0A0<br>Also $(x \pm 5)^2 + (y - 9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0<br>**But** $(x - 0)^2 + (y - 9)^2 = 5^2$ gains M0M1A0 | |
| **(b)** | M1: Attempts to find distance from their **centre of circle** to *P* (or square of this value). If this is called *PT* and given as answer this is M0. Solution may use letter other than *X* , as centre was not labelled in the question.<br>N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0.<br>dM1**:** Applies the **subtraction** form of Pythagoras to find $PT$ or $PT^2$ (depends on previous method mark for distance from **centre to *P***) or uses appropriate complete method involving trigonometry<br>A1: 20 cso | |

| Question Number | Scheme | Marks |
|---|---|---|
| ***Aliter*** | **Gradient Test Method:** | |
| **9. (b)** | $\frac{dy}{dx} = 2x - 16x^{-\frac{1}{2}}$ | |
| ***Way 2*** | ***Helpful table!*** | |

| $x$ | $\frac{dy}{dx}$ |
|---|---|
| 3 | -3.2376 |
| 3.1 | -2.88739 |
| 3.2 | -2.54427 |
| 3.3 | -2.20771 |
| 3.4 | -1.87722 |
| 3.5 | -1.55236 |
| 3.6 | -1.23274 |
| 3.7 | -0.918 |
| 3.8 | -0.60783 |
| 3.9 | -0.30191 |
| 4 | 0 |
| 4.1 | 0.298163 |
| 4.2 | 0.592799 |
| 4.3 | 0.884115 |
| 4.4 | 1.172299 |
| 4.5 | 1.457528 |
| 4.6 | 1.739962 |
| 4.7 | 2.01975 |
| 4.8 | 2.297033 |
| 4.9 | 2.571937 |
| 5 | 2.844582 |

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