

1. (a) Using the factor theorem, show that $(x + 3)$ is a factor of

$$x^3 - 3x^2 - 10x + 24. \quad (2 \text{ marks})$$

- (b) Factorise $x^3 - 3x^2 - 10x + 24$ completely. (4 marks)
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2. $f(n) = n^3 + pn^2 + 11n + 9$, where p is a constant.

- (a) Given that $f(n)$ has a remainder of 3 when it is divided by $(n + 2)$, prove that $p = 6$. (2 marks)

- (b) Show that $f(n)$ can be written in the form $(n + 2)(n + q)(n + r) + 3$, where q and r are integers to be found. (3 marks)

- (c) Hence show that $f(n)$ is divisible by 3 for all positive integer values of n . (2 marks)
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3. Find the values of θ , to 1 decimal place, in the interval $-180 \leq \theta < 180$ for which

$$2 \sin^2 \theta^\circ - 2 \sin \theta^\circ = \cos^2 \theta^\circ. \quad (8 \text{ marks})$$

4. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £ y , where

$$y = 1.04^x.$$

- (a) Sketch the graph of $y = 1.04^x$, $x \geq 0$. (2 marks)

- (b) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years. (2 marks)

- (c) Use logarithms to find the number of years it takes to double the total value of any initial investment. (3 marks)
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5. The curve C with equation $y = p + qe^x$, where p and q are constants, passes through the point $(0, 2)$. At the point $P(\ln 2, p + 2q)$ on C , the gradient is 5.

(a) Find the value of p and the value of q . (5 marks)

The normal to C at P crosses the x -axis at L and the y -axis at M .

(b) Show that the area of $\triangle OLM$, where O is the origin, is approximately 53.8 (5 marks)

6. **Figure 3**

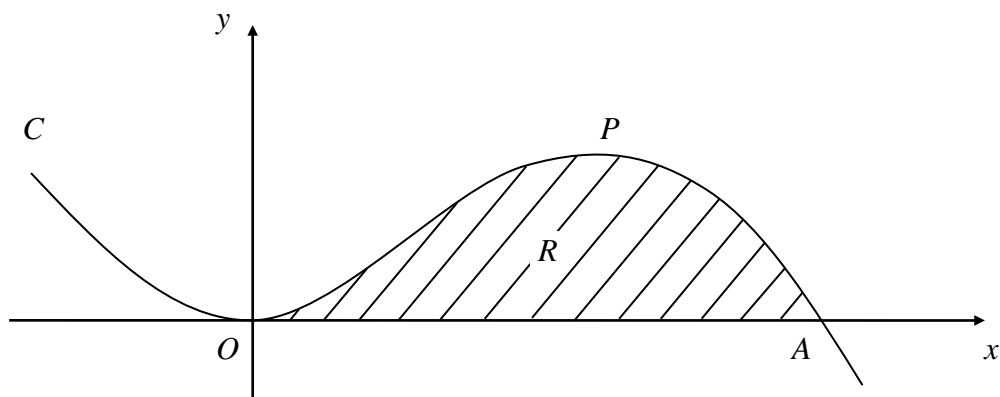


Figure 3 shows part of the curve C with equation

$$y = \frac{3}{2}x^2 - \frac{1}{4}x^3.$$

The curve C touches the x -axis at the origin and passes through the point $A(p, 0)$.

(a) Show that $p = 6$. (1 marks)

(b) Find an equation of the tangent to C at A . (4 marks)

The curve C has a maximum at the point P .

(c) Find the x -coordinate of P . (2 marks)

The shaded region R , in Fig. 3, is bounded by C and the x -axis.

(d) Find the area of R . (4 marks)

7.

Figure 1

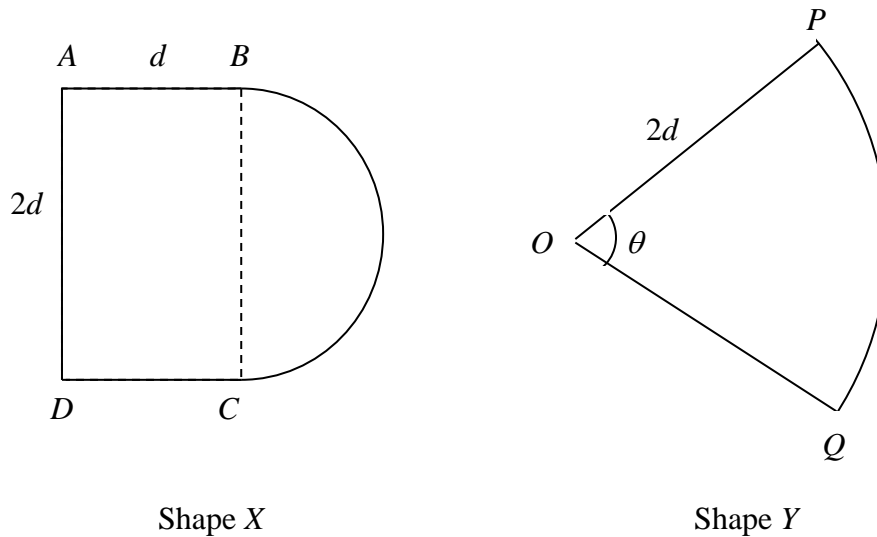


Figure 1 shows the cross-sections of two drawer handles.

Shape X is a rectangle $ABCD$ joined to a semicircle with BC as diameter. The length $AB = d$ cm and $BC = 2d$ cm.

Shape Y is a sector OPQ of a circle with centre O and radius $2d$ cm. Angle POQ is θ radians.

Given that the areas of the shapes X and Y are equal,

(a) prove that $\theta = 1 + \frac{1}{4}\pi$. (5 marks)

Using this value of θ , and given that $d = 3$, find in terms of π ,

(b) the perimeter of shape X , (2 marks)

(c) the perimeter of shape Y . (3 marks)

(d) Hence find the difference, in mm, between the perimeters of shapes X and Y . (2 marks)

8. $f(x) = \left(1 + \frac{x}{k}\right)^n$, $k, n \in \mathbb{N}$, $n > 2$.

Given that the coefficient of x^3 is twice the coefficient of x^2 in the binomial expansion of $f(x)$,

- (a) prove that $n = 6k + 2$. **(3 marks)**
Given also that the coefficients of x^4 and x^5 are equal and non-zero,
- (b) form another equation in n and k and hence show that $k = 2$ and $n = 14$. **(4 marks)**
Using these values of k and n ,
- (c) expand $f(x)$ in ascending powers of x , up to and including the term in x^5 . Give each coefficient as an exact fraction in its lowest terms **(4 marks)**
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END