Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 1

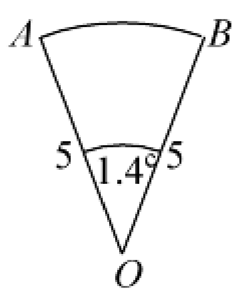
Question:

The sector *AOB* is removed from a circle of radius 5 cm.
The $\angle AOB$ is 1.4 radians and OA = OB.

(a) Find the perimeter of the sector *AOB*. (3)

(b) Find the area of sector *AOB*. (2)

Solution:



(a)

Arc length $= r\theta = 5 \times 1.4 = 7$ cm
Perimeter $= 10 + \text{Arc} = 17$ cm

(b) Sector area $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 5^2 \times 1.4 = 17.5$ cm^2

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 2

Question:

Given that $\log_2 x = p$:

(a) Find $\log_2 \left(8x^2 \right)$ in terms of p. (4)

(b) Given also that $p = 5$, find the value of x. (2)

Solution:

(a) $\log_2 x = p$

$\log_2 \left(8x^2 \right) = \log_2 8 + \log_2 x^2 = 3 + 2 \log_2 x = 3 + 2p$

(b) $\log_2 x = 5 \quad \Rightarrow \quad x = 2^5 \quad \Rightarrow \quad x = 32$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 3

Question:

(a) Find the value of the constant a so that $(x-3)$ is a factor of $x^3 - ax - 6$. (3)

(b) Using this value of a, factorise $x^3 - ax - 6$ completely. (4)

Solution:

(a) Let $f(x) = x^3 - ax - 6$
If $(x-3)$ is a factor then $f(3) = 0$
i.e. $0 = 27 - 3a - 6$
So $3a = 21 \Rightarrow a = 7$

(b) $x^3 - 7x - 6$ has $(x-3)$ as a factor, so
$x^3 - 7x - 6 = (x-3)(x^2 + 3x + 2) = (x-3)(x+2)(x+1)$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Practice paper
Exercise 1, Question 4

Question:

(a) Find the coefficient of x^{11} and the coefficient of x^{12} in the binomial expansion of $(2 + x)^{15}$. (4)

The coefficient of x^{11} and the coefficient of x^{12} in the binomial expansion of $(2 + kx)^{15}$ are equal.

(b) Find the value of the constant k. (3)

Solution:

(a) $(2 + x)^{15} = \ldots \binom{15}{11} 2^4 x^{11} + \binom{15}{12} 2^3 x^{12} + \ldots$

Coefficient of $x^{11} = \binom{15}{11} \times 16 = 1365 \times 16 = 21\,840$

Coefficient of $x^{12} = \binom{15}{12} \times 8 = 455 \times 8 = 3640$

(b) $21\,840k^{11} = 3640k^{12}$

So $k = \frac{21\,840}{3640}$

i.e. $k = 6$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Practice paper
Exercise 1, Question 5

Question:

(a) Prove that:

$\frac{\cos^2\theta}{\sin\theta+\sin^2\theta} \equiv \frac{1-\sin\theta}{\sin\theta}$, $0 < \theta < 180°$. (4)

(b) Hence, or otherwise, solve the following equation for $0 < \theta < 180°$:

$\frac{\cos^2\theta}{\sin\theta+\sin^2\theta} = 2$

Give your answers to the nearest degree. (4)

Solution:

(a) LHS $= \frac{\cos^2\theta}{\sin\theta+\sin^2\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta+\sin^2\theta}$ (using $\sin^2\theta+\cos^2\theta \equiv 1$)

$= \frac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta(1+\sin\theta)}$ (factorising)

$= \frac{1-\sin\theta}{\sin\theta}$ (cancelling $[1+\sin\theta]$)

= RHS

(b) $2 = \frac{\cos^2\theta}{\sin\theta+\sin^2\theta}$

$\Rightarrow \quad 2 = \frac{1-\sin\theta}{\sin\theta}$

$\Rightarrow \quad 2\sin\theta = 1-\sin\theta$ (can multiply by $\sin\theta \because 0 < \theta < 180$)

$\Rightarrow \quad 3\sin\theta = 1$

$\Rightarrow \quad \sin\theta = \frac{1}{3}$

So $\theta = 19.47\ldots°, 160.5\ldots° = 19°, 161°$ (to nearest degree)

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 6

Question:

(a) Show that the centre of the circle with equation $x^2 + y^2 = 6x + 8y$ is (3, 4) and find the radius of the circle. (5)

(b) Find the exact length of the tangents from the point (10, 0) to the circle. (4)

Solution:

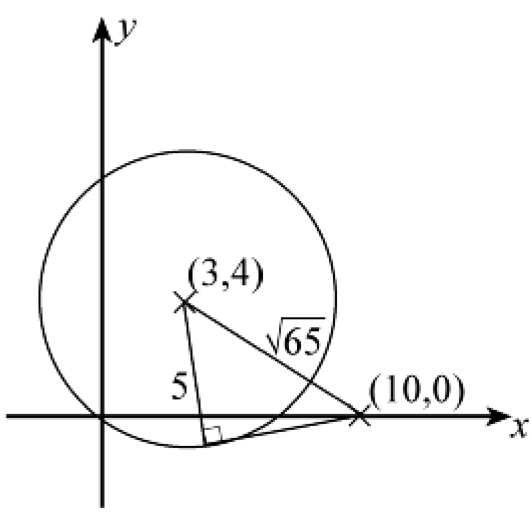
(a) $x^2 - 6x + y^2 - 8y = 0$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 = 9 + 16$

i.e. $(x-3)^2 + (y-4)^2 = 5^2$

Centre (3, 4), radius 5

(b) Distance from (3, 4) to (10, 0) $= \sqrt{7^2 + 4^2} = \sqrt{65}$



Length of tangent $= \sqrt{\sqrt{65}^2 - 5^2} = \sqrt{40} = 2\sqrt{10}$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 7

Question:

A father promises his daughter an eternal gift on her birthday. On day 1 she receives £75 and each following day she receives $\frac{2}{3}$ of the amount given to her the day before. The father promises that this will go on for ever.

(a) Show that after 2 days the daughter will have received £125. (2)

(b) Find how much money the father should set aside to ensure that he can cover the cost of the gift. (3)
After *k* days the total amount of money that the daughter will have received exceeds £200.

(c) Find the smallest value of *k*. (5)

Solution:

(a) Day 1 = £ 75, day 2 = £ 50, total = £ 125

(b) $a = 75$, $r = \frac{2}{3}$—geometric series

$$S_{\infty} = \frac{a}{1-r} = \frac{75}{1-\frac{2}{3}}$$

Amount required = £ 225

(c) $S_k = \frac{a(1-r^k)}{1-r}$

Require $\frac{75\left[1-\left(\frac{2}{3}\right)^k\right]}{1-\frac{2}{3}} > 200$

i.e. $225\left[1-\left(\frac{2}{3}\right)^k\right] > 200$

$\Rightarrow\ 1-\left(\frac{2}{3}\right)^k > \frac{8}{9}$

$\Rightarrow\ \frac{1}{9} > \left(\frac{2}{3}\right)^k$

Take logs: $\log\left(\frac{1}{9}\right) > k\ \log\left(\frac{2}{3}\right)$

Since $\log\left(\frac{2}{3}\right)$ is negative, when we divide by this the inequality will change around.

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

So $k > \dfrac{\log\left(\frac{1}{9}\right)}{\log\left(\frac{2}{3}\right)}$

i.e. $k > 5.419$ …
So need $k = 6$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Practice paper
Exercise 1, Question 8

Question:

Given $I = \int_1^3 \left(\frac{1}{x^2} + 3\sqrt{x} \right) dx$:

(a) Use the trapezium rule with the table below to estimate I to 3 significant figures. (4)

x	1	1.5	2	2.5	3
y	4	4.119	4.493	4.903	5.307

(b) Find the exact value of I. (4)

(c) Calculate, to 1 significant figure, the percentage error incurred by using the trapezium rule as in part (a) to estimate I. (2)

Solution:

(a) $h = 0.5$

$$I \approx \frac{0.5}{2} \left[4 + 2 \left(4.119 + 4.493 + 4.903 \right) + 5.307 \right]$$

$$= \frac{1}{4} \left[36.337 \right]$$

$$= 9.08425$$

(b) $I = \int_1^3 \left(x^{-2} + 3x^{\frac{1}{2}} \right) dx$

$$= \left[\frac{x^{-1}}{-1} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^3$$

$$= \left[-\frac{1}{x} + 2x^{\frac{3}{2}} \right]_1^3$$

$$= \left(-\frac{1}{3} + 2 \times 3\sqrt{3} \right) - \left(-1 + 2 \right)$$

$$= 6\sqrt{3} - \frac{4}{3}$$

(c) Percentage error $= \frac{\left| 6\sqrt{3} - \frac{4}{3} - 9.08425 \right|}{6\sqrt{3} - \frac{4}{3}} \times 100 = 0.279 \ldots \% = 0.3\%$

Dr. Faisal Rana www.biochemtuition.com faisal.rana@biochemtuition.com

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Practice paper
Exercise 1, Question 9

Question:

The curve C has equation $y = 6x^{\frac{7}{3}} - 7x^2 + 4$.

(a) Find $\frac{dy}{dx}$. (2)

(b) Find $\frac{d^2y}{dx^2}$. (2)

(c) Use your answers to parts (a) and (b) to find the coordinates of the stationary points on C and determine their nature. (9)

Solution:

(a) $y = 6x^{\frac{7}{3}} - 7x^2 + 4$

$\frac{dy}{dx} = 6 \times \frac{7}{3}x^{\frac{4}{3}} - 14x$

$\frac{dy}{dx} = 14x^{\frac{4}{3}} - 14x$

(b) $\frac{d^2y}{dx^2} = \frac{56}{3}x^{\frac{1}{3}} - 14$

(c) $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{4}{3}} - x = 0 \Rightarrow x\left(x^{\frac{1}{3}} - 1\right) = 0$

So $x = 0$ or 1

$x = 0 \Rightarrow \frac{d^2y}{dx^2} = -14 < 0 \therefore (0, 4)$ is a maximum

$x = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{56}{3} - 14 > 0 \therefore (1, 3)$ is a minimum