

Review Exercise

- 1** Simplify $\frac{x^2 - 2x - 3}{x^2 - 7x + 12}$.
- 2** In $\triangle ABC$, $AB = \sqrt{5}$ cm, $\angle ABC = 45^\circ$, $\angle BCA = 30^\circ$. Find the length of BC .
- 3**
 - a** Write down the value of $\log_3 81$
 - b** Express $2\log_a 4 + \log_a 5$ as a single logarithm to base a .
- 4** P is the centre of the circle $(x - 1)^2 + (y + 4)^2 = 81$.
 Q is the centre of the circle $(x + 3)^2 + y^2 = 36$.
 Find the exact distance between the points P and Q .
- 5** Divide $2x^3 + 9x^2 + 4x - 15$ by $(x + 3)$.
- 6** In $\triangle ABC$, $AB = 5$ cm, $BC = 9$ cm and $CA = 6$ cm. Show that $\cos \angle TRS = -\frac{1}{3}$.
- 7**
 - a** Find, to 3 significant figures, the value of x for which $5^x = 0.75$.
 - b** Solve the equation $2\log_5 x - \log_5 3x = 1$
- 8** The circle C has equation $(x + 4)^2 + (y - 1)^2 = 25$.
 The point P has coordinates $(-1, 5)$.
 - a** Show that the point P lies on the circumference of C .
 - b** Show that the centre of C lies on the line $x - 2y + 6 = 0$.
- 9**
 - a** Show that $(2x - 1)$ is a factor of $2x^3 - 7x^2 - 17x + 10$.
 - b** Factorise $2x^3 - 7x^2 - 17x + 10$ completely.
- 10** In $\triangle PQR$, $QR = 8$ cm, $PR = 6$ cm and $\angle PQR = 40^\circ$.
 Calculate the two possible values of $\angle QPR$.
- 11**
 - a** Express $\log_2 \left(\frac{4a}{b^2} \right)$ in terms of $\log_2 a$ and $\log_2 b$.
 - b** Find the value of $\log_{27} \frac{1}{9}$.
- 12** The points $L(3, -1)$ and $M(5, 3)$ are the end points of a diameter of a circle, centre N .
 - a** Find the exact length of LM .
 - b** Find the coordinates of the point N .
 - c** Find an equation for the circle.

13 $f(x) = 3x^3 + x^2 - 38x + c$

Given that $f(3) = 0$,

- a** find the value of c ,
- b** factorise $f(x)$ completely,
- c** find the remainder when $f(x)$ is divided by $(2x - 1)$.

14 In $\triangle ABC$, $AB = 5$ cm, $BC = (2x - 3)$ cm, $CA = (x + 1)$ cm and $\angle ABC = 60^\circ$.

- a** Show that x satisfies the equation $x^2 - 8x + 16 = 0$.
- b** Find the value of x .
- c** Calculate the area of the triangle, giving your answer to 3 significant figures.

15 **a** Solve $0.6^{2x} = 0.8$, giving your answer to 3 significant figures.

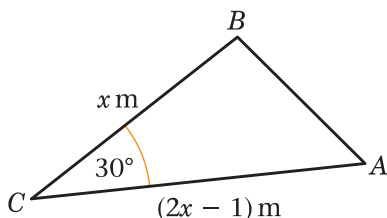
b Find the value of x in $\log_x 243 = 2.5$.

16 Show that part of the line $3x + y = 14$ forms a chord to the circle $(x - 2)^2 + (y - 3)^2 = 5$ and find the length of this chord.

17 $g(x) = x^3 - 13x + 12$

- a** Find the remainder when $g(x)$ is divided by $(x - 2)$.
- b** Use the factor theorem to show that $(x - 3)$ is a factor of $g(x)$.
- c** Factorise $g(x)$ completely.

18 The diagram shows $\triangle ABC$, with $BC = x$ m, $CA = (2x - 1)$ m and $\angle BCA = 30^\circ$. Given that the area of the triangle is 2.5 m²,



- a** find the value of x ,
- b** calculate the length of the line AB , giving your answer to 3 significant figures.

19 **a** Solve $3^{2x-1} = 10$, giving your answer to 3 significant figures.

b Solve $\log_2 x + \log_2(9 - 2x) = 2$

20 Prove that the circle $(x + 4)^2 + (y - 5)^2 + 8^2$ lies completely inside the circle $x^2 + y^2 + 8x - 10y = 59$.

21 $f(x) = x^3 + ax + b$, where a and b are constants.

When $f(x)$ is divided by $(x - 4)$ the remainder is 32.

When $f(x)$ is divided by $(x + 2)$ the remainder is -10 .

- a** Find the value of a and the value of b .
- b** Show that $(x - 2)$ is a factor of $f(x)$.

22 Ship B is 8 km, on a bearing of 030° , from ship A .
Ship C is 12 km, on a bearing of 140° , from ship B .

- a** Calculate the distance of ship C from ship A .
- b** Calculate the bearing of ship C from ship A .

23 **a** Express $\log_p 12 - \left(\frac{1}{2}\log_p 9 + \frac{1}{3}\log_p 8\right)$ as a single logarithm to base p .

b Find the value of x in $\log_4 x = -1.5$.

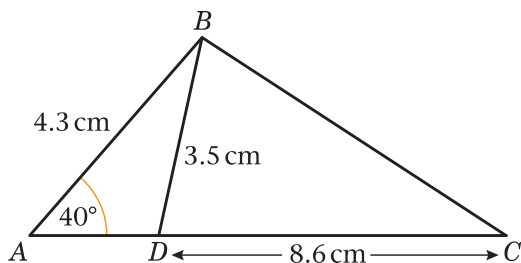
24 The point $P(4, -2)$ lies on a circle, centre $C(1, 5)$.

- a** Find an equation for the circle.
- b** Find an equation for the tangent to the circle at P .

25 The remainder when $x^3 - 2x + a$ is divided by $(x - 1)$ is equal to the remainder when $2x^3 + x - a$ is divided by $(2x + 1)$. Find the value of a .

[Hint: Use the remainder theorem.]

- 26** The diagram shows $\triangle ABC$.
Calculate the area of $\triangle ABC$.



- 27** Solve $3^{2x+1} + 5 = 16(3^x)$.

[Hint: Let $y = 3^x$]

- 28** The coordinates of the vertices of $\triangle ABC$ are $A(2, 5)$, $B(0, 2)$ and $C(4, 0)$.
Find the value of $\cos \angle ABC$.

- 29** Solve the simultaneous equations

$$4\log_9 x + 4\log_3 y = 9$$

$$6\log_3 x + 6\log_{27} y = 7$$

[Hint: Change all the logarithms to base 3].

- 30** The line $y = 5x - 13$ meets the circle $(x - 2)^2 + (y + 3)^2 = 26$ at the points A and B .

- a** Find the coordinates of the points A and B .

M is the mid-point of the line AB .

- b** Find the equation of the line which passes through M and is perpendicular to the line AB . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

- 31** The circle C has equation

$$x^2 + y^2 - 10x + 4y + 20 = 0.$$

Find the length of the tangent to C from the point $(-4, 4)$.