

3 Review Exercise

1 Find the values of x for which $f(x) = x^3 - 3x^2$ is a decreasing function.

2 Given that A is an acute angle and $\cos A = \frac{2}{3}$, find the exact value of $\tan A$.

3 Evaluate $\int_1^3 x^2 - \frac{1}{x^2}\,dx$.

4 Given that $y = \frac{x^3}{3} + x^2 - 6x + 3$, find the values of x when $\frac{dy}{dx} = 2$.

5 Solve, for $0 \leqslant x < 180°$, the equation $\cos 2x = -0.6$, giving your answers to 1 decimal place.

6 Find the area between the curve $y = x^3 - 3x^2$, the x-axis and the lines $x = 2$ and $x = 4$.

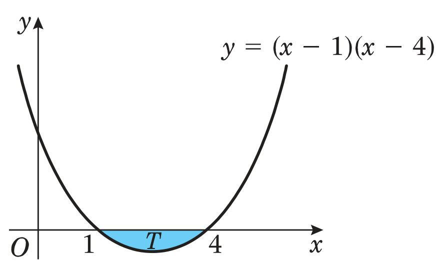
7 Given $f(x) = x^3 - 2x^2 - 4x$,

a find **i** $f(2)$ **ii** $f'(2)$ **iii** $f''(2)$

b interpret your answer to **a**.

8 Find all the values of θ in the interval $0 \leqslant \theta < 360°$ for which $2\sin(\theta - 30°) = \sqrt{3}$.

9 The diagram shows the shaded region T which is bounded by the curve $y = (x - 1)(x - 4)$ and the x-axis. Find the area of the shaded region T.



10 Find the coordinates of the stationary points on the curve with equation $y = 4x^3 - 3x + 1$.

11 **a** Given that $\sin\theta = \cos\theta$, find the value of $\tan\theta$.

b Find the value of θ in the interval $0 \leqslant \theta < 2\pi$ for which $\sin\theta = \cos\theta$, giving your answer in terms of π.

12 **a** Sketch the graph of $y = \frac{1}{x}$, $x > 0$.

b Copy and complete the table, giving your values of $\frac{1}{x}$ to 3 decimal places.

x	1	1.2	1.4	1.6	1.8	2
$\frac{1}{x}$	1					0.5

c Use the trapezium rule, with all the values from your table, to find an estimate for the value of $\int_1^2 \frac{1}{x}\,dx$.

d Is this an overestimate or an underestimate for the value of $\int_1^2 \frac{1}{x}\,dx$? Give a reason for your answer.

13 Show that the stationary point on the curve $y = 4x^3 - 6x^2 + 3x + 2$ is a point of inflexion.

14 Find all the values of x in the interval $0 \leqslant x < 360°$ for which $3\tan^2 x = 1$.

15 Evaluate $\int_1^8 x^{\frac{1}{3}} - x^{-\frac{1}{3}}\,dx$.

16 The curve C has equation $y = 2x^3 - 13x^2 + 8x + 1$.

a Find the coordinates of the turning points of C.

b Determine the nature of the turning points of C.

17 The curve S, for $0 \leqslant x < 360°$, has equation $y = 2\sin\left(\frac{2}{3}x - 30°\right)$.

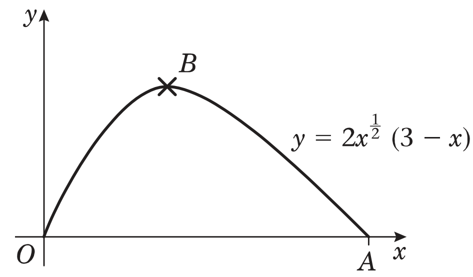
a Find the coordinates of the point where S meets the y-axis.

b Find the coordinates of the points where S meets the x-axis.

18 Find the area of the finite region bounded by the curve $y = (1 + x)(4 - x)$ and the x-axis.

[Hint: Find where the curve meets the x-axis.]

19 The diagram shows part of the curve with equation $y = 2x^{\frac{1}{2}}(3 - x)$. The curve meets the x-axis at the points O and A. The point B is the maximum point of the curve.



a Find the coordinates of A.

b Show that $\frac{dy}{dx} = 3x^{-\frac{1}{2}}(1 - x)$.

c Find the coordinates of B.

20 **a** Show that the equation $2\cos^2 x = 4 - 5\sin x$ may be written as $2\sin^2 x - 5\sin x + 2 = 0$.

b Hence solve, for $0 \leqslant \theta < 360°$, the equation $2\cos^2 x = 4 - 5\sin x$.

21 Use the trapezium rule with 5 equal strips to find an estimate for $\int_0^1 x\sqrt{(1 + x)}\,dx$.

22 A sector of a circle, radius r cm, has a perimeter of 20 cm.

a Show that the area of the sector is given by $A = 10r - r^2$.

b Find the maximum value for the area of the sector.

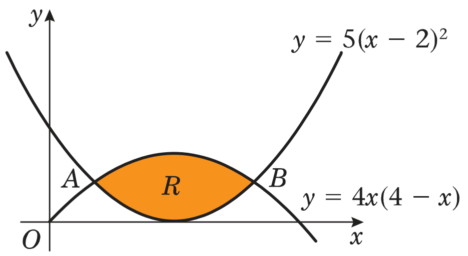
23 Show that, for all values of x:

a $\cos^2 x(\tan^2 x + 1) = 1$

b $\sin^4 x - \cos^4 x = (\sin x - \cos x)(\sin x + \cos x)$

[Hint: Use $a^2 - b^2 = (a - b)(a + b)$.]

24 The diagram shows the shaded region R which is bounded by the curves $y = 4x(4 - x)$ and $y = 5(x - 2)^2$. The curves intersect at the points A and B.



a Find the coordinates of the points A and B.

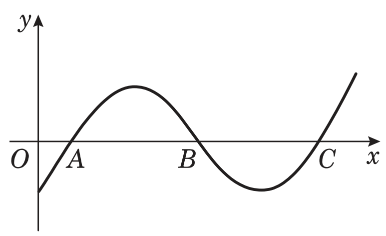
b Find the area of the shaded region R.

25 The volume of a solid cylinder, radius r cm, is 128π.

a Show that the surface area of the cylinder is given by $S = \dfrac{256\pi}{r} + 2\pi r^2$.

b Find the minimum value for the surface area of the cylinder.

26 The diagram shows part of the curve $y = \sin(ax - b)$, where a and b are constants and $b < \frac{\pi}{2}$.



Given that the coordinates of A and B are $\left(\frac{\pi}{6}, 0\right)$ and $\left(\frac{5\pi}{6}, 0\right)$ respectively,

a write down the coordinates of C,

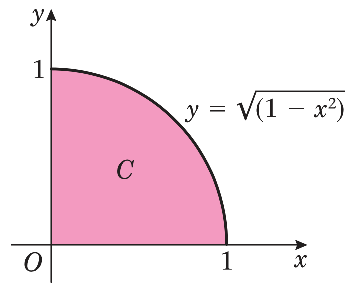
b find the value of a and the value of b.

27 Find the area of the finite region bounded by the curve with equation $y = x(6 - x)$ and the line $y = 10 - x$.

28 A piece of wire of length 80 cm is cut into two pieces. Each piece is bent to form the perimeter of a rectangle which is three times as long as it is wide. Find the lengths of the two pieces of wire if the sum of the areas of the rectangles is to be a maximum.

[Hint: Let the width of each rectangle be x cm and y cm respectively.]

29 The diagram shows the shaded region C which is bounded by the circle $y = \sqrt{(1 - x^2)}$ and the coordinate axes.



a Use the trapezium rule with 10 strips to find an estimate, to 3 decimal places, for the area of the shaded region C.

The actual area of C is $\frac{\pi}{4}$.

b Calculate the percentage error in your estimate for the area of C.

30 The area of the shaded region A in the diagram is 9 cm^2. Find the value of the constant a.