

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 1

#### Question:

Find the values of  $x$  for which  $f(x) = x^3 - 3x^2$  is a decreasing function.

#### Solution:

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x$$

$$3x^2 - 6x < 0$$

$$3x(x - 2) < 0$$

$f(x)$  is a decreasing function for  
 $0 < x < 2$ .

Find  $f'(x)$  and put this expression  $< 0$ .

Solve the inequality by factorisation, consider the three regions  $x < 0$ ,  $0 < x < 2$  and  $x > 2$ , looking for sign changes.

$$\frac{dy}{dx} < 0 \text{ for } 0 < x < 2$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

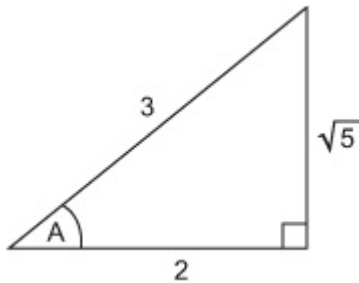
#### Exercise A, Question 2

#### Question:

Given that  $A$  is an acute angle and  $\cos A = \frac{2}{3}$ , find the exact value of  $\tan A$ .

#### Solution:

$$\cos A = \frac{2}{3}$$



$$\text{so } \tan A = \frac{\sqrt{5}}{2}$$

Draw a diagram and put in the information for  $\cos A$ .

Use Pythagoras theorem:  $a^2 + b^2 = c^2$  with  $b = 2$  and  $c = 3$ .  
so

$$a^2 + 2^2 = 3^2$$

$$a^2 + 4 = 9$$

$$a^2 = 5$$

$$a = \sqrt{5}$$

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## Edexcel Modular Mathematics for AS and A-Level

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#### Exercise A, Question 3

#### Question:

Evaluate  $\int_1^3 x^2 - \frac{1}{x^2} dx$ .

#### Solution:

Remember  $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

$$\int_1^3 x^2 - \frac{1}{x^2} dx$$

Change  $\frac{-1}{x^2}$  into index form and integrate:

$$\frac{-1}{x^2} = -1x^{-2}$$

$$\begin{aligned} \int -1x^{-2} dx &= \frac{-1x^{-2+1}}{-2+1} \\ &= \frac{-1x^{-1}}{-1} \\ &= x^{-1} \\ &= \frac{1}{x} \end{aligned}$$

$$\begin{aligned} &= \left[ \frac{x^3}{3} + \frac{1}{x} \right]_1^3 \\ &= \left( \frac{(3)^3}{3} + \frac{1}{(3)} \right) - \left( \frac{(1)^3}{3} + \frac{1}{(1)} \right) \\ &= \left( 9 + \frac{1}{3} \right) - \left( \frac{1}{3} + 1 \right) \\ &= 8 \end{aligned}$$

Evaluate the integral: substitute  $x = 3$ , then  $x = 1$ , and subtract.

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#### Exercise A, Question 4

#### Question:

Given that  $y = \frac{x^3}{3} + x^2 - 6x + 3$ , find the values of  $x$  when  $\frac{dy}{dx} = 2$ .

#### Solution:

$$y = \frac{x^3}{3} + x^2 - 6x + 3$$

$$\frac{dy}{dx} = \frac{3x^2}{3} + 2x - 6$$

$$x^2 + 2x - 6 = 2$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$\text{so } x = -4, \quad x = 2$$

$$\text{Remember } \frac{d}{dx} (ax^n) = anx^{n-1}$$

Put  $\frac{dy}{dx} = 2$  and solve the equation.

Factorise  $x^2 + 2x - 8 = 0$  :

$$(+4) \times (-2) = -8$$

$$(+4) + (-2) = +2$$

$$\text{so } x^2 + 2x - 8 = (x + 4)(x - 2)$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 5

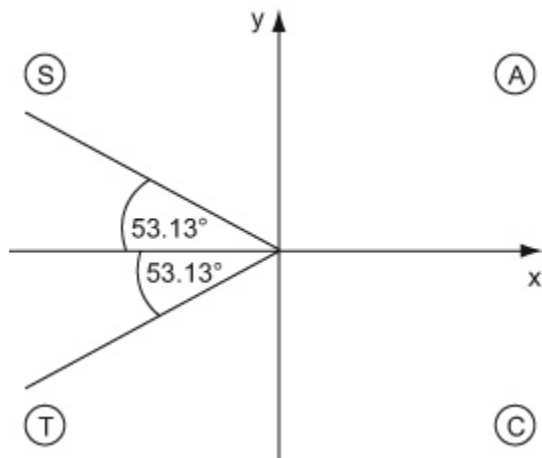
#### Question:

Solve, for  $0 \leq x < 180^\circ$ , the equation  $\cos 2x = -0.6$ , giving your answers to 1 decimal place.

#### Solution:

$$\cos 2x = -0.6$$

$$2x = 126.87^\circ$$



$\cos 2x$  is negative, so you need to look in the 2nd and 3rd quadrants. Here the angle in the 2nd quadrant is  $180^\circ - 126.87^\circ = 53.13^\circ$

$$2x = 126.87^\circ, 233.13^\circ$$

$$\text{so } x = 63.4^\circ, 116.6^\circ$$

Read off the solutions from your diagram

Find  $x$ : divide each value by 2.

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 6

#### Question:

Find the area between the curve  $y = x^3 - 3x^2$ , the  $x$ -axis and the lines  $x = 2$  and  $x = 4$ .

#### Solution:

$$\text{Area} = \int_2^4 x^3 - 3x^2 dx$$

$$\text{Remember } \int ax^n dx = \frac{ax^{n+1}}{n+1}$$

$$= \left[ \frac{x^4}{4} - x^3 \right]_2^4$$

$$= \left( \frac{(4)^4}{4} - (4)^3 \right) - \left( \frac{(2)^4}{4} - (2)^3 \right)$$

Use the limits: Substitute  $x = 4$  and  $x = 2$  into  $\frac{x^4}{4} - x^3$  and subtract.

$$= (64 - 64) - \left( \frac{16}{4} - 8 \right)$$

$$= 0 - (4 - 8)$$

$$= 0 - (-4)$$

$$= 4$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 7

#### Question:

Given  $f(x) = x^3 - 2x^2 - 4x$ ,

(a) find (i)  $f(2)$ , (ii)  $f'(2)$ , (iii)  $f''(2)$

(b) interpret your answer to part (a).

#### Solution:

$$f(x) = x^3 - 2x^2 - 4x$$

$$f'(x) = 3x^2 - 4x - 4$$

$$f''(x) = 6x - 4$$

$$\text{Remember } f'(x) = \frac{d}{dx}f(x), f''(x) = \frac{d^2}{dx^2}f(x)$$

(a)

(i)

$$\begin{aligned} f(2) &= (2)^3 - 2(2)^2 - 4 \\ &= 8 - 8 - 8 \\ &= -8 \end{aligned}$$

Find the value of  $f(x)$  where  $x = 2$ ; substitute  $x = 2$  into  $x^3 - 2x^2 - 4x$

(ii)

$$\begin{aligned} f'(2) &= 3(2)^2 - 4(2) - 4 \\ &= 12 - 8 - 4 \\ &= 0 \end{aligned}$$

Find the value of  $f'(x)$  where  $x = 2$ ; substitute  $x = 2$  into  $3x^2 - 4x - 4$

(iii)

$$\begin{aligned} f''(2) &= 6(2) - 4 \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

Find the value of  $f''(x)$  when  $x = 2$ ; Substitute  $x = 2$  into  $6x - 4$ .

(b)

On the graph of  $y = f(x)$ , the point  $(2, -8)$  is a minimum point.

$f'(2) = 0$  means there is a stationary point at  $x = 2$

$f''(2) = 8 > 0$  means the stationary point is a minimum.

$f(2) = -8$  means the graph of  $y = f(u)$  passes through the point  $(2, -8)$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 8

#### Question:

Find all the values of  $\theta$  in the interval  $0 \leq \theta < 360^\circ$  for which  $2\sin(\theta - 30^\circ) = \sqrt{3}$ .

#### Solution:

$$2\sin(\theta - 30^\circ) = \sqrt{3}$$

$$\sin(\theta - 30^\circ) = \frac{\sqrt{3}}{2}$$

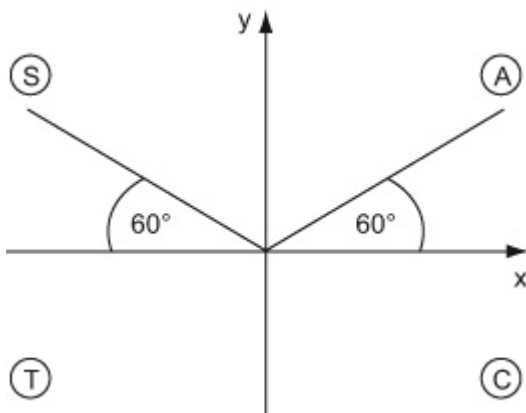
$$\theta - 30^\circ = 60^\circ$$

Divide each side by 2.

Solve the equation: let  $X = \theta - 30^\circ$   $\sin X = \frac{\sqrt{3}}{2}$ , so

$X = 60^\circ$  i.e.  $\theta - 30^\circ = 60^\circ$

$\sin(\theta - 30^\circ)$  is positive so you need to look in the 1st and 2nd quadrants.



$$\theta - 30^\circ = 60^\circ, 120^\circ$$

$$\text{so } \theta = 60 + 30^\circ$$

$$= 90^\circ$$

$$\text{and } \theta = 120 + 30^\circ$$

$$= 150^\circ$$

$$\theta = 90^\circ, 150^\circ$$

Read off the solutions from your diagram

Find  $\theta$ : add  $30^\circ$  to each value.



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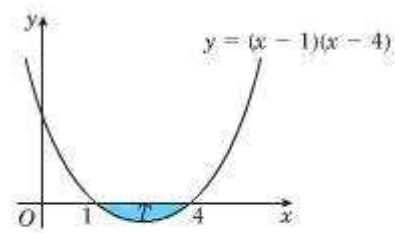
## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 9

#### Question:

The diagram shows the shaded region  $T$  which is bounded by the curve  $y = (x - 1)(x - 4)$  and the  $x$ -axis. Find the area of the shaded region  $T$ .



#### Solution:

$$\begin{aligned} \text{Area} &= \int_1^4 (x - 1)(x - 4) \, dx \\ &= \int_1^4 x^2 - 5x + 4 \, dx \end{aligned}$$

$$= \left[ \frac{x^3}{3} - \frac{5x^2}{2} + 4x \right]_1^4$$

$$\begin{aligned} &= \left( \frac{(4)^3}{3} - \frac{5(4)^2}{2} + 4(4) \right) \\ &\quad - \left( \frac{(1)^3}{3} - \frac{5(1)^2}{2} + 4(1) \right) \end{aligned}$$

$$= -4 \frac{1}{2}$$

$$\text{Area} = 4 \frac{1}{2}$$

Expand the brackets so that

$$\begin{aligned} (x - 1)(x - 4) &= x^2 - 4x - x + 4 \\ &= x^2 - 5x + 4 \end{aligned}$$

Remember  $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

Evaluate the integral. Substitute  $x = 4$ , then  $x = 1$ , into  $\frac{x^3}{3} - \frac{5x^2}{2} + 4x$  and subtract.

The negative value means the area is below the  $x$ -axis, as can be seen in the diagram.

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 10

#### Question:

Find the coordinates of the stationary points on the curve with equation  $y = 4x^3 - 3x + 1$ .

#### Solution:

$$y = 4x^3 - 3x + 1$$

$$\frac{dy}{dx} = 12x^2 - 3$$

$$12x^2 - 3 = 0$$

$$12x^2 = 3$$

$$x^2 = \frac{3}{12}$$

$$= \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}}$$

$$= \pm \frac{1}{2}$$

Remember  $\frac{dy}{dx} = 0$  at a stationary point.

When  $x = \frac{1}{2}$ ,

$$y = 4 \left( \frac{1}{2} \right)^3 - 3 \left( \frac{1}{2} \right) + 1$$

$$= 4 \left( \frac{1}{8} \right) - \frac{3}{2} + 1$$

$$= 0$$

Find the coordinates of the stationary points. Substitute  $x = \frac{1}{2}$  and  $x = -\frac{1}{2}$  into the equation for  $y$ .

When  $x = -\frac{1}{2}$

$$y = 4 \left( -\frac{1}{2} \right)^3 - 3 \left( -\frac{1}{2} \right) + 1$$

$$= 4 \left( -\frac{1}{8} \right) + \frac{3}{2} + 1$$

$$= -\frac{1}{2} + \frac{3}{2} + 1$$

$$= 2$$

Find the coordinates of the stationary points. Substitute  $x = \frac{1}{2}$  and  $x = -\frac{1}{2}$  into the equation for  $y$ .

So the coordinates of the stationary points

are  $\left( \frac{1}{2}, 0 \right)$  and  $\left( -\frac{1}{2}, 2 \right)$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 11

#### Question:

- (a) Given that  $\sin \theta = \cos \theta$ , find the value of  $\tan \theta$ .
- (b) Find the value of  $\theta$  in the interval  $0 \leq \theta < 2\pi$  for which  $\sin \theta = \cos \theta$ , giving your answer in terms of  $\pi$ .

#### Solution:

(a)

$$\sin \theta = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\cos \theta}{\cos \theta}$$

$$\tan \theta = 1$$

$$\text{Remember } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

Divide each side by  $\cos \theta$ .

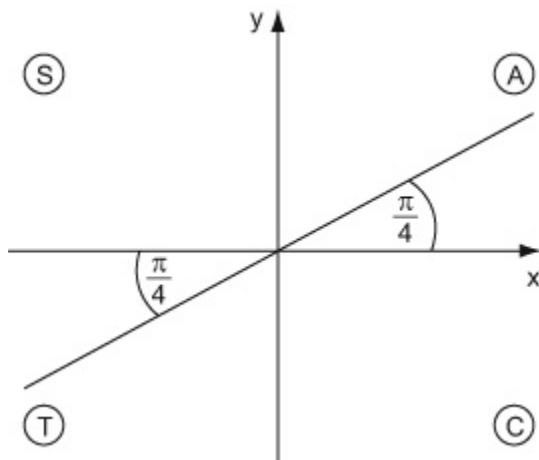
(b)

$$\theta = \frac{\pi}{4}$$

$\tan \theta = 1$ , so  $\theta = 45^\circ$ . Remember  $\pi$  (radians)

$$= 180^\circ \text{ so } 45^\circ = \frac{\pi}{4} \text{ (radians).}$$

$\tan \theta$  is positive in the 1st and 3rd quadrants. Read off the solutions, in  $0 \leq \theta < 2\pi$ , from your diagram.



$$\theta = \frac{\pi}{4}, \quad \frac{5\pi}{4}$$

$$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 12

#### Question:

(a) Sketch the graph of  $y = \frac{1}{x}$ ,  $x > 0$ .

(b) Copy and complete the table, giving your values of  $\frac{1}{x}$  to 3 decimal places.

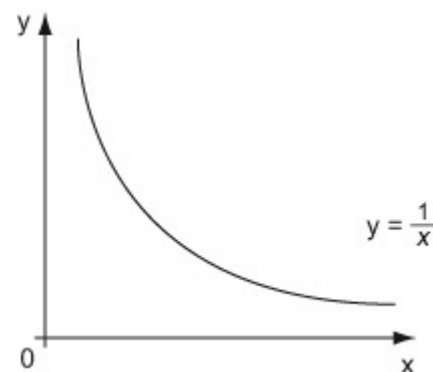
$x$	1	1.2	1.4	1.6	1.8	2
$\frac{1}{x}$	1					0.5

(c) Use the trapezium rule, with all the values from your table, to find an estimate for the value of  $\int_1^2 \frac{1}{x} dx$ .

(d) Is this an overestimate or an underestimate for the value of  $\int_1^2 \frac{1}{x} dx$ ? Give a reason for your answer.

#### Solution:

(a)



Draw the sketch for  $x > 0$ .

(b)

$x$	1	1.2	1.4	1.6	1.8	2
$\frac{1}{x}$	1	0.833	0.714	0.625	0.556	0.5

(c)

$$\begin{aligned}
 \text{Area} &\simeq \frac{1}{2} \times 0.2 \times [1 + 2(0.833 + 0.714 + 0.625 + 0.556) + 0.5] \\
 &\simeq 0.6956 \\
 &\simeq 0.70
 \end{aligned}$$

The width of each strip is 0.2

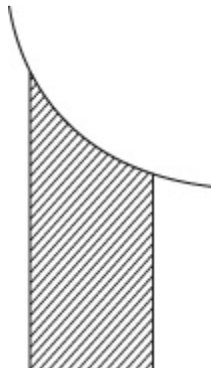
The values of  $\frac{1}{x}$  are to 3 decimal places, so give the final answer to 2

decimal places.

(d)

This is an overestimate.

Due to the shape of the curve, each trapezium will give an area slightly larger than the area under the graph.



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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 13

#### Question:

Show that the stationary point on the curve  $y = 4x^3 - 6x^2 + 3x + 2$  is a point of inflexion.

#### Solution:

$$y = 4x^3 - 6x^2 + 3x + 2$$

$$\frac{dy}{dx} = 12x^2 - 12x + 3$$

$$12x^2 - 12x + 3 = 0$$

$$4x^2 - 4x + 1 = 0$$

$$(2x - 1)(2x - 1) = 0$$

$$x = \frac{1}{2}$$

When  $x = 0$ ,

$$\begin{aligned}\frac{dy}{dx} &= 12(0) - 12(0) + 3 \\ &= 3 > 0\end{aligned}$$

When  $x = 1$

$$\begin{aligned}\frac{dy}{dx} &= 12(1)^2 - 12(1) + 3 \\ &= 3 > 0\end{aligned}$$

Find the stationary point. Put  $\frac{dy}{dx} = 0$

Simplify. Divide throughout by 4. Factorise

$$4x^2 - 4x + 1$$




$$ac = 4, \text{ and } (-2) + (-2) = -4$$

$$\text{so } 4x^2 - 2x - 2x + 1$$

$$= 2x(2x - 1) - 1(2x - 1)$$

$$= (2x - 1)(2x - 1)$$

Find the gradient of the tangent when  $x = 0$  and  $x = 1$ .

x	0	$\frac{1}{2}$	1
$\frac{dy}{dx}$	>0	0	>0
Shape of curve			

Look at the gradient of the tangent on either side of the stationary point.

The Stationary point is a point of inflexion.

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 14

#### Question:

Find all the values of  $x$  in the interval  $0 \leq x < 360^\circ$  for which  $3\tan^2 x = 1$ .

#### Solution:

$$3\tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \sqrt{\frac{1}{3}}$$

Rearrange the equation for  $\tan x$ . Divide each side by 3.

Take the square root of each side.

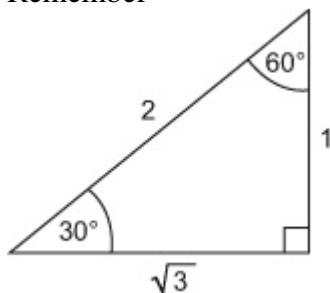
(i)

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = 30^\circ$$

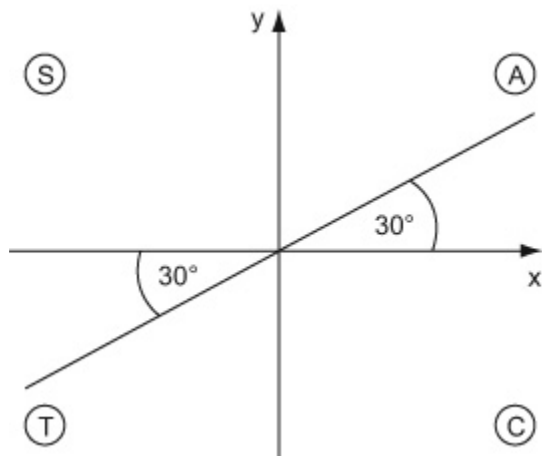
$$\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Remember



$$\text{so } \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Tan  $x$  is positive in the 1st and 3rd quadrants.

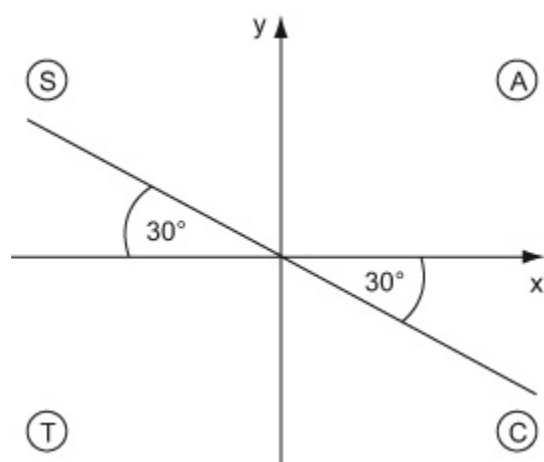


$$\text{so, } x = 30^\circ, 210^\circ$$

(ii)

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$x = 330^\circ \text{ (i.e. } -30^\circ \text{)}$$



Tan  $x$  is negative in the 2nd and 4th quadrants.

$$x = 330^\circ, 150^\circ$$

$$\text{so, } x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

Write down all the solutions in  $0 \leq x < 360^\circ$  in order of size.



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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 15

#### Question:

Evaluate  $\int_1^8 x^{\frac{1}{3}} - x^{-\frac{1}{3}} dx$ .

#### Solution:

$$\int_1^8 x^{\frac{1}{3}} - x^{-\frac{1}{3}} dx$$

$$= \left[ \frac{3}{4} x^{\frac{4}{3}} - \frac{3}{2} x^{\frac{2}{3}} \right]_1^8$$

$$\text{Remember } \int ax^n dx = \frac{ax^{n+1}}{n+1}$$

$$\int x^{\frac{1}{3}} dx = \frac{x^{\frac{4}{3}}}{(\frac{4}{3})} = \frac{3}{4} x^{\frac{4}{3}}$$

$$\int x^{-\frac{1}{3}} dx = \frac{x^{\frac{2}{3}}}{(\frac{2}{3})} = \frac{3}{2} x^{\frac{2}{3}}$$

$$\begin{aligned} &= \left( \frac{3}{4} (8)^{\frac{4}{3}} - \frac{3}{2} (8)^{\frac{2}{3}} \right) - \left( \frac{3}{4} (1)^{\frac{4}{3}} - \frac{3}{2} (1)^{\frac{2}{3}} \right) \\ &= \left( \frac{3}{4} (16) - \frac{3}{2} (4) \right) - \left( \frac{3}{4} (1) - \frac{3}{2} (1) \right) \\ &= (12 - 6) - \left( \frac{3}{4} - \frac{3}{2} \right) \\ &= 6 \frac{3}{4} \end{aligned}$$

$$\begin{aligned} &= \left( 8^{\frac{1}{3}} \right)^4 \\ &= 2^4 \\ &= 16 \end{aligned}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 16

#### Question:

The curve  $C$  has equation  $y = 2x^3 - 13x^2 + 8x + 1$ .

(a) Find the coordinates of the turning points of  $C$ .

(b) Determine the nature of the turning points of  $C$ .

#### Solution:

$$y = 2x^3 - 13x^2 + 8x + 1$$

Find the  $x$ -coordinate. Solve  $\frac{dy}{dx} = 0$ .

(a)

$$\frac{dy}{dx} = 6x^2 - 26x + 8$$

$$6x^2 - 26x + 8 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(3x - 1)(x - 4) = 0$$

Divide throughout by 2.

Factorize  $3x^2 - 13x + 4 = 0$ .  $ac = 12$ ,  $(-12)$   
 $+ (-1) = -13$

$$\begin{aligned} \text{so } 3x^2 - 12x - x + 4 \\ = 3x(x - 4) - 1(x - 4) \\ = (3x - 1)(x - 4) \end{aligned}$$

$$x = \frac{1}{3}, \quad x = 4$$

When  $x = \frac{1}{3}$ ,

$$\begin{aligned} y &= 2\left(\frac{1}{3}\right)^3 - 13\left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) + 1 \\ &= 2\frac{8}{27} \end{aligned}$$

Find the  $y$ -coordinates. Substitute  $x = \frac{1}{3}$  and  $x = 4$

into  $y = 2x^3 - 13x^2 + 8x + 1$

When  $x = 4$

$$\begin{aligned} y &= 2(4)^3 - 13(4)^2 + 8(4) + 1 \\ &= -47 \end{aligned}$$

so  $\left(\frac{1}{3}, 2\frac{8}{27}\right)$ ,  $(4, -47)$ .

Give your answer as coordinates

(b)

$$\frac{d^2y}{dx^2} = 12x - 26$$

Remember  $\frac{d^2y}{dx^2} < 0$  is a maximum stationary point, and

When  $x = \frac{1}{3}$  ,

$\frac{d^2y}{dx^2} > 0$  is a minimum stationary point.

$$\frac{d^2y}{dx^2} = 12 \left( \frac{1}{3} \right) - 26$$

$$= -22 < 0$$

$\left( \frac{1}{3}, 2\frac{8}{27} \right)$  is a maximum.

When  $x = 4$  ,

$$\frac{d^2y}{dx^2} = 12 ( 4 ) - 26$$

$$= 22 > 0$$

$( 4, -47 )$  is a minimum.

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## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 17

#### Question:

The curve  $S$ , for  $0 \leq x < 360^\circ$ , has equation  $y = 2\sin \left( \frac{2}{3}x - 30^\circ \right)$ .

(a) Find the coordinates of the point where  $S$  meets the  $y$ -axis.

(b) Find the coordinates of the points where  $S$  meets the  $x$ -axis.

#### Solution:

$$y = 2\sin \left( \frac{2}{3}x - 30^\circ \right)$$

(a)

$$x = 0$$

$$\begin{aligned} y &= 2\sin \left( \frac{2}{3}(0) - 30^\circ \right) \\ &= 2\sin (-30^\circ) \\ &= -2 \end{aligned}$$

so,  $(0, -2)$

The curve  $y = 2\sin \left( \frac{2x}{3} - 30^\circ \right)$  meets the  $y$ -axis when  $x = 0$ , so substitute  $x = 0$  into  $y = 2\sin \left( \frac{2x}{3} - 30^\circ \right)$ .

(b)

$$y = 0$$

$$2\sin \left( \frac{2}{3}x - 30^\circ \right) = 0$$

$$\sin \left( \frac{2}{3}x - 30^\circ \right) = 0$$

$$\frac{2}{3}x - 30^\circ = 0^\circ, 180^\circ, 360^\circ,$$

The curve  $y = 2\sin \left( \frac{2x}{3} - 30^\circ \right)$  meets the  $x$ -axis when  $y = 0$ , so substitute  $y = 0$  into  $y = 2\sin \left( \frac{2x}{3} - 30^\circ \right)$ .

(i)

$$\frac{2}{3}x - 30^\circ = 0$$

$$\frac{2}{3}x = 30^\circ$$

$$x = 45^\circ$$

Let  $\frac{2}{3}x - 30^\circ = X$  so  $\sin X = 0$  Now,

$X = 0^\circ, 180^\circ, 360^\circ$  Solve for  $x$ :  $X = 0$ , so

$$\frac{2x}{3} - 30^\circ = 0.$$

(ii)

$$X = 180^\circ, \text{ so } \frac{2x}{3} - 30^\circ = 180^\circ.$$

$$\frac{2}{3}x - 30^{\circ} = 180^{\circ}$$

$$\frac{2}{3}x = 210^{\circ}$$

$$x = 315^{\circ}$$

(iii)

$$\frac{2}{3}x - 30^{\circ} = 360^{\circ}$$

$$\frac{2}{3}x = 390^{\circ}$$

$$x = 585^{\circ}$$

so  $(45^{\circ}, 0)$  ,  $(315^{\circ}, 0)$

$$X = 360^{\circ}, \text{ so } \frac{2x}{3} - 30^{\circ} = 360^{\circ}.$$

Solution not in  $0 \leq x < 360^{\circ}$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

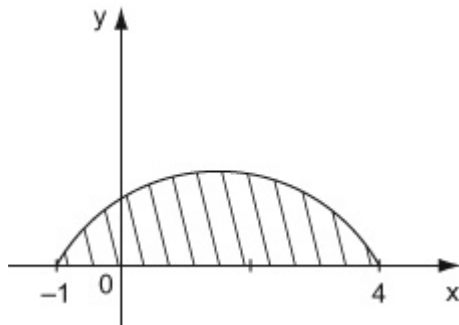
### Algebra and functions

#### Exercise A, Question 18

#### Question:

Find the area of the finite region bounded by the curve  $y = (1 + x)(4 - x)$  and the  $x$ -axis.

#### Solution:



$$\text{Area} = \int_{-1}^4 (1 + x)(4 - x) \, dx$$

$$= \int_{-1}^4 4 + 3x - x^2 \, dx$$

$$= \left[ 4x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^4$$

$$= \left( 4(4) + \frac{3}{2}(4)^2 - \frac{(4)^3}{3} \right) - \left( 4(-1) + \frac{3}{2}(-1)^2 - \frac{(-1)^3}{3} \right)$$

$$= 18 \frac{2}{3} - \left( -2 \frac{1}{6} \right)$$

$$= 20 \frac{5}{6}$$

Sketch a graph of the curve  $y = (1 + x)(4 - x)$ .

Find where the curve meets the  $x$ -axis.

The curve meets the  $x$ -axis when  $y = 0$ , so substitute  $y = 0$  into  $y = (1 + x)(4 - x)$

$(1 + x)(4 - x) = 0$  so  $x = -1$  and  $x = 4$ .

Find the area under the graph and the  $x$ -axis. Integrate  $y = (1 + x)(4 - x)$  using  $x = -1$  and  $x = 4$  as the limits of the integration.

Expand  $(1 + x)(4 - x)$ .

$$\begin{aligned} (1 + x)(4 - x) &= 4 - x + 4x - x^2 \\ &= 4 + 3x - x^2 \end{aligned}$$

Remember  $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

Evaluate the integral. Substitute  $x = 4$ ,

then  $x = -1$  into  $4x + \frac{3x^2}{2} - \frac{x^3}{3}$  and

subtract.

$$\begin{aligned} 18 \frac{2}{3} - \left( -2 \frac{1}{6} \right) &= 18 \frac{2}{3} + 2 \frac{1}{6} \\ &= 20 \frac{5}{6} \end{aligned}$$

# Solutionbank C1

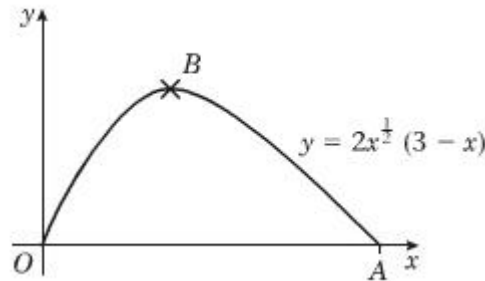
## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 19

#### Question:

The diagram shows part of the curve with equation  $y = 2x^{\frac{1}{2}}(3 - x)$ . The curve meets the  $x$ -axis at the points  $O$  and  $A$ . The point  $B$  is the maximum point of the curve.



(a) Find the coordinates of  $A$ .

(b) Show that  $\frac{dy}{dx} = 3x^{-\frac{1}{2}}(1 - x)$ .

(c) Find the coordinates of  $B$ .

#### Solution:

$$y = 2x^{\frac{1}{2}}(3 - x)$$

(a)

$$2x^{\frac{1}{2}}(3 - x) = 0$$

The curve meets the  $x$ -axis when  $y = 0$ , so substitute

$$y = 0 \text{ into } y = 2x^{\frac{1}{2}}(3 - x).$$

(i)

$$x^{\frac{1}{2}} = 0$$

$$x = 0$$

(ii)

$$3 - x = 0$$

$$x = 3$$

$$\text{so } A(3, 0).$$

$$\text{Remember } \frac{d}{dx}(ax^n) = anx^{n-1}$$

(b)

$$y = 2x^{\frac{1}{2}}(3 - x)$$

$$= 6x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$$

Expand the brackets.

$$2x^{\frac{1}{2}} \times 3 = 6x^{\frac{1}{2}}$$

$$\begin{aligned} 2x^{\frac{1}{2}} \times x &= 2x^{\frac{1}{2}} \times x^1 \\ &= 2x^{\frac{1}{2} + 1} \\ &= 2x^{\frac{3}{2}} \end{aligned}$$

$$\frac{dy}{dx} = 3x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$$

Differentiate.

$$\begin{aligned} \frac{d}{dx} (6x^{\frac{1}{2}}) &= 6 \times \frac{1}{2} \times x^{\frac{1}{2} - 1} \\ &= 3x^{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} (2x^{\frac{3}{2}}) &= 2 \times \frac{3}{2} \times x^{\frac{3}{2} - 1} \\ &= 3x^{\frac{1}{2}} \end{aligned}$$

$$= 3x^{-\frac{1}{2}} (1 - x) \text{ as required}$$

Factorise. Divide each term by  $3x^{-\frac{1}{2}}$  so that

$$3x^{-\frac{1}{2}} \div 3x^{-\frac{1}{2}} = 1$$

$$\begin{aligned} 3x^{\frac{1}{2}} \div 3x^{-\frac{1}{2}} &= \frac{3x^{\frac{1}{2}}}{3x^{-\frac{1}{2}}} \\ &= x^{\frac{1}{2} - (-\frac{1}{2})} \\ &= x^{\frac{1}{2} + \frac{1}{2}} \\ &= x^1 = x \end{aligned}$$

(c)

$$3x^{-\frac{1}{2}} (1 - x) = 0$$

$$1 - x = 0$$

$$x = 1$$

When  $x = 1$ ,

$$\begin{aligned} y &= 2(1)^{\frac{1}{2}} (3 - (1)) \\ &= 2 \times 1 \times 2 \\ &= 4 \end{aligned}$$

so  $B(1, 4)$ .





# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 20

#### Question:

(a) Show that the equation  $2\cos^2 x = 4 - 5\sin x$  may be written as  $2\sin^2 x - 5\sin x + 2 = 0$ .

(b) Hence solve, for  $0 \leq \theta < 360^\circ$ , the equation  $2\cos^2 x = 4 - 5\sin x$ .

#### Solution:

(a)

$$2 \cos^2 x = 4 - 5\sin x$$

$$2(1 - \sin^2 x) = 4 - 5\sin x$$

$$2 - 2\sin^2 x = 4 - 5\sin x$$

$$2\sin^2 x - 5\sin x + 2 = 0 \text{ (as required)}$$

Remember  $\cos^2 x + \sin^2 x = 1$  so  
 $\cos^2 x = 1 - \sin^2 x$ .

(b)

Let  $\sin x = y$

$$2y^2 - 5y + 2 = 0$$

$$(2y - 1)(y - 2) = 0$$

Factorise  $2y^2 - 5y + 2 = 0$

$$ac = 4, (-1) + (-4) = -5$$

$$\text{so } 2y^2 - 5y + 2 = 2y^2 - y - 4y + 2$$

$$= y(2y - 1) - 2(2y - 1)$$

$$= (2y - 1)(y - 2)$$

$$\text{so } y = \frac{1}{2}, y = 2$$

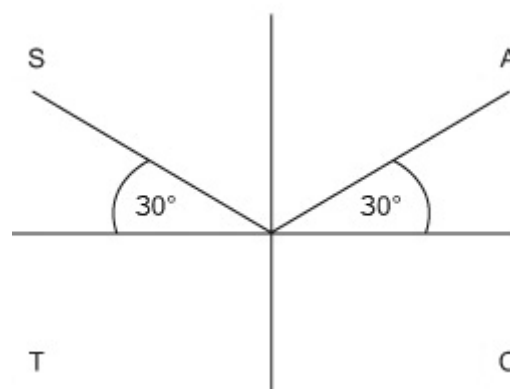
(i)

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ, 150^\circ$$

Solve for  $x$ .

Substitute (i)  $y = \frac{1}{2}$  and (ii)  $y = 2$  into  $\sin x = y$ .



$\sin x$  is positive in the 1st and 2nd quadrants. Read off the solutions in  $0 \leq x < 360^\circ$ .

(ii)

$\sin x = 2$  (Impossible)

No solutions exist as  $-1 \leq \sin x \leq 1$ .

so  $x = 30^\circ$ ,  $150^\circ$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 21

#### Question:

Use the trapezium rule with 5 equal strips to find an estimate for  $\int_0^1 x\sqrt{1+x} \, dx$ .

#### Solution:

x	0	0.2	0.4	0.6	0.8	1
$x\sqrt{1+x}$	0	0.219	0.473	0.759	1.073	1.414

Divide the interval into 5 equal strips.

Use  $h = \frac{b-a}{n}$ . Here  $b = 1$ ,  $a = 0$  and

$n = 5$ . So that  $h = \frac{1-0}{5} = 0.2$

The trapezium rule gives an approximation to the area of the graph. Here we work to an accuracy of 3 decimal places.

Remember  $A \approx \frac{1}{2}h [ y_0 + 2 ( y_1 + y_2 + \dots ) + y_n ]$

$$\begin{aligned} \int_0^1 x\sqrt{1+x} \, dx &\approx \frac{1}{2} \times 0.2 \times [ 0 + 2 ( 0.219 \\ &\quad + 0.473 + 0.759 + 1.073 ) \\ &\quad + 1.414 ] \\ &\approx 0.6462 \text{ or } 0.65 \end{aligned}$$

The values of  $x\sqrt{1+x}$  are to 3 decimal places, so give your final answer to 2 decimal places.

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 22

#### Question:

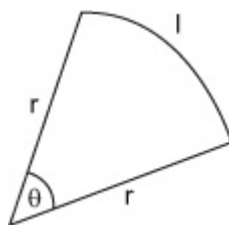
A sector of a circle, radius  $r$  cm, has a perimeter of 20 cm.

(a) Show that the area of the sector is given by  $A = 10r - r^2$ .

(b) Find the maximum value for the area of the sector.

#### Solution:

(a)



$$2r + l = 20$$

$$l = r\theta$$

$$\text{so } 2r + r\theta = 20$$

$$r\theta = 20 - 2r$$

$$\theta = \frac{20}{r} - 2$$

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}r^2 \left( \frac{20}{r} - 2 \right)$$

$$= \frac{1}{2}r^2 \times \frac{20}{r} - \frac{1}{2}r^2 \times 2$$

$$= 10r - r^2 \text{ (as required)}$$

Remember: The length of an arc of a circle is  $l = r\theta$ . The area of a sector of a circle is  $A = \frac{1}{2}r^2\theta$ .

Draw a diagram. Let  $\theta$  be the center angle and  $l$  be the arc length.

The perimeter of the sector is  $r + r + l = 2r + l$  so  $2r + l = 20$

Expand the brackets and simplify.

$$\frac{1}{2}r^2 \times \frac{20}{r} = \frac{1}{2} \times 20 \times \frac{r^2}{r}$$

$$= 10r^2 - 1$$

$$= 10r^1 = 10r$$

$$\frac{1}{2}r^2 \times 2 = 2 \times \frac{1}{2}r^2$$

$$= r^2$$

(b)

Find the value of  $r$  for the area to have a maximum. Solve

$$\frac{dA}{dr} = 0.$$

$$\frac{dA}{dr} = 10 - 2r$$

$$10 - 2r = 0$$

$$2r = 10$$

$$r = 5$$

$$\text{when } r = 5$$

$$\begin{aligned}\text{Area} &= 10(5) - 5^2 \\ &= 50 - 25 \\ &= 25 \text{ cm}^2\end{aligned}$$

Find the maximum area. Substitute  $r = 5$  into  $A = 10r - r^2$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 23

#### Question:

Show that, for all values of  $x$ :

(a)  $\cos^2 x (\tan^2 x + 1) = 1$

(b)  $\sin^4 x - \cos^4 x = (\sin x - \cos x)(\sin x + \cos x)$

#### Solution:

(a)

$$\begin{aligned} \cos^2 x (\tan^2 x + 1) &= \cos^2 x \left( \frac{\sin^2 x}{\cos^2 x} + 1 \right) \\ &= \cos^2 x \times \frac{\sin^2 x}{\cos^2 x} + \cos^2 x \times 1 \\ &= \sin^2 x + \cos^2 x \\ &= 1 \text{ (as required)} \end{aligned}$$

Remember  $\tan x = \frac{\sin x}{\cos x}$  so  $\tan^2 x = \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos^2 x}$

Expand the brackets and simplify.

$$\begin{aligned} \cos^2 x \times \frac{\sin^2 x}{\cos^2 x} &= \cancel{\cos^2 x} \times \frac{\sin^2 x}{\cancel{\cos^2 x}} \\ &= \sin^2 x \end{aligned}$$

Remember  $\sin^2 x + \cos^2 x = 1$

(b)

$$\begin{aligned} \sin^4 x - \cos^4 x &= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) \\ &= (\sin^2 x - \cos^2 x) \times 1 \\ &= \sin^2 x - \cos^2 x \\ &= (\sin x - \cos x)(\sin x + \cos x) \end{aligned}$$

(as required)

Remember  $a^2 - b^2 = (a - b)(a + b)$ . Here  $a = \sin^2 x$  and  $b = \cos^2 x$

Remember  $\sin^2 x + \cos^2 x = 1$

Use  $a^2 - b^2 = (a - b)(a + b)$  again. Here  $a = \sin x$  and  $b = \cos x$ .

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

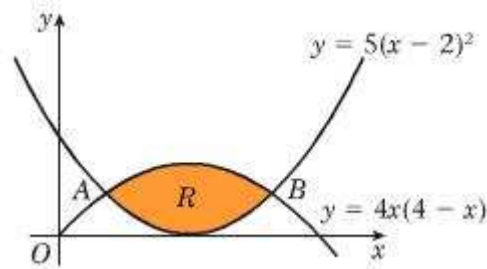
### Algebra and functions

#### Exercise A, Question 24

#### Question:

The diagram shows the shaded region  $R$  which is bounded by the curves  $y = 4x(4 - x)$  and  $y = 5(x - 2)^2$ .

The curves intersect at the points  $A$  and  $B$ .



(a) Find the coordinates of the points  $A$  and  $B$ .

(b) Find the area of the shaded region  $R$ .

#### Solution:

$$y = 4x(4 - x), y = 5(x - 2)^2$$

$$4x(4 - x) = 5(x - 2)^2$$

$$16x - 4x^2 = 5(x^2 - 4x + 4)$$

$$16x - 4x^2 = 5x^2 - 20x + 20$$

$$9x^2 - 36x + 20 = 0$$

$$(3x - 10)(3x - 2) = 0$$

$$x = \frac{10}{3}, x = \frac{2}{3}$$

(i)

When  $x = \frac{10}{3}$ ,

$$y = 4\left(\frac{10}{3}\right)\left(4 - \frac{10}{3}\right)$$

$$= 4 \times \frac{10}{3} \times \frac{2}{3}$$

$$= \frac{80}{9}$$

(ii)

When  $x = \frac{2}{3}$

Solve the equations  $y = 4x(4 - x)$  and  $y = 5(x - 2)^2$  simultaneously. Eliminate  $y$  so that  $4x(4 - x) = 5(x - 2)^2$ .

Expand the brackets and simplify.

Rearrange the equation into the form  $ax^2 + bx + c = 0$

$$\text{Factorise } 9x^2 - 36x + 20 = 0$$

$$ac = 180, (-6) + (-30) = -36$$

$$9x^2 - 6x - 30x + 20$$

$$= 3x(3x - 2) - 10(3x - 2)$$

$$= (3x - 2)(3x - 10).$$

Find the coordinator of  $A$  and  $B$ . Substitute (i)  $x = \frac{10}{3}$  and (ii)  $x = \frac{2}{3}$ , into  $y = 4x(4 - x)$ .

Find the coordinator of  $A$  and  $B$ . Substitute (i)  $x =$



$$\begin{aligned}
 y &= 4 \left( \frac{2}{3} \right) \left( 4 - \frac{2}{3} \right) \\
 &= 4 \times \frac{2}{3} \times \frac{10}{3} \\
 &= \frac{80}{3}
 \end{aligned}$$

$$\text{so } A \left( \frac{2}{3}, \frac{80}{3} \right), B \left( \frac{10}{3}, \frac{80}{3} \right)$$

(b)

$$\begin{aligned}
 \text{Area} &= \int_{\frac{2}{3}}^{\frac{10}{3}} 4x(4-x) - 5(x-2)^2 dx \\
 &= \int_{\frac{2}{3}}^{\frac{10}{3}} 16x - 4x^2 - 5(x^2 - 4x + 4) dx
 \end{aligned}$$

$$\begin{aligned}
 &= \int_{\frac{2}{3}}^{\frac{10}{3}} 16x - 4x^2 - 5x^2 + 20x - 20 dx \\
 &= \int_{\frac{2}{3}}^{\frac{10}{3}} 36x - 9x^2 - 20 dx \\
 &= \left[ 18x^2 - 3x^3 - 20x \right]_{\frac{2}{3}}^{\frac{10}{3}}
 \end{aligned}$$

$$\begin{aligned}
 &= \left( 18 \left( \frac{10}{3} \right)^2 - 3 \left( \frac{10}{3} \right)^3 - 20 \left( \frac{10}{3} \right) \right) \\
 &\quad - \left( 18 \left( \frac{2}{3} \right)^2 - 3 \left( \frac{2}{3} \right)^3 - 20 \left( \frac{2}{3} \right) \right) \\
 &= \left( 18 \left( \frac{100}{9} \right) - 3 \left( \frac{1000}{27} \right) - \frac{200}{3} \right) \\
 &\quad - \left( 18 \left( \frac{4}{9} \right) - 3 \left( \frac{8}{27} \right) - \frac{40}{3} \right) \\
 &= \left( 22 \frac{2}{9} \right) - \left( -6 \frac{2}{9} \right) \\
 &= 28 \frac{4}{9}
 \end{aligned}$$

$$\frac{10}{3} \text{ and (ii) } x = \frac{2}{3}, \text{ into } y = 4x(4-x).$$

Remember Area =  $\int_a^b (y_1 - y_2) dx$ . Here

$$y_1 = 4x(4-x), y_2 = 5(x-2)^2, a = \frac{2}{3} \text{ and } b = \frac{10}{3}.$$

Expand the brackets and simplify.

Evaluate the integral. Substitute  $x = \frac{10}{3}$ , then  $x = \frac{2}{3}$ , into  $18x^2 - 3x^3 - 20x$  and subtract.

$$\begin{aligned}
 22 \frac{2}{9} - \left( -6 \frac{2}{9} \right) &= 22 \frac{2}{9} + 6 \frac{2}{9} \\
 &= 28 \frac{4}{9}
 \end{aligned}$$



# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 25

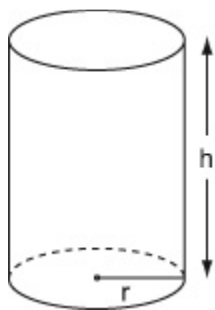
#### Question:

The volume of a solid cylinder, radius  $r$  cm, is  $128\pi$ .

(a) Show that the surface area of the cylinder is given by  $S = \frac{256\pi}{r} + 2\pi r^2$ .

(b) Find the minimum value for the surface area of the cylinder.

#### Solution:



Draw a diagram. Let  $h$  be the height of cylinder.

(a)

Surface area,  $S = 2\pi rh + 2\pi r^2$

(volume  $\Rightarrow$ )  $128\pi = \pi r^2 h$

$$\begin{aligned} h &= \frac{128\pi}{\pi r^2} \\ &= \frac{128}{r^2} \end{aligned}$$

$$\begin{aligned} \text{so } S &= 2\pi r \times \frac{128}{r^2} + 2\pi r^2 \\ &= \frac{256\pi}{r} + 2\pi r^2 \text{ (as required)} \end{aligned}$$

Find expressions for the surface area and volume of the cylinder in terms of  $\pi$ ,  $r$  and  $h$ .

Eliminate  $h$  between the expressions  $S = 2\pi rh + 2\pi r^2$  and  $128\pi = \pi r^2 h$ . Rearrange  $128\pi = \pi r^2 h$  for  $h$  so that

$$\pi r^2 h = 128\pi$$

$$\begin{aligned} h &= \frac{128\pi}{\pi r^2} \\ &= \frac{128}{r^2} \end{aligned}$$

Substitute  $h = \frac{128}{r^2}$  into  $S = 2\pi rh + 2\pi r^2$  and simplify the expression.

(b)

$$\frac{ds}{dr} = 4\pi r - \frac{256\pi}{r^2}$$

Find the value of  $r$  for which  $S = \frac{256\pi}{r} + 2\pi r^2$  has a

stationary value. Solve  $\frac{ds}{dr} = 0$ . Differentiate

$$\frac{256\pi}{r} + 2\pi r^2 \text{ with respect to } r, \text{ so that}$$

$$4\pi r - \frac{256\pi}{r^2} = 0$$

$$4\pi r = \frac{256\pi}{r^2}$$

$$r^3 = 64$$

$$r = 4 \text{ cm}$$

$$\begin{aligned} \frac{d}{dr} \left( \frac{256\pi}{r} \right) &= \frac{d}{dr} 256\pi r^{-1} \\ &= -256\pi r^{-1-1} \\ &= -256\pi r^{-2} \\ &= \frac{-256\pi}{r^2} \end{aligned}$$

$$\begin{aligned} \frac{d}{dr} (2\pi r^2) &= 2 \times 2\pi r^{2-1} \\ &= 4\pi r^1 \\ &= 4\pi r \end{aligned}$$

When  $r = 4$ ,

$$\begin{aligned} S &= \frac{256\pi}{(4)} + 2\pi (4)^2 \\ &= 64\pi + 32\pi \\ &= 96\pi \text{ cm}^2 \end{aligned}$$

Find the value of  $S$  when  $r = 4$ . Substitute  $r = 4$  into  $S =$

$$\frac{256\pi}{r} + 2\pi r^2.$$

Give the exact answer. Leave your answer in terms of  $\pi$ .

# Solutionbank C1

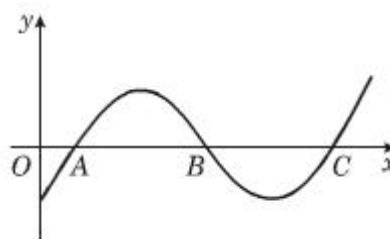
## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 26

#### Question:

The diagram shows part of the curve  $y = \sin(ax - b)$ , where  $a$  and  $b$  are constants and  $b < \frac{\pi}{2}$ .



Given that the coordinates of  $A$  and  $B$  are  $(\frac{\pi}{6}, 0)$  and  $(\frac{5\pi}{6}, 0)$  respectively,

- write down the coordinates of  $C$ ,
- find the value of  $a$  and the value of  $b$ .

#### Solution:

(a)

$$AB = BC$$

$$\begin{aligned} AB &= \frac{5\pi}{6} - \frac{\pi}{6} \\ &= \frac{4\pi}{6} \\ &= \frac{2\pi}{3} \end{aligned}$$

$$\begin{aligned} \text{so, } OC &= \frac{5\pi}{6} + \frac{2\pi}{3} \\ &= \frac{5\pi}{6} + \frac{4\pi}{6} \\ &= \frac{9\pi}{6} \\ &= \frac{3\pi}{2} \end{aligned}$$

$$\text{so, } C \left( \frac{3\pi}{2}, 0 \right)$$

(b)

(i)

$$\sin \left( a \left( \frac{\pi}{6} \right) - b \right) = 0$$

$$\text{so } a \left( \frac{\pi}{6} \right) - b = 0$$

$AB$  is half the period, so  $AB = BC$

Find the coordinates of  $C$ . Work out the length of  $AB$ ,  $AB = OB - OA$ . Work with exact values. Leave your answer in terms of  $\pi$ .

$OC = OB + BC$  and  $AB = BC$ . So,  $OC = OB + AB$ .

$\sin(0) = 0$  and  $\sin(\pi) = 0$ . So, at  $A$ ,  $x = \frac{\pi}{6}$  and  $a$

$$\frac{\pi}{6}) - b = 0 \text{ and at B, } x = \frac{5\pi}{6} \text{ and } a\left(\frac{5\pi}{6}\right) - b = \pi.$$

(ii)

$$\sin\left(a\left(\frac{5\pi}{6}\right) - b\right) = 0$$

$$\text{so } a\left(\frac{5\pi}{6}\right) - b = \pi$$

Solving Simultaneously

$$a\left(\frac{5\pi}{6}\right) - b = \pi$$

$$-a\left(\frac{\pi}{6}\right) - b = 0$$

$$a\left(\frac{4\pi}{6}\right) = \pi$$

$$a = \frac{\pi}{\left(\frac{4\pi}{6}\right)}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$\text{When } a = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)\left(\frac{\pi}{6}\right) - b = 0$$

$$b = \frac{\pi}{4}$$

check

$$\text{sub } a = \frac{3}{2}, \quad b = \frac{\pi}{4} \text{ into}$$

$$a\left(5\frac{\pi}{6}\right) - b$$

$$\left(\frac{3}{2}\right)\left(\frac{5\pi}{6}\right) - \frac{\pi}{4} = \frac{5\pi}{4} - \frac{\pi}{4} \\ = \pi \text{ (as required)}$$

$$\text{so } a = \frac{3}{2} \text{ and } b = \frac{\pi}{4}.$$

Solve the equations  $a\left(\frac{\pi}{6}\right) - b = 0$  and  $a\left(\frac{5\pi}{6}\right) - b = \pi$  simultaneously. Subtract the equations.

$$\text{Find } b. \text{ Substitute } a = \frac{3}{2} \text{ into } a\left(\frac{\pi}{6}\right) - b = 0.$$

$$\text{Check answer by substituting } a = \frac{3}{2} \text{ and } b = \frac{\pi}{4} \text{ into } a\left(\frac{5\pi}{6}\right) - b.$$

$$\left(\frac{3}{2}\right)\left(\frac{5\pi}{6}\right) = \frac{1}{2} \times \frac{5\pi}{2} = \frac{5\pi}{4}$$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

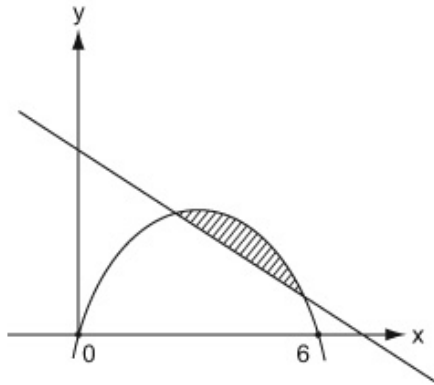
#### Exercise A, Question 27

#### Question:

Find the area of the finite region bounded by the curve with equation  $y = x(6 - x)$  and the line  $y = 10 - x$ .

#### Solution:

$$y = x(6 - x), \quad y = 10 - x$$



$$x(6 - x) = 10 - x$$

$$6x - x^2 = 10 - x$$

$$x^2 - 7x + 10 = 0$$

$$(x - 5)(x - 2) = 0$$

so  $x = 2$  and  $x = 5$ .

$$\text{Finite area} = \int_2^5 x(6 - x) - (10 - x) \, dx$$

$$= \int_2^5 6x - x^2 - 10 + x \, dx$$

$$= \int_2^5 7x - x^2 - 10 \, dx$$

$$= \left[ \frac{7x^2}{2} - \frac{x^3}{3} - 10x \right]_2^5$$

$$= \left( \frac{7(5)^2}{2} - \frac{(5)^3}{3} - 10 \left( \frac{5^3}{3} \right) - 10x \right) \text{ and subtract.}$$

$$- \left( \frac{7(2)^2}{2} - \frac{(2)^3}{3} - 10 \right)$$

$$(2)$$

$$= \left( -4 \frac{1}{6} \right) - \left( -8 \frac{2}{3} \right) - 4 \frac{1}{6} - \left( -8 \frac{2}{3} \right) = -4 \frac{1}{6} + 8 \frac{2}{3}$$

$$= 4 \frac{1}{2} \qquad \qquad \qquad = 4 \frac{1}{2}$$

Find the  $x$ -coordinates of the points where the line  $y = 10 - x$  meets the curve  $y = x(6 - x)$ . Solve the equations simultaneously, eliminate  $y$ .

Factorise  $x^2 - 7x + 10$ .  $(-5) \times (-2) = 10$  and  $(-5) + (-2) = -7$  so  $x^2 - 7x + 10 = (x - 5)(x - 2)$ .

Use area  $= \int_a^b (y_1 - y_2) \, dx$ . Have  $y_1 = x(6 - x)$ ,  $y_2 = 10 - x$ ,  $a = 2$  and  $b = 5$ .

Remember  $\int ax^n \, dx = \frac{ax^{n+1}}{n+1}$

Evaluate the integral. Substitute  $x = 5$ , then  $x = 2$ , into  $\frac{7x^2}{2} -$

# Solutionbank C1

## Edexcel Modular Mathematics for AS and A-Level

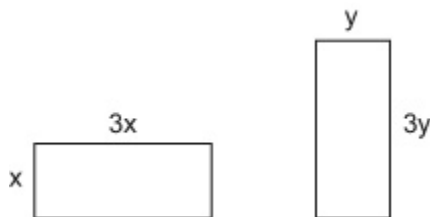
### Algebra and functions

#### Exercise A, Question 28

#### Question:

A piece of wire of length 80 cm is cut into two pieces. Each piece is bent to form the perimeter of a rectangle which is three times as long as it is wide. Find the lengths of the two pieces of wire if the sum of the areas of the rectangles is to be a maximum.

#### Solution:



Total length of wire = 80

$$\text{so } 80 = 8x + 8y$$

$$x + y = 10$$

Total Area = A

$$A = 3x^2 + 3y^2$$

$$\begin{aligned} A &= 3x^2 + 3(10 - x)^2 \\ &= 3x^2 + 3(100 - 20x + x^2) \\ &= 3x^2 + 300 - 60x + 3x^2 \\ &= 6x^2 - 60x + 300 \end{aligned}$$

$$\frac{dA}{dx} = 12x - 60$$

$$12x - 60 = 0$$

$$12x = 60$$

$$x = 5 \text{ cm}$$

The length of each piece of wire is  
(8x = ) 40 cm.

Draw a diagram. Let the width of each rectangle be  $x$  and  $y$  respectively.

Write down an equation in terms of  $x$  and  $y$  for the total length of the wire.

Divide throughout by 8.

Write down an equation in terms of  $x$  and  $y$  for the total area enclosed by the two pieces of wire.

Solve the equations  $x + y = 10$  and

$A = 3x^2 + 3y^2$  simultaneously. Eliminate  $y$ . Rearrange  $x + y = 10$ , so that  $y = 10 - x$ , and substitute into

$$A = 3x^2 + 3y^2$$

Find the value of  $x$  for which  $A$  is a maximum. Solve

$$\frac{dA}{dx} = 0.$$

Total length is 80 cm, so  $40 + 40 = 80$



# Solutionbank C1

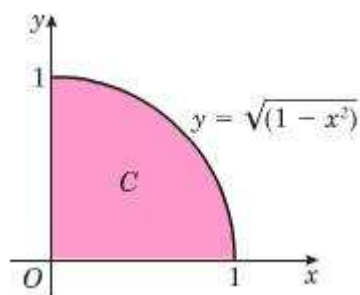
## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 29

#### Question:

The diagram shows the shaded region  $C$  which is bounded by the circle  $y = \sqrt{1 - x^2}$  and the coordinate axes.



(a) Use the trapezium rule with 10 strips to find an estimate, to 3 decimal places, for the area of the shaded region  $C$ .

The actual area of  $C$  is  $\frac{\pi}{4}$ .

(b) Calculate the percentage error in your estimate for the area of  $C$ .

#### Solution:

$$\text{Remember } A \approx \frac{1}{2}h [ y_0 + 2 ( y_1 + y_2 + \dots ) + y_n ]$$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\sqrt{1-x^2}$	1	0.9950	0.9798	0.9539	0.9165	0.8660	0.8	0.7141	0.6	0.4359	0

$$\text{Area} \approx \frac{1}{2} \times 0.1 \times [ 1 + 2 ( 0.9950 + 0.9798 + \dots + 0.4359 ) + 0 ]$$

$$\approx 0.77612 \text{ or } 0.776$$

Divide the interval into 10 equal strips. Use  $h = \frac{b-a}{n}$ . Here  $a = 0$ ,  $b = 1$  and  $n = 10$ , so that  $\frac{1-0}{10} = \frac{1}{10} = 0.1$

The trapezium rule gives an approximation to the area under the graph. Here we round to 4 decimal places.

The values of  $\sqrt{1-x^2}$  are rounded to 4 decimal place. Give your final answer to 3 decimal places.

(b)

$$\frac{\pi}{4} =$$

$$\begin{aligned}\% \text{ error} &= \frac{\frac{\pi}{4} - 0.776}{\left(\frac{\pi}{4}\right)} \times 100 \\ &= 1.2 \%\end{aligned}$$

$$\text{Use percentage error} = \frac{\quad}{\text{True value}} \times 100$$

# Solutionbank C1

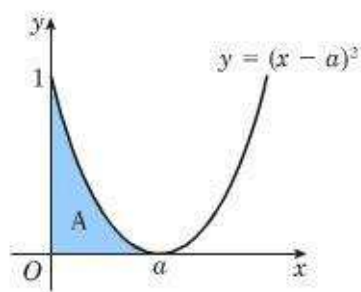
## Edexcel Modular Mathematics for AS and A-Level

### Algebra and functions

#### Exercise A, Question 30

#### Question:

The area of the shaded region A in the diagram is  $9 \text{ cm}^2$ . Find the value of the constant  $a$ .



#### Solution:

$$\int_0^a (x - a)^2 dx = 9$$

$$\int_0^a x^2 - 2ax + a^2 dx = 9$$

$$\left[ \frac{x^3}{3} - ax^2 + a^2x \right]_0^a = 9$$

Write down an equation in terms of  $a$  for the area of region A.

Expand  $(x - a)^2$  so that

$$\begin{aligned} (x - a)(x - a) &= x^2 - ax - ax + a^2 \\ &= x^2 - 2ax + a^2 \end{aligned}$$

Remember  $\int ax^n dx = \frac{ax^{n+1}}{n+1}$ .

Here

$$\int x^2 dx = \frac{x^3}{3}$$

$$\begin{aligned} \int 2ax \, dx &= \frac{2ax^2}{2} \\ &= ax^2 \end{aligned}$$

$$\int a^2 dx = a^2x$$

Evaluate the integral. Substitute  $x = a$ , then  $x = 0$ , into

$\frac{x^3}{3} - ax^2 + a^2x$ , and subtract.

$$\left( \frac{(a)^3}{3} - a(a)^2 + a^2(a) \right)$$

–

$$\left( \frac{(0)^3}{3} - a(0)^2 + a^2(0) \right)$$

$$= 9$$

$$\left( \frac{a^3}{3} - a^3 + a^3 \right) - 0 = 9$$

$$\frac{a^3}{3} = 9$$

$$a^3 = 27$$

$$a = 3$$

