

1 Differentiate with respect to  $x$

a  $e^x$

b  $3e^x$

c  $\ln x$

d  $\frac{1}{2} \ln x$

2 Differentiate with respect to  $t$

a  $7 - 2e^t$

b  $3t^2 + \ln t$

c  $e^t + t^5$

d  $t^{\frac{3}{2}} + 2e^t$

e  $2 \ln t + \sqrt{t}$

f  $2.5e^t - 3.5 \ln t$

g  $\frac{1}{t} + 8 \ln t$

h  $7t^2 - 2t + 4e^t$

3 Find  $\frac{d^2y}{dx^2}$  for each of the following.

a  $y = 4x^3 + e^x$

b  $y = 7e^x - 5x^2 + 3x$

c  $y = \ln x + x^{\frac{5}{2}}$

d  $y = 5e^x + 6 \ln x$

e  $y = \frac{3}{x} + 3 \ln x$

f  $y = 4\sqrt{x} + \frac{1}{4} \ln x$

4 Find the value of  $f'(x)$  at the value of  $x$  indicated in each case.

a  $f(x) = 3x + e^x, \quad x = 0$

b  $f(x) = \ln x - x^2, \quad x = 4$

c  $f(x) = x^{\frac{1}{2}} + 2 \ln x, \quad x = 9$

d  $f(x) = 5e^x + \frac{1}{x^2}, \quad x = -\frac{1}{2}$

5 Find, in each case, any values of  $x$  for which  $\frac{dy}{dx} = 0$ .

a  $y = 5 \ln x - 8x$

b  $y = 2.4e^x - 3.6x$

c  $y = 3x^2 - 14x + 4 \ln x$

6 Find the value of  $x$  for which  $f'(x)$  takes the value indicated in each case.

a  $f(x) = 2e^x - 3x, \quad f'(x) = 7$

b  $f(x) = 15x + \ln x, \quad f'(x) = 23$

c  $f(x) = \frac{x^2}{8} - 2x + \ln x, \quad f'(x) = -1$

d  $f(x) = 30 \ln x - x^2, \quad f'(x) = 4$

7 Find the coordinates and the nature of any stationary points on each of the following curves.

a  $y = e^x - 2x$

b  $y = \ln x - 10x$

c  $y = 2 \ln x - \sqrt{x}$

d  $y = 4x - 5e^x$

e  $y = 7 + 2x - 4 \ln x$

f  $y = x^2 - 26x + 72 \ln x$

8 Given that  $y = x + ke^x$ , where  $k$  is a constant, show that

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

9 Find an equation for the tangent to each curve at the point on the curve with the given  $x$ -coordinate.

a  $y = e^x, \quad x = 2$

b  $y = \ln x, \quad x = 3$

c  $y = 0.8x - 2e^x, \quad x = 0$

d  $y = 5 \ln x + \frac{4}{x}, \quad x = 1$

e  $y = x^{\frac{1}{3}} - 3e^x, \quad x = 1$

f  $y = \ln x - \sqrt{x}, \quad x = 9$

10 Find an equation for the normal to each curve at the point on the curve with the given  $x$ -coordinate.

a  $y = \ln x, \quad x = e$

b  $y = 4 + 3e^x, \quad x = 0$

c  $y = 10 + \ln x, \quad x = 3$

d  $y = 3 \ln x - 2x, \quad x = 1$

e  $y = x^2 + 8 \ln x, \quad x = 1$

f  $y = \frac{1}{10}x - \frac{3}{10}e^x - 1, \quad x = 0$