

- 1 **a** $= 5(x+3)^4$ **b** $= 3(2x-1)^2 \times 2$
 $= 6(2x-1)^2$ **c** $= 7(8-x)^6 \times (-1)$
 $= -7(8-x)^6$ **d** $= 12(3x+4)^5 \times 3$
 $= 36(3x+4)^5$
- e** $= 4(6-5x)^3 \times (-5)$ **f** $= -(x-2)^{-2}$ **g** $= -12(2x+3)^{-4} \times 2$ **h** $= -2(7-3x)^{-3} \times (-3)$
 $= -20(6-5x)^3$ $= -24(2x+3)^{-4}$ **h** $= 6(7-3x)^{-3}$
- 2 **a** $= 6e^{3t}$ **b** $= \frac{1}{2}(4t-1)^{-\frac{1}{2}} \times 4$ **c** $= \frac{5}{t}$ **d** $= \frac{3}{2}(8-3t)^{\frac{1}{2}} \times (-3)$
 $= 2(4t-1)^{-\frac{1}{2}}$ $= -\frac{9}{2}(8-3t)^{\frac{1}{2}}$
- e** $= \frac{3}{6t+1} \times 6$ **f** $= \frac{1}{2}e^{5t+4} \times 5$ **g** $= \frac{d}{dx}[6(2t-5)^{-\frac{1}{3}}]$ **h** $= \frac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4})$
 $= \frac{18}{6t+1}$ $= \frac{5}{2}e^{5t+4}$ $= -2(2t-5)^{-\frac{4}{3}} \times 2$ $= \frac{2}{t-12}$
 $= -4(2t-5)^{-\frac{4}{3}}$
- 3 **a** $\frac{dy}{dx} = 4(3x-1)^3 \times 3$ **b** $\frac{dy}{dx} = \frac{4}{1+2x} \times 2$ **c** $\frac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2)$
 $= 12(3x-1)^3$ $= 8(1+2x)^{-1}$ $= -(5-2x)^{-\frac{1}{2}}$
 $\frac{d^2y}{dx^2} = 36(3x-1)^2 \times 3$ $\frac{d^2y}{dx^2} = -8(1+2x)^{-2} \times 2$ $\frac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2)$
 $= 108(3x-1)^2$ $= \frac{-16}{(1+2x)^2}$ $= -(5-2x)^{-\frac{3}{2}}$
- 4 **a** $f'(x) = 2x - \frac{6}{x}$ **b** $f'(x) = 2 - e^{x-2}$
 $f'(3) = 6 - 2 = 4$ $f'(2) = 2 - 1 = 1$
- c** $f'(x) = 4(2-5x)^3 \times (-5) = -20(2-5x)^3$ **d** $f'(x) = -4(x+5)^{-2}$
 $f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$ $f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$
- 5 **a** $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$ **b** $f'(x) = 2x - \frac{1}{x-2} = 5$
 $\frac{6}{\sqrt{3x+15}} = 2$ $2x(x-2) - 1 = 5(x-2)$
 $\sqrt{3x+15} = 3$ $2x^2 - 9x + 9 = 0$
 $3x + 15 = 9$ $(2x-3)(x-3) = 0$
 $x = -2$ for real $f(x)$, $x > 2 \therefore x = 3$
- 6 **a** $= 3(x^2-4)^2 \times 2x$ **b** $= 12(3x^2+1)^5 \times 6x$ **c** $= \frac{1}{3+2x^2} \times 4x$ **d** $= \frac{d}{dx}[(4-x^2)^3]$
 $= 6x(x^2-4)^2$ $= 72x(3x^2+1)^5$ $= \frac{4x}{3+2x^2}$ $= 3(4-x^2)^2 \times (-2x)$
 $= -6x(4-x^2)^2$
- e** $= \frac{d}{dx}[(\frac{1}{2}x^4+3)^8]$ **f** $= \frac{d}{dx}[(3-x^2)^{-\frac{1}{2}}]$ **g** $= 7e^{x^2} \times 2x$ **h** $= 4(1-5x+x^3)^3 \times (-5+3x^2)$
 $= 8(\frac{1}{2}x^4+3)^7 \times 2x^3$ $= -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x)$ $= 14xe^{x^2}$ $= 4(3x^2-5)(1-5x+x^3)^3$
 $= 16x^3(\frac{1}{2}x^4+3)^7$ $= x(3-x^2)^{-\frac{3}{2}}$

$$\begin{aligned} \mathbf{i} &= \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}}) & \mathbf{j} &= 7(e^{4x} + 2)^6 \times 4e^{4x} & \mathbf{k} &= -(5+4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}} & \mathbf{l} &= 5(\frac{2}{x}-x)^4 \times (-2x^{-2}-1) \\ &= \frac{3}{2x-8\sqrt{x}} & &= 28e^{4x}(e^{4x}+2)^6 & &= \frac{-2}{\sqrt{x}(5+4\sqrt{x})^2} & &= -5(\frac{2}{x^2}+1)(\frac{2}{x}-x)^4 \end{aligned}$$

$$\begin{aligned} 7 \quad \mathbf{a} \quad \frac{dy}{dx} &= 5(2x-3)^4 \times 2 & \mathbf{b} \quad \frac{dy}{dx} &= 3(x^2-4)^2 \times 2x & \mathbf{c} \quad \frac{dy}{dx} &= 8-2e^{2x} \\ \text{SP: } 10(2x-3)^4 &= 0 & \text{SP: } 6x(x^2-4)^2 &= 0 & \text{SP: } 8-2e^{2x} &= 0 \\ x &= \frac{3}{2} & x &= 0 \text{ or } x^2 = 4 & e^{2x} &= 4 \\ \therefore (\frac{3}{2}, 0) & & x &= 0, \pm 2 & x &= \frac{1}{2} \ln 4 = \ln 2 \\ & & \therefore &(-2, 0), (0, -64), (2, 0) & \therefore &(\ln 2, 8 \ln 2 - 4) \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad \frac{dy}{dx} &= \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \times 4x & \mathbf{e} \quad \frac{dy}{dx} &= \frac{2}{x-x^2} \times (1-2x) & \mathbf{f} \quad \frac{dy}{dx} &= 4-(x-3)^{-2} \\ \text{SP: } \frac{2x}{\sqrt{1+2x^2}} &= 0 & \text{SP: } \frac{2(1-2x)}{x-x^2} &= 0 & \text{SP: } 4-\frac{1}{(x-3)^2} &= 0 \\ x &= 0 & x &= \frac{1}{2} & (x-3)^2 &= \frac{1}{4}, x-3 = \pm \frac{1}{2} \\ \therefore (0, 1) & & \therefore &(\frac{1}{2}, -4 \ln 2) & x &= \frac{5}{2}, \frac{7}{2} \\ & & & & \therefore &(\frac{5}{2}, 8), (\frac{7}{2}, 16) \end{aligned}$$

$$\begin{aligned} 8 \quad \mathbf{a} \quad x &= 2 \therefore y = 1 \\ \frac{dy}{dx} &= 4(3x-7)^3 \times 3 = 12(3x-7)^3 \\ \text{grad} &= -12 \\ \therefore y-1 &= -12(x-2) \\ [y &= 25-12x] \\ \mathbf{b} \quad x &= 0 \therefore y = 2 \\ \frac{dy}{dx} &= \frac{1}{1+4x} \times 4 = \frac{4}{1+4x} \\ \text{grad} &= 4 \\ \therefore y &= 4x+2 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad x &= 1 \therefore y = 3 \\ \frac{dy}{dx} &= -9(x^2+2)^{-2} \times 2x = -18x(x^2+2)^{-2} \\ \text{grad} &= -2 \\ \therefore y-3 &= -2(x-1) \\ [y &= 5-2x] \\ \mathbf{d} \quad x &= \frac{1}{4} \therefore y = \frac{1}{2} \\ \frac{dy}{dx} &= \frac{1}{2}(5x-1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x-1)^{-\frac{1}{2}} \\ \text{grad} &= 5 \\ \therefore y-\frac{1}{2} &= 5(x-\frac{1}{4}) \\ [y &= 5x-\frac{3}{4}] \end{aligned}$$

$$\begin{aligned} 9 \quad \mathbf{a} \quad x &= -2 \therefore y = -9 \\ \frac{dy}{dx} &= e^{4-x^2} \times (-2x) = -2xe^{4-x^2} \\ \text{grad} &= 4 \therefore \text{grad of normal} = -\frac{1}{4} \\ \therefore y+9 &= -\frac{1}{4}(x+2) \\ [y &= -\frac{1}{4}x - \frac{19}{2}] \\ \mathbf{b} \quad x &= \frac{1}{2} \therefore y = \frac{1}{8} \\ \frac{dy}{dx} &= 3(1-2x^2)^2 \times (-4x) = -12x(1-2x^2)^2 \\ \text{grad} &= -\frac{3}{2} \therefore \text{grad of normal} = \frac{2}{3} \\ \therefore y-\frac{1}{8} &= \frac{2}{3}(x-\frac{1}{2}) \\ [16x-24y-5 &= 0] \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad x &= 1 \therefore y = \frac{1}{2} \\ \frac{dy}{dx} &= -(2-\ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2-\ln x)^2} \\ \text{grad} &= \frac{1}{4} \therefore \text{grad of normal} = -4 \\ \therefore y-\frac{1}{2} &= -4(x-1) \\ [y &= \frac{9}{2}-4x] \\ \mathbf{d} \quad x &= 3 \therefore y = 6e \\ \frac{dy}{dx} &= 2e^{\frac{x}{3}} \\ \text{grad} &= 2e \therefore \text{grad of normal} = -\frac{1}{2e} \\ \therefore y-6e &= -\frac{1}{2e}(x-3) \\ [x+2ey-12e^2-3 &= 0] \end{aligned}$$