C3 DIFFERENTIATION *Worksheet I*

1 A curve has the equation $x = \sqrt{y}$.

a Write down $\frac{dx}{dy}$ in terms of y.

b Express the equation of the curve in the form $y = f(x)$.

c Write down $\frac{dy}{dx}$ in terms of x.

d Hence verify that for this curve, $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

2 Verify the relationship $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ when

a $y = e^{2x-1}$,

b $y = x^3 + 2$,

c $x = \sqrt{\ln y}$.

3 Find expressions for $\frac{dy}{dx}$ in terms of y in each case.

a $x = y^2 + 3$ **b** $x = (y - 1)^3$ **c** $x = \tan y$

d $x = \ln (3y + 2)$ **e** $x = \sin^2 y$ **f** $x = \frac{y-2}{e^y}$

4 The curve C has the equation $x = y^3 - 4y^2$.

a Find $\frac{dx}{dy}$ in terms of y.

b Find an equation for the tangent to C at the point on the curve with y-coordinate 3.

5 Given that $y = \ln (ax + b)$, where a and b are constants,

a express x as a function of y,

b find $\frac{dx}{dy}$ in terms of y.

c Hence, prove that $\frac{d}{dx}[\ln (ax + b)] = \frac{a}{ax + b}$.

6 A curve has the equation $y = 3^x$.

a Express the equation of the curve in the form $x = f(y)$.

b Find $\frac{dx}{dy}$ in terms of y.

c Hence, find $\frac{dy}{dx}$ in terms of x.

d Find an equation for the tangent to the curve at the point (2, 9).