

$$1 \quad a \quad \frac{dy}{dx} = -\frac{1}{4}x^{-2} - \frac{1}{x}$$

$$x = 1 \quad \therefore \text{grad} = -\frac{5}{4}$$

$$b \quad \text{grad of normal} = \frac{4}{5}$$

$$\therefore y - \frac{1}{4} = \frac{4}{5}(x - 1)$$

$$16x - 20y - 11 = 0$$

$$3 \quad a \quad y = 0 \Rightarrow x = \sqrt{3}$$

$$\therefore (\sqrt{3}, 0)$$

$$b \quad = \frac{1}{2}(e^y + 2)^{-\frac{1}{2}} \times e^y$$

$$= \frac{e^y}{2\sqrt{e^y + 2}}$$

$$c \quad \frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{2\sqrt{e^y + 2}}{e^y}$$

$$\text{grad} = 2\sqrt{3}$$

$$\therefore y - 0 = 2\sqrt{3}(x - \sqrt{3})$$

$$\text{at } Q, x = 0 \quad \therefore y = -6$$

$$\text{area} = \frac{1}{2} \times \sqrt{3} \times 6 = 3\sqrt{3}$$

$$5 \quad a \quad = \frac{1}{2}(\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)$$

$$= \frac{\cos x - \sin x}{2\sqrt{\sin x + \cos x}}$$

$$b \quad = \frac{d}{dx} [\ln(x - 1) - \ln(2x + 1)]$$

$$= \frac{1}{x-1} - \frac{1}{2x+1} \times 2$$

$$= \frac{1}{x-1} - \frac{2}{2x+1}$$

$$2 \quad a \quad \frac{dy}{dx} = 1 \times e^{-2x} + x \times (-2e^{-2x})$$

$$= e^{-2x}(1 - 2x)$$

$$\frac{d^2y}{dx^2} = -2e^{-2x} \times (1 - 2x) + e^{-2x} \times (-2)$$

$$= 4e^{-2x}(x - 1)$$

$$b \quad \text{SP: } e^{-2x}(1 - 2x) = 0$$

$$x = \frac{1}{2}$$

$$\therefore (\frac{1}{2}, \frac{1}{2}e^{-1})$$

$$\text{when } x = \frac{1}{2}, \frac{d^2y}{dx^2} = -2e^{-1}$$

$$\frac{d^2y}{dx^2} < 0 \quad \therefore \text{maximum}$$

$$4 \quad a \quad t = 0, m = 680$$

$$t = 100, m = 653.63$$

$$\% \text{ red'n} = \frac{680 - 653.63}{680} \times 100\% = 3.88\% \text{ (3sf)}$$

$$b \quad 640 = 600 + 80e^{-0.004t}$$

$$t = \frac{-1}{0.004} \ln \frac{1}{2} = 173 \text{ (3sf)}$$

$$c \quad \frac{dm}{dt} = 80 \times (-0.004)e^{-0.004t} = -0.32e^{-0.004t}$$

$$t = 150, \frac{dm}{dt} = -0.176$$

$$\therefore \text{mass decreasing at } 0.176 \text{ g yr}^{-1} \text{ (3sf)}$$

$$6 \quad a \quad \frac{dy}{dx} = 5(2x - 3)^4 \times 2 = 10(2x - 3)^4$$

$$x = 1 \quad \therefore \text{grad} = 10$$

$$\therefore y + 1 = 10(x - 1)$$

$$[y = 10x - 11]$$

$$b \quad \text{at } Q \quad 10(2x - 3)^4 = 10$$

$$2x - 3 = \pm 1$$

$$x = 1 \text{ (at } P) \text{ or } 2$$

$$\therefore Q(2, 1)$$

$$7 \quad a \quad \frac{dy}{dx} = -2(x^2 - 5)^{-2} \times 2x = \frac{-4x}{(x^2 - 5)^2}$$

$$\text{SP: } \frac{-4x}{(x^2 - 5)^2} = 0$$

$$x = 0$$

$$\therefore (0, -\frac{2}{5})$$

$$b \quad x = 3, y = \frac{1}{2}$$

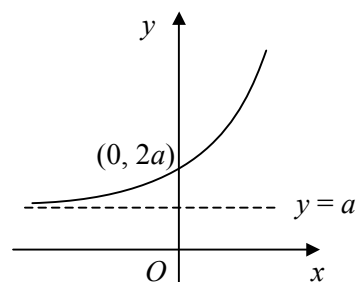
$$\text{grad} = -\frac{3}{4}$$

$$\therefore y - \frac{1}{2} = -\frac{3}{4}(x - 3)$$

$$4y - 2 = -3x + 9$$

$$3x + 4y - 11 = 0$$

8 a



$$b \quad y = ae^x + a$$

$$\text{swap } x = ae^y + a$$

$$y = \ln \frac{x-a}{a}$$

$$f^{-1}: x \rightarrow \ln \frac{x-a}{a}, \quad x \in \mathbb{R}, \quad x > a$$

$$c \quad x = 1 \quad \therefore y = ae + a$$

$$f'(x) = ae^x, \quad \text{grad} = ae$$

$$\therefore y - (ae + a) = ae(x - 1)$$

$$[y = aex + a]$$

$$9 \quad a \quad \frac{d}{dx}(\cot x) = \frac{d}{dx} \left(\frac{\cos x}{\sin x} \right)$$

$$= \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} = -\text{cosec}^2 x$$

$$b \quad \frac{dy}{dx} = e^x \times \cot x + e^x \times (-\text{cosec}^2 x)$$

$$= e^x(\cot x - \text{cosec}^2 x)$$

$$\text{SP: } e^x(\cot x - \text{cosec}^2 x) = 0$$

$$e^x \neq 0 \quad \therefore \cot x = \text{cosec}^2 x$$

$$\frac{\cos x}{\sin x} = \frac{1}{\sin^2 x}$$

$$\sin x \cos x = 1$$

$$\sin 2x = 2$$

$$|\sin 2x| \leq 1 \quad \therefore \text{no solutions}$$

$$\therefore \text{no turning points}$$

$$10 \quad a \quad \frac{dy}{dx} = 3(2 + \ln x)^2 \times \frac{1}{x}$$

$$= \frac{3}{x}(2 + \ln x)^2$$

$$b \quad \text{SP: } \frac{3}{x}(2 + \ln x)^2 = 0$$

$$\ln x = -2$$

$$x = e^{-2}$$

$$\therefore (e^{-2}, 0)$$

$$c \quad x = e, \quad y = 27$$

$$\text{grad} = \frac{27}{e}$$

$$\therefore y - 27 = \frac{27}{e}(x - e)$$

$$y = \frac{27}{e}x$$

$$\text{when } x = 0, \quad y = 0$$

$$\therefore \text{passes through origin}$$

$$\begin{aligned} 11 \quad \mathbf{a} \quad &= \frac{1}{9-x^2} \times (-2x) \\ &= \frac{-2x}{9-x^2} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \text{SP:} \quad &\frac{-2x}{9-x^2} = 0 \\ &x = 0 \end{aligned}$$

$$\therefore (0, \ln 9)$$

$$\begin{aligned} \mathbf{c} \quad x = 1, \quad y = \ln 8 = \ln 2^3 = 3 \ln 2 \\ \text{grad} = -\frac{1}{4} \end{aligned}$$

$$\therefore \text{grad of normal} = 4$$

$$\therefore y - 3 \ln 2 = 4(x - 1)$$

$$y = 4x - 4 + 3 \ln 2$$

$$12 \quad \mathbf{a} \quad \text{model } A: t = 3, M = 764 \text{ (3sf)}$$

$$\text{model } B: t = 3, M = 732 \text{ (3sf)}$$

$$\mathbf{b} \quad \text{model } A:$$

$$\frac{dM}{dt} = 1500(3t + 2)^{-2} \times 3 = \frac{4500}{(3t + 2)^2}$$

$$t = 3, \frac{dM}{dt} = 37.2$$

$$\therefore \text{increasing at } 37.2 \text{ tonnes yr}^{-1} \text{ (3sf)}$$

$$\text{model } B:$$

$$\begin{aligned} \frac{dM}{dt} &= 1500[2 + 5 \ln(t + 1)]^{-2} \times \frac{5}{t + 1} \\ &= \frac{7500}{(t + 1)[2 + 5 \ln(t + 1)]^2} \end{aligned}$$

$$t = 3, \frac{dM}{dt} = 23.5$$

$$\therefore \text{increasing at } 23.5 \text{ tonnes yr}^{-1} \text{ (3sf)}$$