C3 DIFFERENTIATION Worksheet K

1 The curve C has equation $y = \frac{1}{4x} - \ln x$.

a Find the gradient of C at the point $(1, \frac{1}{4})$. **(3)**

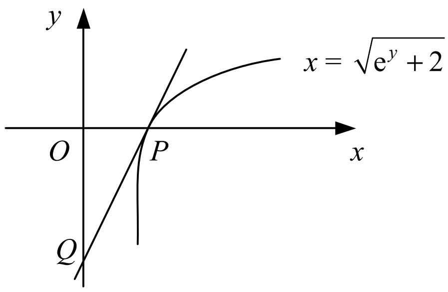
b Find an equation for the normal to C at the point $(1, \frac{1}{4})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

2 A curve has the equation $y = x\mathrm{e}^{-2x}$.

a Find and simplify expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **(4)**

b Find the exact coordinates of the turning point of the curve and determine its nature. **(4)**

3



The diagram shows the curve $x = \sqrt{\mathrm{e}^y + 2}$ which crosses the x-axis at the point P.

a Find the coordinates of P. **(1)**

b Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **(2)**

The tangent to the curve at P crosses the y-axis at the point Q.

c Show that the area of triangle OPQ, where O is the origin, is $3\sqrt{3}$. **(5)**

4 A rock contains a radioactive substance which is decaying.

The mass of the rock, m grams, at time t years after initial observation is given by

$$m = 600 + 80\mathrm{e}^{-0.004t}.$$

a Find the percentage reduction in the mass of the rock over the first 100 years. **(3)**

b Find the value of t when $m = 640$. **(2)**

c Find the rate at which the mass of the rock will be decreasing when $t = 150$. **(3)**

5 Differentiate with respect to x

a $\sqrt{\sin x + \cos x}$, **(3)**

b $\ln\left(\frac{x-1}{2x+1}\right)$. **(3)**

6 A curve has the equation $y = (2x - 3)^5$.

a Find an equation for the tangent to the curve at the point P $(1, -1)$. **(4)**

Given that the tangent to the curve at the point Q is parallel to the tangent at P,

b find the coordinates of Q. **(3)**

7 A curve has the equation $y = \frac{2}{x^2 - 5}$.

a Find the coordinates of the stationary point of the curve. **(4)**

b Show that the tangent to the curve at the point with x-coordinate 3 has the equation

$$3x + 4y - 11 = 0.$$ **(3)**

8 $$f : x \to ae^x + a, \quad x \in \mathbb{R}.$$

Given that a is a positive constant,

a sketch the graph of $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. **(2)**

b Find the inverse function f^{-1} in the form $f^{-1} : x \to \ldots$ and state its domain. **(4)**

c Find an equation for the tangent to the curve $y = f(x)$ at the point on the curve with x-coordinate 1. **(4)**

9 a Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x.$$ **(4)**

b Show that the curve with equation

$$y = e^x \cot x$$

has no turning points. **(5)**

10 A curve has the equation $y = (2 + \ln x)^3$.

a Find $\frac{dy}{dx}$. **(2)**

b Find, in exact form, the coordinates of the stationary point on the curve. **(3)**

c Show that the tangent to the curve at the point with x-coordinate e passes through the origin. **(3)**

11 $$f : x \to \ln(9 - x^2), \quad -3 < x < 3.$$

a Find $f'(x)$. **(2)**

b Find the coordinates of the stationary point of the curve $y = f(x)$. **(2)**

c Show that the normal to the curve $y = f(x)$ at the point with x-coordinate 1 has equation

$$y = 4x - 4 + 3 \ln 2.$$ **(4)**

12 A botanist is studying the regeneration of an area of moorland following a fire.

The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M.

Model A is given by

$$M = 900 - \frac{1500}{3t + 2}.$$

Model B is given by

$$M = 900 - \frac{1500}{2 + 5\ln(t + 1)}.$$

For each model, find

a the value of M when $t = 3$, **(2)**

b the rate at which the biomass is increasing when $t = 3$. **(6)**