

1 a $42 = 60e^{100k}$
 $100k = \ln 0.7$
 $k = \frac{1}{100} \ln 0.7 = -0.00357$ (3sf)

b $30 = 60e^{kt}$
 $kt = \ln 0.5$
 $t = \frac{100 \ln 0.5}{\ln 0.7} = 194$ (3sf)

2 a $e^{3x} = 5.7$
 $x = \frac{1}{3} \ln 5.7 = 0.58$ (2dp)

b $\ln \frac{x}{x-1} = \frac{1}{2}$
 $\frac{x}{x-1} = e^{\frac{1}{2}}$
 $x = e^{\frac{1}{2}}(x-1)$
 $x(e^{\frac{1}{2}} - 1) = e^{\frac{1}{2}}$
 $x = \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}} - 1} = 2.54$ (2dp)

3 a $\ln(4x-3) = 0$
 $4x-3 = 1$
 $x = 1 \quad \therefore A(1, 0)$
 $1 + \ln x = 0$
 $\ln x = -1$
 $x = e^{-1} \quad \therefore B(e^{-1}, 0)$

b $\ln(4x-3) = 1 + \ln x$
 $\ln(4x-3) - \ln x = 1$
 $\ln \frac{4x-3}{x} = 1$
 $\frac{4x-3}{x} = e$
 $4x-3 = ex$
 $x(4-e) = 3$
 $x = \frac{3}{4-e}$

4 $2e^{2x} - 7e^x + 3 = 0$
 $(2e^x - 1)(e^x - 3) = 0$
 $e^x = \frac{1}{2}, 3$
 $x = \ln \frac{1}{2}, \ln 3$

5 a $t = 0 \Rightarrow N = 800$
b $t = 20 \Rightarrow N = 800e^{0.2}$
 $= 977$ (nearest unit)

c $800e^{0.01t} > 2000$
 $e^{0.01t} > 2.5$
 $0.01t > \ln 2.5$
 $t > 91.6 \quad \therefore 92$ days

6 a $1 + e^{2x+1} = 10$
 $e^{2x+1} = 9$
 $2x+1 = \ln 9$
 $x = \frac{1}{2}(-1 + \ln 9)$
 $x = -\frac{1}{2} + \ln 3$

b $1 + e^{2x+1} = 3 - e^x$
 $e(e^{2x}) + e^x - 2 = 0$
 $e^x = \frac{-1 \pm \sqrt{1+8e}}{2e}$
 $x = \ln \frac{-1 - \sqrt{1+8e}}{2e}$ (not real)
or $\ln \frac{-1 + \sqrt{1+8e}}{2e}$
 $\therefore x = -0.366$ (3sf)

$$7 \quad \mathbf{a} \quad 4x - 1 = e^2 \\ x = \frac{1}{4}(e^2 + 1)$$

$$\mathbf{b} \quad 7 = e^{1-3y} \\ 1 - 3y = \ln 7 \\ y = \frac{1}{3}(1 - \ln 7)$$

$$8 \quad \mathbf{a} \quad a = 800$$

$$\mathbf{b} \quad 7200 = 800e^{2b}$$

$$b = \frac{1}{2} \ln 9 = \ln 3$$

$$\mathbf{c} \quad 1600 = 800e^{t \ln 3}$$

$$t = \frac{\ln 2}{\ln 3} = 0.631 \text{ hours}$$

$$\therefore 60 \times 0.631 = 38 \text{ minutes}$$

$$9 \quad \mathbf{a} \quad = \frac{(x-1)(x-3)}{(x+2)(x-1)} \\ = \frac{x-3}{x+2}$$

$$\mathbf{b} \quad \ln(x^2 - 4x + 3) - \ln(x^2 + x - 2) = 1$$

$$\ln \frac{x^2 - 4x + 3}{x^2 + x - 2} = \ln \frac{x-3}{x+2} = 1$$

$$\frac{x-3}{x+2} = e$$

$$x - 3 = e(x + 2)$$

$$x(1 - e) = 2e + 3$$

$$x = \frac{2e+3}{1-e}$$

$$10 \quad e^y + 5 - 9x = 0 \Rightarrow y = \ln(9x - 5)$$

$$\text{sub. } \ln(9x - 5) - \ln(x + 4) = 2$$

$$\frac{9x-5}{x+4} = e^2$$

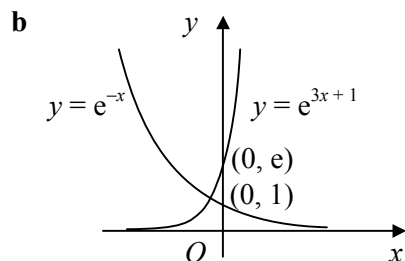
$$9x - 5 = e^2(x + 4)$$

$$x(9 - e^2) = 4e^2 + 5$$

$$x = \frac{4e^2 + 5}{9 - e^2} = 21.4509$$

$$\therefore x = 21.5, y = 5.24 \text{ (3sf)}$$

$$11 \quad \mathbf{a} \quad \text{reflection in } y\text{-axis}$$



$$\mathbf{c} \quad e^{-x} = e^{3x+1}$$

$$1 = e^{4x+1}$$

$$4x + 1 = 0$$

$$x = -\frac{1}{4}$$

$$\therefore \left(-\frac{1}{4}, e^{\frac{1}{4}}\right)$$

$$12 \quad \mathbf{a} \quad \mathbf{i} \quad = \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x = \frac{1}{2} t$$

$$\mathbf{ii} \quad = \ln e^2 + \ln x = 2 + t$$

$$\mathbf{b} \quad 5 + \frac{1}{2} t = 2 + t$$

$$t = \ln x = 6$$

$$x = e^6$$

$$13 \quad \mathbf{a} \quad \text{when } t = 0, v = 13$$

$$\therefore 13 = c - 2$$

$$c = 15$$

$$\mathbf{b} \quad 7 = 15e^{-5.1k} - 2$$

$$e^{-5.1k} = \frac{3}{5}$$

$$k = \frac{\ln \frac{3}{5}}{-5.1} = 0.1002$$

$$\mathbf{c} \quad 10 = 15e^{-0.1002t} - 2, \quad 4 = 15e^{-0.1002T} - 2$$

$$t = \frac{\ln \frac{4}{5}}{-0.1002} = 2.2278, \quad T = \frac{\ln \frac{2}{5}}{-0.1002} = 9.1481$$

$$T - t = 6.92 \text{ seconds (3sf)}$$