

- 1 **a** = $g(5) = -3$ **b** = $g(6) = -4$ **c** = $f(5) = 17$ **d** = $h(9) = 86$
 e = $g(-3) = 5$ **f** = $f(-1) = -7$ **g** = $h(-6) = 41$ **h** = $f(\frac{29}{4}) = 26$
- 2 **a** = $f(\ln 20) = 17.0$ **b** = $g(\ln 3) = 0.455$ **c** = $f(\cos 5) = 3.42$ **d** = $g(\cos(-4)) = 0.794$
 e = $g(\frac{43}{4}) = -0.243$ **f** = $h(\cos 6.7) = -0.0895$ **g** = $h(\ln 50) = 1.36$ **h** = $h(0.5) = -0.693$
- 3 **a** = $f(1-3x)$
 = $2(1-3x) + 1$
 $fg : x \rightarrow 3-6x,$
 $x \in \mathbb{R}$
 b = $f(2x+1)$
 = $2(2x+1) + 1$
 $ff : x \rightarrow 4x+3,$
 $x \in \mathbb{R}$
 c = $f(x^2+4)$
 = $2(x^2+4) + 1$
 $fh : x \rightarrow 2x^2+9,$
 $x \in \mathbb{R}$
 d = $h(2x+1)$
 = $(2x+1)^2 + 4$
 $hf : x \rightarrow 4x^2+4x+5,$
 $x \in \mathbb{R}$
 e = $g(x^2+4)$
 = $1-3(x^2+4)$
 $gh : x \rightarrow -3x^2-11,$
 $x \in \mathbb{R}$
 f = $g(1-3x)$
 = $1-3(1-3x)$
 $gg : x \rightarrow 9x-2,$
 $x \in \mathbb{R}$
 g = $h(1-3x)$
 = $(1-3x)^2 + 4$
 $hg : x \rightarrow 9x^2-6x+5,$
 $x \in \mathbb{R}$
 h = $g(2x+1)$
 = $1-3(2x+1)$
 $gf : x \rightarrow -6x-2,$
 $x \in \mathbb{R}$
- 4 **a** = $g(4-x)$
 = e^{4-x}
 $gf : x \rightarrow e^{4-x},$
 $x \in \mathbb{R}$
 b = $h(e^x)$
 = $2(e^x)^2 + 7$
 $hg : x \rightarrow 2e^{2x} + 7,$
 $x \in \mathbb{R}$
 c = $f(2x^2+7)$
 = $4-(2x^2+7)$
 $fh : x \rightarrow -2x^2-3,$
 $x \in \mathbb{R}$
 d = $g(e^x)$
 = e^{e^x}
 $gg : x \rightarrow e^{e^x},$
 $x \in \mathbb{R}$
 e = $g(2x^2+7)$
 = e^{2x^2+7}
 $gh : x \rightarrow e^{2x^2+7},$
 $x \in \mathbb{R}$
 f = $f(4-x)$
 = $4-(4-x)$
 $ff : x \rightarrow x,$
 $x \in \mathbb{R}$
 g = $f(e^x)$
 = $4-e^x$
 $fg : x \rightarrow 4-e^x,$
 $x \in \mathbb{R}$
 h = $h(4-x)$
 = $2(4-x)^2 + 7$
 $hf : x \rightarrow 2x^2-16x+39,$
 $x \in \mathbb{R}$
- 5 **a** $ff(x) = f(5x-3)$
 = $5(5x-3) - 3$
 = $25x - 18$
 $25x - 18 = -8$
 $x = \frac{2}{5}$
 b $hf(x) = h(5x-3)$
 = $\frac{1}{5x-3-2}$
 = $\frac{1}{5x-5}$
 $\frac{1}{5x-5} = 2$
 $1 = 10x - 10$
 $x = \frac{11}{10}$
 c $gf(x) = g(5x-3)$
 = $3(5x-3)^2 + 1$
 = $75x^2 - 90x + 28$
 $75x^2 - 90x + 28 = 28$
 $15x(5x-6) = 0$
 $x = 0, \frac{6}{5}$
 d $hg(x) = h(3x^2+1)$
 = $\frac{1}{3x^2+1-2}$
 = $\frac{1}{3x^2-1}$
 $\frac{1}{3x^2-1} = \frac{1}{2}$
 $3x^2 - 1 = 2$
 $x^2 = 1$
 $x = \pm 1$
 e $fh(x) = f(\frac{1}{x-2})$
 = $\frac{5}{x-2} - 3$
 $\frac{5}{x-2} - 3 = 7$
 $\frac{5}{x-2} = 10$
 $5 = 10x - 20$
 $x = \frac{5}{2}$
 f $fg(x) = f(3x^2+1)$
 = $5(3x^2+1) - 3$
 = $15x^2 + 2$
 $15x^2 + 2 = 32$
 $x^2 = 2$
 $x = \pm\sqrt{2}$
 g $gh(x) = g(\frac{1}{x-2})$
 = $\frac{3}{(x-2)^2} + 1$
 $\frac{3}{(x-2)^2} + 1 = 4$
 $(x-2)^2 = 1$
 $x-2 = \pm 1$
 $x = 1, 3$
 h $hh(x) = h(\frac{1}{x-2})$
 = $\frac{1}{\frac{1}{x-2}-2} = \frac{x-2}{1-2(x-2)}$
 = $\frac{x-2}{5-2x}$
 $\frac{x-2}{5-2x} = -2$
 $x-2 = -10+4x$
 $x = \frac{8}{3}$

6 a $gh(x) = g(e^x)$ $= 3 + 2e^x$ $3 + 2e^x = 9$ $e^x = 3$ $x = \ln 3$ $x = 1.10$	b $fg(x) = f(3 + 2x)$ $= \ln(3 + 2x)$ $\ln(3 + 2x) = 3.6$ $3 + 2x = e^{3.6}$ $x = \frac{1}{2}(e^{3.6} - 3)$ $x = 16.80$	c $hg(x) = h(3 + 2x)$ $= e^{3+2x}$ $e^{3+2x} = 4$ $3 + 2x = \ln 4$ $x = \frac{1}{2}(\ln 4 - 3)$ $x = -0.81$	d $gf(x) = g(\ln x)$ $= 3 + 2 \ln x$ $3 + 2 \ln x = 10.4$ $\ln x = 3.7$ $x = e^{3.7}$ $x = 40.45$
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7 **a** $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8 **a** $= f(4) = 7$

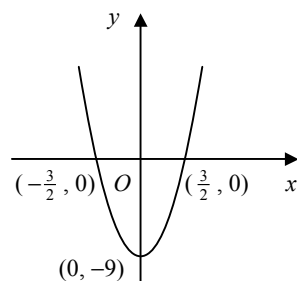
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9 **a** $= g(1) = 4$

b $= h(2) = e^3$

c $= g(e^{-3}) = 1$

d $= f(\tan 1) = 74.7 \text{ (3sf)}$

e $= h(\tan 0.2)$
 $= 0.552 \text{ (3sf)}$

f $= f(4 + \ln 7)$
 $= -0.351 \text{ (3sf)}$

g $= h(e^{-\frac{1}{2}})$
 $= 1.24 \text{ (3sf)}$

h $= f(4 + e)$
 $= 0.465 \text{ (3sf)}$

10 **a** $= f(4x + 1)$
 $= 3e^{4x+1} + 2$

b $= g(3e^x + 2)$
 $= 4(3e^x + 2) + 1$

c $= h(3e^x + 2)$
 $= \frac{1}{3e^x + 2 + 1}$

d $= g(4x + 1)$
 $= 4(4x + 1) + 1$

$$fg : x \rightarrow 3e^{4x+1} + 2,$$

 $x \in \mathbb{R}$

$$gf : x \rightarrow 12e^x + 9,$$

 $x \in \mathbb{R}$

$$hf : x \rightarrow \frac{1}{3e^x + 3},$$

 $x \in \mathbb{R}$

$$gg : x \rightarrow 16x + 5,$$

 $x \in \mathbb{R}$

e $= h(4x + 1)$
 $= \frac{1}{4x+1+1}$

$$hg : x \rightarrow \frac{1}{4x+2},$$

 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

f $= g\left(\frac{1}{x+1}\right)$
 $= \frac{4}{x+1} + 1$

$$= \frac{4+x+1}{x+1}$$

 $gh : x \rightarrow \frac{x+5}{x+1},$
 $x \in \mathbb{R}, x \neq -1$

g $= h\left(\frac{1}{x+1}\right)$
 $= \frac{1}{\frac{1}{x+1} + 1}$

$$= \frac{x+1}{1+x+1}$$

 $hh : x \rightarrow \frac{x+1}{x+2},$
 $x \in \mathbb{R}, x \neq -1, -2$

h $= g(16x + 5)$
 $= 4(16x + 5) + 1$

$$ggg : x \rightarrow 64x + 21,$$

 $x \in \mathbb{R}$

11 a $fh(x) = f\left(\frac{x+1}{3}\right)$ b $fg(x) = f(e^{1+2x})$ c $gh(x) = g\left(\frac{x+1}{3}\right)$ d $hh(x) = h\left(\frac{x+1}{3}\right)$

$$= \sqrt{\frac{x+1}{3} + 4}$$

$$= \sqrt{\frac{x+13}{3}}$$

$$\sqrt{\frac{x+13}{3}} = 3$$

$$\frac{x+13}{3} = 9$$

$$x+13 = 27$$

$$x = 14$$

$$= \sqrt{e^{1+2x} + 4}$$

$$\sqrt{e^{1+2x} + 4} = 7$$

$$e^{1+2x} + 4 = 49$$

$$e^{1+2x} = 45$$

$$1+2x = \ln 45$$

$$x = \frac{1}{2}(\ln 45 - 1)$$

$$x = 1.40 \text{ (3sf)}$$

$$= e^{1+\frac{2(x+1)}{3}}$$

$$= e^{\frac{2x+5}{3}}$$

$$e^{\frac{2x+5}{3}} = 11$$

$$\frac{2x+5}{3} = \ln 11$$

$$2x+5 = 3 \ln 11$$

$$x = \frac{1}{2}(3 \ln 11 - 5)$$

$$x = 1.10 \text{ (3sf)}$$

$$= \frac{\frac{x+1}{3} + 1}{3}$$

$$= \frac{x+1+3}{9}$$

$$= \frac{x+4}{9}$$

$$\frac{x+4}{9} = \frac{2}{3}$$

$$3x+12 = 18$$

$$x = 2$$

e $hg(x) = h(e^{1+2x})$ f $hf(x) = h(\sqrt{x+4})$ g $ff(x) = f(\sqrt{x+4})$ h $ghh(x) = g\left(\frac{x+4}{9}\right)$

$$= \frac{e^{1+2x} + 1}{3}$$

$$\frac{e^{1+2x} + 1}{3} = 1.2$$

$$e^{1+2x} = 2.6$$

$$1+2x = \ln 2.6$$

$$x = \frac{1}{2}(\ln 2.6 - 1)$$

$$x = -0.0222 \text{ (3sf)}$$

$$= \frac{\sqrt{x+4} + 1}{3}$$

$$\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$$

$$\sqrt{x+4} = \frac{1}{2}$$

$$x+4 = \frac{1}{4}$$

$$x = -3\frac{3}{4}$$

$$= \sqrt{\sqrt{x+4} + 4}$$

$$\sqrt{\sqrt{x+4} + 4} = 3$$

$$\sqrt{x+4} = 5$$

$$x+4 = 25$$

$$x = 21$$

$$= e^{1+\frac{2(x+4)}{9}}$$

$$= e^{\frac{2x+17}{9}}$$

$$e^{\frac{2x+17}{9}} = \frac{1}{2}$$

$$\frac{2x+17}{9} = \ln \frac{1}{2}$$

$$x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$$

$$x = -11.6 \text{ (3sf)}$$

12 a $h(x) = fg(x)$

b $h(x) = gf(x)$

c $h(x) = gg(x)$

d $h(x) = ff(x)$

e $h(x) = gff(x)$

f $h(x) = gfg(x)$

13 a $j(x) = fg(x)$

b $j(x) = hf(x)$

c $j(x) = gh(x)$

d $j(x) = gg(x)$

e $j(x) = fhg(x)$

f $j(x) = hfg(x)$

14 a $gf(x) = g(5^x - 7)$
 $= 2(5^x - 7) + 3$
 $= 2(5^x) - 11$
 $gf: x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$

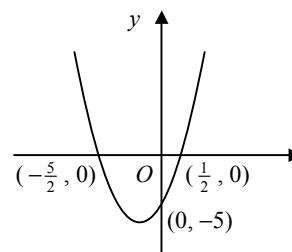
b $2(5^x) - 11 = 10$

$$5^x = \frac{21}{2}$$

$$x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46 \text{ (3sf)}$$

15 a $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$
 $gf: x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$
range: $gf(x) \geq -9$

b $gf(x) = (2x+5)(2x-1)$



c $gf(x) - 2f(x) = a$

$$4x^2 + 8x - 5 - 2[2(x+1)] = a$$

$$4x^2 + 4x - (a+9) = 0$$

no real roots $\therefore b^2 - 4ac < 0$

$$16 + 16(a+9) < 0$$

$$1 + a + 9 < 0$$

$$a < -10$$