C3 FUNCTIONS — Answers - Worksheet E

Note: For this worksheet especially, there may be alternative correct answers

1 **a** translated 3 units in negative x-direction and translated 2 units in positive y-direction

b reflected in the y-axis and stretched by a factor of 2 in y-direction

c translated 1 unit in positive x-direction and stretched by a factor of 3 in y-direction

d reflected in the x-axis and then translated 4 units in positive y-direction

2 **a** $= (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$

b translation by 3 units in negative x-direction and translation by 7 units in negative y-direction

3 **a** $y = 2[2(x - 3) + 7] \quad \Rightarrow \quad y = 4x + 2$

b $y = 2[3e^{(x-3)}] \quad \Rightarrow \quad y = 6e^{x-3}$

c $y = 2[(x - 3)^2 - 3(x - 3) + 1] \quad \Rightarrow \quad y = 2x^2 - 18x + 38$

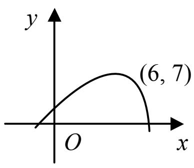
d $y = 2[\frac{1}{(x-3)}] \quad \Rightarrow \quad y = \frac{2}{x-3}$

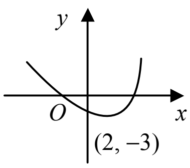
4 **a** stretch by a factor of $\frac{1}{3}$ in x-direction and reflection in the x-axis (either first)

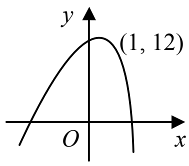
b reflection in the y-axis and translation by 5 units in positive y-direction (either first)

c translation by 4 units in negative x-direction and stretch by a factor of 3 in y-direction (either first)

d stretch by a factor of 3 in y-direction, then translation by 2 units in positive y-direction

5 **a**


b


c


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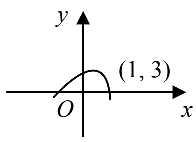


6
first $\Rightarrow \quad y = (x + 2)^2 + 4(x + 2) - 2 \quad \Rightarrow \quad y = x^2 + 8x + 10$
second $\Rightarrow \quad y = 3[x^2 + 8x + 10] \quad \Rightarrow \quad y = 3x^2 + 24x + 30$
third $\Rightarrow \quad y = 3(-x)^2 + 24(-x) + 30 \quad \Rightarrow \quad y = 3x^2 - 24x + 30$

7 **a** $= 2[x^2 - 2x] + 7 = 2[(x - 1)^2 - 1] + 7 = 2(x - 1)^2 + 5$

b translation by 5 units in negative y-direction,
then stretch by a factor of $\frac{1}{2}$ in y-direction,
then translation by 1 unit in negative x-direction

8 **a** $f'(x) = 3x^2 - 6x$

SP: $3x^2 - 6x = 0$

$3x(x - 2) = 0$

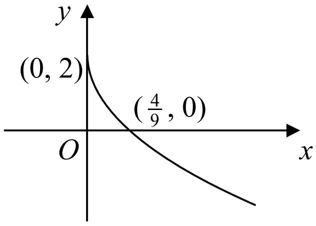
$x = 0, 2$

$\therefore$ (0, 4) and (2, 0)

b **i** (0, –8) and (2, 0) **ii** (0, 7) and (4, 3) **iii** (2, 1) and (4, 0)

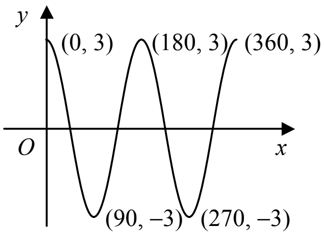
9 **a** stretch by factor of 3 in y-direction,
then reflection in x-axis,
then translation by 2 units in +ve y-dir'n

b

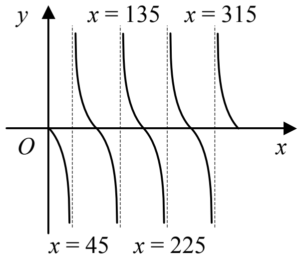


10 **a** 180°
b (0, 1)
c (90, 3) and (270, 3)

11 **a**



b



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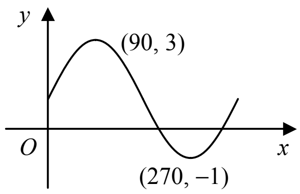

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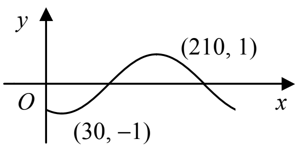

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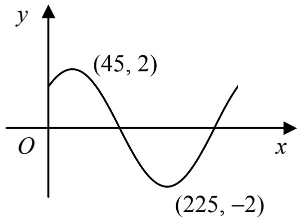

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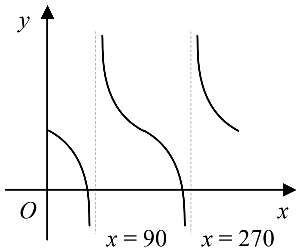

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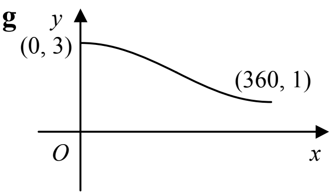

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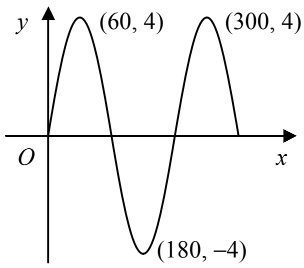
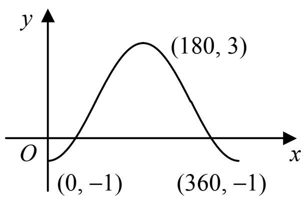
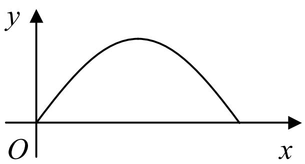


i



12 **a** 60°
b $\dfrac{360°}{k}$

13 **a**



b (π, 2)
c $2\sin\tfrac{1}{2}x = \sqrt{2}$

$\sin\tfrac{1}{2}x = \dfrac{1}{\sqrt{2}}$

$\tfrac{1}{2}x = \tfrac{\pi}{4}, \pi - \tfrac{\pi}{4}$

$= \tfrac{\pi}{4}, \tfrac{3\pi}{4}$

$x = \tfrac{\pi}{2}, \tfrac{3\pi}{2}$

14 **a** max. value 4 $\therefore a = 4$

max. occurs at $x = 45$ $\therefore b = 2$

b (135, −4)