C3 FUNCTIONS Worksheet E

1 Describe how the graph of $y = \mathrm{f}(x)$ is transformed to give the graph of

a $y = 2 + \mathrm{f}(x + 3)$ **b** $y = 2\mathrm{f}(-x)$ **c** $y = 3\mathrm{f}(x - 1)$ **d** $y = 4 - \mathrm{f}(x)$

2 **a** Express $x^2 + 6x + 2$ in the form $a(x + b)^2 + c$.

b Hence, describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 + 6x + 2$.

3 Each of the following graphs is translated by 3 units in the positive x-direction and then stretched by a factor of 2 in the y-direction, about the x-axis.

Find and simplify an equation of the graph obtained in each case.

a $y = 2x + 7$ **b** $y = 3e^x$ **c** $y = x^2 - 3x + 1$ **d** $y = \frac{1}{x}$

4 Describe in order two transformations that would map the graph of

a $y = |x|$ onto the graph of $y = -|3x|$ **b** $y = e^x$ onto the graph of $y = 5 + e^{-x}$

c $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x+4}$ **d** $y = \ln x$ onto the graph of $y = 2 + 3 \ln x$

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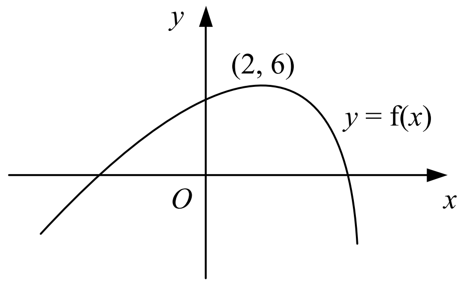

The diagram shows the curve with equation $y = \mathrm{f}(x)$ which is stationary at the point (2, 6).

Showing the coordinates of the stationary point in each case, sketch on separate diagrams the graphs of

a $y = 1 + \mathrm{f}(x - 4)$ **b** $y = 3 - \mathrm{f}(x)$ **c** $y = 2\mathrm{f}(x + 1)$ **d** $y = \frac{1}{2}\mathrm{f}(2x)$

6 The graph of $y = x^2 + 4x - 2$ undergoes the following three transformations:

first: translation by -2 units in the positive x-direction,
second: stretch by a factor of 3 in the y-direction, about the x-axis,
third: reflection in the y-axis.

Find and simplify an equation of the graph obtained.

7 **a** Express $2x^2 - 4x + 7$ in the form $a(x + b)^2 + c$.

b Hence, describe in order a sequence of transformations that would map the graph of $y = 2x^2 - 4x + 7$ onto the graph of $y = x^2$.

8 $$\mathrm{f}(x) \equiv x^3 - 3x^2 + 4, \; x \in \mathbb{R}.$$

a Find the coordinates of the stationary points on the graph of $y = \mathrm{f}(x)$.

b Hence, find the coordinates of the stationary points on each of the following graphs.

i $y = -2\mathrm{f}(x)$ **ii** $y = 3 + \mathrm{f}(\frac{1}{2}x)$ **iii** $y = \frac{1}{4}\mathrm{f}(x - 2)$

9 **a** Describe clearly, in order, the sequence of transformations that would map the graph of $y = \sqrt{x}$ onto the graph of $y = 2 - 3\sqrt{x}$.

b Sketch the graph of $y = 2 - 3\sqrt{x}$ showing the coordinates of any points where the graph meets the coordinate axes.

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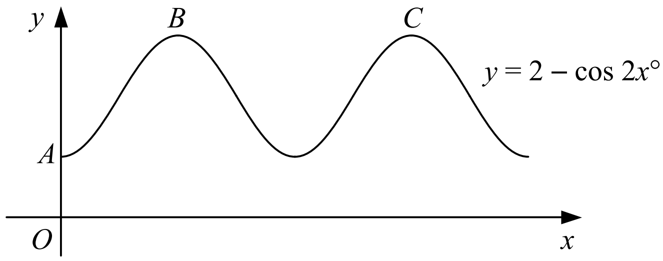

The diagram shows part of the curve with equation $y = 2 - \cos 2x°$, $x > 0$.

a State the period of the curve.

b Write down the coordinates of the point A where the curve meets the y-axis.

c Write down the coordinates of B and C, the first two maximum points on the curve.

11 Sketch each of the following curves for x in the interval $0 \leq x \leq 360$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = 3 \cos 2x°$ **b** $y = \tan(-2x°)$ **c** $y = 1 + 2 \sin x°$

d $y = -\sin(x + 60)°$ **e** $y = 2 \cos(x - 45)°$ **f** $y = 3 - \tan x°$

g $y = 2 + \cos \frac{1}{2}x°$ **h** $y = 4 \sin \frac{3}{2}x°$ **i** $y = 1 - 2 \cos x°$

12 State the period of the curves with the equations

a $y = 2 \tan 3x°$,

b $y = 1 + \sin kx°$, giving your answer in terms of k.

13 $$f(x) \equiv 2 \sin \tfrac{1}{2}x, \quad 0 \leq x \leq 2\pi.$$

a Sketch the graph $y = f(x)$.

b State the coordinates of the maximum point of the curve.

c Solve the equation $f(x) = \sqrt{2}$, giving your answers in terms of π.

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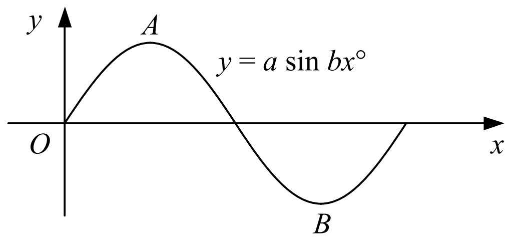

The graph shows the curve $y = a \sin bx°$, $0 \leq x \leq 180$.

The curve has a maximum at the point A with coordinates (45, 4).

a Find the values of the constants a and b.

b Write down the coordinates of the minimum point of the curve, B.