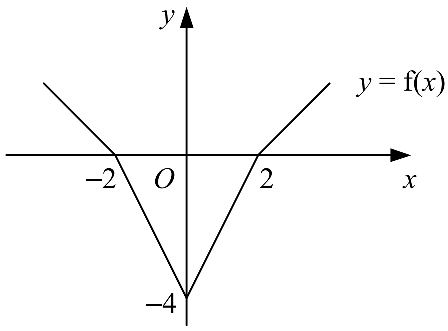
C3 FUNCTIONS Worksheet F

1 **a** Express $x^2 - 8x + 18$ in the form $(x + a)^2 + b$.

b Find the distance of the vertex of the curve $y = x^2 - 8x + 18$ from the origin, giving your answer in the form $k\sqrt{5}$.

c Describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 18$.

2



The diagram shows the graph of $y = \mathrm{f}(x)$ which meets the coordinate axes at the points $(-2, 0)$, $(0, -4)$ and $(2, 0)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the graphs of

a $y = \frac{1}{2}|\mathrm{f}(x)|$,

b $y = 4 + \mathrm{f}(x + 2)$.

3 Sketch the curve with equation $y = 2 - 2\sin x$ for x in the interval $0 \le x \le 2\pi$.

Label on your sketch the coordinates of any maximum or minimum points and any points where the curve meets the coordinate axes.

4 $$\mathrm{f}(x) \equiv |2x + 5|, \ x \in \mathbb{R}.$$

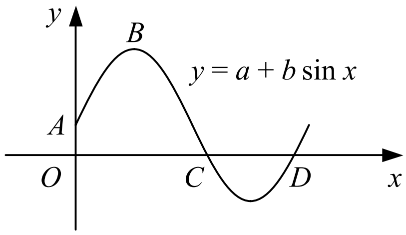
a Sketch the graph $y = \mathrm{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes.

b Evaluate $\mathrm{ff}(-4)$.

$$\mathrm{g}(x) \equiv \mathrm{f}(x + k), \ x \in \mathbb{R}.$$

c State the value of the constant k for which $\mathrm{g}(x)$ is symmetrical about the y-axis.

5



The diagram shows the curve $y = a + b\sin x$, $0 \le x \le 360°$.

The curve meets the y-axis at the point A (0, 2) and has a maximum at the point B (90°, 7).

a Find the values of the constants a and b.

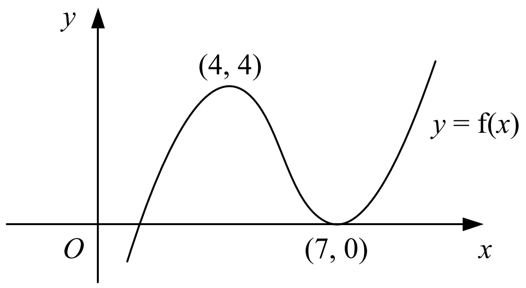
b Find the x-coordinates of the points C and D, where the curve crosses the x-axis.

6 **a** Sketch the curve $y = 3\cos 2x°$ for x in the interval $0 \le x \le 360$.

b Write down the coordinates of the points where the curve intersects the x-axis.

c Write down the coordinates of the turning points of the curve.

7



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which has two stationary points with coordinates (4, 4) and (7, 0).

Showing the coordinates of any stationary points, sketch on separate diagrams the curves

a $y = 1 + 2\mathrm{f}(x)$,

b $y = \mathrm{f}(-3x)$.

8 **a** Sketch the curve $y = \frac{1}{2} + \sin 3x$ for x in the interval $0 \le x \le 180°$.

b Write down the coordinates of the turning points of the curve.

c Find the x-coordinates of the points where the curve crosses the x-axis.

9 The function f is defined by

$$\mathrm{f} : x \to x^{\frac{1}{2}} - 2,\ x \in \mathbb{R},\ x \ge 0.$$

Showing the coordinates of any points where each graph meets the coordinate axes, sketch on separate diagrams the graphs of

a $y = \mathrm{f}(x)$,

b $y = 2 + |\mathrm{f}(x)|$,

c $y = 3\mathrm{f}(x + 1)$.

10 Sketch the curve $y = 4\sin(x + \frac{\pi}{3})$ for x in the interval $0 \le x \le 2\pi$.

Label on your sketch

i the value of x at each point where the curve intersects the x-axis,

ii the coordinates of the maximum and minimum points of the curve.

11
$$\mathrm{f}(x) \equiv \frac{3x - 5}{x - 2},\ x \in \mathbb{R},\ x \ne 2.$$

a Find $\mathrm{f}^{-1}(x)$ and state its domain.

b Hence, or otherwise, solve the equation $\mathrm{f}(x) = 4$.

c Find the values of a and b such that

$$\mathrm{f}(x) = a + \frac{b}{x - 2}.$$

d Hence, describe two transformations that map the graph of $y = \frac{1}{x}$ onto the graph of $y = \mathrm{f}(x)$.