C3 NUMERICAL METHODS — Answers - Worksheet A

1 **a** f(1) = −3 f(2) = 7
sign change, f(x) continuous ∴ root

b f(0.5) = 2.89 f(1) = −0.298
sign change, f(x) continuous ∴ root

c f(−6) = −0.995 f(−5) = 0.0135
sign change, f(x) continuous ∴ root

d f(2.1) = −1.60 f(2.2) = 0.226
sign change, f(x) continuous ∴ root

e f(0.4) = −0.351 f(0.5) = 0.25
sign change, f(x) continuous ∴ root

f f(10) = 6.00 f(11) = −9.00
sign change, f(x) continuous ∴ root

2 **a** f(0) = −4
f(3) = 17.8
f(1) = −6
f(2) = −0.243
∴ $N = 2$

b f(1) = −12
f(5) = 5.65
f(3) = −0.704
f(4) = 2.55
∴ $N = 3$

c f(0) = 15
f(−2) = −57
f(−1) = 9
∴ $N = -2$

d f(0) = −1.63
f(1) = 3
∴ $N = 0$

e f(0) = 1
f(−5) = −2.87
f(−4) = −2.25
f(−3) = 0.473
∴ $N = -4$

f f(0) = −6
f(4) = −1.58
f(5) = −0.454
f(6) = 0.684
∴ $N = 5$

3 **a** let $f(x) = x^3 - 12 + \frac{x}{4}$
f(2) = −3.5 f(3) = 15.75
sign change, f(x) continuous ∴ root

b let $f(x) = 12e^x - 9 + 4x$
f(−1) = −8.59 f(0) = 3
sign change, f(x) continuous ∴ root

c let $f(x) = 10 \ln 3x - 5 + 7x^2$
f(0.47) = −0.0178 f(0.48) = 0.259
sign change, f(x) continuous ∴ root

d let $f(x) = \sin 4x - 7e^x$
f(−6.5) = −0.773 f(−6) = 0.888
sign change, f(x) continuous ∴ root

e let $f(x) = 4^x - 3x - 10$
f(−4) = 2.00 f(−3) = −0.984
sign change, f(x) continuous ∴ root

f let $f(x) = \tan(\tfrac{1}{2}x) - 2x + 1$
f(2.6) = −0.598 f(2.7) = 0.0552
sign change, f(x) continuous ∴ root

4 **a** f(1) = −1
f(2) = 12.5
f(1.1) = −0.809
f(1.2) = −0.426
f(1.3) = 0.164
∴ $a = 12$

b f(2) = −0.303
f(3) = 0.292
f(2.5) = −0.00553
f(2.6) = 0.0537
∴ $a = 25$

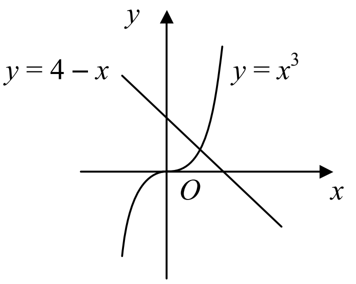
c f(−2) = −41
f(−1) = 3
f(−1.1) = 0.715
f(−1.2) = −1.96
∴ $a = -12$

d f(11) =0.723
f(12) = −0.177
f(11.7) = 0.0362
f(11.8) = −0.0425
∴ $a = 117$

e f(5) = 1.19
f(6) = −1.13
f(5.5) = 0.928
f(5.8) = 0.256
f(5.9) = −0.246
∴ $a = 58$

f f(−3) = 6.42
f(−2) = −15.0
f(−2.7) = 2.60
f(−2.6) = 1.03
f(−2.5) = −0.75
∴ $a = -26$

5 **a**



b $x^3 + x - 4 = 0 \Rightarrow x^3 = 4 - x$

the graphs $y = x^3$ and $y = 4 - x$

intersect at exactly one point

$\therefore$ one real root

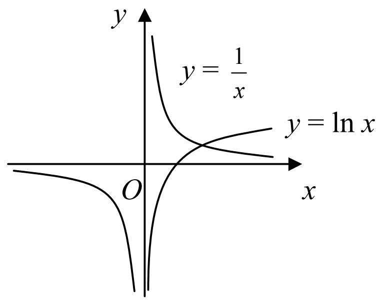
c let $f(x) = x^3 + x - 4$

$f(1) = -2$

$f(1.5) = 0.875$

sign change, $f(x)$ continuous $\therefore$ root

6 **a**



b $x \ln x - 1 = 0 \Rightarrow x \ln x = 1 \Rightarrow \ln x = \frac{1}{x}$

the graphs $y = \ln x$ and $y = \frac{1}{x}$

intersect at exactly one point

$\therefore$ one real root

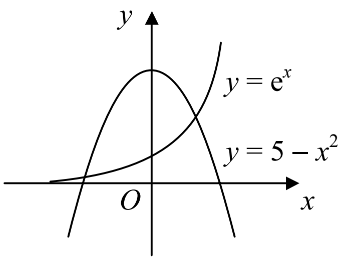
c $f(1) = -1$

$f(2) = 0.386$

$\therefore 1 < \alpha < 2$

$\therefore n = 1$

7 **a**



b $e^x + x^2 - 5 = 0 \Rightarrow e^x = 5 - x^2$

the graphs $y = e^x$ and $y = 5 - x^2$

intersect at two points,

one for $x < 0$ and one for $x > 0$

$\therefore$ one negative and one positive real root

c let $f(x) = e^x + x^2 - 5$

$f(-3) = 4.05$

$f(-2) = -0.865$

sign change, $f(x)$ continuous $\therefore$ root

d $f(1) = -1.28$

$f(2) = 6.39$

$f(1.2) = -0.240$

$f(1.3) = 0.359$

$\therefore 1.2 < \alpha < 1.3$

$\therefore n = 12$