## **NUMERICAL METHODS**

For each equation, show that it can be rearranged into the given iterative form. Use this and the given value of  $x_0$  to find  $x_1$ ,  $x_2$  and  $x_3$ . Give your value of  $x_3$  correct to 4 decimal places.

a	$9 + 4x - 2x^3 = 0$	$x_{n+1} = \sqrt[3]{2x_n + 4.5}$	$x_0 = 2$
b	$e^x - 8x + 5 = 0$	$x_{n+1} = \ln\left(8x_n - 5\right)$	$x_0 = 3$
c	$\tan x - 5x + 13 = 0$	$x_{n+1} = \arctan(5x_n - 13)$	$x_0 = -1.2$
d	$\ln x + \sqrt{x} + 1.4 = 0$	$x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$	$x_0 = 0.16$

2 For each equation, show that it can be rearranged into the given iterative form and state the values of the constants *a* and *b*. Use this and the given value of  $x_0$  to find  $x_1$ ,  $x_2$  and  $x_3$ . Give your value of  $x_3$  correct to 3 decimal places.

a	$e^{2x-1} - 6x = 0$	$x_{n+1} = a(\ln bx_n + 1)$	$x_0 = 1.7$
b	$\frac{2}{x} + \cos x - 3 = 0$	$x_{n+1} = \frac{a}{b - \cos x_n}$	$x_0 = 0.8$
c	$2x^3 - 6x - 11 = 0$	$x_{n+1} = \sqrt{a + \frac{b}{x_n}}$	$x_0 = 2$
d	$15\ln(x+3) - 4x = 0$	$x_{n+1} = e^{ax_n} + b$	$x_0 = -2.5$

- 3 In each case, use the given iteration formula and value of  $x_0$  to find a root of the equation f(x) = 0 to the stated degree of accuracy. Justify the accuracy of your answers.
  - **a**  $f(x) = 10^x + 3x 4$  $x_{n+1} = \log_{10} (4 3x_n)$  $x_0 = 0.44$ 3 decimal places**b**  $f(x) = x^2 + \frac{1}{x-5}$  $x_{n+1} = \sqrt{\frac{x_n^3 + 1}{5}}$  $x_0 = 0.5$ 2 significant figures**c**  $f(x) = 30 5x + \sin 2x$  $x_{n+1} = 6 + 0.2 \sin 2x_n$  $x_0 = 6$ 3 significant figures**d**  $f(x) = e^{4-x} \ln x$  $x_{n+1} = 4 \ln (\ln x_n)$  $x_0 = 3.7$ 3 decimal places

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**C**3

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 $f(x) = x^5 - 10x^3 + 4.$ 

The equation f(x) = 0 has a root in the interval -4 < x < -3.

**a** Use the iteration formula  $x_{n+1} = \sqrt[5]{10x_n^3 - 4}$  and the starting value  $x_0 = -3.2$  to find the value of this root correct to 2 decimal places.

The equation f(x) = 0 can be rearranged into the iterative form  $x_{n+1} = \sqrt[3]{\frac{a}{b-x_n^2}}$ .

**b** Find the values of the constants *a* and *b* in this formula.

The equation f(x) = 0 has another root in the interval 0 < x < 1.

**c** Using the iteration formula with your values from part **b** and the starting value  $x_0 = 1$ , find the value of this root correct to 3 decimal places.

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$$f: x \to \arcsin 2x - 0.5x - 0.7, x \in \mathbb{R}, |x| \le 0.5$$

The equation f(x) = 0 can be rearranged into the iterative form  $x_{n+1} = a \sin(bx_n + c)$ .

**a** Find the values of the constants *a*, *b* and *c* in this formula.

The equation f(x) = 0 has a solution in the interval (0.3, 0.4).

**b** Using the iterative formula with your values from part **a** and the starting value  $x_0 = 0.4$ , find this solution correct to 3 decimal places.