C3 NUMERICAL METHODS — Answers - Worksheet C

1 **a** let $f(x) = x^3 - 7x - 11$
$f(3) = -5$
$f(4) = 25$
sign change, f(x) continuous ∴ root

b $x_1 = 3.230712$
$x_2 = 3.225651$
$x_3 = 3.226479 = 3.23$ (2dp)

2 **a** $f(4) = -2.29$ (3sf)
$f(5) = 0.829$ (3sf)

b sign change, f(x) continuous ∴ root

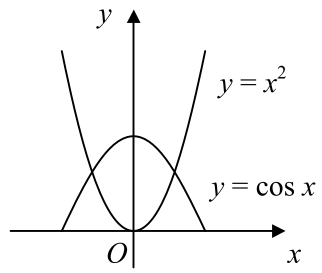
c $4 \operatorname{cosec} x - 5 + 2x = 0$
$2x = 5 - 4 \operatorname{cosec} x$
$x = 2.5 - \frac{2}{\sin x}$, $a = 2.5$, $b = -2$

d $x_1 = 4.545973$
$x_2 = 4.528018$
$x_3 = 4.534481 = 4.534$ (3dp)

3 **a** $f(0.4) = -0.809$
$f(0.5) = 0.307$
sign change, f(x) continuous ∴ root
∴ $0.4 < \alpha < 0.5$

b $x_1 = 0.468857$
$x_2 = 0.463841$
$x_3 = 0.465157$
$x_4 = 0.464810$
∴ $\alpha = 0.465$ (3dp)

4 **a**



b $\cos x - x^2 = 0 \Rightarrow \cos x = x^2$
the graphs $y = \cos x$ and $y = x^2$ intersect at 2 points, one for $x < 0$ and one for $x > 0$
∴ one negative and one positive real root

c let $f(x) = \cos x - x^2$
$f(0.8) = 0.0567$
$f(0.9) = -0.188$
sign change, f(x) continuous ∴ root

d $x_1 = 0.834690$
$x_2 = 0.819395$
$x_3 = 0.826235$
$x_4 = 0.823195$
$x_5 = 0.824550$
∴ root = 0.82 (2dp)

5 **a** $f(1.4) = 3.65$
$f(1.5) = -0.205$
sign change, f(x) continuous ∴ root

b $e^{5-2x} - x^5 = 0 \Rightarrow x^5 = e^{5-2x}$
$\Rightarrow x = (e^{5-2x})^{\frac{1}{5}}$
$\Rightarrow x = e^{1-\frac{2}{5}x}$, $k = \frac{2}{5}$

c $x_1 = 1.491825$
$x_2 = 1.496711$
$x_3 = 1.493789 = 1.494$ (3dp)

6 **a** $f(1.3) = -0.341$
$f(1.4) = 0.383$
sign change, f(x) continuous ∴ root

b $x_1 = 1.331571$
$x_2 = 1.354168$
$x_3 = 1.346907$
$x_4 = 1.349261$

c 1.35 (3sf)

d diverges leading to ln of a –ve which is not real

7 **a** $f'(x) = 6x^2 + 4$

b for all real x, $x^2 \geq 0$

$\Rightarrow \quad 6x^2 + 4 > 0$

$\therefore$ f(x) increasing for all x

$\therefore$ y = f(x) only crosses x-axis once so exactly 1 real root

c f(1.2) = –0.744

f(1.3) = 0.594

sign change, f(x) continuous $\therefore$ root

d $x_1 = 1.280579$

$x_2 = 1.246945$

$x_3 = 1.261203$

$x_4 = 1.255199$

$\therefore$ root = 1.26 (2dp)

e f(1.255) = –0.0267

f(1.265) = 0.109

sign change, f(x) continuous $\therefore$ root

8 **a** $3x + \ln x - x^2 = x \Rightarrow \ln x = x^2 - 2x$

$\Rightarrow x = e^{x^2 - 2x}$

b let $f(x) = 2x + \ln x - x^2$

f(0.4) = –0.276

f(0.5) = 0.0569

sign change, f(x) continuous $\therefore$ root

c f(2.3) = 0.143

f(2.4) = –0.0845

sign change, f(x) continuous $\therefore$ root

d $x_1 = 0.472367$

$x_2 = 0.485973$

$x_3 = 0.479134$

$x_4 = 0.482537$

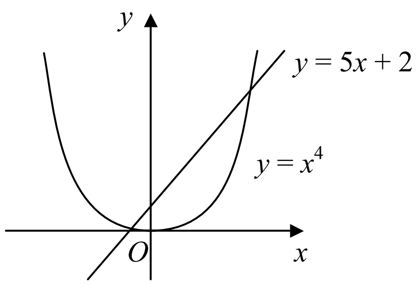
$\therefore$ x-coord of A = 0.48 (2dp)

e f(0.475) = –0.0201

f(0.485) = 0.0112

sign change, f(x) continuous $\therefore$ root

9 **a**



b $x^4 - 5x - 2 = 0 \Rightarrow x^4 = 5x + 2$

the graphs $y = x^4$ and $y = 5x + 2$ intersect at 2 points, one for $x < 0$ and one for $x > 0$

$\therefore$ one negative and one positive real root

c $x_1 = 1.821160$

$x_2 = 1.825524$

$x_3 = 1.826420$

$x_4 = 1.826603 = 1.827$ (3dp)

d $x^4 - 5x - 2 = 0 \Rightarrow x^4 - 5x = 2$

$\Rightarrow x(x^3 - 5) = 2$

$\Rightarrow x = \dfrac{2}{x^3 - 5}$, $a = 2$, $b = -5$

e $x_1 = -0.394945$

$x_2 = -0.395132$

$x_3 = -0.395125$

$\therefore$ root = –0.3951 (4dp)