

1 **a** $\frac{dy}{dx} = e^x + 2x$

b at A , $x = 0 \therefore y = -3$, grad = 1
 $\therefore y = x - 3$

c SP: $e^x + 2x = 0$

let $f(x) = e^x + 2x$

$f(-0.4) = -0.130$

$f(-0.3) = 0.141$

sign change, $f(x)$ continuous \therefore root
 \therefore x-coord of B in interval $[-0.4, -0.3]$

d $x_1 = -0.34694$

$x_2 = -0.35126$

$x_3 = -0.35169$

$x_4 = -0.35173$

\therefore x-coord of $B = -0.352$ (3dp)

2 **a** $f(0) = 0.279$

$f(5) = -4.10$

$f(1) = 0.266$

$f(3) = -2.44$

$f(2) = -0.853$

$\therefore k = 1$

b $x_0 = 1$

$x_1 = 1.2684$

$x_2 = 1.3106$

$x_3 = 1.3106$

3 **a** area of segment = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin \theta$

$$= \frac{1}{2}r^2(\theta - \sin \theta)$$

$$\therefore \frac{1}{2}r^2\sin \theta = 4 \times \frac{1}{2}r^2(\theta - \sin \theta)$$

$$\sin \theta = 4(\theta - \sin \theta)$$

$$\sin \theta = 4\theta - 4 \sin \theta$$

$$4\theta - 5 \sin \theta = 0$$

b $\theta_1 = 1.11401$

$\theta_2 = 1.12184$

$\theta_3 = 1.12613$

$\theta_4 = 1.12844$

$\theta_5 = 1.12968$

$\therefore \theta = 1.13$ (2dp)

4 **a** $e^{x^2} - x - 3 = 0$

$$e^{x^2} = x + 3$$

$$x^2 = \ln(x + 3)$$

$$x = \sqrt{\ln(x + 3)} \quad \therefore a = 1, b = 3$$

b e.g. $x_0 = 1.5$

$x_1 = 1.226408$

$x_2 = 1.200563$

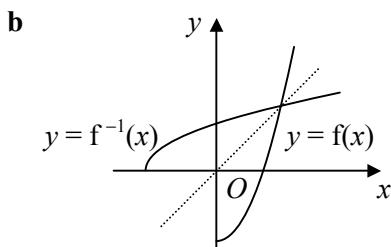
$x_3 = 1.198006$

$x_4 = 1.197752$

$x_5 = 1.197727$

\therefore solution = 1.198 (3dp)

5 a $y = x^2 - 9$
 swap $x = y^2 - 9$
 $y = \pm\sqrt{x+9}$
 (domain $\Rightarrow +$)
 $f^{-1}(x) = \sqrt{x+9}, x \in \mathbb{R}, x \geq -9$
 range: $f^{-1}(x) \geq 0$



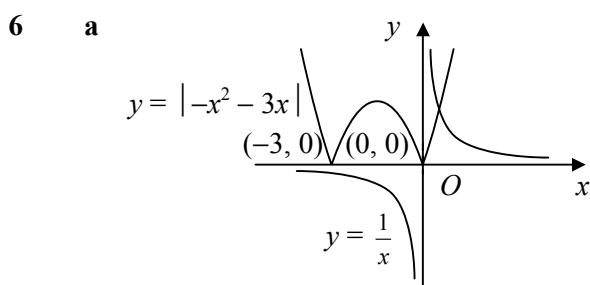
c let $h(x) = f^{-1}(x) + g(x) = \sqrt{x+9} + x^3$
 $h(-2) = -5.35$
 $h(-1) = 1.83$
 sign change, $h(x)$ continuous \therefore root
 d $x_1 = -1.41421, x_2 = -1.40174,$
 $x_3 = -1.40212, x_4 = -1.40211$
 \therefore root = -1.402 (3dp)

7 a at $A, x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x = 0$

let $f(x) = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$
 $f(4) = -2, f(5) = 14.2$
 sign change, $f(x)$ continuous \therefore root
 $\therefore 4 < \alpha < 5$

b $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$
 at $B, \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$
 let $g(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$
 $g(2) = -0.990, g(3) = 5.12$
 sign change, $g(x)$ continuous \therefore root
 $\therefore 2 < \beta < 3$

c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$
 $5x^2 - 3 - 14x^{\frac{1}{2}} = 0$
 $x^2 = 0.6 + 2.8x^{\frac{1}{2}}$
 $x > 0 \therefore x = \beta$ is a soln to $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$
 d $x_1 = 2.158144$
 $x_2 = 2.171031$
 $x_3 = 2.173853$
 $x_4 = 2.174470$
 $x_5 = 2.174604$
 $\therefore \beta = 2.175$ (4sf)



b $-(-x^2 - 3x) = \frac{1}{x}$
 $x^2 + 3x = \frac{1}{x}$
 $x^3 + 3x^2 = 1$
 $x^3 + 3x^2 - 1 = 0$
 c $x_1 = 0.57735$
 $x_2 = 0.52871$
 $x_3 = 0.53234$
 $x_4 = 0.53207$
 \therefore x-coord of $P = 0.532$ (3dp)

8 a $\frac{dy}{dx} = 3 - \frac{1}{x}$

grad = 2
 \therefore grad of normal = $-\frac{1}{2}$
 $y - 3 = -\frac{1}{2}(x - 1)$
 $[y = \frac{7}{2} - \frac{1}{2}x]$

b $3x - \ln x = \frac{7}{2} - \frac{1}{2}x$

$6x - 2 \ln x = 7 - x$

$2 \ln x - 7x + 7 = 0$

c $2 \ln x = 7x - 7$
 $\ln x = \frac{7}{2}(x - 1)$
 $x = e^{\frac{7}{2}(x-1)} \therefore k = \frac{7}{2}$

d $x_1 = 0.173774$
 $x_2 = 0.055477$
 $x_3 = 0.036669$
 $x_4 = 0.034333$
 $x_5 = 0.034053$
 \therefore x-coord of $Q = 0.034$ (3dp)
 e let $f(x) = 2 \ln x - 7x + 7$
 $f(0.0335) = -0.027$
 $f(0.0345) = 0.025$
 sign change, $f(x)$ continuous \therefore root