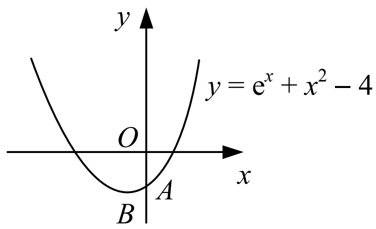
C3 NUMERICAL METHODS Worksheet D

1



The diagram shows the curve $y = e^x + x^2 - 4$. The curve intersects the y-axis at the point A and has a stationary point at B.

a Find $\frac{dy}{dx}$. **(1)**

b Find an equation for the tangent to the curve at A. **(2)**

c Show that the x-coordinate of B lies in the interval [–0.4, –0.3]. **(3)**

d Using the iteration formula $x_{n+1} = \frac{1}{3}(x_n - e^{x_n})$, with $x_0 = -0.3$, find the x-coordinate of B correct to 3 decimal places. **(4)**

2 The function f is defined by

$$f(x) \equiv \sin(x - 6) - \ln(x^2 + 1), \quad x \in \mathbb{R},$$

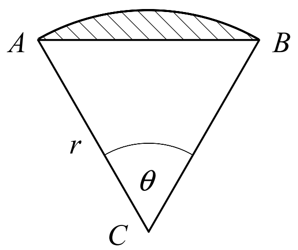
where x is measured in radians.

The equation $f(x) = 0$ has a root in the interval $k < x < k + 1$, where k is a positive integer.

a Find the value of k. **(3)**

b Use the iteration formula $x_{n+1} = \sqrt{e^{\sin(x_n - 6)} - 1}$, with $x_0 = k$, to find three further approximations for this root, giving your answers to 4 decimal places. **(3)**

3



The diagram shows a sector ABC of a circle, centre C, radius r. Angle ACB is θ radians.

Given that the ratio of the area of the shaded segment to the area of triangle ABC is 1 : 4,

a show that $4\theta - 5\sin\theta = 0$, **(4)**

b use the iterative formula $\theta_{n+1} = \frac{5}{4}\sin\theta_n$, with $\theta_0 = 1.1$, to find the value of θ correct to 2 decimal places. **(4)**

4 $$f : x \to e^{x^2} - x - 3, \quad x \in \mathbb{R}.$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt{\ln(ax_n + b)}$.

a Find the values of the constants a and b in this formula. **(3)**

The equation $f(x) = 0$ has a solution in the interval (1, 2).

b Using the iterative formula with your values from part **a** and a suitable starting value, find this solution correct to 3 decimal places. **(4)**

5 $$\text{f} : x \to x^2 - 9,\ x \in \mathbb{R},\ x \geq 0,$$
$$\text{g} : x \to x^3,\ x \in \mathbb{R}.$$

a Find $\text{f}^{-1}(x)$ and state its domain and range. **(4)**

b On the same set of axes, sketch the curves $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$. **(2)**

c Show that the equation $\text{f}^{-1}(x) + \text{g}(x) = 0$ has a root in the interval $[-2, -1]$. **(3)**

d Use the iterative formula $x_{n+1} = -(x_n + 9)^{\frac{1}{6}}$, with $x_0 = -1$, to find this root correct to 3 decimal places. **(4)**

6 **a** On the same diagram, sketch the curves $y = \frac{1}{x}$ and $y = |-x^2 - 3x|$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The curves intersect at the point P.

b Show that the x-coordinate of P can be found by solving the equation $x^3 + 3x^2 - 1 = 0$. **(3)**

c Use the iteration formula $x_{n+1} = \frac{1}{\sqrt{x_n + 3}}$, with $x_0 = 0$, to find the x-coordinate of P correct to 3 decimal places. **(4)**

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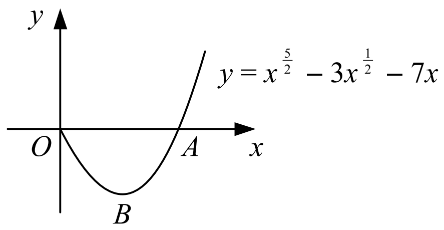

The diagram shows the curve $y = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x,\ x \geq 0$, which crosses the x-axis at the point A, where $x = \alpha$, and has a stationary point at B, where $x = \beta$.

Show that

a $4 < \alpha < 5$, **(2)**

b $2 < \beta < 3$, **(4)**

c $x = \beta$ is a solution to the equation $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$. **(3)**

d Use the iterative formula $x_{n+1} = \sqrt{0.6 + 2.8x_n^{\frac{1}{2}}}$, with $x_0 = 2.1$, to find β correct to 4 significant figures. **(4)**

8 The curve with equation $y = 3x - \ln x$ passes through the point $P(1, 3)$.

a Find an equation for the normal to the curve at P. **(4)**

The normal to the curve at P intersects the curve again at the point Q.

b Show that the x-coordinate of Q satisfies the equation

$$2 \ln x - 7x + 7 = 0.$$ **(1)**

The x-coordinate of Q is to be found using an iteration of the form $x_{n+1} = e^{k(x_n - 1)}$.

c Find the value of the constant k. **(2)**

d Using $x_0 = 0.5$, find the x-coordinate of Q correct to 3 decimal places. **(4)**

e Justify the accuracy of your answer to part **d**. **(2)**