Practice paper

(Marks are shown in brackets.)

1 The curve C, with equation $y = x^2 \ln x$, $x > 0$, has a stationary point P. Find, in terms of e, the coordinates of P. (7)

2 $f(x) = e^{2x-1}, \quad x \geqslant 0$

The curve C with equation $y = f(x)$ meets the y-axis at P.
The tangent to C at P crosses the x-axis at Q.

a Find, to 3 decimal places, the area of triangle POQ, where O is the origin. (5)

The line $y = 2$ intersects C at the point R.

b Find the exact value of the x-coordinate of R. (3)

3 $f(x) = \dfrac{3x}{x+1} - \dfrac{x+7}{x^2-1}, \quad x > 1$

a Show that $f(x) = 3 - \dfrac{4}{x-1}, \quad x > 1.$ (5)

b Find $f^{-1}(x)$. (4)

c Write down the domain of $f^{-1}(x)$. (1)

4 **a** Sketch, on the same set of axes, for $x > 0$, the graphs of

$$y = -1 + \ln 3x \quad \text{and} \quad y = \frac{1}{x}$$ (2)

The curves intersect at the point P whose x-coordinate is p.
Show that

b p satisfies the equation

$$p \ln 3p - p - 1 = 0$$ (1)

c $1 < p < 2$ (2)

The iterative formula

$$x_{n+1} = \tfrac{1}{3}e^{\left(1 + \frac{1}{x_n}\right)}, \quad x_0 = 2$$

is used to find an approximation for p.

d Write down the values of x_1, x_2, x_3 and x_4 giving your answers to 4 significant figures. (3)

e Prove that $p = 1.66$ correct to 3 significant figures. (2)

5 The curve C_1 has equation

$$y = \cos 2x - 2\sin^2 x$$

The curve C_2 has equation

$$y = \sin 2x$$

a Show that the x-coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$2\cos 2x - \sin 2x = 1 \quad (3)$$

b Express $2\cos 2x - \sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact value of R and giving α in radians to 3 decimal places. (4)

c Find the x-coordinates of the points of intersection of C_1 and C_2 in the interval $0 \leqslant x < \pi$, giving your answers in radians to 2 decimal places. (5)

6 a Given that $y = \ln \sec x$, $-\frac{\pi}{2} < x \leqslant 0$, use the substitution $u = \sec x$, or otherwise, to show that $\frac{dy}{dx} = \tan x$. (3)

The curve C has equation $y = \tan x + \ln \sec x$, $-\frac{\pi}{2} < x \leqslant 0$.

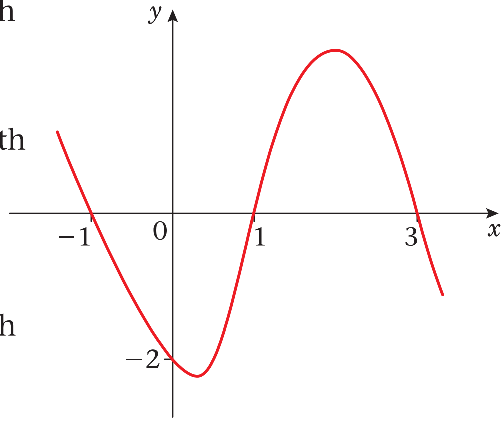
At the point P on C, whose x-coordinate is p, the gradient is 3.

b Show that $\tan p = -2$. (6)

c Find the exact value of $\sec p$, showing your working clearly. (2)

d Find the y-coordinate of P, in the form $a + k\ln b$, where a, k and b are rational numbers. (2)

7 The diagram shows a sketch of part of the curve with equation $y = f(x)$. The curve has no further turning points.



On separate diagrams show a sketch of the curve with equation

a $y = 2f(-x)$ (3)

b $y = |f(2x)|$ (3)

In each case show the coordinates of points in which the curve meets the coordinate axes.

The function g is given by

$$g: x \rightarrow |x + 1| - k, \quad x \in \mathbb{R}, k > 1$$

c Sketch the graph of g, showing, in terms of k, the y-coordinate of the point of intersection of the graph with the y-axis. (3)

Find, in terms of k,

d the range of $g(x)$ (1)

e $gf(0)$ (2)

f the solution of $g(x) = x$ (3)