1	Prove, by counter-example, that each of the following statements is false. a For all positive real values of x , $\sqrt[3]{x} \le x$. b For all positive integer values of n , $(n^3 - n + 7)$ is prime.	(2) (2)
2	Use proof by contradiction to prove that $\sqrt{\pi}$ is irrational. (You may assume that π is irrational).	(4)
3	Find a counter-example to prove that the statement $"15x^2 - 11x + 2 \ge 0$ for all real values of x" is false	(4)
4	 a Given that n = 2m + 1, find and simplify an expression in terms of m for n² + 2 b Hence, use proof by contradiction to prove that if (n² + 2n) is even, where n is an integer, then n is even. 	. <i>n.</i> (1) 1 (5)
5	a Prove that if the equation $k \cos x - \csc x = 0$, where k is a constant, has real solutions, then $ k \ge 2$.	(5)
	b Find the values of x in the interval $0 \le x \le 360$ for which $3 \cos x^\circ - \operatorname{cosec} x^\circ = 0.$	(3)
6	Use proof by contradiction to prove that there are no positive integers, x and y, such $x^2 - y^2 = 1$.	that (6)
7	For each statement, either prove that it is true or find a counter-example to prove th it is false.	at
	a If a and b are irrational and $a \neq b$, then $(a + b)$ is irrational.	(2)
	b If <i>m</i> and <i>n</i> are consecutive odd integers, then $(m + n)$ is divisible by 4.	(3)
	c For all real values of x , $\cos x \le 1 + \sin x$.	(2)
8	a Show that if $\log_2 3 = \frac{p}{q}$, then	
	$2^p = 3^q.$	(2)
	b Use proof by contradiction to prove that $\log_2 3$ is irrational.	(4)
	c Prove, by counter-example, that the statement	
	is false.	(2)
9	The function f is defined by	
,	f: $x \to \frac{x-2}{x}, x \in \mathbb{R}, x \neq 0.$	
	4x a Find an expression for the inverse function $f^{-1}(x)$ and state its domain	(5)
	b Prove that there are no real values of x for which	
	$\mathbf{f}(\mathbf{x}) = \mathbf{f}^{-1}(\mathbf{x}).$	(4)

C3

PROOF

$$f(x) = f^{-1}(x).$$
 (4)

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